The 17-tone Puzzle — And the Neo-medieval Key That Unlocks It

by George Secor

A Grave Misunderstanding

The 17 division of the octave has to be one of the most misunderstood alternative tuning systems available to the microtonal experimenter. In comparison with divisions such as 19, 22, and 31, it has two major advantages: not only are its fifths better in tune, but it is also more manageable, considering its very reasonable number of tones per octave. A third advantage becomes apparent immediately upon hearing diatonic melodies played in it, one note at a time: 17 is wonderful for melody, outshining both the twelve-tone equal temperament (12-ET) and the Pythagorean tuning in this respect.

The most serious problem becomes apparent when we discover that diatonic harmony in this system sounds highly dissonant, considerably more so than is the case with either 12-ET or the Pythagorean tuning, on which we were hoping to improve. Without any further thought, most experimenters thus consign the 17-tone system to the discard pile, confident in the knowledge that there are, after all, much better alternatives available.

My own thinking about 17 started in exactly this way. In 1976, having been a microtonal experimenter for thirteen years, I went on record, dismissing 17-ET in only a couple of sentences:

> The 17-tone equal temperament is of questionable harmonic utility. If you try it, I doubt you'll stay with it for long.[1]

Since that time I have become aware of some things which have caused me to change my opinion completely. I now realize that, had music history taken a different turn during the later Middle Ages, it is plausible that we would now be using 17-ET instead of 12-ET. Furthermore, if we were now in the position of evaluating 12-ET as a possible alternative to 17-ET in the search for new tonal resources, we would probably dismiss 12-ET just as readily, declaring it to be melodically and harmonically bland and crude.

Does this sound a bit far-fetched? If so, then permit me to make my case.

A Bit of History

During the Middle Ages the Pythagorean tuning had a virtual monopoly as the tonal system in use in western Europe. Constructed as a series of just perfect fifths (having tones produced by string lengths in the exact ratio of 3:2, or in modern terms,

[1] *Xenharmonikôn 5*, Spring 1976 - George Secor, "Notes and Comments", p. 2

frequencies of 2:3), it was more suitable for melody than triadic harmony, and this was clearly reflected in the musical practice of the time, particularly in the perception of thirds and sixths as semi-consonant intervals that should resolve to true consonances, i.e., fourths, fifths, and octaves.

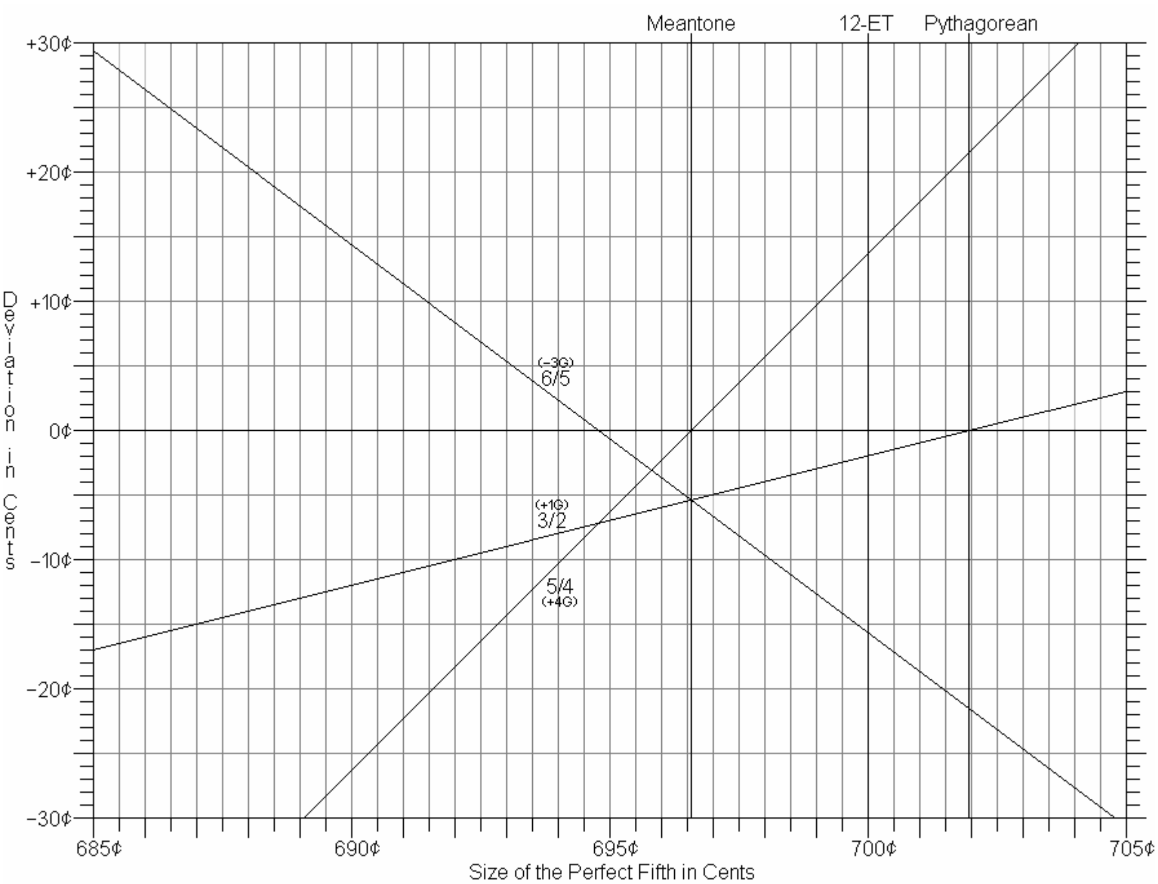

Figure 1 - Deviation of Tones from Just Ratios with 1/1 as a Function of the Generating Interval

Once tones related by simple ratios involving the prime number 5 were discovered to be harmonically consonant (beginning in the 13th century), it was found that if the fifths were altered (or tempered) slightly narrow, the resulting scale would be better for triadic harmony. The meantone temperament, devised in 1523 by Pietro Aron, has a fifth tempered by one-quarter of Didymus' comma (the amount by which four just fifths less two octaves exceeds a just major third), of approximately 696.6 cents. A fifth of this size will generate the most consonant major and minor triads in a tempered diatonic scale in which all whole tones are the same size. This is demonstrated in Figure 1, where the deviations of tones representing the three intervals occurring in the root position of these triads are plotted as a function of the size of the generating interval. Notice that the change in deviation of the major and minor thirds is +4 and -3 times as great, respectively, as the change in deviation of the perfect fifth, as indicated by the slope of their representative lines. This is a direct result of the number of fifths one must move in the series to arrive at each of those intervals. As a consequence of the recognition of thirds and sixths as consonant intervals, the principal roles of the fourth and third in the scale became reversed, with the former resolving to the latter.

The meantone temperament was the tuning predominantly in use for several centuries, and recent efforts to achieve historically authentic performances of older music have revived its use, giving more of us the opportunity to hear and appreciate the charming subtleties of this tuning, in which enharmonic sharps and flats differ significantly in pitch. We not only enjoy the greater consonance of its triads, but we also notice a difference in its melodic effect, since its diatonic semitone is somewhat larger than that to which we are accustomed.

As we gradually moved toward universal adoption of the 12-tone equal temperament, we gave up over one-half of the harmonic improvement in the ratios of 5 that we had gained with the meantone temperament (as shown in Figure 1), thus arriving at a compromise in which the requirements of both melody and harmony are met about equally well (or badly, depending on your point of view).

A Bit of Honesty

It is often observed that the larger semitones of the meantone temperament (~117.1 cents) are less effective melodically, this being a part of the price that must be paid in favoring the harmonic element over the melodic. For many years I suspected that this judgment was purely subjective, a prejudice resulting from our habituation to the smaller semitones of 12-ET (100 cents). To support my contention I needed to look no further than the diatonic semitone of just intonation (15:16), which (at ~111.7 cents) is both larger than that of 12-ET and nearer in size to that of the meantone temperament.

It has been over 35 years since I first tried the meantone temperament. Since that time I have made extensive use of many different sorts of tonal systems – both just (up to the 19 limit, and occasionally beyond) and tempered (with narrow fifths, wide fifths, and no fifths; equal, regular, and irregular; near-equal, near-just, and in-between). Having found a distinct preference for most of these over 12-ET, I feel that it is safe to say that by now I should have become free of any prejudice caused by extensive exposure to 12-ET. In spite of all this, I am forced to admit that, while I do not find the larger semitones of 19-ET, 31-ET, and the meantone temperament unacceptable, I still do not perceive them as being as effective melodically as those of 12-ET or the Pythagorean tuning.

Instead, I have found that the diatonic scales that are most melodically effective are those that have wide fifths, resulting in diatonic semitones significantly smaller than those in 12-ET. There is considerable evidence to indicate that I am not alone in making this judgment, which serves as a premise upon which the following line of reasoning is based.

But, What If …

Once theorists in the later Middle Ages and Renaissance realized that intervals based on ratios of 5 resulted in more consonant thirds and sixths, they justified their attempts to use modifications of or alternatives to the Pythagorean tuning by investigating scales proposed by the ancient Greeks. Two scales were of particular interest in this regard, the first being the diatonic scale of Didymus (c. 60 A.D.), in which one of the Pythagorean whole tones (8:9) in each tetrachord was replaced with 9:10, giving a major third of 4:5 (shown here in the Greek Dorian mode, using disjunct tetrachords; the tones should be read from *right to left*, since these tetrachords were constructed with the tones *descending*):

E		F		G		A		B		C		D		E
1/1		16/15		32/27		4/3		3/2		8/5		16/9		2/1
	15:16		9:10		8:9		8:9		15:16		9:10		8:9	

There is a false fourth in this scale (between G and C) that differs from a true 3:4 by Didymus' comma (which is also the difference in size between the two whole tones, 8:9 and 9:10).

Claudius Ptolemy of Alexandria (b. 139 A.D.) assembled a collection of scales which included not only his own, but also those of his predecessors. Among these was the Pythagorean tuning, which, being generated by a single series of just fifths (2:3), possesses a major third twice as large as a whole tone (8:9), i.e., a ditone (64:81). He called this scale the diatonic *ditoniaion*, from which we get the term *ditonic* (or Pythagorean) comma.

In his diatonic *syntonon* he reversed the order of the whole tones in Didymus' diatonic tuning:

E		F		G		A		B		C		D		E
1/1		16/15		6/5		4/3		3/2		8/5		9/5		2/1
	15:16		8:9		9:10		8:9		15:16		8:9		9:10	

From Ptolemy's designation for this scale we have the term *syntonic* comma, another name for Didymus' comma. If these tones are rearranged in ascending order so that they begin on C, they produce a so-called just major scale (for which Harry Partch preferred the term *Ptolemaic sequence*), with Didymus' comma occurring in the fifth between D and A. This scale, with its three just major triads, thus served as a theoretical basis for the development of triadic harmony in the centuries that followed.

Theorists in the later Middle Ages and Renaissance were necessarily selective in singling out the foregoing Greek scales to justify the direction that was to be taken, but, seeking a different objective, they might have selected instead the diatonic scale of Archytas of Tarentum (c. 400 B.C.), a contemporary of Plato. In his diatonic tetrachord, Archytas replaced the lower 8:9 in the Pythagorean tetrachord with 7:8, the septimal whole tone (or supermajor second), which resulted in the following scale:

E	F	G	A	B	C	D	E
1/1	28/27	32/27	4/3	3/2	14/9	16/9	2/1
27:28	7:8	8:9	8:9	27:28	7:8	8:9	

The two different sizes of whole tone differ in size by 63:64 (27 cents), which Alexander Ellis referred to as the *septimal comma*. Ptolemy listed Archytas' diatonic tuning as the diatonic *toniaion*, from which we might be a little hesitant to coin the term *toniaic comma*. Instead, I believe it would be fitting to honor the originator of this scale by calling this Archytas' comma. (For many years I have felt that the use of the names Pythagoras and Didymus in association with their respective commas is a clearer and more memorable way of identifying them than the adjectives *ditonic* and *syntonic*, which only a scholar could love. Confusion between these two terms can happen to the best of us: Even as knowledgeable an authority as J. Murray Barbour slipped up in this regard in the beginning of the first chapter of his book, *Tuning and Temperament*.[2]

We find in Archytas' diatonic scale a rather small semitone (27:28) which, at approximately 63 cents, could more accurately be called a third-tone. Archytas must have been greatly impressed with the melodic effect of this interval, for he also used it in *both* his chromatic and enharmonic tetrachords. Writing over two millenniums later, Feruccio Busoni also described how well suited he found the third-tone for melody.[3] Of course, in our own time it is well known that string players are often instructed to sharpen the leading tone slightly for a better melodic effect, which leads us to the question: What is the size of the diatonic semitone that is best for melody?

As with the meantone temperament (with fifths tempered narrow) for harmony, there is an optimum size to which the fifths can be tempered (wide) so as to achieve the best melodic result in a diatonic scale, but, unlike the meantone solution, there is no way to arrive at this mathematically. It seems to be dependent on psychological factors (i.e., on how our brains are "wired" to perceive melody), and it therefore must be determined experimentally. I don't know whether anyone has tried this with a scientifically valid number of test subjects, so the best I can do is to give my own conclusion, which is the result of careful observation using the precise pitch and instant retuning capability of the Motorola Scalatron.

It is fairly obvious that the most melodically effective semitone is definitely smaller than the equal-tempered semitone of 100 cents. It is also definitely larger than a quartertone (50 cents), which is sufficiently small that it has a distinctly different quality, putting it in a different interval class; in other words, a quartertone does not sound like a semitone. Once I had made a sufficient number of comparisons, I concluded that the optimum size is around 24:25, or 70 cents, which is approximately the third-tone of 17-ET, and I believe that my observations regarding Archytas and Busoni serve to corroborate this conclusion. Further confirmation can be found in the recommendation of Ivor Darreg that, when using 31-ET, a chromatic semitone (of ~77 cents) may be

[2] On page 1 Barbour states, "In this [Pythagorean] tuning the major thirds are a ditonic comma (about 1/9 tone) sharper than the pure thirds of the harmonic series."
[3] *Sketch of a New Esthetic of Music*, pp. 93-94

substituted for a diatonic semitone (of ~116 cents) for a better melodic effect in certain instances.[4]

Given our present perspective, the harmonic element would overshadow any melodic considerations in selecting a tonal system, but we should not assume that this would necessarily be the case with western European musicians of the 13th and 14th centuries. For them the thirds and sixths of the Pythagorean tuning were not truly consonant, and their music treated these intervals as such. They had no idea that their efforts might ultimately result in the major-minor harmonic system with which we are so familiar, so, unlike us, they had no reason to reject the 17-tone system solely on the basis of its allegedly unsuitable thirds and sixths.

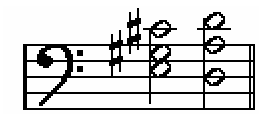

Figure 2
A Medieval Cadence

Let us suppose that medieval theorists had taken as great an interest in the requirements of melody as they had for harmony. Seeking a scale with optimal melodic properties, they would have discovered that, with a whole-tone-to-semitone ratio of 3:1, the *harmonic* effect of the tuning would also have enhanced or intensified the resolution of the highly dissonant thirds and sixths to the consonant open fifths and octaves. (A 14th-century cadence of this sort is shown in Figure 2.) In this they would have thereby accomplished no small feat in getting both the harmonic and melodic characteristics of the tuning working in cooperation with one another, rather than in opposition (as occurs with the meantone temperament). Should anyone have any doubts about this, I would advise not jumping to any conclusions until you have had a chance to hear this. The effect of 14th-century style (*ars nova*) passages played in the 17-tone system are nothing short of amazing, making 12-ET and even the Pythagorean tuning sound lackluster by comparison.

There is no point in speculating about the probability of whether hypothetical medieval experiments with a 17-tone system could have made a difference in the path that the music of the West might have taken, because it simply did not happen. We could just as easily imagine that in the 18th century the meantone temperament might have been expanded, resulting in the adoption of 19-ET or 31-ET instead of 12-ET. The fact that it didn't happen has not deterred us from seeking new tonal materials in 19 and 31 in our own time, using the similarities of those systems with the meantone temperament as historical justification for doing so.

A similar justification could be used to explore the harmonic resources of the 17-tone system, the difference being that it would be necessary to go back a bit farther in time – to the 14th century, to be specific – and, strange as it sounds, in true *xenharmonic* spirit this is exactly what Margo Schulter has done in her neo-medieval approach to composition. She even found a precedent for this in the writings of certain medieval theorists, most notably "Marchettus of Padua, who seems to describe cadential semitones somewhat narrower than Pythagorean, and vertical major thirds and sixths somewhat wider."[5]

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[4] Ivor Darreg, "The Calmer Mood: 31 Tones/Octave," in *Xenharmonic Bulletin* 9, October 1978, p. 13; this was included in *Xenharmonikôn 7 & 8*, Spring 1979

[5] Margo Schulter, letter to George Secor, September 17, 2001

Although it is open to question exactly how much Marchettus altered these thirds, sixths, and semitones, it is clear that the amount was significant. And, whereas these melodically enhanced intervals were historically restricted only to cadences, Schulter has sought to make them full-fledged members of a musical scale, finding the 17-tone division of the octave to be the most practical *closed system* in which these intervals are available. She has thus attempted, at the turn of a new century, to complete a task for which medieval theorists seven centuries earlier had taken only a first step: reconciliation of the apparent conflict between the requirements of melody and harmony in a simple diatonic scale. In this she faced one not-so-small problem: While musicians in the Middle Ages could be perfectly content with 3-limit harmony, in which the perfect fourth, fifth, and octave are the only consonant intervals, aren't those of the 21st century going to expect a bit more than that?

All in the Family

It was noted at the outset that the 17-tone system can be a bit daunting to anyone desiring to employ a harmonic vocabulary above the 3-limit. While an abundance of intervals that approximate ratios of 11 and 13 are present, prime numbers as high as these do not create chords that are very consonant in combination with ratios of 3. And the consonant open fifth of the Middle Ages is not really a chord, since it has only two different tones. If there are no chords in the system that can be recognized as consonant, then it appears that this approach leads to a dead end.

The key to unlocking the harmony in 17 lies in a proper understanding of the acoustical basis for the family of temperaments to which it belongs. Just as 12-ET eventually became acceptable by a gradual process in which musicians in prior centuries became accustomed to the meantone temperament and irregular 12-tone systems (including well-temperaments), once we realize how a similar path can be taken for 17, it is possible for us to explore a new harmonic system quite different from any other.

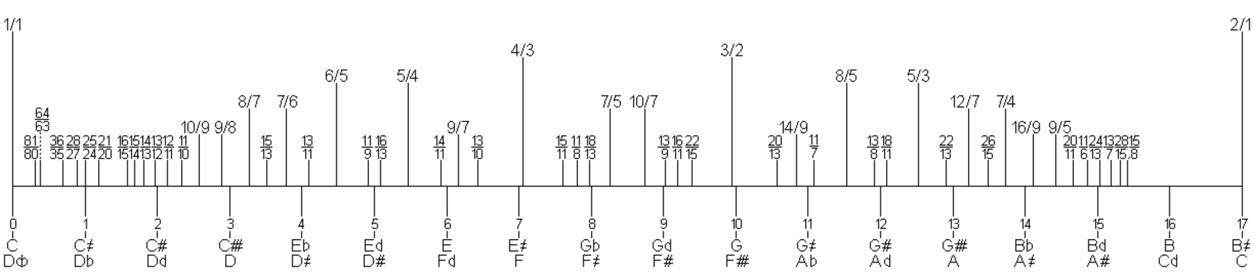

Figure 3 - Just Intonation Compared with 17-tone Equal Temperament

The first clue is supplied by Figure 3, which compares the intervals in 17-ET with those in just intonation. To say that the ratios of 5 are badly represented is patently false – they are not represented at all. To call the intervals of 4, 6, 11, and 13 degrees major and minor thirds and sixths is at best misleading. The alleged major third actually falls between 14/11 and 9/7, and the small minor third is between 7/6 and 13/11, so we are more justified in calling these ratios of 7 than of 5, i.e., supermajor and subminor intervals.

Another fruitful observation is to evaluate the interval of 3 degrees of the 17-tone system (3º17), the whole tone. In 12-ET, 19-ET, and 31-ET the whole tone falls between 9/8 and 10/9, which differ by Didymus' comma; in 17-ET it falls between 9/8 and 8/7, which differ by Archytas' comma. If the graph in Figure 1 is modified so that the fifths depicted for the Pythagorean tuning and meantone temperament are removed and those for 19-ET and 31-ET are added, and if the domain of values for the x-axis (representing the size of the generating interval) is shifted to include wider fifths, and if two other intervals are plotted which are generated by these wider fifths, the result will be the graph shown in Figure 4.

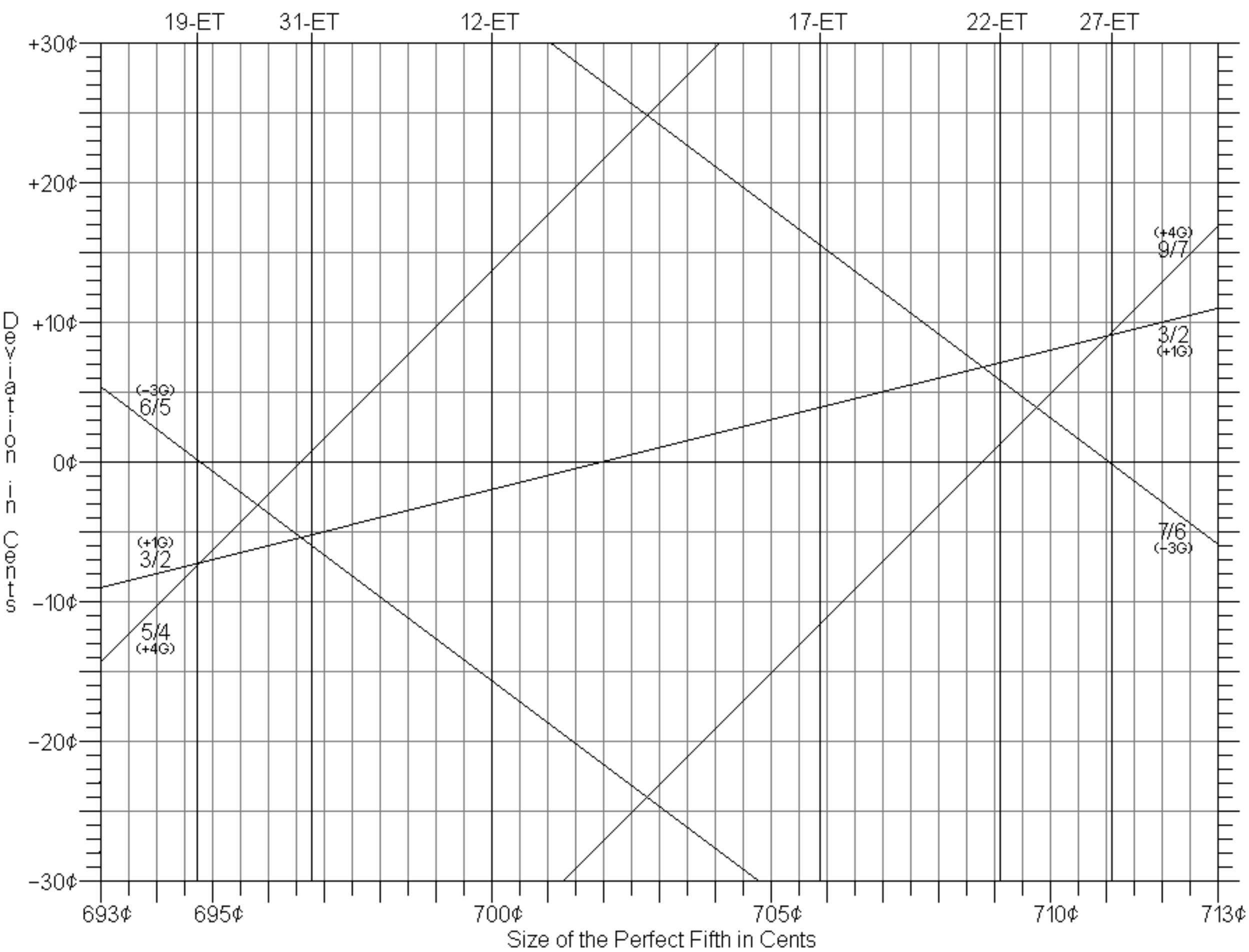

Figure 4 - Deviation of Tones from Just Ratios with 1/1 as a Function of the Generating Interval

From this new graph, it is evident that a parallel exists for the tones that make up the major and minor triads (4:5:6 and 10:12:15) in narrow-fifth temperaments with the tones that make up the subminor and supermajor triads (6:7:9 and 14:18:21) in wide-fifth temperaments. Each family of temperaments has an equal division that approximates its harmonically optimal “meantone” system (31-ET vs. 22-ET), in which the tempered whole tone is approximately midway between two just ratios separated by the comma defining that family of temperaments. Each has an equal division that represents almost exactly the intervals generated in either direction by three fifths: the major sixth and minor third in 19-ET, and the supermajor sixth and subminor third in 27-ET. And each has an equal division (12-ET vs. 17-ET) that has, within its family, the fifth of least error, the lowest number of tones, and the best melodic properties; in the latter respect, 17-ET is unexcelled.

The parallels are not exact in every detail. For example, 12 plus 19 equals 31, but 17 plus 27 equals 44, not 22 (although 22 is a subset of 44). There would not be much incentive to explore the resources of 27-ET, since it has not only more tones than 22-ET, but also inferior intonation, particularly in that its fifth is tempered by more than 9 cents.

The tone representing 11/8 is generated in all three wide-fifth temperaments by six fifths taken in the negative (flatwise) direction. In 22-ET 5/4 is represented by nine tempered fifths taken in the positive direction, but 13/8 is not represented at all. In 17-ET it has already been mentioned that 5/4 is not represented at all, but 13/8 is represented by nine tempered fifths taken in the negative direction (or eight in the positive). One particularly nice feature that 17-ET shares with 31-ET (and also with 41-ET and 53-ET) is that the prime harmonic factors are encountered in ascending order by moving along the tones in the circle of fifths in the appropriate direction, which establishes a complete correlation between two different measures of harmonic remoteness in these systems.

Thus 17-ET is a 13-limit system without ratios of 5. Given the number of chords that could be built from combinations of four prime numbers, the possibilities for harmony should be considerable.

All Things Being Unequal

I became aware of the harmonic potential of the 17-tone system around the time that I obtained a copy of Owen Jorgensen’s book, *Tuning the Historical Temperaments by Ear*,[6] in which he set forth guidelines for improving various historical unequal 12-tone tunings, specifically those that were intended to be usable in all keys, which he called “well temperaments.” Having previously produced a less-than-satisfactory unequal 19-tone tuning, in which I sought to improve the consonance of chords in the more common keys at the expense of the more remote keys, I applied some of Jorgensen’s principles and achieved a much better result, my 19+3 temperament, which became one of my favorite tunings. Buoyed by my success, I decided it would be fun to try the same thing with the 17-tone system, and within a matter of days I had produced a 17-tone well temperament (17-WT) that was even better than my 19-tone effort.

[6] This was published as a limited edition by the Northern Michigan University Press, Marquette, 1977. A new edition (1991) was published by the Michigan State University Press (East Lansing) and is described at <http://mmd.foxtail.com/Tech/jorgensen.html>

The requirements for a temperament having an irregular closed circle of fifths in 17 and 19 are somewhat different from those for 12 in that many of the tempered intervals are used to represent two different just ratios. For example, 4 system degrees of 19-ET (4º19) falls between 7:8 and 6:7, while 6 system degrees of 17-ET (6º17) falls between 11:14 and 7:9. In a temperament with a circle of fifths of varying size, certain intervals will more closely approximate one of these ratios in one part of the circle and the other ratio in another part of the circle. The objective is to construct the temperament in such a way that the best approximations of these ratios will occur, in the desired keys, simultaneously in chords in which these intervals (or their inversions) are used in combination, e.g., 6:7:8 in 19 or 7:9:11 in 17.

In 19, using primes 3, 5, 7, and 13, I was able to achieve the best intonation in three different keys (with F, C, and G as fundamental tones), adding three extra tones to supply the 11 factor in those three keys to arrive at the 19+3 temperament. In these three keys, the overall harmonic effect within the 13 limit is at least as good as for 31-ET, while the general melodic characteristics of 19-ET are retained.

In my 17-WT, using primes 3, 7, 11, and 13, the best intonation occurs in *five* different keys (with B-flat, F, C, G, and D as fundamental tones), with the effect of the tempered 6:7:9:11:13 chord being comparable to that of the best keys in the 19+3 temperament. While the 6:7:9 (subminor) triad in 17-WT is not as good as in 22-ET, the best 6:7:9:11 tetrads in 17-WT are considerably better than those in 22-ET. Even though a significant improvement is made in the harmonic effect of 17-WT over 17-ET, the former retains the general melodic characteristics of the latter.

The improvement in intonation in 17-WT and 19+3 over their respective equal temperaments is every bit as effective as in the very best well temperament possible in the twelve-tone system. In both of these tunings, a very favorable configuration of first-order difference tones does much to stabilize chords having tones in 7:9:11 and 9:11:13 relationships, making it possible to produce consonant chords containing ratios of 11 and 13. (A complete description of 17-WT is given at the conclusion of this article.)

In summary, the groundwork that would be needed to unlock the full harmonic resources of the 17-tone system at the outset of the 21st century was quietly established in the space of a couple of months in the winter of 1978. About a year later the papers relating to this work were permanently packed in a cardboard box, along with various other microtonal notes, sketches, and diagrams; these included the graph shown in Figure 4 and the two moment-of-symmetry (MOS) scales which will be discussed below under a separate heading.

An Avalanche of Ideas

My review of Jorgensen's book, along with a couple of short articles describing my 19+3 and 17-WT tonal systems, appeared in the first issue of *Interval* in May 1978.[7] For over 20 years I was unaware that anyone else was seriously interested in either of those tunings until September 2001, when I received an e-mail from Margo Schulter profusely thanking me for 17-WT, "a very beautiful tuning for the kind of 'neo-medieval'

[7] Information regarding back issues is available at <http://interval.xentonic.org>

style in which [she] composed and improvised," having tried it for the first time only three days before. (As the cartoon character Bullwinkle the moose once remarked, "Flattery will get you somewhere.")

Pleasantly surprised and puzzled, I spent the next week trying to figure out why a tuning specifically designed for one purpose (new harmonies) should be so suitable for an entirely different one (medieval music). Upon realizing that the classic conflict between the requirements of melody and harmony was elegantly resolved in the 17-tone system, I found new ideas taking shape, each leading to another, and could give only an incomplete reply at first, until my train of thought had reached some final conclusions.

Our subsequent correspondence raised as many new questions and issues as it answered, as each of us, approaching the tonal materials from different perspectives, kept coming up with new ideas for chord progressions in 17-WT (in various musical styles, including medieval) in response to ideas suggested by the other. At one point I made the following observation:

> What a contrast this is with the plight of composers in the 20th century, searching in vain to find something harmonically new in a worn-out twelve-tone scale! In 17, almost every chord you try is new, but the challenge is to discover how to make use of consonance and dissonance in combination with good voice leading to achieve the most effective results. And you can't revert back to the old major-minor system either – it just isn't there, so you are forced to do something new and different, hoping that you won't fall flat on your face (not an easy task in unfamiliar territory). These things are learned one step at a time, all in good time, and, as any journey must begin with a first step, I think you are already off to a good start.[8]

What started out as a few provocative ideas quickly snowballed into an avalanche. It became evident that our discoveries would need to be organized in a form suitable for publication, and we agreed to write complementary articles for the upcoming issue of *Xenharmonikôn*.

Remaking History

The analysis that I had done of the wide-fifth temperaments in 1978 treated them only from a harmonic perspective, and I did not attempt to tie this alternative approach to harmony to any particular point in history, other than that it would have been a logical consequence of using Archytas' diatonic scale as a theoretical basis for one or more equal-tempered or well-tempered alternative tuning systems. Schulter's approach is novel in that it not only takes both the harmonic and melodic elements into account, but also identifies the particular period in history at which the adoption of a wide-fifth temperament would have been most likely to occur, finding support in both the musical practice and theoretical writings of the 14th century.

[8] George Secor, letter to Margo Schulter, September 26, 2001

It is a tricky undertaking to speculate about the path along which an alternative development of harmony might occur or to extrapolate from events of the past to predict what the harmony of the future should be. Approaches based on even the most brilliant ideas and creative insights must be taken to their logical conclusions and tested for consistency and validity. In the face of compelling contradictory evidence or data, one cannot afford to be so enamored with a theory as to refuse to modify it, or, if necessary, to discard it entirely. (In this regard, I cannot help thinking of Joseph Yasser's book, *A Theory of Evolving Tonality*, from which I hope that I have learned something.)

In her paper, *Enharmonic Excursion to Padua, 1318: Marchettus, the cadential diesis, and neo-Gothic tunings*[9], Margo Schulter not only documents the use of very wide thirds and sixths by Marchettus of Padua in the 14th century to enhance the melodic and harmonic effect of cadences (such as the one shown in Figure 2), but she also attempts to determine the most likely sizes of those intervals from his somewhat ambiguous specification for a cadential major sixth, which he describes as differing equally from a perfect fifth and an octave.

The modern interpretation is to take the average value of the fifth and octave, described in previous centuries as a *geometric* mean between the string lengths on the monochord for these two intervals. This gives an interval of approximately 951 cents, which poses two serious difficulties. In the first place, the resulting ratio is an irrational number, which is both atypical for the time and unnecessary for the purpose of the writer, being more suited to the description of a temperament that might be proposed in a later century. In the second place, this interval is so large that it has a very different character from anything that we would expect to interpret as a major sixth, so it is highly doubtful that this is what Marchettus had in mind.

The most probable interpretation of the cadential major sixth is arrived at by taking an *arithmetic* mean between the string lengths for the octave and perfect fifth, resulting in the ratio 12:7 (~933 cents). The cadential major third would then have a ratio of 9:7 (~435 cents). Schulter tested this on an arrangement of two keyboards tuned to separate sets of just fifths separated by 64:63 (Archytas' comma), with the cadential leading tone being 27:28, or ~63 cents, from its resolution.[10] She found the result so melodically and harmonically satisfying that this has become one of her favorite tunings.[11] As with Archytas' diatonic tetrachord, this tuning contains whole tones of two different sizes, 8:9 and 7:8, confirming the former as a theoretical basis for the latter. The cadences of Marchettus can thereby be identified as a probable first step toward the introduction of ratios of 7 into the harmonic vocabulary of the Middle Ages.

In concluding this section of her article, Schulter notes that the *regular* diatonic scale (i.e., having all whole tones of the same size) found in the division of the octave into 22 equal parts would have major (or, if you prefer, supermajor) thirds and sixths of 8º22 and 17º22, respectively, closely approximating ratios of 9:7 and 12:7, a tempered solution having "the charm of simplicity," while deviating significantly from the Pythagorean tuning in order to favor ratios of 7. It was previously mentioned that 22-ET

[9] March 2001, available at <http://value.net/~mschulter/marchetmf.txt> (ASCII text) and <http://value.net/~mschulter/marchetmf.zip> (text and PostScript)
[10] *Ibid*, Section III
[11] Letter of October 4, 2001

is the division approximating the harmonically optimal "meantone" temperament for wide fifths, analogous to 31-ET for the family of temperaments with narrow fifths. Had she followed through on this observation, she would have taken a path paralleling Nicola Vicentino's experiments with a 31-tone octave in the 16th century, and 22-ET might well have been the end of this microtonal odyssey.

Just as different theorists proposed various alternatives to the harmonically optimal meantone temperament of Aron and 31-division of Vicentino, so could we imagine that successors to their counterparts in a hypothetical wide-fifth alternative tuning history might entertain a variety of opinions and thereby propose many such alternatives. They might even propose the same alternative, but for entirely different reasons, which was exactly the case in our arriving at a 17-tone system in preference to 22-ET. In my judgment a diatonic semitone of 55 cents (1º22) is not only somewhat less than the optimal melodic range of 70±10 cents, but is also at the borderline between what we might perceive either as a semitone or quartertone (depending on the musical context). Seeking a system better suited for melody, I would expect that the majority opinion would be in favor of the 17 division of the octave, which (like 12-ET in the family of narrow-fifth temperaments) gives up some harmonic consonance to achieve a better melodic effect, with the added benefit of having fewer tones per octave. And, as the musicians and theorists of earlier centuries devised tunings having irregular circles of 12 fifths in order to make certain intervals less dissonant in the most common keys, I would expect those in our alternative history to express a dissatisfaction with the rather dissonant thirds and sixths in the diatonic scale of 17-ET, thus arriving at a well-tempered (i.e., closed unequal) tuning of 17 tones.

Schulter's path and priorities proceeded somewhat differently from mine, as she recounted in a personal communication:

> As it happens, 17-ET and a regular 24-note Pythagorean tuning were the first two systems of this kind I tried in 1998, with 22-ET coming much later, in June of 2000. As I'll discuss below, my main mixed feelings about 22-ET are "harmony vs. harmony" issues, for example the heavy temperament of the fifth by over 7 cents. When I tuned it up, I found that the 55-cent semitone was no problem for me as a diatonic step, although I agree that 70±10 is a likely optimal range. One of my favorite tunings has regular semitones of 77 cents, and narrow cadential semitones or dieses of 55 cents.
>
> By the way, to give credit where credit is due: John Chalmers gave me the idea for both of my first two "neo-medieval" tunings, telling me about 17-ET, and also pointing out that Pythagorean when carried far enough emulates ratios of 7. With that hint, I quickly realized that a 24-note Pythagorean tuning would give major thirds and sixths a Pythagorean comma wider than the regular ones — very close indeed to 7:9 and 7:12, about 3.80 cents off. It seemed a very attractive way of doing the kind of thing that Marchettus describes, and late last year [2000] I came up with the refinement of spacing the two 12-note Pythagorean chains an Archytas comma apart for pure ratios of 3 and 7.

> … I tend to regard 22-ET as a charming tuning near the "far end" of the conventional neo-medieval spectrum, rather than a standard solution for combining ratios of 3 and 7.
>
> My mixed feelings about this system as a standard solution are mainly harmonic:
>
> (1) The fifths are tempered by over 7 cents, when there are lots of other solutions for combining ratios of 3 and 7 which treat the fifths more gently;
>
> (2) The tuning doesn't include thirds close to Pythagorean, or to ratios of 11:14 and 11:13; and
>
> (3) We don't have "submajor/supraminor" thirds rather close to 14:17:21, or neutral thirds.[12]

Most of the "other solutions for combining ratios of 3 and 7" are open systems. Besides 17 and 22, the only other possible closed systems having a reasonably small number of tones (with fifths that are not tempered considerably narrower than just) are the 24 and 29 divisions of the octave. While both of these have fifths that are tempered more gently than in 22-ET, both have very large cadential major sixths that are arrived at by taking the average of the fifth and octave (950 cents for 24-ET, and 952 cents for 29-ET). As was previously noted, intervals near this size have a very different character from what we would expect for a major sixth, so the choice of a closed system comes down to the 17 and 22 divisions. (The equal division of lowest number that approximates both the 3rd and 7th harmonic with a relatively small error is 36-ET, but this would be more cumbersome than using an open system in just or near-just intonation.)

It may be somewhat surprising to observe that, even though 22-ET presents an opportunity to introduce ratios of 5 into the harmonic vocabulary, this was not a determining factor for either of us in deciding whether 17 or 22 tones per octave would be a more suitable choice. It is entirely plausible that a school of thought favoring 22-ET in our hypothetical alternative history might have prevailed, but the fact is that this did not occur in the present neo-medieval approach. Instead, the preferred system was found to be the one that more closely resembles the original Pythagorean tuning in *both* its melodic and harmonic characteristics. As an added feature, ratios of 11 and 13 are available to provide some truly different harmonies that would be highly appropriate for the music of a new millennium.

Schulter also developed an irregular closed 17-tone temperament before learning of my 17-WT. She now uses both of these, but says that mine is her favorite. Inasmuch as I did not consider melodic requirements in the design of my 17-WT, I can claim nothing other than dumb luck on my part for the diatonic semitones being in the range of 64 to 78 cents, which, in Schulter's own words is "very neatly optimized."[13]

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[12] Margo Schulter, Letter of October 31, 2001
[13] Letter of September 25, 2001

In this parallel history-in-the-making, I feel very strongly that a final step from 17-WT to 17-ET is one that should not be taken, just as I believe that our abandonment of 12-WT for 12-ET on instruments of fixed pitch was a great tragedy. The dimension of “mood” or “color” imparted by the variation of melodic and harmonic characteristics of the intervals in different keys is a property that may be used to good effect in a piece written for a “well temperament,” but since it is completely absent in our present system of twelve equal semitones, this idea would not even occur to most contemporary composers or musicians. Besides, the uniform restlessness of chords that we now experience with 12-ET would be all the more prevalent in 17-ET.

Is 17-WT, then, the final step in this alternate history of tuning and temperament? At this point I think not. Once the resources of the 17-tone system were fully exploited, we could expect that other options with better intonation would be sought. I have tried a number of the progressions that we have discovered in 17-WT in other tuning systems, and there is a near-just 13-limit system (that includes ratios of 5) into which virtually everything that we have tried can be transferred; the progressions not only work, but they sound even better than in 17-WT! Hopefully, this will be a topic for a follow-up article.

Four-part Xenharmony

Whereas the 12-tone system has two different triads containing a perfect fifth (with major and minor thirds), the 17-tone system has three, with subminor, neutral, and supermajor thirds. Of these three, the subminor triad (6:7:9) is by far the most consonant, so this has been used as the tonic triad in most of our experiments.

In dealing with intervals involving prime numbers above 5 or 7 in any tonal system, I have found that the most consonant or stable chord structures are those that are *isoharmonic*, i.e., in which at least two first-order difference tones coincide. (For the origin of this term, as well as numerous examples of its application, see Leigh Gerdine’s translation of Adriaan Fokker’s *New Music with 31 Notes*, pp. 79-81.) To illustrate how this principle can be applied in 17-WT, I devised the following basic 13-limit scale (with semiflats indicated using flat symbols written backwards):

C	Dd	Eb	F	G	Ad	Bd	C	(d = semiflat)
0	2	4	7	10	12	15	17	º17
1/1	13/12	7/6	4/3	3/2	13/8	11/6	2/1	just ratios

With the exception of A-semiflat, all of the tones of this scale represent simple harmonics of a lower octave of F. There is therefore no comma of the usual sort in this scale, even in just intonation.

The ninth chord with root on C is a tempered 6:7:9:11:13 chord. The dominant triad is unusual in that it does not have a perfect fifth, but its seventh chord has a relatively consonant sound, particularly in the third inversion (F, G, Bd, Dd), where the tones approximate 8:9:11:13. The three top tones of this inversion are isoharmonic, producing first-order difference tones of 2. The supertonic and subdominant have neutral triads (with perfect fifths and neutral thirds).

One enharmonic alteration of this basic 13-limit scale (chromatic alteration being not quite the right term) is to substitute tone 13 (A, 12/7) for tone 12 (A-semiflat, 13/8), making a supermajor triad on the subdominant. This also makes the scale symmetrical in that intervals of the same number of degrees are mirrored above and below the tonic. With the presence of fewer neutral intervals, this scale sounds similar to the medieval Dorian mode (which, due to a misinterpretation, is different from the ancient Greek Dorian mode).

Another alteration of the basic scale is to substitute tone 11 (A-flat, 14/9) for tone 12 (A-semiflat, 13/8), making a subminor triad on the subdominant; this makes a relatively large interval between the sixth and seventh scale degrees, analogous to the harmonic minor scale of our major-minor harmonic system. (Figure 3 may be helpful in determining the number of system degrees for various tones or intervals discussed here.)

The first example in Figure 5 illustrates a progression that, lacking the sixth degree of the above scale, is playable in all three versions. It consists of a third-inversion dominant seventh chord moving to a first-inversion tonic (subminor) triad, followed by a second-inversion dominant seventh chord resolving to a root-position tonic triad. While these examples are voiced as in conventional four-part harmony, these chords sound more consonant and are most easily analyzed if the two lower voices are transposed an octave higher, putting them in a closed spacing. The tones in the respective chords will then represent ratios of 8:9:11:13, 7:9:12, 13:16:18:22, and 6:7:9:12. Half of the melodic intervals in this progression are neutral seconds of 2º17 (as are 4 of the 7 steps in the original scale), all of which contribute to a strange, yet harmonious sound, *xenharmonic* in the truest sense of the word.

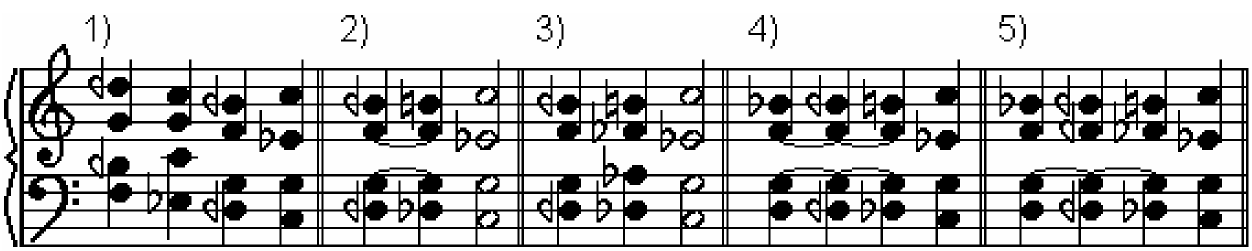

Figure 5 - Chord Progressions in the 17-tone Well Temperament

Examples 2 through 5 of Figure 5 illustrate several chord progressions that are derived from the last two chords of the first example. In Example 2 a chord is introduced between these two chords so that the outer voices each move by successive third-tones, producing an *enharmonic* progression. Not only is this melodically desirable, but the increase in harmonic dissonance from the first to the second chord is effectively resolved to the much more consonant tonic (subminor) triad.

Example 3 differs from Example 2 only in the inner voices of the second chord, which in closed position has the tones approximating 6:7:9:11. This is not one of the "best" (i.e., most consonant) keys for this chord, so it has a slightly more restless sound (or mood) than it would have if it were built on the tonic. This should not be considered a defect, however, since the resulting contrast of dissonance to consonance is a desirable component of any resolution. Also, there are parallel fifths between the last two chords, but this would be allowable under the rules of traditional harmony for the resolution of an augmented sixth chord (as with a German sixth chord in 12-ET resolving to the tonic).

The last three chords of Example 4 are the same as the three chords in Example 2, with the new beginning chord being a subminor seventh chord (12:14:18:21) in its second inversion. Here the outer voices each move by three successive third-tones in an expanded enharmonic progression.

Example 5 is the same as Example 4, except for the alto voice in the two middle chords, which moves in parallel subminor tenths with the bass. This progression is highly effective, not only because of the movement by $1^{o}17$ in three of the voices, but also in that the dissonance builds with the first three chords until the final resolution to the consonant subminor triad. The melodic refinement of enharmonic voice movement makes chromatic progressions in 12-ET seem crude by comparison.

In experimenting with progressions such as these, we concluded that it isn't necessary to analyze every single note of every chord, inasmuch as it is possible to find all sorts of dissonant combinations of tones that are just that. But I also observed that it would be very good to have at least a few relatively consonant chords in one's harmonic vocabulary to which these may be effectively resolved.

Starting at the End

Neutral and supermajor triads are by nature more dissonant than subminor triads, so it may take some time to get accustomed to them, even when they are in just intonation. However, once this has taken place, I have found that these triads (either just or slightly tempered) do not sound any more dissonant than the major triads of 12-ET. While the dissonance of 12-ET triads can be attributed to heavy tempering, in the best neutral and supermajor triads in 17-WT both the amount of tempering and the prominence of beating harmonics is relatively small, with the perception of dissonance due primarily to disturbances involving difference tones.

In just intonation, neutral and supermajor triads both occur in more than one form. Neutral triads may have ratios of 14:17:21, 18:22:27, or 26:32:39, while supermajor triads may occur as 14:18:21 or 22:28:33. In both my 17-WT and Schulter's irregular 17-tone temperament, the ratio which a particular triad most closely resembles will vary with the key (or tonal center).

The idea of using neutral intervals in a medieval style occurred to me when I first used 17-WT in 1978, but I experimented with this only briefly, using two voice parts. As I recall, the scale that I used for this purpose is the following MOS scale, which is generated by a neutral third ($5^{o}17$):

C	Dd	Ed	F	G	Ad	Bd	C	
0	2	5	7	10	12	15	17	º17
1/1	13/12	11/9	4/3	3/2	13/8	11/6	2/1	just ratios

This differs from my basic 13-limit scale by only one tone, and it also has intervals of the same number of degrees mirrored above and below the tonic. In addition, its just version consists of identical disjunct tetrachords separated by the whole tone between F and G.

The neo-medieval two-voice cadence that most impressed me has the 2nd and 7th degrees of the scale (a supermajor sixth apart) progressing in contrary motion by neutral seconds (2º17) to an octave. When Schulter learned of this, she modified the medieval cadence shown in Example 1 of Figure 6 (copied from Figure 2) to create a three-voice cadence that was completely new to both of us (see Example 2). I subsequently transposed this back into D and rewrote it using semisharp and semiflat notation (see Example 3). This has a very unusual sound, which results from the melodic and harmonic elements involving completely different families of intervals.

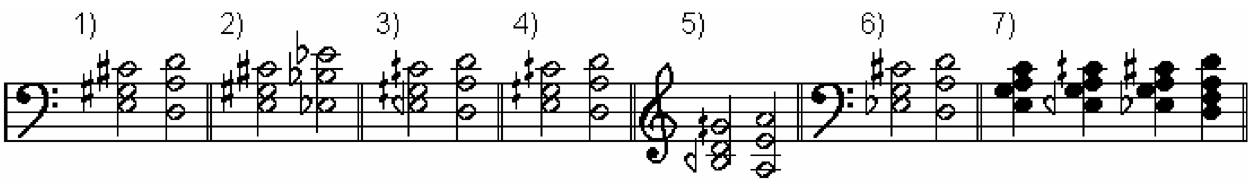

Figure 6 - The Development of Neo-medieval Cadences

She then showed me another cadence that she had previously used, in which the voices move by neutral seconds, but with different vertical intervals, which I renotated as Example 4 of Figure 6. From this I derived the following scale, which requires no enharmonic alteration to achieve the cadence:

D	E	F≠	G≠	A	Bd	C≠	D	(≠ = semisharp)
0	3	5	8	10	12	15	17	º17
1/1	9/8	39/32	11/8	3/2	13/8	11/6	2/1	just ratios

This is a transposition of a mode of the MOS scale given above, starting on the fourth degree (F). With this scale it is also possible to make a cadence to the dominant (shown in Example 5 of Figure 6) that we considered successful without raising the D to D-semisharp.

In response to a remark that I liked the effect of voices separated by 15º17 (an augmented 6th) moving in contrary motion, each by a single degree, to resolve to an octave, Schulter agreed, illustrating this with the three-voice cadence shown in Example 6 of Figure 6. I found the first chord rather dissonant and felt inspired to rework the cadence into the four-voice progression in Example 7 (which is the original version of Figure 5, Example 4). At this point, I could not help noticing that suddenly we had left the Middle Ages and were now having a *jamais vu* experience – that of hearing bits of music belonging to a time or place that neither of us had ever been before.

These examples, all of which were shared in less than 3 weeks from the time Schulter sent me her first e-mail (which time included the several days each of us had suspended our microtonal activity in reaction to the terrible events of September 11), only begin to hint at the possibilities of the 17-tone system. Much more experimentation needs to be done before any compositions can be written. Cadential progressions such as these are but a start, or should I say a tentative end (inasmuch as we have chosen to start at the end), for which an appropriate beginning and middle are still lacking. It takes time to become familiar with the new tonal materials and, just as the major-minor system wasn't invented overnight, it will take both patience and perseverance to discover the harmonies that will work most successfully for different sorts of melodies and musical styles.

The Resolution of the Problem of the Resolution

It has already been mentioned how highly effective the medieval cadence of Figure 2 is when played in 17 (in either the ET or WT), where the resolution is made to an open fifth. Equally effective, for both melodic and harmonic reasons, is a dominant-to-tonic (V-I) resolution where the dominant consists of a (rather dissonant) supermajor triad and the tonic a (much more consonant) subminor triad. Five weeks after Margo Schulter's initial e-mail to me, while in the process of simply enjoying the experience of hearing resolutions such as this, I made an unexpected discovery, about which I wrote the following to her:

> For once I have not come up with any new progressions to share with you. Taking my own advice, I have spent some time just playing some of the things we have found up to this point, just to allow them to sink in a bit more. As a result of something I tried the other day, I am now going to have to qualify my statement that "in 17 … you can't revert back to the old major-minor system … – it just isn't there, so you are forced to do something new and different." Having discussed the possibilities of dissonant-to-consonant V-I resolutions in [another tuning], it finally occurred to me that the same principle would apply to a Renaissance style played in 17-WT, as long as the tonic has a subminor triad (thus giving us something familiar which we can fall back on, even if it limits us to three modes). I was having such a good time finding new progressions that, until now, I had neglected trying out something much more obvious to hear how well it worked. What a surprise I got! – It's fantastic! What really got me excited was hearing a very ordinary suspension that I should have expected to be effective, but instead it caught me completely off guard. [See Figure 7.]

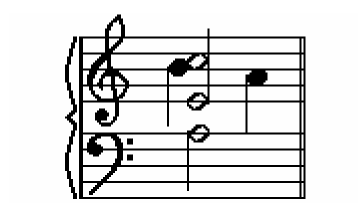

Figure 7
An "Ordinary" Suspension

> I have been so focused on how *melodically* effective those 1º17 semitones are that I was absolutely astounded to hear how *harmonically* effective they are, as evidenced by the huge dissonance-consonance contrast between the suspension and its resolution. I immediately had to try the same thing in 12-ET and 31-ET; in 12 it sounds bland by comparison, whereas in 31 you can generate the same excitement only if you substitute B-semisharp for B, or else keep the B and substitute C-semiflat for C (which puts you in subminor mode). In trying this resolution in various tunings (having various sizes of semitones, using both diatonic and chromatic versions) in various keys, I have finally had the opportunity to verify something that I had only suspected previously: The ideal *melodic* size for the semitone is the same as the ideal (i.e., most dissonant or most effective) *harmonic* size. (As the saying goes, it doesn't get any better than this!) After journeying through progressions from out of this world, suddenly I find myself back [home], only to find that the familiar landscape has suddenly become more vivid than anything I had ever dreamed. And all along I was led to believe that the requirements of melody and harmony were in conflict; I think that this could be the one crucial piece of evidence that will finally lay that notion to rest.[14]

In light of this discovery, how is the apparent conflict between the requirements of melody and harmony in the family of narrow-fifth temperaments to be explained?

Put in the simplest possible terms, it is observed that *melodic effectiveness* (or melodic "dissonance") and *harmonic dissonance* are completely correlated with one another, with the melodic effectiveness in a resolution being determined by the size of the interval (or intervals) of resolution. The *harmonic effectiveness* of a resolution is a bit more complex in that it is determined by the *difference* in the relative dissonance between the resolving interval (or chord) and its resolution. In other words, the most effective resolutions will have a large dissonance-to-consonance *contrast* between the two chords.

This observation is in direct opposition to the belief that chords involving ratios with relatively large numbers should be *avoided* as much as possible in just intonation. It is also in conflict with the objective that led to the adoption of the meantone temperament: to *minimize* the overall dissonance of the intervals contained in *all* major and minor triads.

Placing two major triads *in just intonation* in succession, such as we would have in an *idealized* dominant-to-tonic progression, results in a situation in which the dissonance-to-consonance contrast between the two triads is minimal. From a melodic perspective, we have already seen that the interval by which the leading tone resolves to the tonic, a just minor second (15:16), is not overwhelmingly effective. Having heard some highly effective things in 17-WT, I cannot help but characterize this as a rather bland progression.

[14] George Secor, Letter to Margo Schulter, October 12, 2001

In the meantone temperament, the situation is not much different, but in 12-ET one might expect that the tempering of the thirds and sixths would spice things up a bit, while the smaller semitone would be more melodically effective. This is not the case, however, since the resolution to the tonic chord offers no relief from the tense mood of 12-ET, which offsets any melodic advantage that the smaller semitone might have. It is a *lack of contrast* between the resolving chord and its resolution that makes a progression bland, not a lack of dissonance in the progression, and the increase in dissonance that occurs in both chords by changing the intonation from meantone temperament to 12-ET only results in progressions that are, relatively speaking, both bland and tiresome, as anyone who has had the opportunity to make the comparison can testify.

Only when the leading tone is significantly raised in pitch to make it harmonically more dissonant with respect to the dominant (ideally in the range of 14:11 to 9:7), and only when it is then resolved to a considerably more consonant tonic triad (either major, minor, or subminor, not heavily tempered), is the best contrast achieved. As a result of raising the leading tone, the semitone becomes smaller, thereby improving the melodic effect of the resolution in cooperation with the harmonic effect. So both the progression and the melodic-harmonic problem are very nicely resolved!

This technique can be put to good use in systems such as 31-ET, 41-ET, or just intonation (given a sufficient number of tones), where there is a good variety of intervals available. The great advantage of the 17-tone well temperament is that highly effective resolutions can be easily achieved using a very reasonable number of tones in the octave.

A Final Major Problem

Conventional compositions in the minor mode sometimes conclude with a final major triad, a highly effective device known as a Picardy third. Something of this sort would be desirable in pieces employing a tonic subminor triad in the 17-tone system, but both the neutral and supermajor triads are more dissonant than the subminor triad, making them unsuitable for this purpose.

The solution rests in an enharmonic alteration of the basic 13-limit scale described previously. The replacement of B-semiflat (11/6) with B-flat (7/4) changes intervals involving ratios of 11 into others that are more consonant:

C	Dd	Eb	F	G	Ad	Bb	C	
0	2	4	7	10	12	14	17	º17
1/1	13/12	7/6	4/3	3/2	13/8	7/4	2/1	just ratios

This also produces identical disjunct tetrachords separated by the whole tone between F and G. If the ratios of 13 are omitted, the result will be a 7-limit pentatonic scale (or 5-note chord) similar in character to the Javanese *slendro* scale, which is noted for its stability or restful quality. This could be used to provide a satisfying alternative to a simple subminor triad (6:7:9) at the close of a composition, either in closed position (perhaps as an arpeggio) or with the tones spaced by fourths (similar to the open strings of a guitar). Another possibility is to end with a subminor seventh chord consisting of C, E-flat, G, and B-flat, with the C repeated at the octave (12:14:18:21:24 in closed position).

Each of these choices contain a subminor and supermajor triad, yet they possess a sweetness and stability that makes them more desirable in ending a composition than either of those triads used alone. This is due not only to the increased number of perfect fourths or fifths present, but also to the inclusion of isoharmonically related tones in the closed positions of these chords, as 6:7:8 or 6:7:8:9.

A New Generation of Scales

Since 17 is a prime number, any of the intervals in the 17-tone system used in a series will pass through the entire system of tones before returning to an octave of the original tone. The excessive symmetry of 12-ET imposes the limitation that this is possible only with the semitone and fifth (or their inversions) as generating intervals, another way in which it may be characterized as crude. There are therefore a number of different ways to construct moment-of-symmetry (MOS) scales (i.e., scales that are structurally consistent) in 17.[15]

In various divisions of the octave, the perfect fifth may be used as a generating interval to form both pentatonic and heptatonic MOS scales. The conventional diatonic scale (i.e., the heptatonic scale generated by fifths of the Pythagorean tuning, the meantone temperament, 12-ET, 19-ET, 31-ET, etc.) is elegant in that each of its tones is a member of at least one of the three major triads in the scale (or of the three minor triads, for that matter). The same is true with regard to the three subminor (or supermajor) triads for the heptatonic scale generated by wide fifths, such as those found in 17-ET or 22-ET. The discovery of similar properties in scales generated by other intervals in various tonal systems could provide completely different approaches to tonality, not only in that new intervals would produce new harmonies, but also because the functional relationships between the various tones in the scale would be new. There are at least two new scales of this type in the 17-tone system.

One of these is a 9-tone scale generated by a neutral second (2º17):

0	2	4	6	8	10	12	14	15	17	º17
D	**Ed**	**F**	Gd	Ab	A	Bd	C	C≠	D	
6	13	7			9			11	6	Harmonic
	6	13	7			9			11	functions
	11	**6**	13	7			9			of tones

This creates three pentads approximating 6:7:9:11:13 on three different tones, D, E-semiflat, and F. (Note that D and F are among the five best root tones for this chord in 17-WT; choosing A, B-semiflat, and C would also have accomplished this, with A and C being the best root tones.) This scale may also be regarded as producing three

[15] Moment-of-symmetry scales were first described by Ervin M. Wilson in his two articles in *Xenharmonikôn 3*, Spring 1975; a concise formal definition (as well as a more elaborate explanation) may also be found in Joseph Monzo's tuning dictionary at <http://sonic-arts.org/dict/mos.htm>

tetrads approximating 6:7:9:11, which will still result in each tone of the scale being used in at least one chord. Another variation is to eliminate tone 15 (C-semisharp) from the scale, in which case three triads approximating 6:7:9 are produced by an 8-tone MOS scale.

Either version of this scale would be right at home on a 17-tone guitar. Imagine, if you will, a xenharmonic version of “Malagueña” with the chords moving up and down by neutral seconds (instead of major triads on E, F, and G).

In attempting an improvisation in 1978 using this scale as a subset of the 17-WT, I found it well suited for a jazz style, with the neutral intervals providing a good supply of “pre-bent” tones.

The second MOS scale consists of 11 tones generated by an interval of 11º17 (most closely approximating 7:11 or 9:14, depending on the specific tones in 17-WT):

0	3	4	5	8	9	10	11	14	15	16	17	º17
C	D	Eb	**Ed**	F≠	F#/Gd	**G**	**Ab**	A≠/Bb	Bd	**B/Cd**	C	
6		7				9			11		6	Harmonic
		9			11		**6**		7			functions
	11		**6**		7				9			of
	7				9			11		**6**		tones
	9			11		**6**		7				

This creates five different tempered 6:7:9:11 tetrads, of which two in the above example are built on the best tones in 17-WT, C and G. Unlike the situation in the previous scale, two of the root tones are a tempered 2:3 (or heptatonic “fifth”) apart, which could be expected to be a very useful feature. (I suggest that intervals in pentatonic, hexatonic, heptatonic, octatonic, nonatonic, decatonic, unidecatonic, and dodecatonic MOS scales be designated by the number of degrees spanned in those scales, so that a tempered or just 2:3 occurring in these respective scales could be identified as being 3ºP, 3ºX or 4ºX, 4ºH, 5ºO, 5ºN, 6ºD, 6ºU, and 7ºC, respectively, in those scales. Computer programmers will recognize “C” as the hexadecimal notation for “12”. We could then call a 2:3 in this MOS scale a “six-U” rather than a “fifth.”)

This scale may be generated in certain other divisions of the octave using the pattern LSSLSSSLSSS, where L and S are the number of system degrees in the large and small intervals between consecutive tones in the generated scale. The ratio L:S determines the number of tones per octave, N=3L+8S, and the number of degrees in the generating interval, G=2L+5S. The most useful division of the octave other than 17 is given by L=5 and S=2, i.e., 31-ET with the generating interval 20º31.

This second MOS scale poses a problem of complexity. Not only are its harmonic relationships quite complicated, but it may also be questioned whether 11 tones are too many to be successfully comprehended by most listeners as a cohesive scale. Even if this should turn out to be the case, at least it does demonstrate that the harmonic resources of the 17-tone system are capable of highly extensive development – not bad for a tonal system that I once thought to be of “questionable harmonic utility.”

No Turning Back

In conclusion, the 17-tone system is very different from the 12-tone system, having not much more in common with it than simple 3-limit intervals. However, it offers the xenharmonic composer a wealth of new harmonic material in a very reasonable number of tones per octave. It also possesses a melodic effectiveness that is unsurpassed by any other system, opening up a totally new perspective for melodic possibilities.

While the high dissonance of the diatonic thirds and sixths is a major problem of 17-ET, the employment of a well temperament of 17 tones can provide significantly increased consonance in a generous number of keys, not only for those intervals, but also for chords containing the more exotic ratios of 11 and 13. Its lack of ratios of 5, normally considered a disadvantage, makes it an ideal system for learning to use many of these new intervals by allowing the composer to focus on the new tonal materials, free of the temptation to revert back to the familiar major-minor system, thus instilling the discipline to move forward without undue dependence on the heritage of the past. Once experience with these new tonal materials has been gained, there is always the option of progressing to more complex systems such as 31-ET, 41-ET, or just (or near-just) intonation, in which the expertise gained from using the 12-tone and 17-tone systems individually may be applied in combination.

A commonly held belief is that a tonal system for the future should build on rather than discard previously accepted norms. Such a viewpoint often results in rejection, not only of temperaments with wide fifths such as the 17-tone system, but also of just intonation, with its requirement of contending with commas. I find it somewhat ironic that, during much of the 20th century, the musical establishment most readily applied the label of “serious composer” to those who had most completely broken away from the past, while judging those tonal systems that did not maintain a connection with that heritage as least worthy of serious consideration for the music of the future. Let us hope that in this brave new century we can muster a bit more courage and boldness, not dismissing out of hand a tonal system that would transport us to a very different world of tonality.

Technical Description of the 17-tone Well Temperament

The 17-tone well temperament is composed of a circle of 17 tempered fifths (of 10º17), occurring in two different sizes. The fifths in the far side of the circle, from tones C-semiflat to G-semisharp, are approximately 704.37699 cents (or ~2.422 cents wide), such that tones separated by four fifths in the series (less two octaves) will be in the exact ratio of 11:14.

The remaining fifths (in the near side of the circle), from tones A-flat to B (the end-points of the first series renamed and taken in reverse order) are then all made the same size, approximately 707.22045 (or ~5.265 cents wide), which results in tones separated by seven fourths in this part of the circle (less two octaves) being *almost exactly* in a ratio of 6:11. The error of these fifths is about the same as in 31-ET, but in the opposite direction, or slightly less than 1/5 of Archytas' comma.

With a pitch standard of C=264 Hz, no pitch in the 17-tone well temperament deviates from the corresponding pitch of the 17-tone equal temperament by more than 6.7 cents, making it possible to use instruments tuned to the two systems in combination without any major intonation problems.

17-WT Tone	17-WT Degrees	Harmonic Function in C	Cents from C	Frequency
C# / Dd	2		144.856	287.040
F# / Gd	9		640.479	382.185
B / Cd	16		1136.102	508.867
E	6		428.882	338.215
A	13		921.661	449.583
D	3	9	214.441	298.812
G	10	3	707.220	397.206
C	0	1	0.000	264.000
F	7		492.780	350.931
Bb / A≠	14	7	985.559	466.487
Eb / D≠	4		278.339	310.047
Ab / G≠	11		771.118	412.141
Db / C≠	1		66.741	274.376
Gb / F≠	8	11	562.364	365.324
A# / Bd	15		1057.987	486.417
D# / Ed	5		353.610	323.825
G# / Ad	12	13	849.233	431.163

Following is a special section for those who cannot contemplate life without ratios of five.

Seventeen Plus Five

In the event it is desired to add auxiliary tones to supply ratios of 5 in the best keys (as was done for ratios of 11 in my 19+3 temperament), then the following five tones may be added to produce a 17+5 temperament (\! = ½°17 down)[16]:

17+5 Tone	17-WT Degrees	Harmonic Function in C	Cents from C	Frequency
F#\!	8½		600.755	373.515
B\!	15½	15	1093.534	496.508
E\!	5½	5	386.314	330.000
A\!	12½		879.760	438.833
D\!	2½		173.961	291.906

Tones E\! through F#\! are tuned a just 5:4 above C through D, while tones D\! and A\! are tuned to make major thirds of equal error with B-flat (below) and F-semisharp (above) and with F (below) and C-semisharp (above), respectively. No major third has an error exceeding 2.089 cents.

These auxiliary tones not only provide major thirds above tones B-flat through D but also minor thirds above tones B-semiflat through D-semisharp. In addition, each of these auxiliary tones has a tone in the main set of 17 both a major third (F-semisharp through A-semisharp) and minor third (F through A) above it.

To accommodate these auxiliary tones on my generalized keyboard Scalatron (with the arrangement of keys hard-wired to supply up to 31 separate pitches at a time), I put the tones of 17-WT on 26 keys, D-doubleflat through F-doublesharp, which provides 9 duplicate pitches. The five auxiliary tones are then assigned to keys C-doublesharp through E-doublesharp, which in the hard-wired 31-ET-duplicate configuration also makes them available in the positions corresponding to E-tripleflat through G-doubleflat. The auxiliary tones are thus available in positions both closer and farther from the front edge of the keyboard than the main set of 17 tones, with the distance of their duplicate positions from the "naturals" being approximately equal.

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[16] The symbol \! is an ASCII simulation of the Didymus-comma-down symbol in the new multi-system Sagittal notation currently being developed by David C. Keenan and myself. This symbol is used to indicate an alteration of 1 degree in 34-ET (hence ½°17), which is the equivalent of the Didymus comma in that division of the octave.