# Pleasant Sequences by Addition Chords 

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#### Abstract

Summary Having noted Christian Huygens's tricesimoprimal division of the octave and Giuseppe Tartinis discovery of the "terzo suono", which led to new important addition chords, the author gives some examples of sequences constructed by stringing pairs of addition chords in alternative succession. When tested on a 31 -tone instrument these new sequences proved to have a peculiar charm.


Angenehme Sequenzen durch Additionsdreiklänge

## Zusammenfassung

Nach Hinweisen auf die Christian Huygenssche 31 -stufige Oktavteilung und auf die auf Giuseppe Tartinis Entdeckung des „terzo suone" basierenden Additionsdreiklänge verknüpft der Verfasser beispielsweise einige Additionsdreiklänge paarweise abwechselnd zu steigenden Sequenzen, die sich auf einem 31-Ton-Instrument als eigenartig bezaubernd bewährten.

Séquences agréables par cordes additionnelles

## Sommaire

Utilisant la division en 31 parties de l'octave par Christiann Huygens et les accords par addition qui découlent de la découverte du «terzo suono» par Giuseppe Tartini, l'auteur construit quelques exemples de séquences montantes au moyen de cordes ajoutées en séquences alternées. Jouées sur un instrument à 31 sons, ces nouvelles séquences se sont montrées d'un charme tout spécial.

In earlier times the role of sound in physics was mainly associated with music and the names of Helmholtz and Huygens come readily to mind in this connection. It is well known that Christian Huygens objected strongly against the equal temperament with its twelve semitones which had been so strongly advocated by Simon Stevin and he pointed out that a temperament should divide the octave in thirty-one diësises. The exact figures could be computed thanks to the logarithms which had been invented some time before. Huygens wanted the finer division in order to conserve the perfect major third in the common chords, and to retain the refined expressive qualities of major and minor semitones. In addition he remarked that the harmonic seventh could be rendered pretty well in that temperament, and this opened the way to new developments.

The discovery of Huygens was not new, but he was not aware that Nicolo Vicentino, in trying to solve tuning problems of organs, had already found out, a century before, that the 31 -division would do very well. Even in the 15 th century

[^0]Marchettus from Padua had already found that the ratio of major and minor semitones should be as 3 to 2 , so that in consequence the octave had to contain 31 elementary steps.

In the eighteenth century the violinist Giuseppe Tartini emphasised in his theory the phenomenon of substraction or difference tones. Producing two tones at a time with frequencies $n$ and $m$, our ear perceives what Tartini called "il terzo suono", a tone with frequency $l=n-m$.

This fact leads to the paramount importance of what we shall from now call "addition chords". They consist of three notes, the frequency of the highest being the sum of the frequencies of the other two.

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\(1+2=3 \quad 2+3=5 \quad 3+4=7 \quad 4+5=9 \quad \ldots\)
\(1+3=4 \quad 3+5=8 \quad 5+7=12 \quad 7+9=16 \quad \ldots\)
\(1+4=5 \quad 2+5=7 \quad \ldots\)
\(1+5=6 \quad 3+7=10 \quad 5+9=14 \quad \ldots\)
\(1+6=7 \quad 2+7=9 \quad \ldots\)
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It is obvious that combinations with a factor 7 cannot be played in duodecimal temperament, as can be done in Huygens's temperament of 31 .

I have curtailed the list at the point where factors 11 and 13 would come in. Anyhow, in fact, these too might well be played in tricesimoprimal, that is 31 -tone temperament, but it would take me too long to enter into the discussion of these chords with $11,13,17$, or higher prime numbers.

I want to call attention to some pleasant sequences, resulting from the connections of pairs of addition chords.

The intervals $3: 4$ and 2:3 have a difference of one whole tone. As the example shows we get a sequence of ascending whole tones by connecting addition chords $3: 4: 7$ and $2: 3: 5$ (Fig. 1).


Fig. 1. Sequence nr. 1.
The lower and the middle notes proceed in seconds of $8: 9$. In the upper line one hears minor and major semitones.
Likewise there is a difference of a whole tone between the intervals $2: 3$ and $3: 5$. This time it is a whole tone $9: 10$. Again we make a sequence with alternating addition chords 2:3:5 and 3:5:8 (Fig. 2).


Fig. 2. Sequence nr. 2.
Listening to the upper lines, again we hear an alternate order of succession of major and minor semitones, but in a way opposite to the case before. In much the same way we get a chromatic line with alternating major and minor semitones by connecting the addition chords $4: 5: 9$ and $5: 7: 12$. This time the whole tones have a measure of $25: 28$, between $8: 9$ and $9: 10$ (Fig. 3).


Fig. 3. Sequence nr. 3.
Still, a fourth sequence comes out with a whole tone of $9: 10$ with the addition chords $5: 7: 12$ and $9: 14: 23$ (Fig. 4).


Fig. 4. Sequence nr. 4.

The major semitones in the chromatic upper line should have the uncommon value 108:115, and the minor semitones $23: 24$.

The procedure also leads to sequences ascending by major semitones of $15: 16$. Take the addition chords $4: 5: 9$ and $3: 4: 7$. The result is shown in Fig. 5.


Fig. 5. Sequence nr. 5.
The upper line now is subchromatic with alternating steps of minor semitone 7:28 and a diësis $35: 36$.

Likewise the connection of the chords 5:7:12 and $2: 3: 5$ too offers a sequence ascending by major semitones, this time semitones 14:15 (Fig. 6) .


Fig. 6. Sequence nr. 6.
The steps in the subchromatic upper line are minor semitones $24: 25$ and diësises $35: 36$.

As a last example we shall connect the chords $3: 4: 7$ and $5: 7: 12$. The ascending steps will be minor semitones $20: 21$ (Fig. 7).


Fig. 7. Sequence nr. 7.
Now all intervals in the upper line come out to be diësises. A real extremely microtonal line indeed! The diësises, however, are not equal. One might say that major diësises $35: 36$ alternate with mean diësises 48 : 49 .

The step $d$-sharp/e-flat is a major diësis $35: 36$. The other step e-flat/e-semiflat is a mean diësis $48 / 49$. So far we stop the search for more sequences.

It is obvious that these sequences have nothing to do with ordinary gamuts. Each one is in itself uniform. Every part is equivalent to any other part. It may be interesting to professor Kosten, as perhaps to others too, to know that the sequences have been tested, with success, on an electronic 31keyed instrument, which has been constructed by Mr. H. Van der Horst, Neonvo Inc., Buddezand, at Wilp (Gldl), The Netherlands. The instrument has been baptized "archiphone".
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[^0]:    $\dagger$ Deceased on September 24 ${ }^{\text {th }}, 1972$.

