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<td>35 ( \gamma )</td>
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</table>

\( \gamma \) fused in 87

\( \gamma \) fused in 87

\( \gamma \) fused in 87

\( \gamma \) fused in 87
NEOUS IDENTITIES
1-3-5-7-9-11-13-15 HEBDOMEKONTANY

ISSUED BY ERV WILSON, 1969
This is a geometric expression of one of the "hyper-spherical" patterns occurring in the set of 70 combinations of 4-out-of-8.

1, 3, 5, 7, 9, 11, 13, 15 refer to members of the harmonic series. Each combination of 4 is multiplied together (and then divide by 15 to place it in a chosen key). 70 new pitches are thus derived expressing this unusual and lovely musical geometry. E.g., abgh.

Hebdomekontany
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 Partitioned Cross-sets of the Hebdomekontany
© 1989 by Erv Wilson

Item 1. In the 4|8 Hebdomekontany the 1|2 dyany subset has 25 varieties, each of which occurs 20 times, in a 3|6 Eikosany partitioned cross-set.

Example; the \( (3) \) 3 dyany \( \times \) the \( (3) \) 5 7 9 11 13 15
Eikosany \( \{ (3) 1 \ 3 \} \times \{ (3) 5 7 9 11 13 15 \} = \)

\[
\begin{array}{ccc}
5.7.9 & 1.5.7.9 & 3.5.7.9 \\
5.7.11 & 1.5.7.11 & 3.5.7.11 \\
5.7.13 & 1.5.7.13 & 3.5.7.13 \\
5.7.15 & 1.5.7.15 & 3.5.7.15 \\
5.9.11 & 1.5.9.11 & 3.5.9.11 \\
5.9.13 & 1.5.9.13 & 3.5.9.13 \\
5.9.15 & 1.5.9.15 & 3.5.9.15 \\
5.11.13 & 1.5.11.13 & 3.5.11.13 \\
5.11.15 & 1.5.11.15 & 3.5.11.15 \\
5.13.15 & 1.5.13.15 & 3.5.13.15 \\
7.9.11 & 1.7.9.11 & 3.7.9.11 \\
7.9.13 & 1.7.9.13 & 3.7.9.13 \\
7.9.15 & 1.7.9.15 & 3.7.9.15 \\
7.11.13 & 1.7.11.13 & 3.7.11.13 \\
7.11.15 & 1.7.11.15 & 3.7.11.15 \\
7.13.15 & 1.7.13.15 & 3.7.13.15 \\
9.11.13 & 1.9.11.13 & 3.9.11.13 \\
9.11.15 & 1.9.11.15 & 3.9.11.15 \\
9.13.15 & 1.9.13.15 & 3.9.13.15 \\
11.13.15 & 1.11.13.15 & 3.11.13.15 \\
\end{array}
\]
Item 2. The $\{\{4\} \mid \{8\} 3 5 7 9 11 13 15\}$ Hebdomekantamy may be partitioned into the following 13 types of cross-sets;

- $\{(8) \emptyset \mid \{8\} \{4\} a b c d e f g h \}$ (1)
- $\{(9) a \{4\} \mid \{8\} b c d e f g h \}$ (35)
- $\{(1) a \{4\} \mid \{8\} b c d e f g h \}$ (35)
- $\{(1) a \{4\} \mid \{8\} b c d e f g h \}$ (35)
- $\{(2) a b \{2\} \mid \{8\} c d e f g h \}$ (15)
- $\{(2) a b \{2\} \mid \{8\} c d e f g h \}$ (28)
- $\{(2) a b \{2\} \mid \{8\} c d e f g h \}$ (15)
- $\{(2) a b \{2\} \mid \{8\} c d e f g h \}$ (28)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (15)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (56)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (10)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (56)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (10)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (56)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (10)
- $\{(3) a b c \{3\} \mid \{8\} d e f g h \}$ (56)
- $\{(4) a b c d \{4\} \mid \{8\} e f g h \}$ (10)
- $\{(4) a b c d \{4\} \mid \{8\} e f g h \}$ (40)
- $\{(4) a b c d \{4\} \mid \{8\} e f g h \}$ (40)
- $\{(4) a b c d \{4\} \mid \{8\} e f g h \}$ (40)
- $\{(4) a b c d \{4\} \mid \{8\} e f g h \}$ (40)
- $\{(4) a b c d \{4\} \mid \{8\} e f g h \}$ (40)
- $\{(4) a b c d \{4\} \mid \{8\} e f g h \}$ (40)

12 remaining cross-sets (total 25) are in effect a re-ordering of the first 12 cross-sets above.
Item 3; In the 4-out-of-8, (4,8) hebdomekonamy matrix, the (1,5) Pentany subset has 56 variations, each of which occurs once, in a (3,3) Monamy partitioned cross-set. Example; the (5) 1 3 5 7 9 Pentany partitioned cross with (3) 11 13 15 Monamy can be expressed,

\[
\begin{bmatrix}
(5) & 1 & 3 & 5 & 7 & 9 \\
(3) & 11 & 13 & 15
\end{bmatrix}
\times
\begin{bmatrix}
1 & 3 & 5 & 7 & 9 \\
\end{bmatrix}
= \\
\begin{bmatrix}
(4,8) & 1 & 3 & 5 & 7 & 9 \\
(0,3) & 1 & 3 & 5 & 7 & 9
\end{bmatrix}
\]

Item 4; In the (4,8) hebdomekonamy matrix, the (4,5) Pentany subset has 56 variations, each of which occurs once, in a cross-set with its partitioned complementary (0,3) Monamy. Note; because the (4,5) Pentany conceals a subharmonic hexad, it is helpful to show that relationship, thus — Example; \[\begin{bmatrix}
(4) & 1 & 3 & 5 & 7 & 9 \\
(0) & 1 & 3 & 5 & 7 & 9
\end{bmatrix}\times\begin{bmatrix}
1 & 3 & 5 & 7 & 9 \\
\end{bmatrix} = \]

The last step of this operation, (multiplication of the (4,5) Pentany by the (0,3) Monamy [an empty set, indicated \(\emptyset\)]) is obviously academic.
Item 5; In the 4-out-of-8, (4, 8) Hebdomekountany matrix the (3, 4) Tetrany forms a partitioned cross-set with the (3, 4) Tetrany. There are 70 combinations by which this partitioning may occur. Note; the (3, 4) Tetrany carries a concealed sub-harmonic tetrad which is identified thus;

Example

\[
\begin{array}{cccc}
1 & 3 & 5 & 7 \\
3 & 5 & 7 & 1 \\
9 & 1 & 5 & 7 \\
13 & 7 & 1 & 5 \\
15 & 7 & 3 & 5 \\
\end{array}
\]

\[
\times \begin{array}{cccc}
3 & 5 & 7 & 1 \\
1 & 5 & 7 & 1 \\
3 & 5 & 7 & 1 \\
1 & 5 & 7 & 1 \\
3 & 5 & 7 & 1 \\
\end{array}
= \begin{array}{cccc}
\text{(3, 4) Tetrany} \\
\text{(3, 4) Tetrany} \\
\text{(3, 4) Tetrany} \\
\text{(3, 4) Tetrany} \\
\text{(3, 4) Tetrany} \\
\end{array}
\]

Now, then — By adding a single tone, 1,3,5,7, to the 4 harmonic tetrads and 2 the 4 subharmonic tetrads, a set of 4 plus 4 pentads will occur, each of which shares the 1,3,5,7 in common. A 17-tone quasi-diamond, providing 8 ways to harmonize the member 1,3,5,7. If the entire set above is divided by 1,3,5,7, this simple relationship emerges:

\[
\begin{array}{cccc}
1 & 3 & 5 & 7 \\
9 & 1 & 3 & 5 \\
11 & 1 & 3 & 5 \\
13 & 1 & 3 & 5 \\
15 & 1 & 3 & 5 \\
\end{array}
\]

\[
\times \begin{array}{cccc}
1 & 3 & 5 & 7 \\
9/1 & 9/3 & 9/5 & 9/7 \\
11/1 & 11/3 & 11/5 & 11/7 \\
13/1 & 13/3 & 13/5 & 13/7 \\
15/1 & 15/3 & 15/5 & 15/7 \\
\end{array}
= \begin{array}{cccc}
\text{add} \\
\text{add} \\
\text{add} \\
\text{add} \\
\text{add} \\
\end{array}
\]

One sees (by \(\frac{1,3,5,7}{1,3,5,7} = \frac{1}{9}\)) the 8 harmonizing roles that 1,3,5,7 can play; 4 harmonic, partitioned w/ 4 subharmonic sense.
Item 5 continued:

Another way to show this is to simply add 1.3.5.7 to each of the two tetrad strands thus:

\[
\begin{align*}
(7) & \quad (3) & \quad (5) & \quad (7) & \quad \text{sequence} \\
3.5.7 & \quad 1.5.7 & \quad 1.3.7 & \quad 1.3.5.7 & \quad \text{add} \\
9 & \quad 3.5.7.9 & \quad 1.5.7.9 & \quad 1.3.7.9 & \quad 1.3.5.7 \\
11 & \quad 3.5.7.11 & \quad 1.5.7.11 & \quad 1.3.7.11 & \quad 1.3.5.11 \\
13 & \quad 3.5.7.13 & \quad 1.5.7.13 & \quad 1.3.7.13 & \quad 1.3.5.13 \\
15 & \quad 3.5.7.15 & \quad 1.5.7.15 & \quad 1.3.7.15 & \quad 1.3.5.15 \\
\end{align*}
\]

The horizontal lines may be seen as partitioned cross-sets \([4,5) \ a \ b \ c \ d \ e \ f \ g \ h \] \times \((0,3) \ f \ g \ h \]

The vertical lines may be seen as partitioned cross-sets \([1,5) \ a \ b \ c \ d \ e \ f \ g \ h \] \times \((3,3) \ f \ g \ h \]

But that doesn't really tie it all together as well as I'd wish.

What we have in this "quasi-diamond" is that set of (4,5) Pentanies \((0,3) \ f \ g \ h \]\, which share 1.3.5.7. This simultaneously gives us the material for that set of (1,5) Pentanies \((3,3) \ f \ g \ h \]\, which also share the 1.3.5.7. These are the 17 points that would be shared by the ogdoadic cross-set \((1.3.5.7.9.11.13.15) \times \(7.3 \circ 3 \circ 3 \times 7.11 \times 13 \times 15\)\) with its \(1/2\) at key 1.3.5.7. [This kind of cross-set is usually called a "Diamond" after Partch's usage]. These 17 points (in their 70 partitioned permutations) are the 70 bonding-sites between centered and centerless modules in ogdoadic tone-space. They are the basis for modulating from Hebdomekantaries to Badic Diamonds & vice versa throughout the infinite realms of open Badic tone-space.
Item 6:

Within the (4,8) hebdomekontany the (2,5) Dekany forms a partitioned cross-set with the (2,3) Triany. There are 56 ways the 8 elements of the master set can be partitioned into 2 sets of 3 and 5 elements.

Example:

\[
\{ (2,5) \ni 3, 5, 7, 9 \} \times \{ (2,3) \ni 11, 13, 15 \} =
\]

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<th>(2,3) Triany</th>
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<td>11,13</td>
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<tr>
<td>7,9</td>
<td>11,13</td>
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</table>
Item 7:

In the (4, 8) Hebdomekontany [70ny] matrix the (3, 5) Dekany [10ny] forms a partitioned cross-set with the (1, 3) Triany [3ny]. There are 56 ways a set of 8 elements may partitioned into 2 sets, of 5 and 3 elements. (Hence 56 cross-sets)

Example: \( \{3, 5\} \times \{1, 3\} \)

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<td>1.3.9.13</td>
<td>1.3.9.15</td>
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<td>1.5.7.11</td>
<td>1.5.7.13</td>
<td>1.5.7.15</td>
</tr>
<tr>
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<td>1.5.9.13</td>
<td>1.5.9.15</td>
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<td>1.7.9.11</td>
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<td>3.5.7.13</td>
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</tr>
<tr>
<td>3.5.9</td>
<td>3.5.9.11</td>
<td>3.5.9.13</td>
<td>3.5.9.15</td>
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<td>3.7.9.11</td>
<td>3.7.9.13</td>
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<td>5.7.9.11</td>
<td>5.7.9.13</td>
<td>5.7.9.15</td>
</tr>
</tbody>
</table>
Item 8:

In the (4,8) Robbo mekantany matrix the (2,4) Hexany forms a partitioned cross-set with the (2,4) Hexany. The master-set of 8 elements may be partitioned into 2 sets, of 4 and 4 elements each, in 35 ways only \((70 = 2 = 35)\), because of the mirroring nature of the entire theoretical group of 70 ways.

Example: \(\{(2,4) 1 3 5 7\} \times \{(2,4) 9 11 13 15\}\)

<table>
<thead>
<tr>
<th>X</th>
<th>1 3 5 7</th>
<th>3 5 3 7</th>
<th>5 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 11</td>
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<td>1 5 9 11</td>
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<td>1 5 11 13</td>
<td>1 7 11 13</td>
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<td>11 15</td>
<td>1 3 11 15</td>
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</tr>
<tr>
<td>13 15</td>
<td>1 3 13 15</td>
<td>1 5 13 15</td>
<td>1 7 13 15</td>
</tr>
</tbody>
</table>

Note: \(4! \times 4! = 576\)

576: the no. of ways this cross-set might be viewed. Shown, is but one of them.
Item 9;
Contained in the hebdomekontany matrix, the \((2,6)\) Pentadekany forms a partitioned cross-set with the \((2,2)\) Monany. The ogdoad (8-ad) partitions into 2 sets, of 6 and 2 elements, in 28 different ways. (Hence 28 different forms of this cross-set.)

Example:

\[
\begin{array}{c|ccc}
\times & 13 & 15 \\
\hline
13 & 13 & 15 \\
15 & 15 & 13 \\
17 & 17 & 13 \\
19 & 19 & 15 \\
11 & 11 & 13 \\
35 & 35 & 13 \\
37 & 37 & 15 \\
39 & 39 & 13 \\
31 & 31 & 13 \\
57 & 57 & 13 \\
59 & 59 & 13 \\
51 & 51 & 13 \\
79 & 79 & 13 \\
71 & 71 & 13 \\
91 & 91 & 13 \\
\end{array}
\]
In the (4,8) Hebdomecontany [70n] the (4,6) Pentadekany forms a partition cross-set with the (0,2) Monany. There are 28 expressions of this cross-set, based on the 28 ways 6 and 2 elements may be partitioned out of 8 elements.

Example: \( \{4,6\} 1357911 \times \{0,2\} 1315 \)

\[
\begin{array}{c|cc}
\times & 1\cdot3\cdot5\cdot7 & 1\cdot3\cdot5\cdot9 \\
1\cdot3\cdot5\cdot7 & 1\cdot3\cdot5\cdot7 & 1\cdot3\cdot5\cdot9 \\
1\cdot3\cdot5\cdot9 & 1\cdot3\cdot5\cdot11 & 1\cdot3\cdot7\cdot9 \\
1\cdot3\cdot7\cdot9 & 1\cdot3\cdot7\cdot11 & 1\cdot5\cdot7\cdot9 \\
1\cdot3\cdot7\cdot11 & 1\cdot3\cdot7\cdot11 & 1\cdot5\cdot7\cdot11 \\
1\cdot3\cdot9\cdot11 & 1\cdot3\cdot9\cdot11 & 1\cdot5\cdot9\cdot11 \\
1\cdot5\cdot7\cdot9 & 1\cdot5\cdot7\cdot11 & 1\cdot5\cdot9\cdot11 \\
1\cdot5\cdot7\cdot11 & 1\cdot5\cdot7\cdot11 & 1\cdot5\cdot9\cdot11 \\
1\cdot5\cdot9\cdot11 & 1\cdot5\cdot9\cdot11 & 1\cdot7\cdot9\cdot11 \\
1\cdot7\cdot9\cdot11 & 1\cdot7\cdot9\cdot11 & 3\cdot5\cdot7\cdot9 \\
3\cdot5\cdot7\cdot9 & 3\cdot5\cdot7\cdot9 & 3\cdot5\cdot7\cdot11 \\
3\cdot5\cdot7\cdot11 & 3\cdot5\cdot7\cdot11 & 3\cdot5\cdot9\cdot11 \\
3\cdot5\cdot9\cdot11 & 3\cdot5\cdot9\cdot11 & 3\cdot7\cdot9\cdot11 \\
3\cdot7\cdot9\cdot11 & 3\cdot7\cdot9\cdot11 & 5\cdot7\cdot9\cdot11 \\
5\cdot7\cdot9\cdot11 & 5\cdot7\cdot9\cdot11 &
\end{array}
\]

Note: Caution - the "empty set" (\( \emptyset \)) is not equivalent to zero (0). This notwithstanding, the cross is academic, and done to maintain a useful format.
Item 11:

In the (4,8) Hebdomecontany $[70\text{my}]$ the (0,4) Monany forms a partitioned cross-set with the (4,4) Monany. There are 70 expressions of this cross-set, according to the number of ways 4 and 4 elements may be partitioned from 8 elements.

Example:

$$\{0,4\} \{3,5,7\} \times \{4,4\} \{9,11,13,15\}$$

$$\times$$

$$\emptyset$$

$$9,11,13,15$$

Note: obviously, going thru all 70 expressions of this cross-set would be a futile exercise. Don't do it.

Item 12:

In the (4,8) Hebdomecontany the (3,7) Triakontacontany forms a partitioned cross-set with the (1,1) Monany. There are 8 expressions of this cross-set, according to the number of ways 7 and 1 elements may be partitioned from 8 elements.

Example:

$$\{3,7\} \{3,5,7,9,11,13\} \times \{1,1\} \{15\}$$

$$\times$$

$$\frac{15}{1,3,5}$$

$$1,3,5,15$$

$$1,3,7,15$$

$$1,3,9,15$$

$$e+c$$

$$e+c$$
Item 13;

In the $(4,8)$ 70ng the $(4,7)$ 35ng forms a partitioned cross-set with the $(0,1)$ 1ng. There are 8 expressions of this cross-set, as there are 8 ways in which a set of 8 elements may be partitioned into 2 sets, of 7 and 1 elements.

Example: $\{ (4,7) | 3 \ 5 \ 7 \ 9 \ 11 \ 13 \}$ $\times$ $\{ (0,1) | 15 \}$

$$\emptyset = \text{(empty)}$$

- 1,3,5,7
- 1,3,5,9
- 1,3,5,11
- etc

Item 14;

And finally, in the $(4,8)$ 70ng, the $(4,8)$ 70ng itself, forms a partitioned cross-set with the $(0,0)$ 1ng. There is only 1 expression of this cross-set. This corresponds to the 1 way a set of 8 elements may be partitioned into 2 sets, of 8 and 1 elements.

Example: $\{ (4,8) | 1 \ 3 \ 5 \ 7 \ 9 \ 11 \ 13 \ 15 \}$ $\times$ $\{ (0,0) | \emptyset = \text{(empty)} \}$

$$\emptyset = \text{(empty)}$$

- 1,3,5,7
- 1,3,5,9
- 1,3,5,11
- etc
Part II

Partitioned Cross-sets of the Hexany

Pascal's Triangle:

\[(0,0) 1 \quad \text{← Zeroadic tone-space} \]
\[(0,1) 1 (0,1) 1 \quad \text{← Monadic tone-space} \]
\[(0,2) 1 (1,2) 2 (2,2) 1 \quad \text{← Dyadic tone-space} \]
\[(0,3) 1 (1,3) 3 (2,3) 3 (3,3) 1 \quad \text{← Triadic tone-space} \]
\[(0,4) 1 (1,4) 4 (2,4) 6 (3,4) 4 (4,4) 1 \quad \text{← Tetradic tone-space} \]

Fig 1.

Pascal's Triangle (Fig 1) may be used to codify the combination-product sets.

2-out-of-4 → \([(2,4) 6] \quad \text{Hexany (the set of 6 tones produced by the 6 combinations of 2 out of 4)} \)

A. Each base combination-product set contains all the combination-product sets obliquely above it in the triangular hierarchy.

B. Further, each "subsequent" combination-product set occurs a predictable number of times in the "base" combination-product set. The pattern of this multiple occurrence is (interestingly enough) also a combination-product set. A cross-set is produced. The second set is labeled "consequent"-set.
C. The partitioned nature of the cross-set is seen in the numerical terms of combination as well as in the elements of the set. To illustrate:

<table>
<thead>
<tr>
<th>Terms</th>
<th>Elements</th>
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<tbody>
<tr>
<td>1st</td>
<td>2nd</td>
</tr>
<tr>
<td>1 out of 3</td>
<td>1 3 5</td>
</tr>
</tbody>
</table>

"Subsequent set" | (1 out of 3) | 1 3 5 |
"Consequent set" | (1 out of 1) | 7 |

The partitioned cross-sets

"Base set" | (2 out of 4) | 1 3 5 7 |

Partitioned into 1 & 1
Partitioned into 3 & 1
Partitioned into (1 3 5) & (7)

D. Finally, the partitioned cross-set, itself, has a predictable number of "expressions" (permutations) based on the no. of ways the elements of the "base set" can be partitioned into the appropriate no. of elements in the partitioned sets. For example: when the "base set" has the 4 elements (1 3 5 7), and is being partitioned into sets of 3 and 1 elements, the "expressions" or permutations are (135)(7), (137)(5), (157)(3), and (357)(1); 4 permutations.

E. When the various "consequent" sets of a "base" set are assigned to their proper niches in the Triangle, they produce, among them, the 180° rotation of the Triangle (see Fig 2). The apex has now become the nadir, which is juxtaposed upon the "base" set, (see Fig 3).
Figure 2
Pascal's Triangle rotated 180°

Figure 3
180° rotation of Pascal's Triangle rising from the (2,4) hexagon. Note: this is the key for all the partitioned cross-sets of the 2-out-of-4, (2,4) Hexagon. Within the outlined rhombus the cross-sets are cross-indexed, as well.
### Notes on Terminology

<table>
<thead>
<tr>
<th>No. of Jones</th>
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<th>Name of Primary Module</th>
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<tr>
<td>0</td>
<td>Monany</td>
<td>Zeroad</td>
</tr>
<tr>
<td>1</td>
<td>Dyany</td>
<td>Monad</td>
</tr>
<tr>
<td>2</td>
<td>Triany</td>
<td>Dyad</td>
</tr>
<tr>
<td>3</td>
<td>Tetrany</td>
<td>Triad</td>
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<tr>
<td>23</td>
<td>Eikosioktany</td>
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</tr>
<tr>
<td>35</td>
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</tr>
<tr>
<td>70</td>
<td>Hebdomekontany</td>
<td></td>
</tr>
</tbody>
</table>

2. The suffix "-any" or "-any" is reserved for combination-product sets.

3. The "Primary Module" is a master set; that set from which the combination-product set is derived.

4. Combination-product Set: a set of combinations taken from the elements of a master set, where the elements of each combination are multiplied. Example: the 2-out-of-4 combination-product set of the master set, 1 3 5 7, is 1 3, 1 5, 1 7, 3 5, 3 7, 5 7.
Notes on Terminology (continued)

5. \((^3_6)\) or \((^3_6) = 3\ out\ of\ 6\)

6. \((^4_8)\) hebdomekontany = a combination-product set of 70 members, derived by taking 4-out-of-8 elements (usually the harmonics 1 3 5 7 9 11 13 15, but they could be any appropriate set of 8) and multiplying together the 4 elements of each combination, thus: 1·3·5·7, 1·3·5·9, 1·3·5·11, etc., to 9·11·13·15.
Partitioned Cross-sets of the Hexany
(cont.)

Refering to Figure 3 the outlined rhombus shows the following "Partitioned Cross-sets:

\[
\begin{align*}
\{ (0,0) \} \times \{ (2,4) a \ b \ c \ d \} & \quad \text{Permutations} \quad 1 \\
\{ (0,1) a \} \times \{ (2,3) b \ c \ d \} & \quad 4 \\
\{ (1,1) a \} \times \{ (1,3) b \ c \ d \} & \quad 4 \\
\{ (0,2) a \ b \} \times \{ (2,2) c \ d \} & \quad 6 \\
\{ (1,2) a \ b \} \times \{ (1,2) c \ d \} & \quad 6 \\
\{ (2,2) a \ b \} \times \{ (0,2) c \ d \} & \quad 6 \\
\{ (1,3) a \ b \ c \} \times \{ (1,1) d \} & \quad 4 \\
\{ (2,3) a \ b \ c \} \times \{ (0,1) d \} & \quad 4 \\
\{ (2,4) a \ b \ c \ d \} \times \{ (0,0) \} & \quad 1
\end{align*}
\]

(in the body of permutations)

In effect the 1st half of the series merely cross-indexes the 2nd half. Further, the sets on the 4 corners of the rhombus are academic. The 3 cross-sets indicated (*) will be discussed below.
Item 15.

In the $(2,4)$ Hexang matrix the $(1;2)$ Dyang forms a partitioned cross-set with the $(1;2)$ Dyang. This cross-set is a rare mirror of itself. Therefore, although normally there would be 6 permutations of the ways 2 can be taken from 4, in this partitioned circumstance 3 of the cross-sets thus derived are merely a re-ordering of the remaining 3.

Example 1: \[
\begin{pmatrix}
1,2,3,5-1,2,3,5
\end{pmatrix} \times \begin{pmatrix}
1,2,3,5-1,2,3,5
\end{pmatrix}
\]

Example 2: \[
\begin{pmatrix}
1,2,3,5-1,2,3,5
\end{pmatrix} \times \begin{pmatrix}
1,2,3,5-1,2,3,5
\end{pmatrix}
\]

Examples of cross-sets; (within the $(2,4)1,3,5,7$ Hexang)

1. \[
\begin{pmatrix}
1,3
\end{pmatrix} \times \begin{pmatrix}
1,3
\end{pmatrix}
\]

2. \[
\begin{pmatrix}
1,5
\end{pmatrix} \times \begin{pmatrix}
1,5
\end{pmatrix}
\]

3. \[
\begin{pmatrix}
1,7
\end{pmatrix} \times \begin{pmatrix}
1,7
\end{pmatrix}
\]
**Example:**

1. \( \{(3,3), 3, 5\} \times \{(1,1), 7\} \)
2. \( \{(3,3), 1, 5, 7, 3\} \times 3 \)
3. \( \{(3,3), 3, 5, 7\} \times \{(1,1), 7\} \)
4. \( \{(3,3), 3, 5, 7\} \times \{(1,1), 1\} \)

**Hexagon (c) Tree**

- 1 out of 3
- 1 out of 2

- 1 out of 3
- (1, 3) B

**Tree**

- 1 out of 1
- (1, 1) B

There are 4 permutations of 3 and/or 1 according to the combinations of (1, 3) Tree. The (1, 3) B forms 2 partitioned cross-set with the (1, 1) Moonway.
Item 17: 2-out-of-3 \rightarrow (2,3) \text{ Triang.} \\
0-out-of-1 \rightarrow (0,1) \text{ Monany}

In the (2,4) Hexany matrix the (2,3) Triang. forms a partitioned cross-set with the (0,1) Monany. 
There are 4 permutations of this cross-set, resulting from the 4 combinations of 3 and/or 1 out of 4.

The 4 permutations; Example

1. \( \{(2,3), 1, 5, 2\} \times \{(0,1), 7\} \)
2. \( \{(2,3), 1, 7, 2\} \times \{(0,1), 5\} \)
3. \( \{(2,3), 1, 5, 7\} \times \{(0,1), 3\} \)
4. \( \{(2,3), 3, 5, 7\} \times \{(0,1), 1\} \)

(1) \[
\begin{array}{ccc}
1.3 & 1.5 & 3.5 \\
\emptyset & 1.3 & 1.5 & 3.5
\end{array}
\rightarrow (x \frac{1.35}{1.35} = \frac{5}{3} \frac{1}{3})
\]

(2) \[
\begin{array}{ccc}
1.3 & 1.7 & 3.7 \\
\emptyset & 1.3 & 1.7 & 3.7
\end{array}
\rightarrow (x \frac{1.37}{1.37} = \frac{7}{3} \frac{1}{3})
\]

(3) \[
\begin{array}{ccc}
1.5 & 1.7 & 5.7 \\
\emptyset & 1.5 & 1.7 & 5.7
\end{array}
\rightarrow (x \frac{1.57}{1.57} = \frac{7}{5} \frac{1}{5})
\]

(4) \[
\begin{array}{ccc}
3.5 & 3.7 & 5.7 \\
\emptyset & 3.5 & 3.7 & 5.7
\end{array}
\rightarrow (x \frac{3.57}{3.57} = \frac{7}{5} \frac{3}{5})
\]

The 4 (2,3) x (0,1) Cross-sets are shown as shaded Triangles.

(1,4) Tetrahex
also Master Set
(Ref 1)
There will be 51 radial species of Dekatesergrams. \[ 7 \times 51 = 357 + 3 = 360 \] (40 \times 14 + 280 + 8 = 360)
(85) 3 concentric species.
Dear John Chalmers,

Using the above code the Hebdomekontany can be transferred to Hanson's generalized keyboard with rather nice results. The keyboard position is derived by summing the 4-out-of-8 combinations of the chain position (template). These 70 combinations all fall within a 72-tone chain of (unequal) minor thirds ($9/72$). There are duplicate entries at positions $+22$ and $+7$, respectively, which require special handling. 16 "empties" are left, which can be filled (as shown in parentheses) to construct a formal 72-tone scale. This can be notated and fingered/malletted in an orderly and predictable (homogeneous) way. Article follows.

References; Sistema Natural del Natural-Aproximado, Augusto Novaro, Mexico D.F. 1927
(Library of Congress call number ML3805.N49 1927)
Development of a 53-Tone Keyboard Layout
Larry Hanson 1989, XenHarmonikon XIII (Frog Peak Music Box A36, Hanover NH 03755)

Yours,

Ervin M. Wilson

Novaro's use of the number 72 to measure the harmonic root makes the placement of the Hebdomekontany on the Hanson generalized keyboard possible. This is a significant step, his amazing what else he was doing (in 1927!).
<table>
<thead>
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<th></th>
<th>6</th>
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<th>68</th>
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(8) 1, 3, 5, 7, 9, 11, 13, 15 Hebdome Kontany,
19/72 scale, 3/11 keyboard
© 1992 by Ervin M. Wilson
(Hanson Keyboard Geometry used by permission)
<table>
<thead>
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<th>coll</th>
<th>duir</th>
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### Harmonics

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<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
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### Scale

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</tbody>
</table>

### Additional Information

- **(8)**: $1,3,5,7,9,11,13,15$ Hébdome Kontany
- **19/72 scale, 3/11 keyboard**
- © 1992 by Ervin M. Wilson
- **Hanson Keyboard geometry used by permission**
72-Tone Hebdomekontaught Scale, on the Hanson (15-rank) Generalized Keyboard

19/72 scale on 4/15 Keyboard
Hanson's Generalized Geometry used by permission. ©1992 by ERV Wilson
72-Tone Hebdomekantary Scale
on the Hanson (19-rank) Generalized Keyboard

19/2 Scale on 5/12 Keyboard
Hanson's Generalized Geometry used by Permission ©1992 by Erv Wilson
HEBDOMEKONTANY, in a 72-tone scale
Shown with Hanson Generalized Geometry, on Starr-switch grid
© 1994 by Erv Wilson
72-Tone Tubulong, with Hebdomakerfany
on 19-34-19 Keyboard
© 1992 by Erv Wilson

(2 pitches at one site require special handling)
<table>
<thead>
<tr>
<th>Note</th>
<th>Octave</th>
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<td>2.0</td>
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</table>

Note: This table represents a 72-tone tuning for the 10-34-19 keyboard with Hebdomekonfand. 

Pascal's Triangle may be used to codify the combination-product sets.
If the nadir of this triangle is juxtaposed over any particular set of the upright triangle, the partitioned cross-sets of that set may be identified. See Fig. 5, 6, & 7.