Letter to John Chalmers from Erv Wilson ①
Apr 4 1971

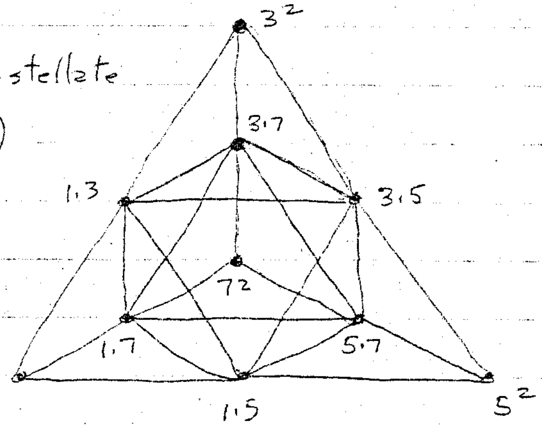
If, as you suggest, ~~B~~, the scale is multiplied by itself, thus:

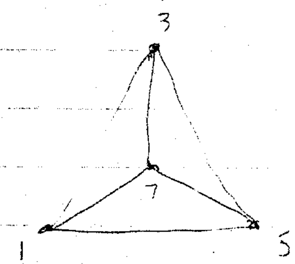
×	1	3	5	7
1	1·1	1·3	1·5	1·7
3	3·1	3·3	3·5	3·7
5	5·1	5·3	5·5	5·7
7	7·1	7·3	7·5	7·7

= A

we get a "tetrad of tetrads":

(or quasi-stellate octohedron)





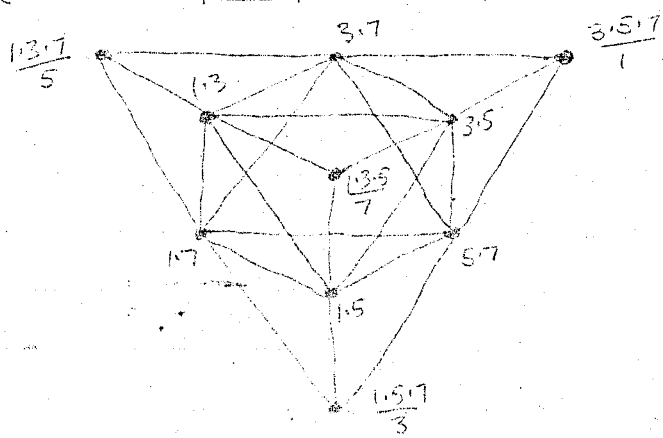
(Fuller makes a big to-do about this arrangement of tetrahedra forming the octohedron.)

The complement to the above figure:

$(1\cdot3\cdot5\cdot7\ \times)$

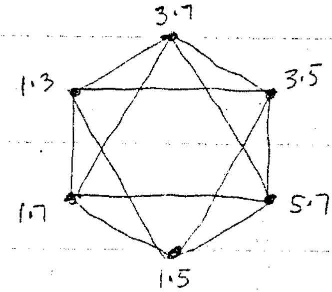
×		$\bar{1}$	$\bar{3}$	$\bar{5}$	$\bar{7}$
$\bar{1}$	3·5·7	3·5·7/1	5·7	3·7	3·5
$\bar{3}$	1·5·7	5·7	1·5·7/3	1·7	1·5
$\bar{5}$	1·3·7	3·7	1·7	1·3·7/5	1·3
$\bar{7}$	1·3·5	3·5	1·5	1·3	1·3·5/7

(point for point complement of above figure) = B



A and B have six members in common, these are the 2)4 hexany:

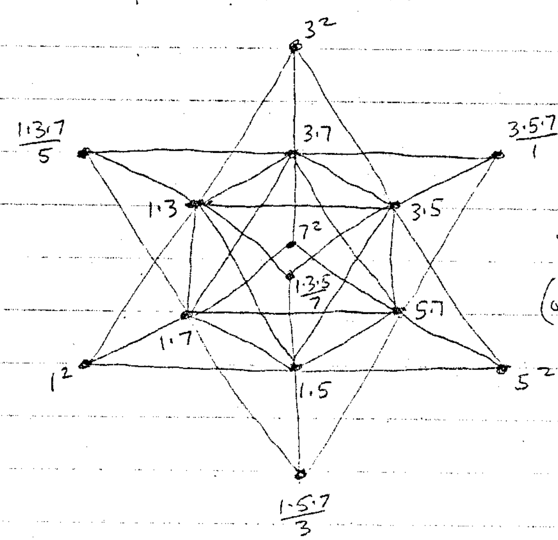
A ∩ B



Hexany
(or octohedron)

The fact that A and B are complements, and share, in common, the hexany, provides the aesthetic justification for uniting all members of A and B into a single set:

A U B



"Mandala"
Stellate Hexany
(or Stellate Octohedron)

In a similar manner the Stellate Eikosany may be constructed, However, a 3-dimensional matrix is required. The set resulting from {1,3,5,7,9,11} × {1,3,5,7,9,11} × {1,3,5,7,9,11} ~~when~~ intersects its point-by-point complement at the Eikosany, and unions with the complement to form the Stellate Eikosany. (not illustrated here)

There is also a possibility, in pentadic space, of a stellated dekateserany (rhombic dodecahedron, stellate & doubly centered):

D

	1	3	5	7	9
1	1.1	1.3	1.5	1.7	1.9
3		3.3	3.5	3.7	3.9
5			5.5	5.7	5.9
7				7.7	7.9
9					9.9

= A

Complement of A = B =

3.5.7.9/1	5.7.9	~~1~~.3.7.9	3.5.9	3.5.7
	$\frac{1.5.7.9}{3}$	1.7.9	1.5.9	1.5.7
		$\frac{1.3.7.9}{5}$	1.3.9	1.3.7
			$\frac{1.3.5.9}{7}$	1.3.5
				$\frac{1.3.5.7}{9}$

÷ 9* = C

D

3.5.7/1	5.7	3.7	3.5	$\frac{3.5.7}{9}$
	$\frac{1.5.7}{3}$	1.7	1.5	$\frac{1.5.7}{9}$
		$\frac{1.3.7}{5}$	1.3	$\frac{1.3.7}{9}$
			$\frac{1.3.5}{7}$	$\frac{1.3.5}{9}$
				$\frac{1.3.5.7}{9^2}$

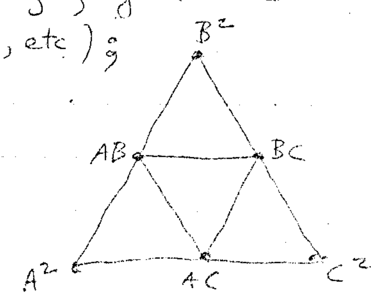
* ÷ 1, 3, 5, 7, or 9. There are 5 possible solutions. ~~These~~

C and A intersect at the 1,3,5,7, hexany, ~~(stellated hexany)~~ (heavy lines). The union of all members of A and C except those in diagonal rows, D, gives the dekateserany 2)5 U 3)5 (or rhombic dodecahedron). The inclusion of members in rows D in this union gives the stellated dekateserany (or stellated rhombic dodecahedron. 2 of the points occur over each other in the center). ~~This figu~~ This figure has 24 points. This is somewhat difficult to represent in-the-flat, but makes a beatiful structure when rendered in full-space, using the centered tetrahedron ~~as~~ to represent the pentad.

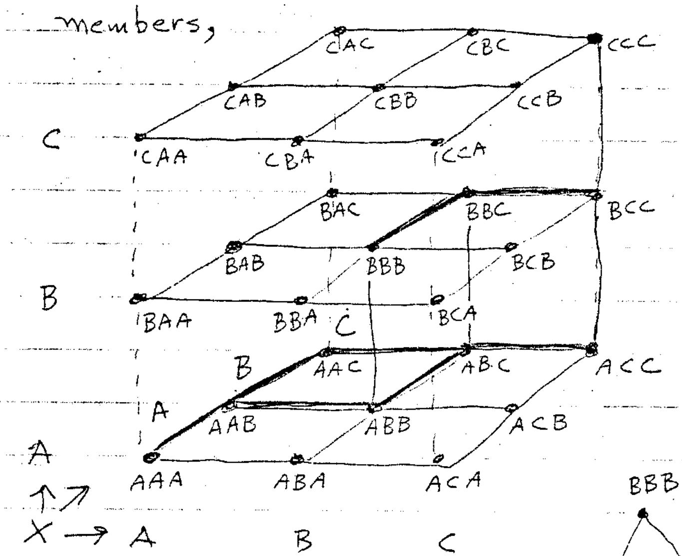
0	1	2	3	4
1	1	1	1	1
+0	+1	+2	+3	+4
1	2	3	4	5
+0	+1	+3	+6	+10
1	3	6	10	15
+0	+1	+4	+10	+20
1	4	10	20	35
+0	+1	+5	+15	+35
1	5	15	35	70
+0	+1	+6	+21	+56
1	6	21	56	126
+0	+1	+7	+28	+84
1	7	28	84	210
+0	+1	+8	+36	+120
1	8	36	120	330

The above progression of sums (which, in retrospect, I find is Pascal's Triangle) determines the number of tones in the resultant set when I raise a set to a power. For example; reading across the horizontal row, 1 3 6 10 15, I find that a triadic set to the ~~to~~ 0th power, $\{ABC\}^0$, yeilds 1 member (altho I dont know what that means); To the 1st power, $\{ABC\}^1$, yeilds 3 members (which is obvious); To the 2nd power $\{ABC\}^2$, yeilds 6 members, (AB = BA, etc);

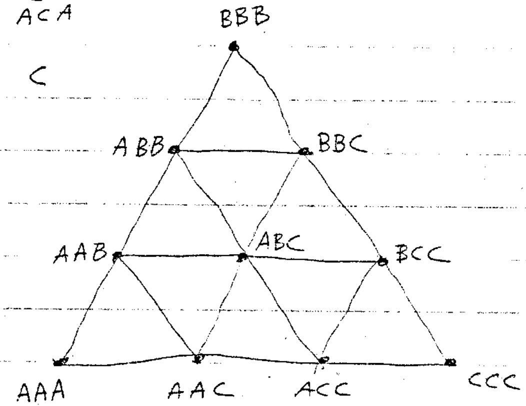
×	A	B	C
A	AA	AB	AC
B	BA	BB	BC
C	CA	CB	CC



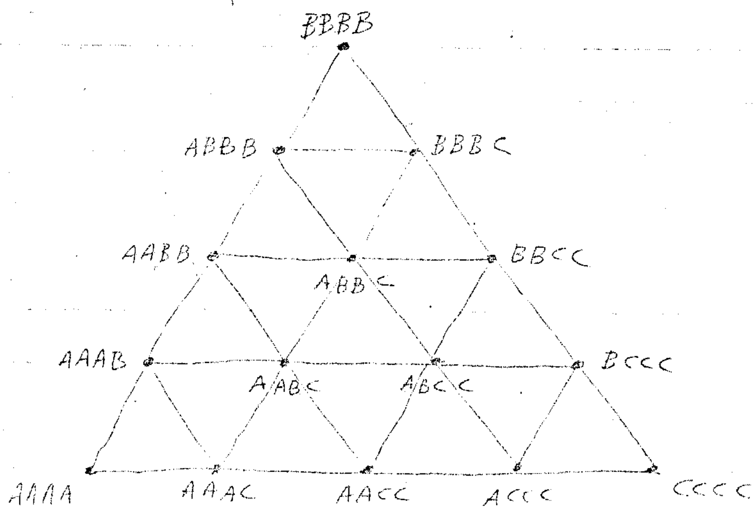
to The 3rd power, $\{ABC\}^3$, yeilds 10 members,



(we're getting close to Fokkers Figure, here)



and, finally to the 4th power, $\{ABC\}^4$, yeilds 15 members, which need not be matriced here. The logical progression of triadic lattices confirms the number 15...



We may read from Table 1, (in the 1 4 <u>10</u> 20 35 line) that the number of tones in the tetrad^2 is 10. Unioning this figure with its complement [in 1,3,5,7 tone-space, the figure by which a member must be multiplied to equal 1·3·5·7] we get 20 members, which, however, intersect on the 6 tones of the hexany. Therefore 2 × 10 − 6 = 14, The number of members in the stellate hexany. The stellate eikosany; ~~2×56~~ : 2×56 − 20 = 92 members. The stellate heptekontany 2×330 − 70 = 590 members. The ~~st~~ stellate dyany continues to have 2 members; 2×2 − 2 = 2

Cross-Sets (ref.* ordered partitions)

You suggest the figure $\{1, 3, 5\} \times \{\bar{7}, \bar{9}, \bar{11}\}$:

×	1	3	5
$\bar{7}$	1/7	3/7	5/7
$\bar{9}$	1/9	3/9	5/9
$\bar{11}$	1/11	3/11	5/11

I will restate this thus;

$\left.\begin{matrix}\bar{7}\\ \bar{9}\\ \bar{11}\end{matrix}\right\} \times 7\cdot9\cdot11 =$

×	1	3	5
9·11	1·9·11	3·9·11	5·9·11
7·11	1·7·11	3·7·11	5·7·11
7·9	1·7·9	3·7·9	5·7·9

In the 1,3,5,7,9,11 eikosany the 1)3 {1,3,5} triany has a complementary relationship to the 2)3 {7,9,11} triany. I call them cross-sets of each other, for lack of a better term.

Multiplied by each other they give a very cohesive (& very musical) block, which as you see, occurs entirely within the limits of the eikosany. There are 20 varieties of this figure, which need not be tabulated here.

The other pair of cross-sets, not involving the monany may be illustrated thus:

2)4 {1,3,5,7} hexany

	U	1,3	1,5	1,7	3,5	3,7	5,7
1)2 {9,11} dyany	9	1,3,9	1,5,9	1,7,9	3,5,9	3,7,9	5,7,9
	11	1,3,11	1,5,11	1,7,11	3,5,11	3,7,11	5,7,11

There are 15 varieties of this figure. I could have multiplied directly here, however in some cases the Union operation must be performed prior to multiplication to get the desired results. If I multiply a 0)0 {Ø} monany by a 3)6 {ABCDEF} ~~hexany~~ eikosany I get a set of 20 nulls*, which is not an answer consistant with the remaining context. I must U first and then multiply in such a case. *I sense intuitively that this is an incorrect answer; I should be able to cross the monany by the hexany and get the hexany, even if my monany is a 0-out-of-0 situation, 0)0. Using the notation of binomial coefficients, and notating 0-out-of-0 and 3-out-of-6 as $\binom{0}{0}$ and $\binom{6}{3}$ instead of 0)0 and 3)6, it would appear that I may perform this operation:

$$\binom{0}{0} \times \binom{6}{3} = 1 \times \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} = 20 \quad *$$

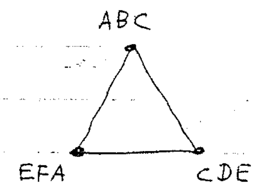
This is the kind of answer I want, ~~but with~~ except for the specific details of the sets. Any suggestions?

Cycles

I've come across an interesting family of cycles, in the combinational sets, that I hadn't appreciated before. I will illustrate from the eikosany;

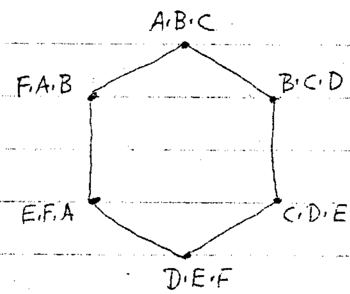
3 tone cycle:

A·B·C
C·D·E
E·F·A
(A·B·C)



6 tone cycle:

A·B·C
B·C·D
C·D·E
D·E·F
E·F·A
F·A·B
(A·B·C)

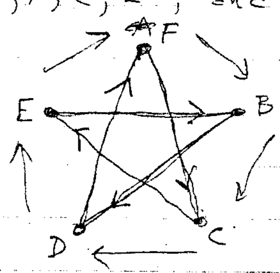


This 6-cycle is correctly mapped over a hexagon, but unlike the hexany, it is not akin to the octahedron. Performing it in a harmonically biased eikosany (like 1·3·5·7·9·11), one appreciates, immediately, its greater dissonance, compared to a hexany, in the same environment. Obviously, by arranging A, B, C, D, E, F in its various sequences, I will get the (????) permutations* of these cycles. I have yet to estimate the ramifications of this cycle, but I expect it will enrich, considerably, the vocabulary of the Eikosany.

*60 for the 6 tone, ? for 3 Tone.

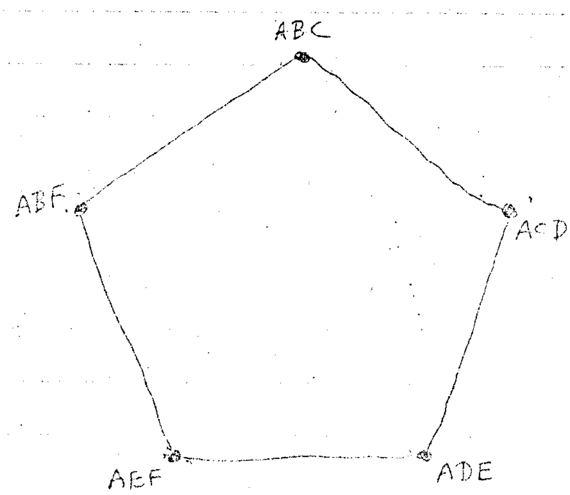
Pentagonal cycles:

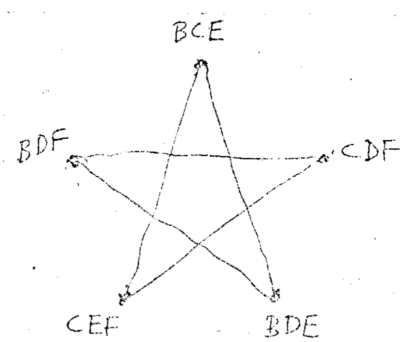
Pentagonal cycles in the 2)5 & 3)5 context are formed as shown. The 3)5, is automatically in 3)6 context, but the 2)5, must elevated to that status by supplying the missing member. ("A") A [1,2] Transform of the 2)5 series is here applied To the 3)5 series to illustrate a relationship. ([1,2] B, C, D, E, F = B, D, F, C, E, and may be visualized thus:



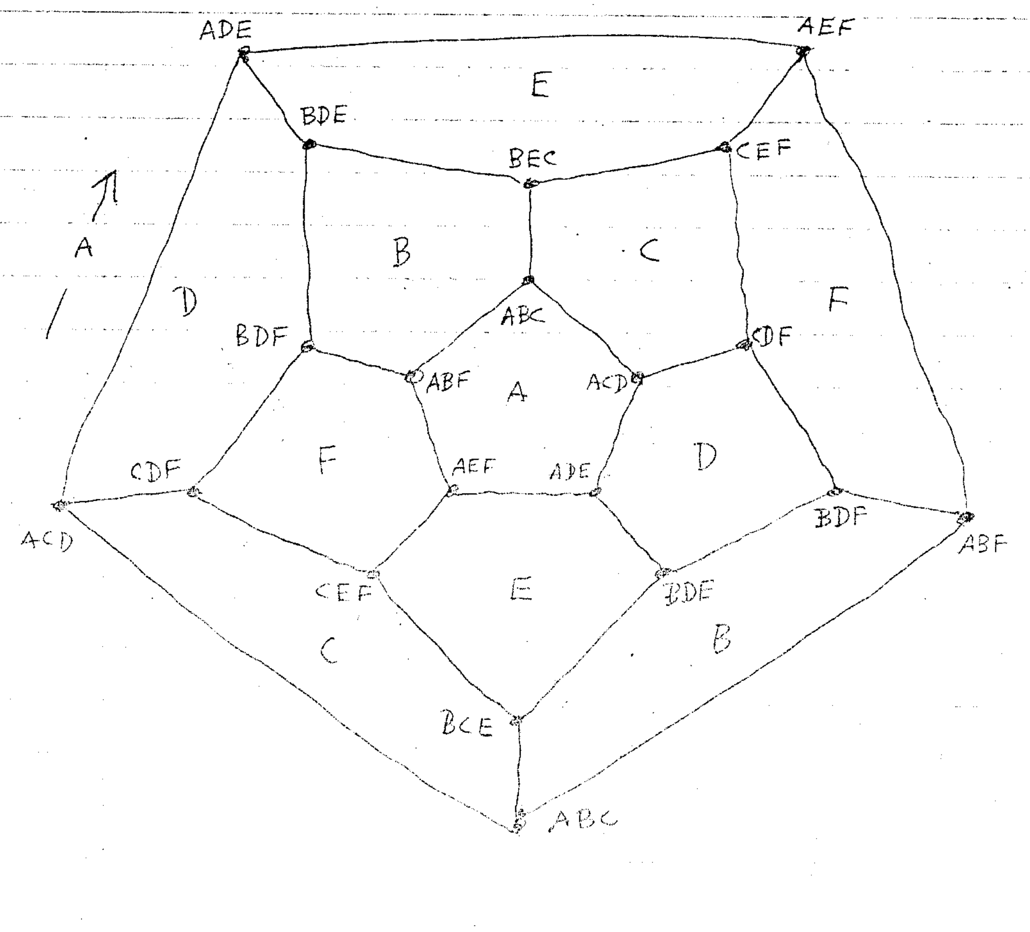
The pentagon represents the "1," and the pentagram represents the "2", of the [1,2] Transform.)

2)5 +A	3)5
A · B · C	B · D · F
A · C · D	D · F · C
A · D · E	F · C · E
A · E · F	B · C · E
A · B · F	B · D · E
(A · B · C)	(B · D · F)

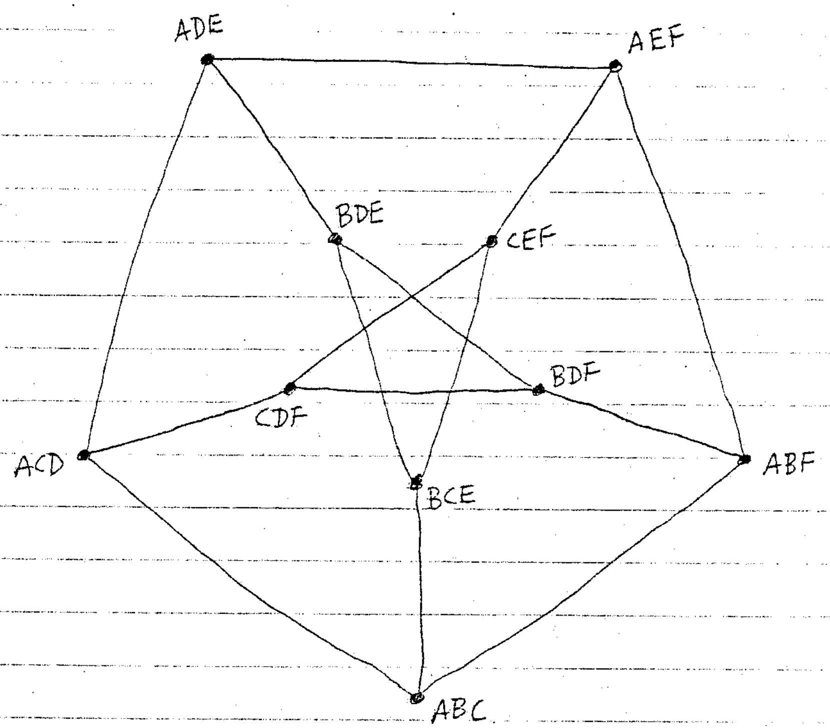




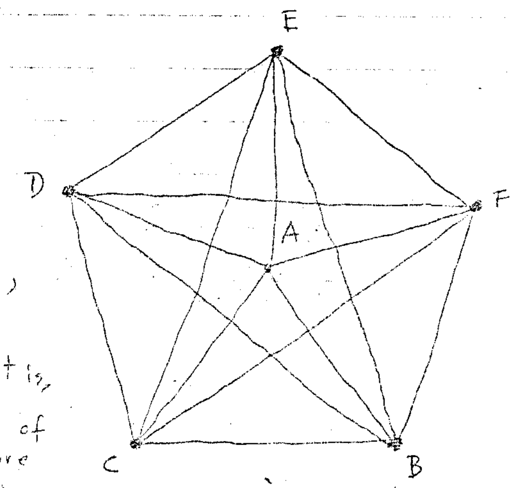
A pentagon of either ~~ver~~ species will intersect at 2 connected points with another(1) pentagon of the same species. It will also, intersect at 1 point with 2 other pentagons of the same species. A mesh of pentagons of either species may be mapped over the pentagonal dodekahedron. Each pentagon returns to its likeness on the opposite side, and each member of the set duplicates on the opposite point. Very strange. The letter in the center of each pentagon identifies the element ~~in~~ ~~intersected by eac~~ common to each member of the pentagon. The "2) 5 plus" species is shown:



An imaginary structure having the same properties of intersection (as above), but with each pentagon, and each member point, appearing only once, may be diagrammed thus:



(The above diagram is consistant with this diagram of the hexad. Each of the 15 relationships, here, occur, also, in the above figure, once. Interesting. That is, from the same no. of sticks of respective lengths & orientation either figure may be constructed)



We find the 6 2/5 pentagons cycles intersecting ~~conjuncting~~

in precisely the same way they did over the dodekahedron. But there is more, that wasnt apparent on the dodekahedron. We find a similar mesh of 3)5 pentagon cycles utterly congruent with the 2)5 pentagon cycle mesh. ~~Each of these are~~ There are six of these, also, each of which complements a 2)5 cycle by omitting the element common to all members of that cycle, and transforming [1,2] the sequence of the remaining 5 elements. Trip! A cycle of either species will intersect, by 3-members-in-sequence, 5 cycles of the other species. The 6th (complementary) cycle will have no point of intersection. Thus, we find that instead of 6 cycles, each occuring twice, over a dodekahedron, we have 12 cycles, each occurring once over a regular imaginary polygon (of a somewhat more elusive nature).

10-tone cycles (dekagram)

10-Tone cycles may be constructed in 2, complementary ways:

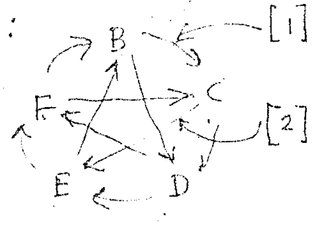
[1] (1)	(2)
A · E · F	A · D · F
· B · E · F	B · D · F
A · B · F	A · B · D
· B · C · F	B · D · E
A · B · C	A · B · E
· B · C · D	B · C · E
A · C · D	A · C · E
· C · D · E	C · E · F
A · D · E	A · C · F
· D · E · F	C · D · F
(A · E · F)	(A · D · F)

If figure (2) is turn top-for-bottom, and the letters B, C, D, E, F rearranged from left to right, in the sequence of their actual introduction from new top to bottom, and we begin the cycle at A·C·E;

[2] (3)

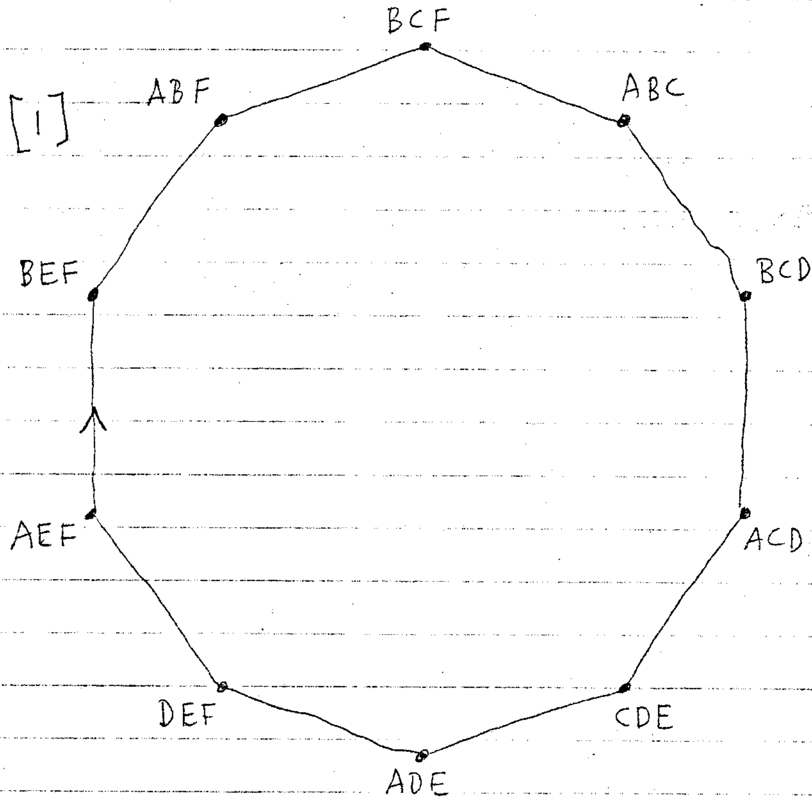
A · C · E
· B · C · E
A · B · E
· B · D · E
A · B · D
· B · D · F
A · D · F
· D · F · C
A · F · C
· F · C · E
(A · C · E)

it can be seen that the structure of cycle [2] is identical to the structure cycle [1]. The difference is that the ~~sequence~~ cycle B, C, D, E, F is [1,2] Transformed to B, D, F, C, E (by taking Taking every other element of the cycle:

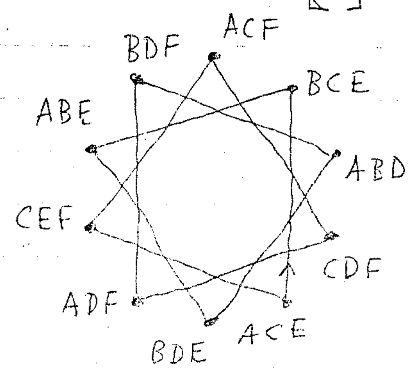


For purposes of showing a relationship* the 2 cycles may be represented like this:

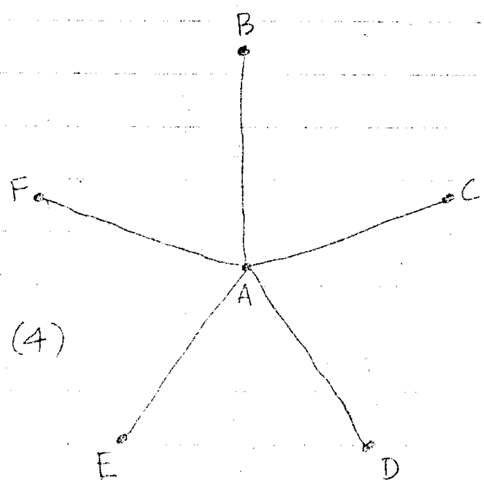
[1]



[2] or (2)



(4)

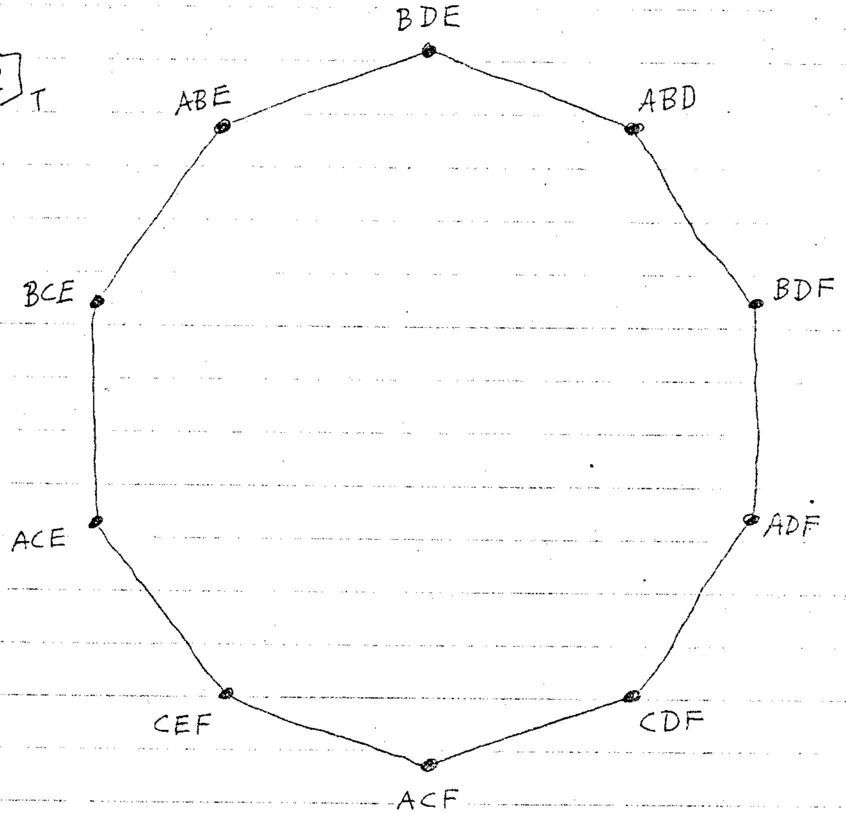


(4) is the hexad lattice consistent with [1] and [2].

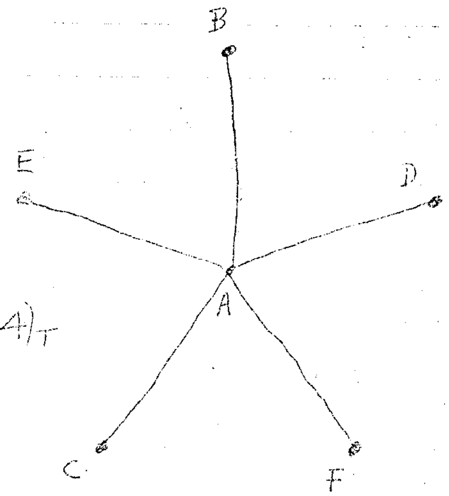
* See page 23

Or, the geometric images of cycles [1] and [2] may by Transformed, one into the other, $[1]_T$ & $[2]_2$, by [1,2] Transforming the sequence of the 5 radial elements in the hexad $(4)_T$,

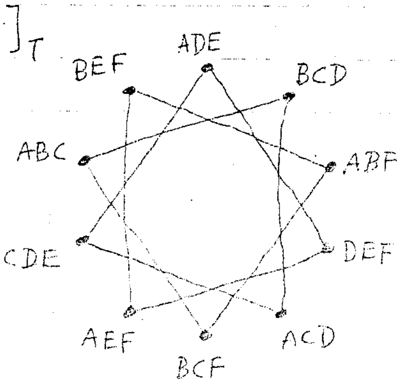
$[2]_T$



$[1]_T$

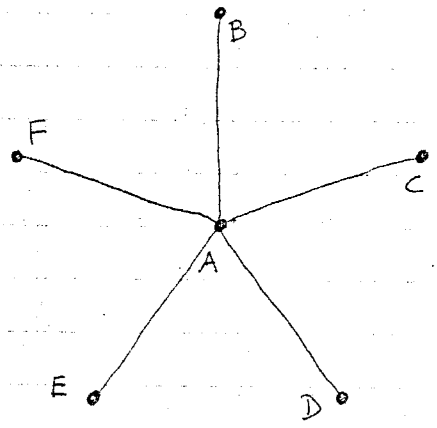


$(4)_T$

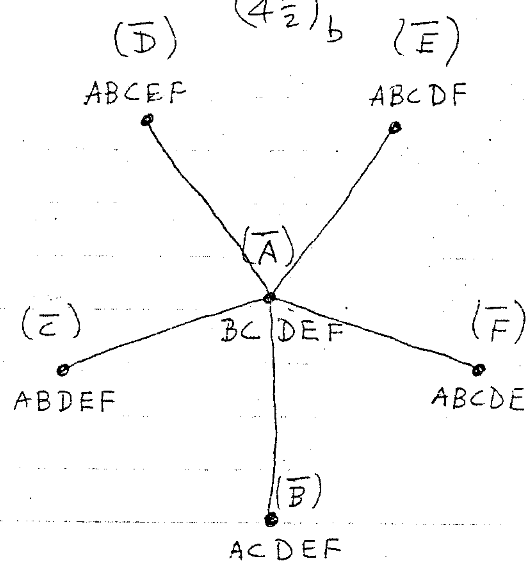


Therefore, structurally, cycles [1] & [2] bear equivalent relationships to each other.

$(4\frac{1}{2})_a$



$(4\frac{1}{2})_b$



The set of elements of the generating hexad ~~are~~ is congruent with the 1)6 hexany (not to be confused with the 3)6 hexany). The antipodal 5)6 hexany is congruent with the inverse of the generating hexad as shown in parentheses, above. These 2 hexad images are biased toward A (and $\bar{A}$); In the A,B,C,D,E,F hexad there are 5 possible dyads having A as one element: A,B ; A,C ; A,D ; A,E ; A,F. These, and only these are outlined in the hexad images. Likewise, these are the only dyads to appear, in connected form, in the dekagrams [1] & [2]. Each of the 5 dyads appears twice in the dekagram. (The only difference between dekagram $[1]_{\ast}$ and $[2]_{T}$ is the sequence in which the dyads occur.)

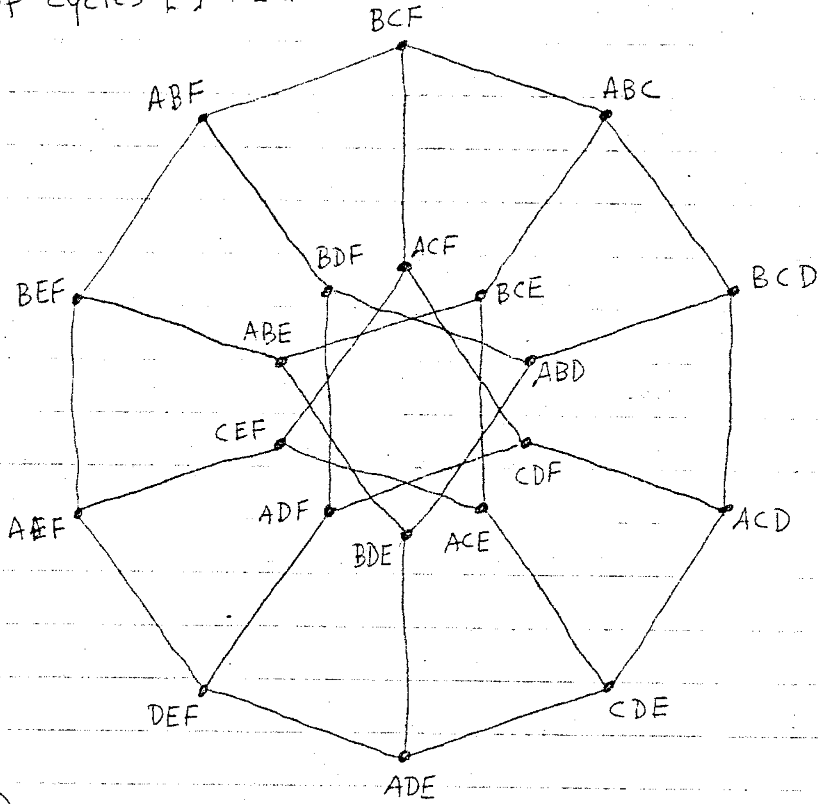
In the hexad images we see two groups of connected triads; those with 72° elbows at A (or $\bar{A}$), and those with 144° elbows at A ($\bar{A}$). In Dekagram [1] The 5 possible 144° triads of the hexad appear, alternately, with the

5 possible 144° Triads of the inverse hexad. Examination shows that every other member of the dekagram contains an A element, and will have an A function in a triadic context. The alternate members of the dekagram are notable by their exclusion of element A. These, a little reflection will show, have a $\overline{A}$ function in an inverse Triad context. The 72° Triads of the hexad and inverse hexad alternate, in a similar manner, in dekagram [2].

I should mention, also, before I go on, that cycles [1] and [2], altho complementary to each other, in the sense that the one contains the members of the eikosany that the other excludes, they are not opposites, or antipodal to each other. I am using the word "antipodal" or "opposites" to describe the members of the subsets of (in this case) ~~...~~, A·B·C·D·E·F whose elements complement each other; i.e. A·B·C and D·E·F are opposites or antipodal because their elements complement each other to make the complete hexadic set. I will in some cases augment the meaning in this manner: A^3 and $\frac{B\cdot C\cdot D\cdot E\cdot F}{A^2}$ are antipodal in a symmetrical, hexadic tone-space context because when multiplied by each other they yield A·B·C·D·E·F. And there are other variations, which will come up. The dekagram, like the eikosany, contains its own opposite, because the antipode of each member is present in the cycle.

(5)

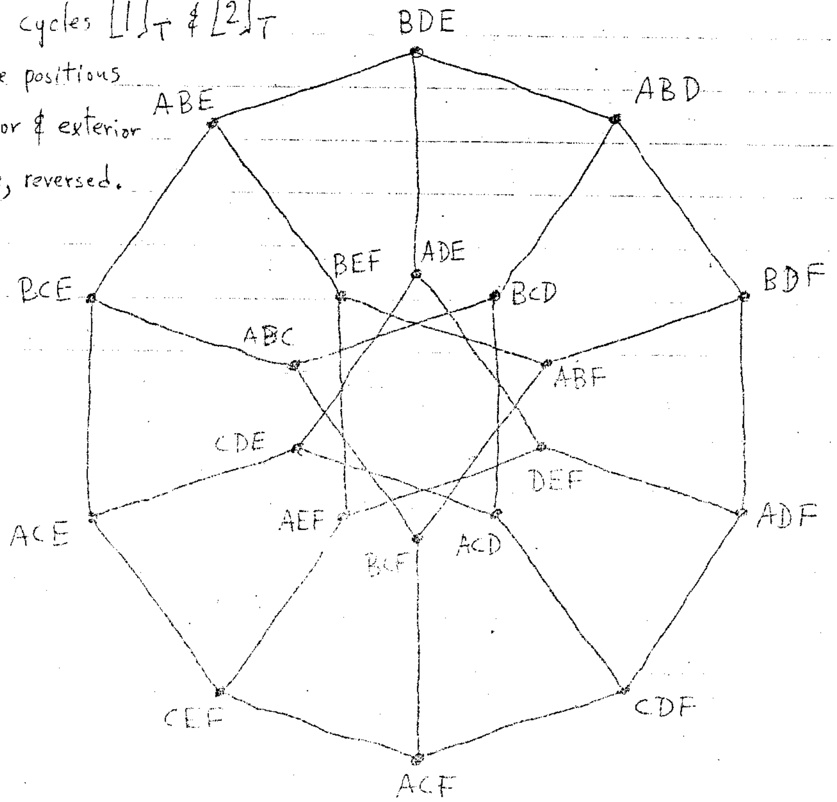
Union of cycles [1] & [2]



(5)$_T$

Union of cycles $[1]_T$ & $[2]_T$

Showing the positions of the interior & exterior cycles, above, reversed.



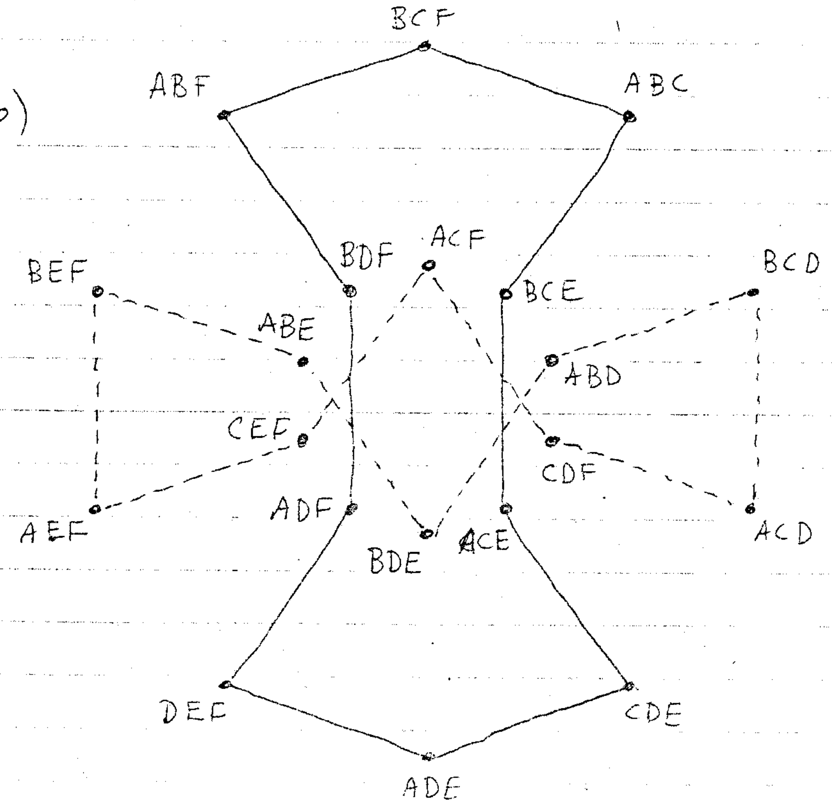
Cycles [1] & [2] may be joined (5) by connecting certain points, as shown, ~~with~~ those dyads already outlined in the dekagrams. Each of the ~~interval~~ 5 dyads in the hexad (4) repeats ~~3~~ 6 times. This figure is the Eikosany, and its structure is the ~~inevitable~~ inevitable result of geometrically projecting the set of triplicate, multiplicative, combinations of the hexad (4). It is a very interesting figure, as it outlines each of the combinational subsets of the A·B·C·D·E·F Eikosany containing the element, A. That is, it is biased toward A, as was the hexad figure (4) from which it generates. No need to digress further, here; back to the dekagrams. By connecting the 2 complementary dekagrams, thus, one may find, outlined, the full set of 12 dekagrams having "A" fixed and the B, C, D, E, F cycle in its 12 permutations (6 of which are [1, 2] transforms of the other 6). The cycles are: [see (6) for typical outlines]

1,	B F A C E D	B F A C F D	B F A C D E	B D A C E F	B F A D E C	B F A E C D
2	B E A D C F	B F A D C E	B D A E C F	B E A F C D	B E A C D F	B C A D E F

The next set in the dekagram heirarchy is one of 72 dekagrams, or 36 dekagram-plus-complement pair. This is derived by holding each element in turn ~~member~~ of the hexad, fixed, and permuting the cyclic sequence of the remaining five elements. This superset continues to have only 20 tones, but it ~~they~~ must be outlined with ~~to~~ each of the 15 dyads (rather than 5), possible in the hexad, These occurring six times, i.e. full eikosany lattice.

Returning to the dek~~ants~~agrams in the 12-dek~~ant~~agram set, 2 of the outlines have been shown, [1] & [2]. The remaining ten dekagrams can be found outlined as shown below (6), and rotating in increments of ~~72°~~ 36°. The dekagram in dotted outlines is complementary to the dekagram in solid outlines, also, typically so thru the rotations.

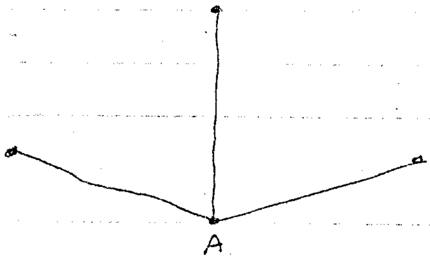
(6)



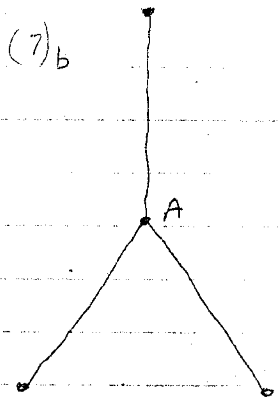
IT is easy enough to visualize 5- or 10-fold cycles of dekagrams formed by rotating either, or both together, of the above patterns by 36° or 72° increments. No need to Tabulate these here.

Consider the union of cycle [1] & [2], (5), in relation to the hexad and its inverse, $(4\frac{1}{2})$. The tetrads, in the hexad, containing the A element, number ten. They appear in either of these two forms $(7)_{a,b}$ and 72° cyclic rotations of them!

$(7)_a$

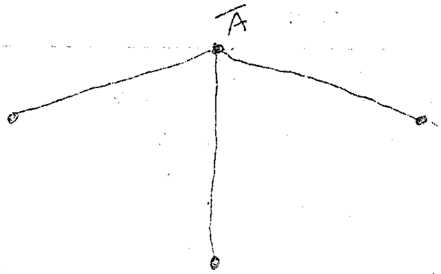


$(7)_b$

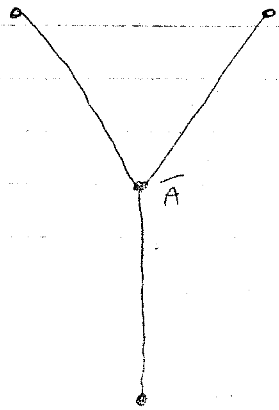


Ten, further, inverse tetrads appear, similarly, in the inverse hexad $(4\frac{1}{2})_b$. These contain element $\overline{A}$, $(8)_{a,b}$:

$(8)_a$



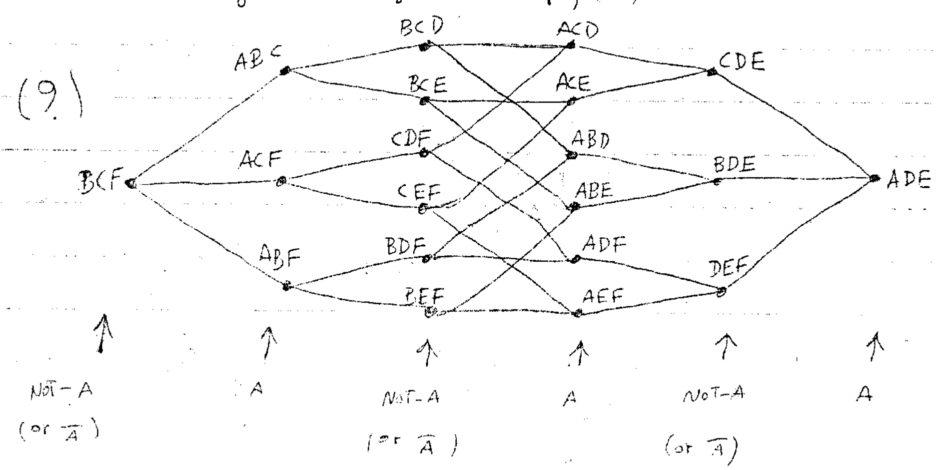
$(8)_b$



~~This sum of~~ Each of these 20 tetrads will appear once in the eikosany diagram (5). Regarding the diagram (5) one may observe that each member, n, is connected to

3 other members, p_2 p_3 p_4. ~~This~~ Any 4 members, thus connected, invariably form a 1)4 tetrany (Tetrad) or a 3)4 Tetrany (Inverse Tetrad). If the member at p_1 contains the element A, p_1 will have an A function in a tetradic context. If, on the other hand, the member at p_1 excludes the element A, p_1 will have a $\overline{A}$ function in context of the inverse tetrad. A meander thru the connected points of the eikosany (5), it follows, will pass thru, alternately, A-inclusive and A-exclusive members. Or, to rephrase, a meander will pass thru, alternately, the A-function of a Tetrad and the $\overline{A}$-function of an inverse Tetrad. This diagram might help, (9):

(9)



The expanding network of ~~A,~~ alternating A, $\overline{A}$ closes itself on the opposite end of the eikosany, or the antipode of its origin.

After 10 pages of laying the groundwork, I will now present a vastly elegant & lovely dual-cycle involving cycles [1] & [2], & $(10)_{a,b}$.

(10) ↓ cycle (2), circulating once ↓ cycle (1), circulating 3 Times

A	B	C	D	E	F	A	B	C	D	E	F	
						A	B				F	
A		C			F		B	C			F	
						A	B	C				
							B	C	D			
		C	D		F	A		C	D			
								C	D	E		1
						A			D	E		
A			D		F				D	E	F	
						A				E	F	
							B			E	F	
	B		D		F	A	B				F	
							B	C			F	
						A	B	C				
A	B		D				B	C	D			
						A		C	D			2
								C	D	E		
	B		D	E		A			D	E		
									D	E	F	
						A				E	F	
A	B			E			B			E	F	
						A	B				F	
							B	C			F	
	B	C		E		A	B	C				
							B	C	D			
						A		C	D			3
A		C		E				C	D	E		
						A			D	E		
									D	E	F	
		C		E	F	A				E	F	
							B			E	F	

Return to Beginning

(10)$_T$ ↓cycle (1), circulating once ↑cycle (2), circulating 3 Times

A	B	D	F	C	E	A	B	D	F	C	E	
						A	B				E	
A		D			E		B	D			E	
						A	B	D				
							B	D	F			
		D	F		E	A		D	F			1
								D	F	C		
						A			F	C		
A			F		E				F	C	E	
						A				C	E	
							B			C	E	
	B		F		E	A	B				E	
							B	D			E	
						A	B	D				
A	B		F				B	D	F			
						A		D	F			
								D	F	C		2
	B		F	C		A			F	C		
									F	C	E	
						A				C	E	
A	B			C			B			C	E	
						A	B				E	
							B	D			E	
	B	D		C		A	B	D				
							B	D	F			
						A		D	F			
A		D		C				D	F	C		3
						A			F	C		
									F	C	E	
		D		C	E	A				C	E	
							B			C	E	

Return to Beginning

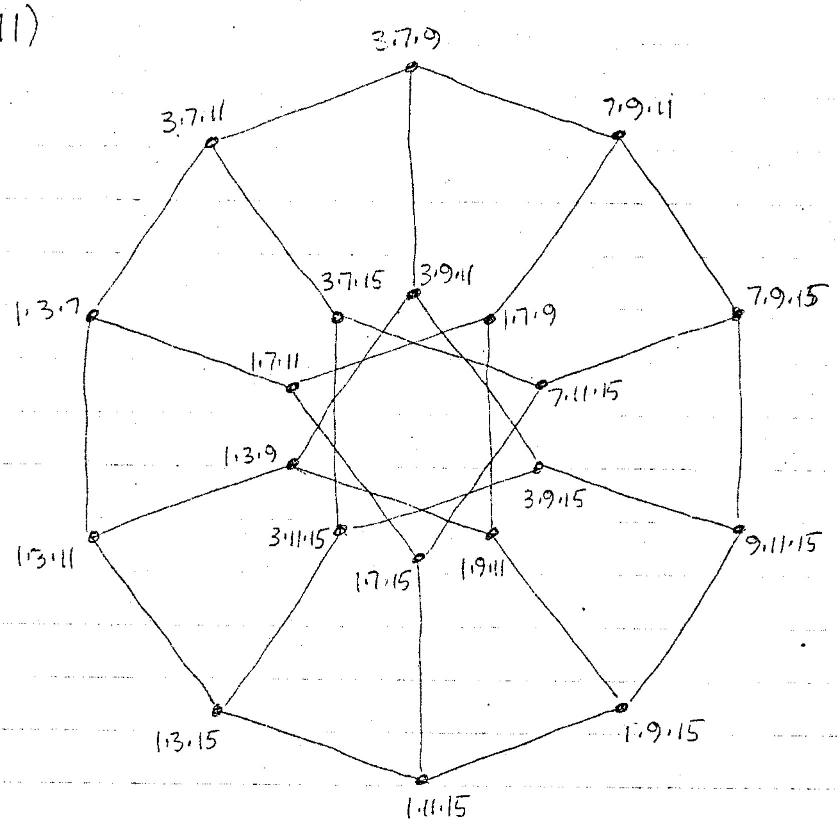
Cycles (1) & (2) are synchronized with each other, one progressing thru 3 members while the other progresses thru 1 ~~one~~ member, as shown in diagrams (10) & $(10)_T$ ~~&~~, synchronized in such a manner that the 3 members of the one cycle are supplemented by the one member of the other cycle to form, amongst the 4 members, alternately, a 1)4 Tetrany (Tetrad) and a 3)4 Tetrany (inverse Tetrad). The duo-cycle has 4 forms; forward & retrograde; sequence (10) & convoluted sequence $(10)_T$. ~~It may start anywhere~~ ~~at~~ The cycle may begin at any ~~point~~ tetrad of ten. Duo-cycles (10) & $(10)_T$ may both be observed visually in either or both eikosany diagrams (5) & $(5)_T$. Something interesting, worth pointing out: in duo-cycle (10), cycle (1) moves at a triple rate forward to cycle (2), moving in same direction; in duo-cycle $(10)_T$, cycle (1) deccelerates to 1/3 rate forward, but cycle (2) has now reversed itself and is moving at a triple rate retrograde. Hmm!

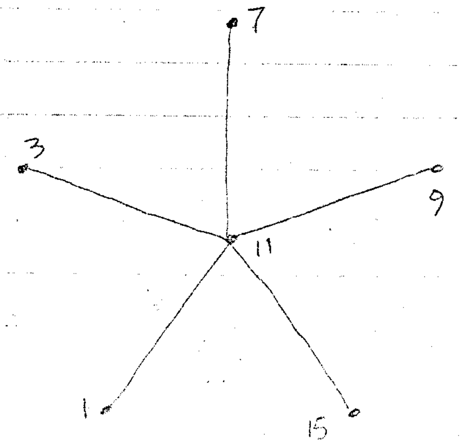
I'll include 3 figures from actual use; lattice of the 11-7, 9, 15, 1, 3 eikosany (11), formation of the double-cycle (12), notating the cycle (13). This notation is for a specific instrument, but it is identified on the top of the sheet.

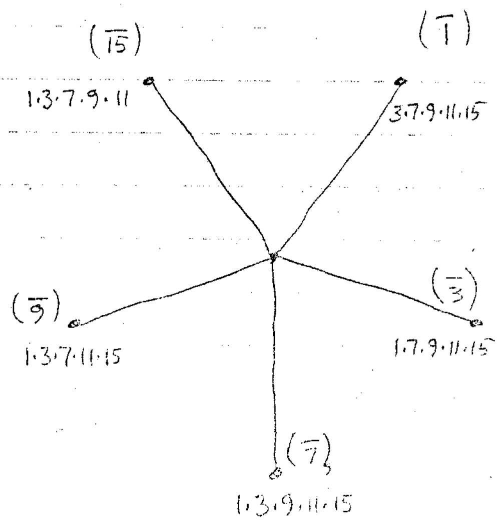
Theres more, but I'll pick it up later.

Erv Wilson Jan 71

(11)

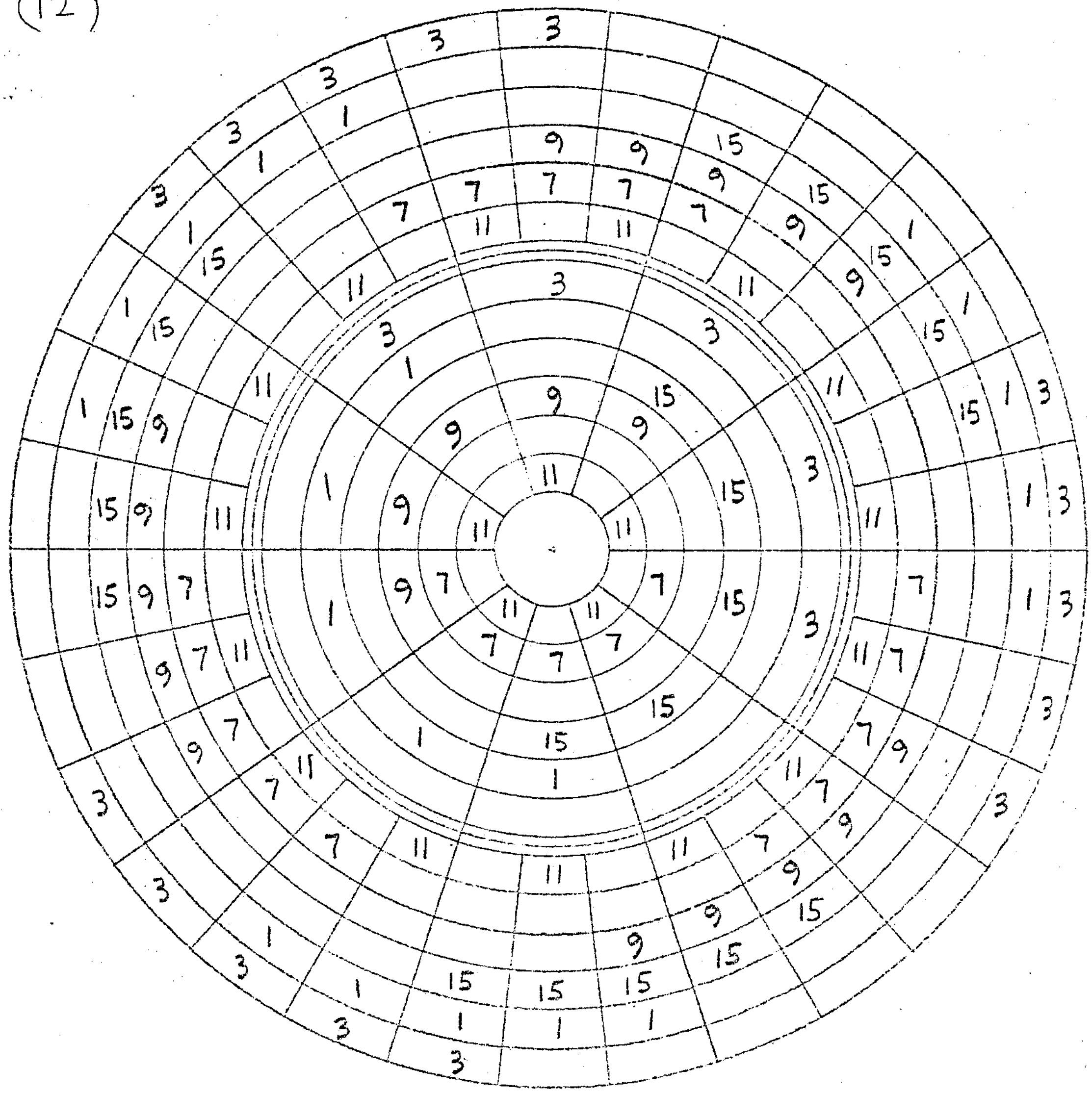






There would be 72, each, of these figures (11), (12), (13) for any given eikosany. I'll not tabulate these here, nor anywhere, much as they deserve to be tabulated.

(12)



7
3 11 9 Aspect, Complementary Dekagrams In
1 15
Duo-Cycle

3 7 11 9
1 15 Aspect Complementary Dekagrams,
1·3·7·9·11·15 Eikosany

(13)

Basis for my improvisation entitled: "Nuptial Flight of the Nocturnal, Three-Toed Tree Toad".

E. Wilson, '71