

①

Generator, $2/5$;

Generator, 3/10;

Generator, 3/7;

Generator, 5/11 ; scale

Generator, $3/8$;

Generator, 4/11;

Generator, 4/9;

Generator, 3/11

Generator, 2/9 :

Gen. 2/11

2	9				$1/2$	<u>Scale</u> 
2	2	7			$1/3$	
2	2	2	5		$1/4$	
2	2	2	2	3	$1/5$	
2	2	2	2	2	$1/6$	
0	1	2	3	4	5	

Moments of Symetry

Generator, 5/12

0	5	3	1	6	4	2	0	
	5			7				2
	5			5		2		3
	3	2		3	2	2		5
	1	2	2	1	2	2	2	7
								12

Generator, 2/13

0	1	2	3	4	5	6	0	
	2			11				2
	2	2		9				3
	2	2	2		7			4
	2	2	2	2		5		5
	2	2	2	2	2	3		6
	2	2	2	2	2	2	1	7

Generator, 6/13;

0	9	7	5	3	1	10	8	6	4	2	0	
	6					7						2
	6					6				1		3
	5			1		5			1	1		5
	4		1	1		4		1	1	1	1	7
	3	1	1	1		3	1	1	1	1	1	9
	2	1	1	1	1	2	1	1	1	1	1	11

Generator, 5/14;

0	3	6	9	1	4	7	10	2	5	8	0	
	5					9						2
	5					5			4			3
	1	4		1		4			4			5
	1	1	3	1	1	3		1	3			8
	1	1	1	2	1	1	1	2	1	1	2	11

3/14;

0	5	1	6	2	7	3	8	4	0	
	3					11				2
	3	3				8				3
	3	3		3		5				4
	3	3		3		3	2			5
	1	2	1	2	1	2	1	2	2	9

5/13;

0	3	5	1	4	6	2	7	0	
	5				8				2
	5				5		3		3
	2	3		2	3		3		5
	2	2	1	2	2	1	2	1	8

4/13;

0	7	4	1	8	5	2	9	6	3	0	
	4					9					2
	4			4		5					3
	4			4		4		1			4
	3	1		3	1	3		1	1		7
	2	1	1	2	1	1	2	1	1	1	10

Generator, 7/15

0	11	9	7	5	3	1	12	10	8	6	4	2	0	
	7						8							2
	7						7					1		3
	6			1			6				1	1		5
	5		1	1			5			1	1	1		7
	4		1	1	1		4			1	1	1	1	9
	3	1	1	1	1		3	1	1	1	1	1	1	11
	2	1	1	1	1	1	2	1	1	1	1	1	1	13

3/13;

0	5	1	6	2	7	3	8	4	0	
	3					10				2
	3	3				7				3
	3	3		3		4				4
	3	3		3		3	1			5
	2	1	2	1	2	1	2	1	1	9

Generator, 4/15;

0	4	8	1	5	9	2	6	10	3	7	0	
	4						11					2
	4		4				7					3
	4		4			4		3				4
	1	3	1	3	1	3		3				7
	1	1	2	1	1	2	1	1	2	1	2	11

Moments of Symmetry

Generator, $7/18$;

0	8	3	11	6	1	9	4	12	7	2	10	5	0	
	7					11								2
	7					7					4			3
	3		4			3		4			4			5
	3		3	1		3		3	1		3	1		8
	2	1	2	1	1	2	1	2	1	1	2	1	1	13

$5/18$;

0	4	8	1	5	9	2	6	10	3	7	0		
5				13								2	
5				5			8					3	
5				5			5			3		4	
2		3		2		3		2		3		3	7
2		2		1	2		2		1	2		1	11

Generator $9/19$;

0	15	13	11	9	7	5	3	1	16	14	12	10	8	6	4	2	0
	9								10								2
	9								9								3
	8								8								5
	7								7								7
	6								6								9
	5								5								11
	4								4								13
	3								3								15
	2								2								17

$8/19$;

0	5	10	3	8	1	6	11	4	9	2	7	0	
	8					11							2
	8					8					3		3
	5					5					3		5
	2	3	3	2	3	3	2	3	3	3			7
	2	2	1	2	1	2	2	1	2	1	2	1	12

$7/19$;

0	3	6	9	1	4	7	10	2	5	8	0		
7					12							2	
7					7					5			3
2	5				2	5			5				5
2	2		3		2	2		3		2		3	8
2	2	2	1		2	2	2	1		2	2	1	11

Moments of Symmetry

Generator 6/19;

0	13	10	7	4	1	14	11	8	5	2	15	12	9	6	3	0
	6										13					2
	6						6						7			3
	6						6					6			1	4
	5		1			5		1			5		1		1	7
	4		1	1		4		1	1		4		1	1	1	10
	3	1	1	1		3		1	1	1	3		1	1	1	13
	2	1	1	1	1	2		1	1	1	2		1	1	1	16

5/19;

0	4	8	12		1	5	9	13		2	6	10	14		3	7	11	0
	5										14							2
	5					5							9					3
	5					5					5				4			4
1		4		1		4		1		4		4						7
1	1		3	1	1		3	1	1		3	1		3				11
1	1	1		2	1	1		2	1	1		2	1	1		2		15

4/19;

0	5	10		1	6	11		2	7	12		3	8	13		4	9	0
	4										15							2
	4				4								11					3
	4				4			4						7				4
	4				4			4				4				3		5
1		3		1		3		1		3		1		3		3		9
1	1		2	1	1		2	1	1		2	1	1		2	1	2	14

3/19;

0	7	1		8	2		9	3		10	4		11	5		12	6	0
	3												16					2
	3		3											13				3
	3		3		3									10				4
	3		3		3		3								7			5
	3		3		3		3		3				3			4		6
	3		3		3		3		3		3			3		1		7
	2	1		2	1		2	1		2	1		2	1		2	1	13

9/20;

0	9		7		5		3		1	10		8		6		4		2	0
	9																		2
	9																		3
	7					2				7						2		2	5
	5				2		2			5			2			2		2	7
	3		2		2		2		3		2		2		2	2		2	9
1		2		2		2		2	1		2		2		2		2		11

Moments of Symmetry

Generator 5/21

0	13	9	5	1	14	10	6	2	15	11	7	3	16	12	8	4	0
	5								16								2
	5				5								11				3
	5				5				5					6			4
	5				5				5					5		1	5
	4	1			4	1			4	1				4	1	1	9
	3	1	1		3	1	1		3	1	1			3	1	1	13
	2	1	1	1	2	1	1	1	2	1	1	1		2	1	1	17

4/21

0	11	6	1	12	7	2	13	8	3	14	9	4	15	10	5	0
	4									17						2
	4				4								13			3
	4				4				4					9		4
	4				4				4					5		5
	4				4				4					4	1	6
	3	1			3	1			3	1				3	1	11
	2	1	1		2	1	1		2	1	1			2	1	16

Generator 9/22

0	5	10	15	3	8	13	1	6	11	16	4	9	14	2	7	12	0	(1)	
9				13															2
9				9										4				3	
5			4			5			4			4			5				
1	4		4		1	4		4		4		7							
1	1	3		1	3		1	1	3		1	3		1	3		12		
1	1	1	2		1	1	2		1	1	1	2		1	1	2		17	
																		(22)	

Generator 7/22;

0	16	13	10	7	4	1	17	14	11	8	5	2	18	15	12	9	6	3	0
	7												15						2
	7						7							8					3
	7						7						7					1	4
	6				1		6			1			6				1	1	7
	5				1	1	5			1	1		5			1	1	1	10
	4				1	1	1	4			1	1	1	4			1	1	13
	3				1	1	1	1	3			1	1	1	1	3		1	16
	2				1	1	1	1	1	2			1	1	1	1	1	1	19

5/22

0	9	5	1	10	6	2	11	7	3	12	8	4	0	
	5							17						2
	5				5					12				3
	5				5			5			7			4
	5				5			5			5		2	5
	3	2		3	2		3	2		3	2	2		9
1	2	2	1	2	2	1	2	2	1	2	2	2		13

Moments of Symmetry

generator 3/22

0	8	1	9	2	10	3	11	4	12	5	13	6	14	7	0
3									19						2
3	3								16						3
3	3	3							13						4
3	3	3	3						10						5
3	3	3	3	3					7						6
3	3	3	3	3	3				4						7
3	3	3	3	3	3	3			3	3					8
2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	15

Generator 11/23

Scale

																11/2
																11/3
																2/5 → means; 5 step scale with
																3/7 generator spanning 2 steps
																4/9
																5/11
																6/13
																7/15
																8/17
																9/19
																10/21

Generator 10/23;

Scale

0	7	5	3	1	8	6	4	2	0			
10					13					1/2		
10					10					3	1/3	
7			3		7			3		3	2/5	
4		3		3		4		3		3	3	3/7
1	3		3		3		1	3		3	3	4/9
1	1	2	1	2	1	2	1	1	2	1	2	7/16

Generator 9/23;

Scale

0	13	8	3	16	11	6	1	14	9	4	17	12	7	2	15	10	5	0	
																			1/2
																			1/3
																			2/5
																			3/8
																			5/13
																			7/18

Moments of Symetry

Generator 8/23

Scale

0 3 6 9 12 15 18 1 4 7 10 13 16 19 2 5 8 11 14 17 0

8						15																
8						8						7										
1	7					1	7					7										
1	1	6				1	1	6				1	6									
1	1	1	5			1	1	1	5			1	1	5								
1	1	1	1	4		1	1	1	1	4		1	1	1	4							
1	1	1	1	1	3		1	1	1	1	1	3		1	1	1	1	3				
1	1	1	1	1	1	2		1	1	1	1	1	1	2		1	1	1	1	1	2	

1/2
1/3
2/5
3/8
4/11
5/14
6/17
7/20

Generator 7/23

Scale

0 10 7 4 1 11 8 5 2 12 9 6 3 0

7				16												
7				7				9								
7				7				7								
5			2		5			2		5			2		2	
3		2		2		3		2		2		3		2		2
1	2	2	2	1	2	2	2	1	2	2	2	2	2			

1/2

1/3

1/4

2/7

3/10

4/13

Generator 6/23

0	4	8	12	16	1	5	9	13	17	2	6	10	14	18	3	7	11	15	0
6					17														
6					6					11									
6					6					6					5				
1 5					1 5					1 5					5				
1 1 4					1 1 4					1 1 4					1 4				
1 1 1 3					1 1 1 3					1 1 1 3					1 1 3				
1 1 1 1 2					1 1 1 1 2					1 1 1 1 2					1 1 1 2				

scale

1/2
1/3
1/4
2/7
3/11
4/15
5/19
(6/23)

Generator 5/23

0	5	10	1	6	11	2	7	12	3	8	13	4	9	0	Scale
5															1/2
5		5				18									1/3
5		5				5					13				1/4
5		5				5					8				1/5
2	3	2	3	2	3	2	3	2	3	2	3	2	3		2/9
2	2	1	2	2	1	2	2	1	2	2	1	2	2	1	3/14
															(5/23)

Moments-of-Symetry

Generator 4/23

0	6	12	1	7	13	2	8	14	3	9	15	4	10	16	5	11	0
4									19								
4			4						15								
4			4			4			11								
4			4			4			4			7					
4			4			4			4			4			3		
1	3		1	3		1	3		1	3		1	3		3		
1	1	2	1	1	2	1	1	2	1	1	2	1	1	2	1	2	

Scale

1/2

1/3

1/4

1/5

1/6

2/11

3/17

(4/23)

Generator 3/23

0	8	1	9	2	10	3	11	4	12	5	13	6	14	7	0
3								20							
3		3						17							
3		3		3				14							
3		3		3		3		11							
3		3		3		3		3		8					
3		3		3		3		3		3		5			
3		3		3		3		3		3		3		2	
1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	

1/2

1/3

1/4

1/5

1/6

1/7

1/8

2/15

(3/23)

Generator 11/24

0	11	9	7	5	3	1	12	10	8	6	4	2	0
	11							13					
	11							11				2	
	9				2		9				2	2	
	7			2	2		7			2	2	2	
	5		2	2	2		5		2	2	2	2	
	3	2	2	2	2		3	2	2	2	2	2	
1	2	2	2	2	2	1	2	2	2	2	2	2	

1/2

1/3

2/5

3/7

4/9

5/11

6/13

(11/24)

Generator 7/24

0	7	14	4	11	1	8	15	5	12	2	9	16	6	13	3	10	0
	7								17								
	7					7			10								
	7					7			7						3		
	4		3		4		3		4		3		3				
1	3		3	1	3		3	1	3		3		3		3		
1	1	2	1	2	1	1	2	1	2	1	1	2	1	2	1	2	

1/2

1/3

1/4

2/7 → means 7 intervals in 8ve; generator subtends 2 intervals)

3/10

5/17

(7/24)

Moments-of-Symetry

Generator 5/24

0	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	0	Scale
	5											19								1/2
	5				5							14								1/3
	5				5				5					9						1/4
	5				5				5			5				4				1/5
1		4		1		4		1		4		1		4			4			2/9
1	1	3		1	1	3		1	1	3		1	1	3		1	3			3/14
1	1	1	2	1	1	1	2	1	1	1	2	1	1	1	2	1	1	2		4/19

Generator 12/25

0	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	0	Scale
				12											13									1/2
				12											12									1/3
				11							1				11						1	1		2/5
				10						1	1				10						1	1	1	3/7
				9						1	1	1			9						1	1	1	4/9
				8						1	1	1	1		8						1	1	1	5/11
				7						1	1	1	1	1	7						1	1	1	6/13
				6						1	1	1	1	1	6						1	1	1	7/15
				5						1	1	1	1	1	5						1	1	1	8/17
				4						1	1	1	1	1	4						1	1	1	9/19
				3						1	1	1	1	1	3						1	1	1	10/21
				2						1	1	1	1	1	2						1	1	1	11/23
																								(12/25)

Generator 11/25

0	7	14	5	12	3	10	1	8	15	6	13	4	11	2	9	0	Scale
				11								14					1/2
				11								11			3		1/3
				8				3			8			3	3		2/5
				5				3			5			3	3	3	3/7
				2				3			2			3	3	3	4/9
				2				2			2			2	1	2	7/16
																	(11/25)

Generator 9/25

0	3	6	9	12	1	4	7	10	13	2	5	8	11	0	Scale
				9											1/2
				9									7		1/3
													7		2/5
													5		3/8
													3		4/11
													2		5/14
													2		(9/25)

Gen 8/25 Moments of Symetry

0	19	16	13	10	7	4	1	20	17	14	11	8	5	2	21	18	15	12	9	6	3	0													
8								17																											
8								8								9																			
8								8								8								1											
7												1	7												1	1									
6												1	1	6												1	1	1							
5												1	1	1	5												1	1	1	1					
4												1	1	1	1	4												1	1	1	1	1			
3												1	1	1	1	1	3												1	1	1	1	1		
2												1	1	1	1	1	1	2												1	1	1	1	1	1

Scale

↓

1/2

1/3

1/4

2/7

3/10

4/13

5/16

6/19

7/22

Generator 7/25

0	11	4	15	8	1	12	5	16	9	2	13	6	10	3	14	7	0
7								18									
7								7								11	
7								7								4	
3	4					3	4	3	4							4	
3	3	1				3	3	1	3							3	1
2	1	2	1	1		2	1	2	1	1						2	1

1/2

1/3

1/4

2/7

3/11

5/18

(7/25)

Generator 6/25

0	17	13	9	5	1	18	14	10	6	2	19	15	11	7	3	20	16	12	8	4	0
6								19													
6								6								13					
6								6								7					
6								6								6					
5								5								5					
4								4								4					
3								3								3					
2								2								2					

1/2

1/3

1/4

1/5

2/9

3/13

4/17

5/21

(6/25)

Generator 4/25

0	13	7	1	14	8	2	15	9	3	16	10	4	17	11	5	18	12	6	1
4								21											
4								4								17			
4								4								13			
4								4								9			
4								4								5			
4								4								4			
3								3								3			
2								2								2			

Scale

↓

1/2

1/3

1/4

1/5

1/6

1/7

2/13

3/19

(4/25)

Moments of Symmetry

Generator 5/26

0	16	11	6	1	17	12	7	2	18	13	8	3	19	14	9	4	20	15	10	5	0				
5					21																				
5					5					16															
5					5					5					11										
5					5					5					5					6					
5					5					5					5					5					1
4		1		4		1		4		1		4		1		4		1		4		1		1	
3		1		1		3		1		1		3		1		1		3		1		1		1	
2		1		1		1		2		1		1		1		2		1		1		1		1	

Scale

→

 $1/2$

173

 $\frac{1}{4}$ $1/5$

1/6

2/11

3/16

4/21

5/26

Generator 3/26

0 9	1 10	2 11	3 12	4 13	5 14	6 15	7 16	8	0
3					23				
3	3					20			
3	3	3				17			
3	3	3	3			14			
3	3	3	3	3			11		
3	3	3	3	3	3			8	
3	3	3	3	3	3	3			5
3	3	3	3	3	3	3	3		2
1	2	1	2	1	2	1	2	1	2

 $1/2$

113

114

1/5

116

1/7

1/8

1/9

2/17

3/26

Generator 13/27

0 23 21 19 17 15 13 11 9 7 5 3 1 24 22 20 18 16 14 12 10 8 6 4 2 0

13												14																			
13												13												1							
12												12												1	1						
11												11												1	1	1					
10												10												1	1	1	1				
9												9												1	1	1	1	1	1		
8												8												1	1	1	1	1	1		
7												7												1	1	1	1	1	1	1	
6												6												1	1	1	1	1	1	1	1
5												5												1	1	1	1	1	1	1	1
4												4												1	1	1	1	1	1	1	1
3												3												1	1	1	1	1	1	1	1
2												2												1	1	1	1	1	1	1	1

 $\frac{1}{2}$

113

215

317

4/9

5/11

6/13

7/12

8/17

9/19

10/2

11/23

12/25

Moments of Symetry

Generator 11/27

0 5 10 15 20 3 8 13 18 1 6 11 16 21 4 9 14 19 2 7 12 17 0

11											16										
11											11										
6						5					6						5				
1		5					5				1		5					5			
1	1		4			1		4			1	1		4			1		4		
1	1	1		3		1	1		3		1	1	1		3		1	1		3	
1	1	1	1		2	1	1	1		2	1	1	1	1		2	1	1	1		2

1/2
1/3
2/5
3/7
5/12
7/17
9/22
(11/27)

Generator 10/27

0 11 3 14 6 17 9 1 12 4 15 7 18 10 2 13 5 16 8 0

10										17																																																	
10										10										7																																							
3					7					3					7					7																																							
3					3					4					3					3					4																																		
3					3					3					1					3					3					1																													
2					1					2					1					2					1					1					2					1					2					1					1				

1/2
1/3
2/5
3/8
4/11
7/19
(10/27)

Generator 8/27

0 7 14 4 11 1 8 15 5 12 2 9 16 6 13 3 10 0

8											19										
8											8										
8											11										
5						3					5						3				
2	3		3		2	3		3		2	3		3		2	3		3		3	
2	2	1	2	1	2	2	1	2	1	2	2	1	2	1	2	2	1	2	1	2	1

scale
↓
1/2
1/3
1/4
2/7
3/10
5/17
(8/27)

Generator 7/27

0 4 8 12 16 20 1 5 9 13 17 21 2 6 10 14 18 22 3 7 11 15 19 0

7					20															
7					7															
7					7					7					6					
1	6				1	6				1	6				6					
1	1	5			1	1	5			1	1	5			1	5				
1	1	1	4		1	1	1	4		1	1	1	4		1	1	4			
1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	3		
1	1	1	1	1	2	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2

1/2
1/3
1/4
2/7
3/11
4/15
5/19
6/23
(7/27)

Moments of Symetry

Generator 5/27

0	11	6	1	12	7	2	13	8	3	14	9	4	15	10	5	0		
5			22															
5			5			17												
5			5			5				12								
5			5			5				5				7				
5			5			5				5				5				2
3		2	3		2	3		2	3		2	3		2	3		2	
1	2	2	1	2	2	1	2	2	1	2	2	1	2	2	1	2	2	

$$\begin{array}{l} 1/2 \\ 1/3 \\ 1/4 \\ 1/5 \\ 1/6 \\ 2/11 \\ 3/16 \\ (5/27) \end{array}$$

Generator 4/27

[illegible]
$$\begin{array}{l} 1/2 \\ 1/3 \\ 1/4 \\ 1/5 \\ 1/6 \\ 1/7 \\ 2/13 \\ 3/20 \\ (4/27) \end{array}$$

Generator 13/28

0	13	11	9	7	5	3	1	13	12	10	8	6	4	2	0
			13								15				
			13							13					2
		11					2			11				2	2
		9				2	2			9			2	2	2
	7				2	2	2		7			2	2	2	2
	5			2	2	2	2		5		2	2	2	2	2
	3		2	2	2	2	2		3		2	2	2	2	2
1	2		2	2	2	2	2	1	2		2	2	2	2	2

$$\begin{array}{l} 1/2 \\ 1/3 \\ 2/5 \\ 3/7 \\ 4/9 \\ 5/11 \\ 6/13 \\ 7/15 \\ (13/28) \end{array}$$

Generator 11/28

0	18	13	8	3	21	16	11	6	1	19	14	9	4	21	17	12	7	2	20	15	10	5	0											
11										17																								
11										11					6																			
5					6					5					6																			
5					5					1					5					1														
4					1					4					1					1														
3					1					1					3					1					1									
2					1					1					1					2					1					1				

$$\begin{array}{l} 1/2 \\ 1/3 \\ 2/5 \\ 3/8 \\ 5/13 \\ 7/18 \\ 9/23 \end{array}$$

GENERATOR 9/28

9								19																1/2							
9								9								10								1/3							
9								9								9								1	1/4						
8				1	8				1	8				1	1	8				1	1	2/7									
7				1	1	7				1	1	7				1	1	1	7				1	1	1	3/10					
6				1	1	1	6				1	1	1	6				1	1	1	1	6				1	1	1	1	4/13	
5				1	1	1	1	5				1	1	1	1	5				1	1	1	1	1	1	1	1	1	5/16		
4				1	1	1	1	1	4				1	1	1	1	1	4				1	1	1	1	1	1	1	1	6/19	
3				1	1	1	1	1	1	3				1	1	1	1	1	3				1	1	1	1	1	1	1	1	7/22
2	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	8/25		

GENERATOR 5/28

5																1/2	
5		5														1/3	
5		5			5											1/4	
5		5			5			5									1/5
5		5			5			5			5				3		1/6
2	3	2	3	2	3	2	3	2	3	2	3	2	3	3	2/11		
2	2	1	2	2	1	2	2	1	2	2	1	2	2	1	2	1	3/17

The Moments-of-Symetry of 31
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Generating interval; 15/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

[illegible]

Generating Interval; 14/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

										31																				1
																				17										2
										14																				3
										14																				3
										11										3										5
										8										3										7
										5										3										9
										2										3										11
										2										2										20
										(31)																				(31)

Generating Interval; 13/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{6}$

[illegible]

Generating Interval; 12/31

0 13 8 3 16 11 6 1 14 9 4 17 12 7 2 15 10 5 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{2}$

[illegible]

Generating Interval; 11/31

0	3	6	9	12	15	1	4	7	10	13	16	2	5	8	11	14	0														
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	$\frac{3}{10}$

						31												1
												20						2
												9						3
2						2						2						5
2	2					2	2					2					8	
2	2	2				2	2	2				2	2				11	
2	2	2	2	3		2	2	2	2	3		2	2	2	3		14	
2	2	2	2	2	1	2	2	2	2	2	1	2	2	2	2	1	17	
(31)																		

Generating Interval; 10/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{10}$

[illegible]

Generating Interval; 9/31

0 7 14 4 11 1 8 15 5 12 2 9 16 6 13 3 10 0

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

[illegible]

Generating Interval; 8/31

0	4	8	12	1	5	9	13	2	6	10	14	3	7	11	0																
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	$\frac{31}{0}$

[illegible]

Generating Interval; 7/31

0 9 18 5 14 1 10 19 6 15 2 11 20 7 16 3 12 21 8 17 4 13 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{10}$

[illegible]

Generating Interval; 6/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 ³¹/₀

										31																			
6															25														
6					6										19														
6					6					6										13									
6					6					6					6					7									
6					6					6					6					6									
5					1					5					1					5					1				
4					1					1					4					1					1				
3					1					1					3					1					1				
2					1					1					2					1					1				

Generating Interval; $5/31$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

[illegible]

Generating Interval; 4/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{10}$

[illegible]

Generating Interval; 3/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

[illegible]

Generating Interval; 2/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{10}$

[illegible]

Generating Interval; $1/31$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{10}$

[illegible]

Address:

ERV WILSON
844 N. AVE 65
LOS Angeles, CA 90042
Phone: (213) 256-2624

9-tone Pseudo-Pelog Cycle, in 15

$\frac{15}{6}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	$\frac{15}{6}$
	5			1	2			5				2			
	5			1			5			2		2			
		2	1			5				2			5		
		2			5			1	2				5		
	2	2			5			1			5				
	2		5				2	1			5				
	2		5				2		5					1	
		5			2		2			5				1	
		5			2			5			2			1	
(	5		1	2			5			2					)

— Same as starting scale

Sequence of Fourths

7 6 7 7
 6 7 7 6
 7 7 6 7
 7 6 7 7
 6 7 7 6
 7 7 6 7
 7 6 7 7
 6 7 7 6
 7 7 6 7
 (7 6 7 7)

Consecutive Pentatonics in an iterated 2,5,2,5 Cycle, in 15

$\frac{15}{6}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	$\frac{15}{6}$
	1	2			5			2				5			
	2	1			5			2				5			
	2		5			1	2			5					
	2		5			2	1			5					
	2		5			2			5				1		
	1		5			2			5				2		
		5			1	2			5				2		
		5			2	1			5				2		
		5			2			5			1	2			
	1		5			2			5			2			
		5			1	2			5			2			
		5			2	1			5			2			
		5			2			5			1	2			
		5			2			5			2		1		
	5		1	2			5			2					
	5		2	1			5			2					
	5		2			5			1	2					
	5		2			5			2		1				
	1	2			5			2				5			
	2	1			5			2				5			
	2		5			1	2			5					
	2		5			2	1			5					
	1	2			5			2				5			
	2	1			5			2				5			
	2		5			1	2			5					
	2		5			2	1			5					
(	1	2			5			2				5			)

← same as beginning scale

2 5 2 5
 5 2 5 2
 2 5 2 5
 5 2 5 2
 etc.

Moments-of-Symetry based on 9/34 Octave

0 +4 +8 +12 +15 +1 +5 +9 +13 +16 +2 +6 +10 +14 +17 +3 +7 +11 +18 0

34															1
9					25										2
					9										3
										9					4
2	7				2	7				2	7				7
2	2	5			2	2	5			2	2	5			11
2	2	2	3		2	2	2	3		2	2	2	3		15
2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	19

(34)

Moments-of-Symetry based on 4/15 8ve

0 +4 +8 +1 +5 +9 +2 +6 +10 +3 +7 0

15										1
4		11								2
4	4		7							3
4	4		4	3						4
1	3	1	3	1	3	3				7
1	1	3	1	1	3	1	1	3	1	11

(15)

Allusion to Harmonic Series in 15

3	3	3	2	2	2
---	---	---	---	---	---

6 7 8 9 10 11 12

Allusion to Sub-harmonic Series in 15

3	3	3	2	2	2
---	---	---	---	---	---

9 8 7 (6) 11 10 9

12

Cycle of 7-tone MOS out of 15. (4/15) is generator.

13	14	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
			1	3	1	3	1	3	1	3	1	3	1	3	1	3	
		3		3		1		3		1		3		1		3	
	3		1		3		3		1		3		1		3		1
3		1		3		1		3		3		1		3		1	
3		1		3		1		3		1		3		1		3	
	3		3		1		3		1		3		1		3		1
	3		1		3		3		1		3		1		3		1
	3		1		3		1		3		3		1		3		1
	3		1		3		1		3		1		3		1		3
1		3		3		1		3		1		3		1		3	
1		3		1		3		3		1		3		1		3	
1		3		1		3		1		3		1		3		3	

same as beginning scale

7 7 6 7
7 6 7 7
6 7 7 6
7 7 6 7
7 6 7 7

Moments of Symmetry (MOS), where the Octave has 19 steps and the Generator has 11 steps

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10DEC98·EW

<u>Root</u>																				<u>Octave</u>
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19%
0																				0
0												1								0
0			2									1								0
0			2			4						1		3						0
0			2			4			6			1		3			5			0
0	7		2	9		4	11		6			1	8	3	10		5			0
0	7	14	2	9	16	4	11	18	6	13	1	8	15	3	10	17	5	12		0
C	C#	D $\flat$	D	D#	E $\flat$	E	(E#) F $\flat$	F	F#	G $\flat$	G	G#	A $\flat$	A	A#	B $\flat$	B	(B#) C $\flat$	C	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19%	

1
2
3
5
7
12
19

MOS

Ref: www.anaphoria.com see wilson, Letter to John Chalmers 26 April, 1975

further: The Three Gap Theorem (Steinhaus Conjecture), Tony Van Ravenstiem 1988,

[J. Austral. Math Soc (Series A) 45 (1988), 360-370]

Φ scales in 36

0	3	6	9	12	14	17	20	23	25	28	31	34	36 (12)
36													1
25										11			2
14						11				11			3
3	11					11				11			4
3	3	8				3	8			3	8		7
3	3	3	5		3	3	5		3	3	5		10
3	3	3	3	2	3	3	3	2	3	3	3	2	13
1	2	1	2	1	2	1	2	2	1	2	1	2	23
1	1	1	1	1	1	1	1	1	1	1	1	1	36
0	3	6	9	12	15	18	21	24	27	30	33	36	0
13	16	19	22	25	28	31	34	37	40	43	46	49	52
26	29	32	35	38	41	44	47	50	53	56	59	62	65

0	5	13	21	29	34	42	50	58	66		
66										1	
29					37					2	
29					29				8	3	
21				8	21			8	8	5	
13			8	8	13		8	8	8	7	
5	8	8	8	5	8	8	8	8	8	9	
5	5	3	5	3	5	3	5	3	5	3	16
2	3	2	3	3	2	3	3	2	3	3	25
2	2	1	2	2	1	2	2	1	2	2	41

It is the duty of the conscientious speculative musical theorist (CSMT) to go where "no man has gone before"; to venture into those spaces between all harmonies and all equal divisions, (Dartun, 4297)

0	5	10	18	23	31	36	41	49	54	62	67	75						
75													1					
31						44							2					
31						31					13		3					
18				13			18			13		13		5				
5	13			13			5	13			13		13		7			
5	5	8		5	8		5	5	8		5	8		5	8		12	
5	5	5	3	5	5	3	5	5	5	3	5	5	3	5	5	3	17	
2	3	2	3	2	3	3	2	3	2	3	3	2	3	2	3	3	29	
2	2	1	2	2	1	2	2	1	2	2	1	2	2	1	2	2	1	46

(27/22)

0	5	13	21	26	34	42	47	55	63	71	
71											1
21				50							2
21				21				29			3
21				21				21			4
13			8	13			8	13			7
5	8	8	5	8	8	5	8	8	5	8	10
5	5	3	5	3	5	5	3	5	3	5	17
2	3	2	3	3	2	3	2	3	3	2	27
2	2	1	2	2	1	2	2	1	2	2	44
											(71)

0	3	8	13	18	21	26	31	36	39	44	49	54	59	
59														1
18				41										2
18				18					23					3
18				18					18				5	4
13			5	13			5	13			5	5	5	7
8		5	5	8		5	5	8		5	5	5	5	10
3	5	5	5	3	5	5	5	3	5	5	5	5	5	13
3	3	2	3	2	3	2	3	2	3	2	3	2	3	23
1	2	1	2	2	1	2	2	1	2	2	1	2	2	36
														(59)

(10/9)

0 5 10 15 20 25 30 33

33										1
5	28									2
5	5	23								3
5	5	5	18							4
5	5	5	5	13						5
5	5	5	5	5	8					6
5	5	5	5	5	5	5	3			7
2	3	2	3	2	3	2	3	2	3	13
2	2	1	2	2	1	2	2	1	2	20

Omega
scales

0 7 1 8 2 9 3 10 4 11 5 12 6 0 } Linear
14 15 16 17 18 19 13

$$\omega 5 = .15110 22762 50004$$

$$\omega 1 = .03567$$

$$\omega 2 = .05771$$

$$\omega 3 = .09338$$

$$\omega 8 = .24448$$

$$\omega 13 = .39559$$

$$18 = .54669$$

$$23 = .69779$$

$$28 = .84889$$

$$33 = 1.00000$$

0 8 16 24 32 40 45

45															1			
8		37													2			
8		8		29											3			
8		8		8		21									4			
8		8		8		8		13							5			
8		8		8		8		8			5			6				
3	5	3	5	3	5	3	5	3	5	3	5	5	11					
3	3	2	3	3	2	3	3	2	3	3	2	3	3	2	17			
1	2	1	2	2	1	2	1	2	2	1	2	1	2	2	1	2	2	28

* save
Wandy carlos

13/57

0	13	26	39	52	57	
57						1
13			44			2
13		13		31		3
13		13		13		4
13		13		13		5
8		5		8		5
3		5		3		5
3		3		2		3
1		2		1		2

0 5 18 23 36 41 54 67

67																													
18							49																						
18							18						31																
18							18						18				13												
5		13					5		13					5		13				13									
5		5		8			5		5		8			5		5		8			5		8						
5		5		5		3		5		5		5		3		5		5		5		3		5		5		3	
2		3		2		3		2		3		3		2		3		2		3		3		2		3		3	
2		2		2		2		2		2		2		2		2		2		2		2		2		2		2	

1

2

3

4

7

11

15

26

41

(67)

Linear series:

0 4 8 12 16 1 5 9 13 17 2 6 10 14 18 3 7 11 15 0

72																			1
19									53										2
19									19					34					3
19									19					19					4
4	15								4	15								4	7
4	4	11							4	4	11							4	11
4	4	4	7						4	4	4	7						4	15
4	4	4	4	3					4	4	4	4	3					4	19
1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	34
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	53
0	4	8	12	16	19	23	27	31	35	38	42	46	50	54	57	61	65	69	72(72)

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$\frac{9}{8}$				$\frac{16}{15}$				$\frac{25}{24}$				$\frac{27}{25}$				$\frac{10}{9}$				$\frac{9}{8}$				$\frac{16}{15}$				$\frac{25}{24}$				$\frac{16}{15}$			
$\frac{25}{24}$	$\frac{648}{625}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{648}{625}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{648}{625}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{648}{625}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{25}{24}$	$\frac{128}{125}$	$\frac{25}{24}$	$\frac{128}{125}$

F F# Gb G G# Ab A A# Bb B Cb C C# Db D Ebb Eb E E# F
30 34 38 42 46 49 53 57 61 65 68 0/72 4 8 12 15 19 23 27 30