C. John Chalmers

Letter to B. McLaren 9-10-89

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B. McLaren,

The triad of 15 ET altho not very well in tune with the 4:5:6 triad, is in a sense, very well in tune with itself. And in non-just tunings that may be what really matters, (rather than the degree of approximation to just). At least, I suspect as much. Putting the major triad in root position, at A 220;

15	Pitch	Difference tones	Beats per second
$A_{0/15}$	220	57.1826	.9076
$C\#_5$	277.1826	56.2750	
E_9	333.4576		

$$\frac{A_0+E_9}{2} = \frac{220+333.4576}{2} = 276.7288 \quad \text{(Third, of the Arithmetic mean)}$$

$C\#_5$ is 2.8367 cents above the arithmetic mean of A_0 and E_9. A very good approximation.

By way of contrast/comparison lets look at the major triad in 12, in root position at A440;

12	Pitch	Difference Tones	Beats per second
$A_{0/12}$	220	57.1826	4.7377
$C\#_4$	277.1826	52.4449	
E_7	329.6275		

$$\frac{A_0+E_7}{2} = \frac{220+329.6275}{2} = 274.81375 \quad \text{(arithmetic mean)}$$

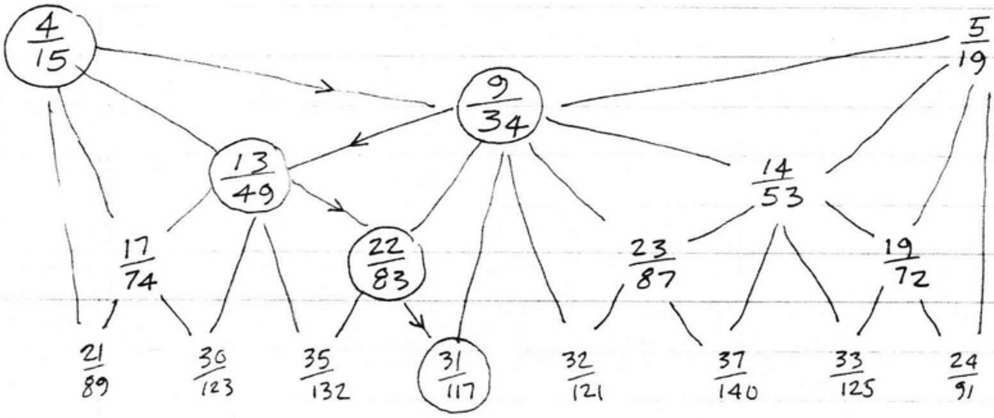
The arithemetic mean of A_0 and E_7 is 14.8590 cents below $C\#_4$. A very poor approximation, for 12.

The point of the arithmetic mean of the Fifth being that the difference tones of the 2 thirds A C#, and C# E will be exactly in tune with each other, and thus not

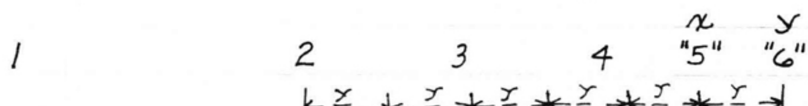
9-12-89

Dear John, B. McLaren has sent me a tape of some very effective music he has composed for 15 equal division of the 8ve., also some other equal scales; 13, 14, 16, 17, 19, 24. His letter is encl., and this is my answer to him. Perhaps you have something to add. get his tape! Also you may wish to see if he has a contribution for XH#XII, yours, Erv beat with each other. (With a pure sine tone these would be the primary source of beating.)

If the 15-tone scale is treated as a chain/cycle of minor thirds (4 units) (especially in the Hansonian* tuning strategy where 6 minor thirds are equal to the Octave-and-a-Fifth), it finds itself in the company of some excellent and highly efficient tonal systems. That section of the scale tree looks like this;



Each of these scales will have 15 as a linear moment-of-symetry. A sequence of scales which give a good approximation to the arithmetic-mean-of-the-Fifth is shown encircled. A scale of 117 tones cycling by 31 units is very nearly ideal in that regard.



The exact value of that minor third, $\frac{y}{x}$, which will give the triad of the arithmetic mean in a linear series of minor thirds is found thus; $x = \frac{y+4}{2}$, and $\frac{x}{2} = \left(\frac{y}{x}\right)^5$ which solving for $y = \sqrt[5]{x^5\left(\frac{x}{2}\right)}$. This can be solved with a hand-held calculater by iteration, and patience. (I am indebted to Larry Hanson for technical assistance in solving this problem.)

* See Development of a 53-tone Keyboard Layout, Larry A. Hanson, Xenharmonikon XII 1989

The value for Y is 6.020138134, and the value for X is 5.010069067. Hence the critical minor third $\frac{Y}{X} = 1.201607813$.

		$\frac{Y}{X}$ to the Power		Pitch	Difference Tones	Beats per second
0	0	0	A	220		
1	1	4		229.32091		
2	2	8		239.03673		
3	3	12	B	249.16418	55.5538	
(4)		(16)		(259.72072)		
5	4	1	C	264.35372		
6	5	5	C#	275.55380		0
7	6	9		287.22840		
8	7	13	D	299.39763		
9		(17)	D#	(312.08244)	55.5538	
10	8	2		317.64949		
11	9	6	E	331.10760		
12	10	10		345.13589		
13	11	14		359.75853		
14		(18)	F#	(375.00070)		
15	12	3		381.69011		
16	13	7	G	397.86148		
17	14	11	G#	414.71798		
18		(15)		(432.28866)		
19/0	15/0	0	A	440		

Moments-of-Symetry (scale-like patterns falling along the chain of minor 3rds), occur having the following number of Tones; 1, 2, 3, 4, 7, 11, 15, 19, 34, 49, 83, 117, 200, 317, 434, 551, 668, 785, 902, 1019, · · · when 1.201607813 is taken as the generator. These are non-equal. However they can be "equalized", and with good approximations, in some cases, to the arithmetic mean of the respective Fifth.

34 is a noteworthy case;

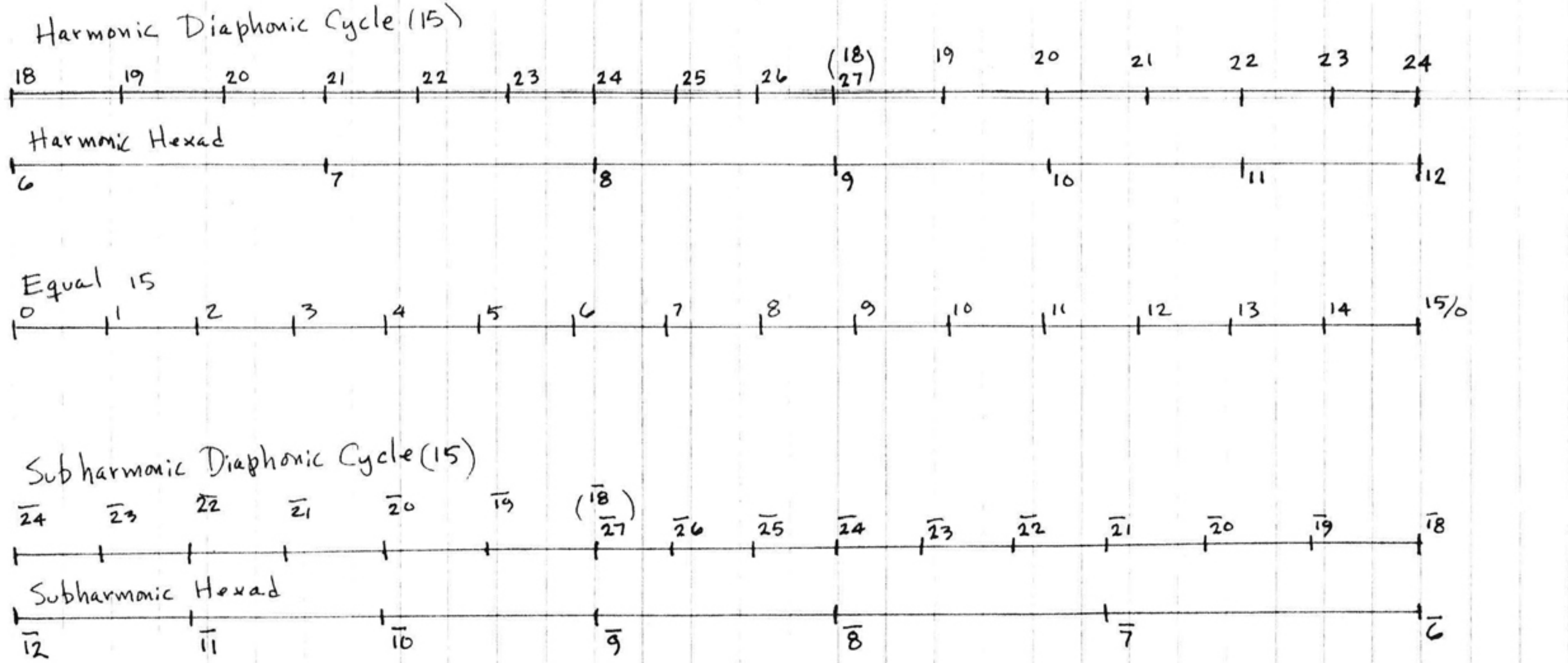
34	Pitch	Difference Tone	Beat Per second
A_0	220	55.3054	.1387
$C\#_{11}$	275.3054	55.4441	
E_{20}	330.7495		

arith. mean — 275.3748 ÷ 275.3054 = 1.000251818 = .4359002 cents (−), (That is $C\#_{11}$ is .436 flat of the arithmetic mean). This puts 34 in an extremely favorable light.

Now then — you can make linear (Moment-of-symetry) scales of 7, 11, 15, & 19 tones based on a generator of 9/34 (minor Third), of 34-tone equal-division. [A chain, of 9-unit intervals/ out of the 34 equal scale]. You may find it interesting to experiment with these.

Or — you can make linear (MOS) scales of 7 and 11 tones out of 15 equal by superimposing a generotor of 4/15 (minor Third). If you havent already discovered these you may find them interesting. (see separate sheet for a diagram). You may, if you wish, try modulating the 7 tone scale about the 15-tone cycle of minor Thirds (4 units). Or harmonizing the 11-tone scale with major and minor triads! I know its amazing, but it can be done.

There are obviously other reasons why you may prefer 15 to 34, tho.



The first 15-tone scale(s) I entertained was constructed with a chain of (sub) harmonics, from 18 to 27 connected to a second chain from 18 to 24, (to complete the 8ve). This gave me a number of nice tetrachordal scales & variations and the (sub) harmonic series thru 6 to 12. Obviously, a 15-tone equal scale alludes very broadly to these "Diaphonic Cycles" as I call them.

In certain voicings the harmonic series might sound quite good in 15 because of a roughly proportional stretching of the sonority. I would expect the voicing; 7 8 10 11 12 15 18 26 on scale degrees 12. 0. 5. 7. 9. 14. 3. 11. respectively to have a rather brilliant stretched quality, (altho I regret to say I've never heard it). The chord,

7 8 10 11 15 , (and its subharmonic mirror)
12. 0. 5 7. 14.

give a stretching, independently of the 8ve, that rather suggests the tones of Pelog. Can it be that you are picking out extended harmonic & subharmonic allusions out of 15? Or that intuitively you are picking out Pelog-like organizations of tone? Whatever you are doing keep it up— it sounds good to me.

I'm sure that John Chalmers will be interested in hearing a copy of your tape. His address is;

John Chalmers
9411 Beckford Drive
Houston TX 77099

If you have a score of a composition in 15 equal that you feel is representative of your intuitions on the subject, you might consider submitting it to him for publication in Xenharmonikon, possibly with some commentary on your compositional techniques.

Are you acquainted with Augusto Novarro's book Sistema Natural de la Música, mexico, 1951 (Library of Congress call no. ML3805,.N69, 1951)? He has a sizeable section given to 15, showing guitar fretting, fingering diagrams, a keyboard design, notation, and a small Prelude.

Here is a sample of his "Escalas fundamentales", based in this case on the non-octave, 4/1;
(Transcription into harmonics is my paraphrasing)

Primera	1					4
Segunda	2			5		8
Tercera	3		6		9	12
Cuarta	4	7	10		13	16
Quinta	5	8	11	14	17	20
etc.						

It was "escalas" like the "cuarta" and "quinta" above that opened my ears to the acoustic validity of arithmetically proportional sequences. Also, the escala 12 17 22 27 32, below;

	3					8
	6			11		16
	9		14		19	24
⟶	12	17		22	27	32
	15	20	25	30	35	40

Also The whole area of summation and/or difference tone series is wide open, and relatively unexplored. This includes the Fibonacci series and Φ (the golden interval that the Fibonacci and all other summation series converge upon). Consider the following 4 voice harmonic changes;

*	3	26	14	30	8	34	18	38	5	
	2	17	9	19	5	21	11	23	3	
	1	9	5	11	3	13	7	15	2	← alternate fixed voice
Fixed → Pitch	1	8	4	8	2	8	4	8	1	
	0	1	1	3	1	5	3	7	1	(First difference tone)

All fine and good — but now try holding the 2nd voice fixed! As you see there is a lot of natural territory to explore before we ever get out to Φ. And look what happens to the first difference tone — (think about it).

Let us suppose now that we make a continuous, 5-voice, sliding gliss thru the 9 chords, and where each voice is maintained in exact arithmetic proportion with all other voices thru-out the gliss. Then let us suppose That at different stages of the gliss we alter our "fixed pitch" from one voice to another, Etc and so on. Now instead of just glissing, let us begin to weave subtle melodies in one voice or another, still maintaining, of course, the exact arithmetic proportion with all other voices. [I dont have the equipment to do this, but it shouldn't be to hard to set up.] What i'm talking about is a rational compositional style in the <u>full</u> <u>pitch</u> <u>continuum</u>.

* Allow me to simplify that chord progression for readability;

24	26	28	30	32	34	36	38	40
16	17	18	19	20	21	22	23	24
8	9	10	11	12	13	14	15	16
8	8	8	8	8	8	8	8	8
0	1	2	3	4	5	6	7	8

Lets do this to the Just Major Scale; (this is all done with primes not exceeding 23)

	3	26	14	11	8	13	38	5	22	12	19	7	23	34	9
	2	17	9	7	5	8	23	3	13	7	11	4	13	19	5
	1	($\underline{9}$)	5	4	3	5	15	2	9	5	8	3	10	15	4
→	1	8	4	3	2	3	8	1	4	2	3	1	3	4	1

Now, the E mode of that scale;

	3	47	17	11	8	21	23	5	
	2	31	11	7	5	13	14	3	
	1	16	6	4	3	8	9	2	← or
→	1	15	5	3	2	5	5	1	

The D mode;

	3	29	91	11	107	13	41	5
	2	19	59	7	67	8	25	3
	1	10	32	4	40	5	16	2
fixed pitch →	1	9	27	3	27	3	9	1

The complex intervals produce summation chords that heighten the effect of complexity, altho the sonorities are very much locked in tune by the Fibonacci effect. The resolutions of complex chords to simple chords are startling in their contrast and beauty. We do not find such progressions in 12 equal.

Another vastly unexplored area is combination-product sets. Dalessandro like a Hurricane in XH12 is a primer in that regard.

And still another area wide open to exploration is Moments-of-Symetry. I've enclosed examples of these in 17 and 31. These are not limited to equal, by any means.

Sincerely,

Erv Wilson

844 N. Ave 65
Los Angeles, CA 90042

phone (213) 256-2624

Linear Tuning of 4:"5":"6" with the "5" as the arithmetic Mean

($\sqrt[6]{3}$ analog)

2	4	"5" (x)	"6" (Y)

$$\frac{Y+4}{2} = x$$

This →
$$\frac{x}{2} = \left(\frac{Y}{x}\right)^5 = \frac{Y^5}{x^5}$$

$$x^5\left(\frac{x}{2}\right) = Y^5 \qquad \sqrt[5]{x^5\left(\frac{x}{2}\right)} = Y$$

$Y = \sqrt[5]{x^5\left(\frac{x}{2}\right)}$	$x = \frac{Y+4}{2}$
	start 5
6.00562217	5.002811085
6.009674153	5.004837077
6.012594761	5.006297381
6.014700038	5.007350019
6.016217671	5.008108836
6.017311729	5.008655865
6.018100453	5.009050227
6.018669068	5.009334534
6.019079005	5.009539502
6.019374547	5.009687274
6.019587619	5.009793809
6.019741234	5.009870617
6.019851984	5.009925992
6.019931831	5.009965915
6.019989397	5.009994698
6.0200309	5.01001545

(Continued)

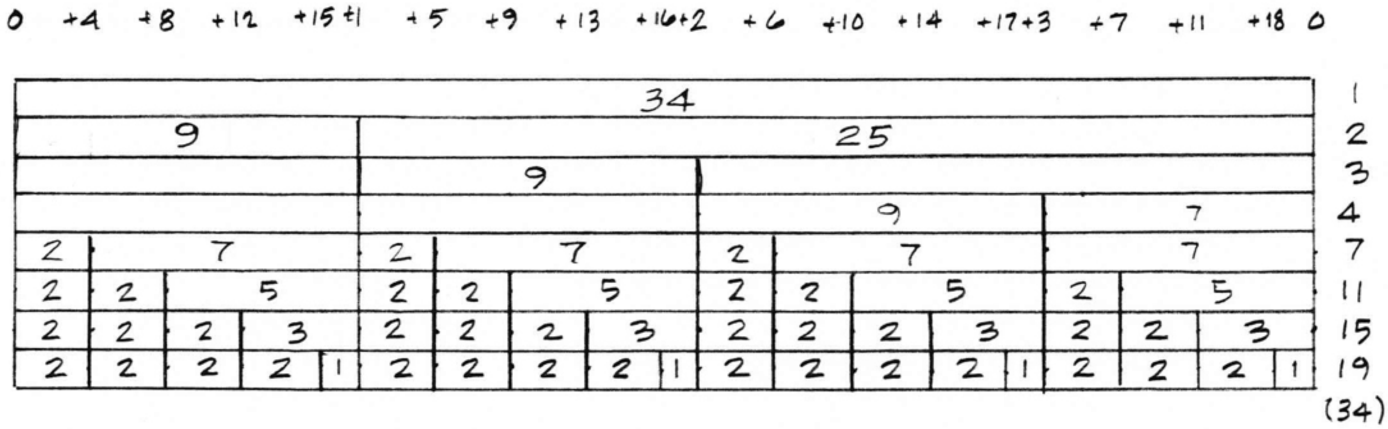
$Y = \sqrt[5]{x^5\left(\frac{x}{2}\right)}$	$x = \frac{Y+4}{2}$
6.020060822	5.01001545
6.020082395	5.010030411
6.020097948	5.010041197
6.020109161	5.010048974
6.020117245	5.010054581
6.020123074	5.010058623
6.020127226	5.010061537
6.020130306	5.010063638
6.02013249	5.010065153
6.020134065	5.010066245
6.0201352	5.010067032
6.020136019	5.01006776
6.020136609	5.01006801
6.020137034	5.010068305
6.020137341	5.010068517
6.020137562	5.010068671
6.020137722	5.010068781
6.020137837	5.010068861
6.020137919	5.010068918
6.020137979	5.01006896
6.020138022	5.01006899
6.020138053	5.010069011
6.020138076	5.010069027
6.020138092	5.010069038
6.020138103	5.010069046
6.020138112	5.010069052
6.020138118	5.010069056
6.020138122	5.010069059
6.020138125	5.010069061
6.020138128	5.010069063
6.020138129	5.010069064
6.02013813	5.010069065
6.020138131	5.010069065
6.020138132	5.010069066
6.020138133	5.010069066
6.020138133	5.010069067
6.020138133	5.010069067
6.020138133	5.010069067
34	

$\frac{Y}{x} = 1.2016607813$, $\text{Log}_2 = .264966098 \times 1200 = 317.9593181$ ¢

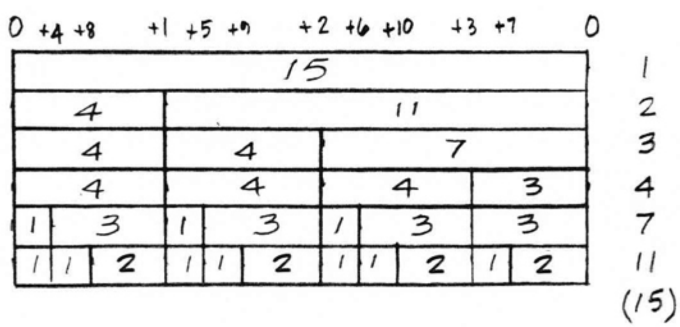
MOS 1, 2, 3, 4, 7, 11, 15, 19, 34, 49, 83, 117, 200, 317, 434, 551, 668, 785, 902, 1019

I am indebted to Larry Hanson for technical assistance in solving this problem.

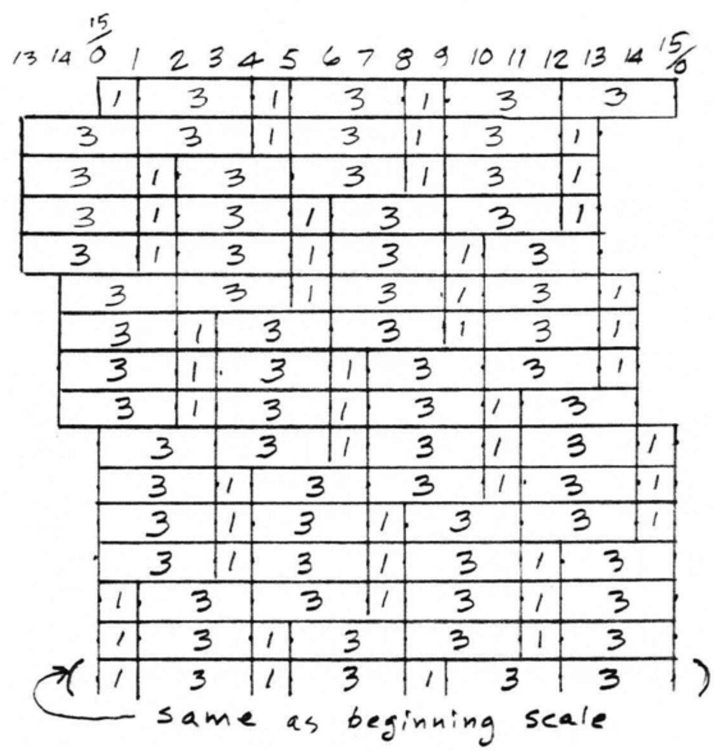
Moments-of-Symetry based on 9/34 8ctave



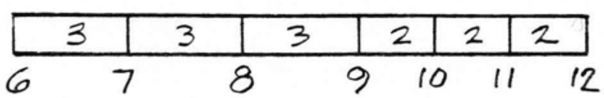
Moments-of-Symetry based on 4/15 8ve



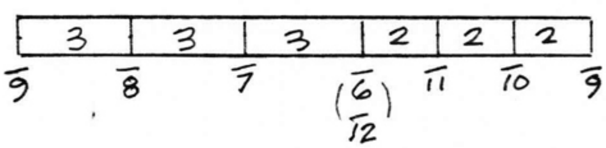
Cycle of 7-tone MOS out of 15. (4/15) is generator.



Allusion to Harmonic Series in 15

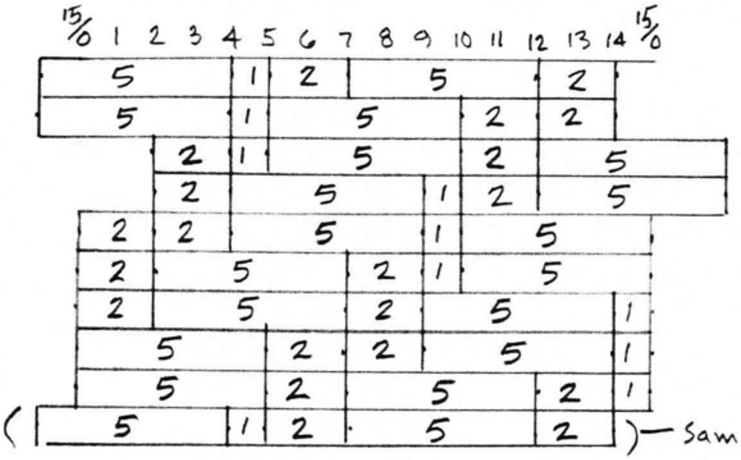


Allusion to Sub-harmonic Series in 15



7 7 6 7
7 6 7 7
6 7 7 6
7 7 6 7
7 6 7 7

9-tone Pseudo-Pelog Cycle, in 15

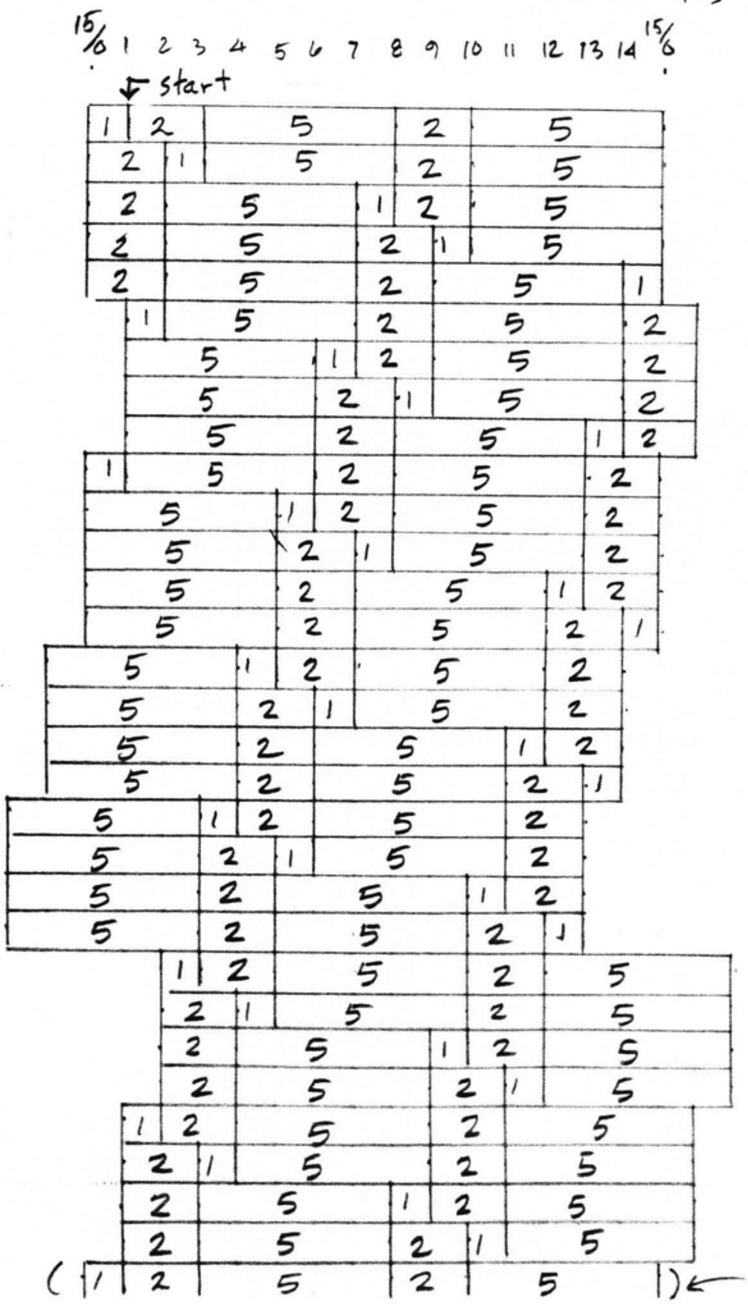


— same as starting scale

Sequence of Fourths

7 6 7 7
6 7 7 6
7 7 6 7
7 6 7 7
6 7 7 6
7 7 6 7
7 6 7 7
6 7 7 6
7 7 6 7
(7 6 7 7)

Consecutive Pentatonics in an iterated 2,5,2,5 Cycle, in 15



← same as beginning scale

2 5 2 5
5 2 5 2
2 5 2 5
5 2 5 2
etc.