

Ervin Wilson's Hexany

by Kraig Grady

In 1967, Ervin Wilson invented a method for organizing tonal space, which he called Combination Product Sets.¹ As the name implies, a Combination Product Set is composed of the products obtained by multiplying each possible group of n elements selected from some larger set. The simplest Combination Product Set is called the *Hexany*. This structure is composed of the six possible combinations of two from a set of four elements. If, for example, we use the four harmonics 1-3-5-7, we obtain the set (1×3 , 1×5 , 1×7 , 3×5 , 3×7 , 5×7). [In a more conventional expression, $3/2$, $5/4$, $7/4$, $15/8$, $21/16$, $35/32$ —Ed.] These products can be multiplied by powers of two to raise the resulting tones into any desired octave.

Examining Wilson's lattices in Figures 1 and 2 (pages 9 and 10), we can begin to see some of the properties of this set of six tones. The most important is that we have eight different triads derived from the original set of four harmonics. Four of these triads are harmonic and four are subharmonic, with the subharmonic triads being the inversions (complements) of the harmonic triads. [The four types of triads have the forms 4:5:6,

4:5:7, 4:6:7, and 5:6:7, respectively—Ed.] Note that each of the subharmonic triads is composed of the three elements not used in the corresponding harmonic triad. This allows the structure to be viewed as four distinct mirroring pairs of triads. These pairs, which Wilson refers to as the facets of the Hexany, are illustrated in the lower half of Figure 2. It is evident that this structure implies no tonic, or, equally, that any tone can serve as the tonic.

Some have compared the Hexany with Schoenberg's 12-tone row, but whereas in the tone row all tones are equal in function, in the Hexany each tone has a unique relationship to the whole. The tone represented by 1×3 , for instance, will function as the 1 identity (4) in the 4:5:7 triad, and as the 3 identity (6) in the 5:6:7 triad. Also, 1×3 will function as the 5 identity in the subharmonic 4:5:6 triad and as the 7 identity in the subharmonic 4:6:7 triad. The three pairs of tones which have no factors in common (e.g., 1×3 and 5×7 , 1×5 and 3×7 , and 1×7 and 3×5) have complementary tonal functions. For example, 1×3 , as stated above, functions harmonically as a 1 or 3 identity and subharmonically as a 5 or 7 identity, whereas 5×7 functions harmonically as a 5 or 7 identity, and subharmonically as a 1 or 3 identity.

(Text concludes on page 11)



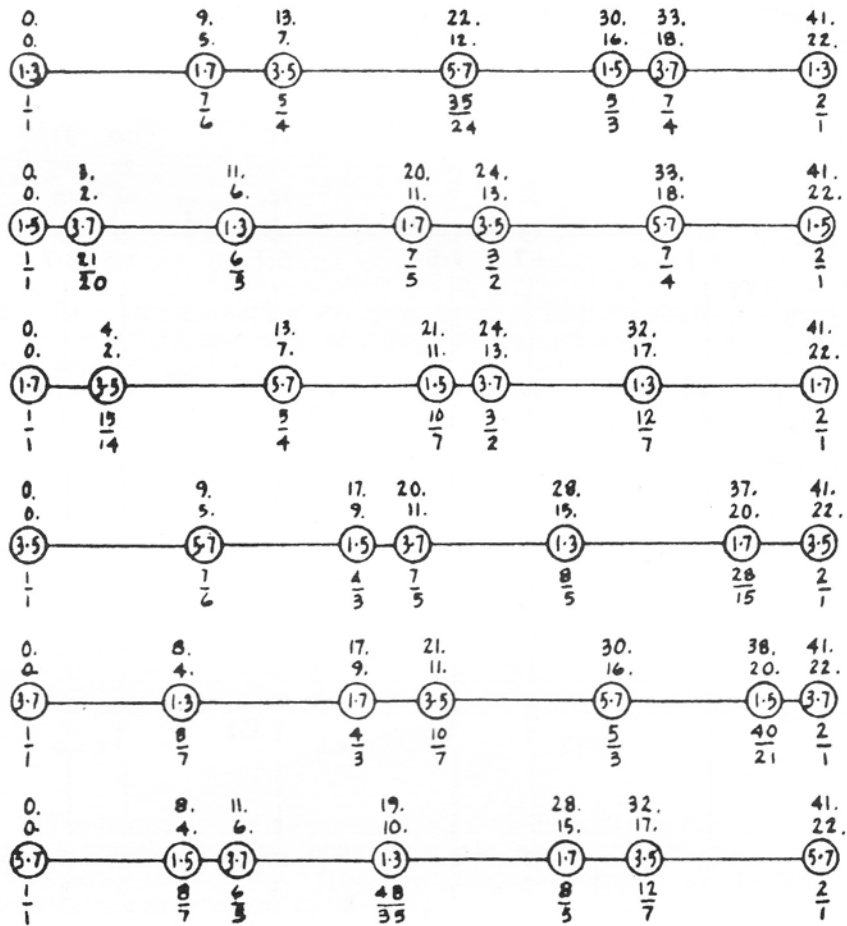
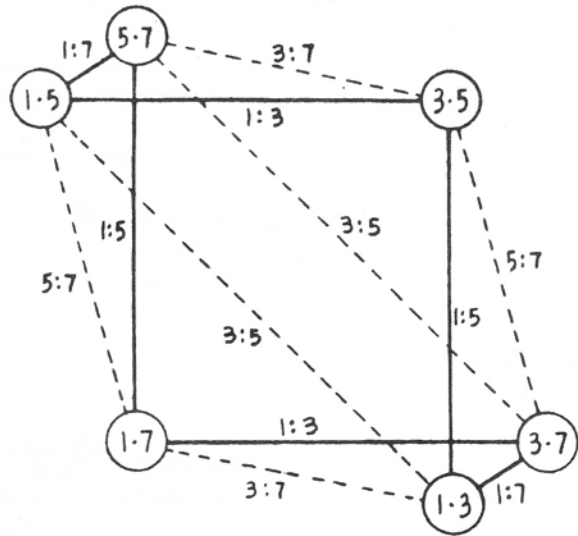
Example 1. The upper voice outlines a harmonic 4:5:7 triad, while the lower voice uses a subharmonic 4:5:7 triad. The interaction of the two voices generates the other six triads contained in the Hexany, as indicated by the labels above and below the staff (overstrikes indicate subharmonic chords). (The pitches used in this and subsequent examples are generated by multiplying each of the tones of the 1-3-5-7 Hexany by 11. The notation follows the conventions illustrated above.)

Figure 1

HEXANY

1-3-5-7

1 3
1 5
1 7
3 5
3 7
5 7

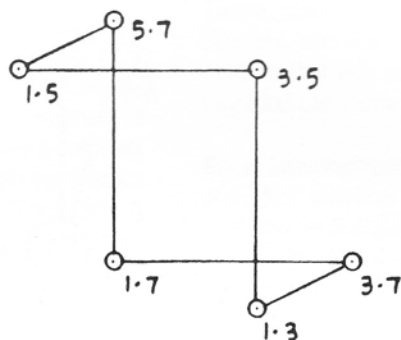
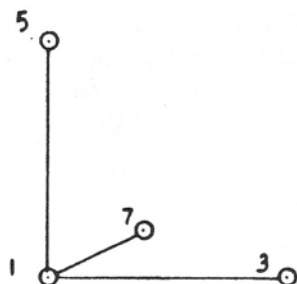


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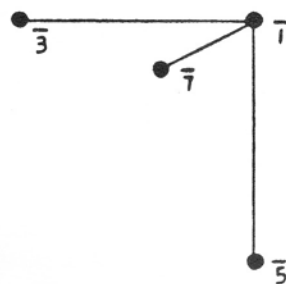
Figure 2

The 1-3-5-7 Hexany & its 8 Facets

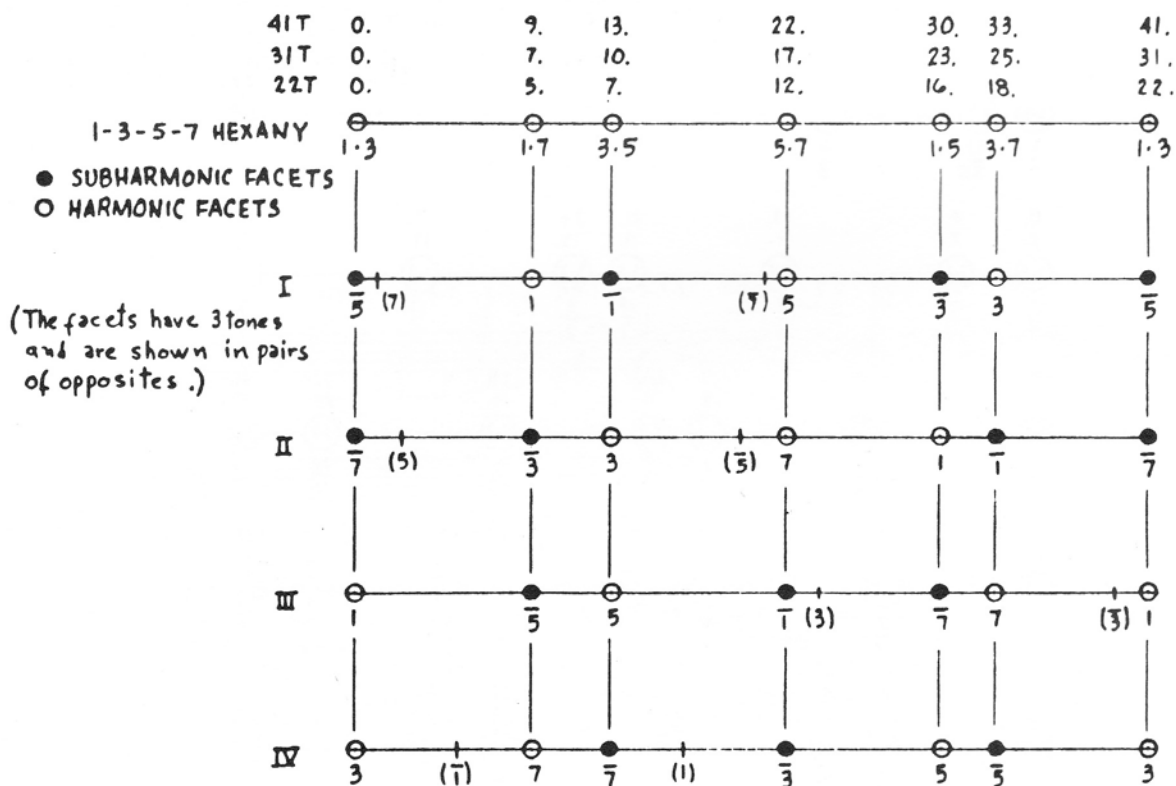
HARMONIC TETRAD 1 3 5 7



1-3-5-7 HEXANY



SUBHARMONIC TETRAD 1̄ 3̄ 5̄ 7̄



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While the presence in the Hexany of relatively high-numbered ratios such as those involving 35 are troublesome to some musicians, they are actually no more complex than the secondary dominants and other borrowed chords found in traditional harmony. In the key of C, nothing sounds more foreign than a B⁷ chord, until we hear it resolve to an E minor triad.

A Hexany can be constructed from any set of four elements. It is by no means limited to the first four prime harmonics. I have used Hexanies based on harmonics as high as the fifteenth, omitting thirteen, which Wilson has investigated. The great advantage of the Hexany over Partch's Tonality Diamond is that it provides a greater variety of chord types with fewer tones. Whereas the Hexany involves only six tones, the 1-3-5-7 diamond, the smallest that provides the same eight chords, has thirteen tones. (In the diamond, there are four instances of each of the eight triads.) If each of the partial triads in the Hexany were completed, the result would be a fourteen-point structure that Wilson calls the Stellate Hexany or Hexany Mandala. This structure allows greater modulatory

possibilities than does the tonality diamond. For instance, it is possible in this structure to move a tetrad between positions having no common tones. Many common-tone modulations are also possible. The Hexany Mandala permits modulations to more remote key centers while retaining many of the advantages of the tonality diamond.

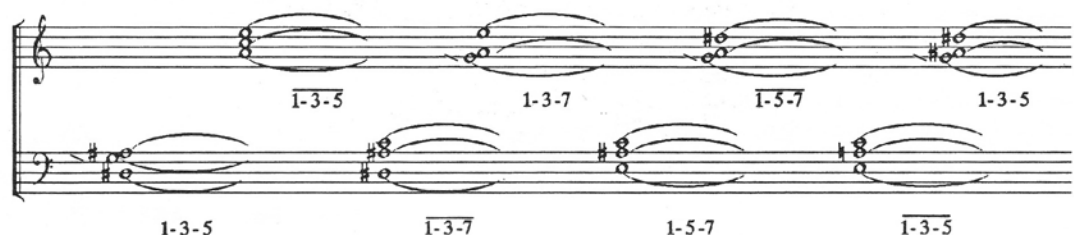
Notes:

¹ On January 27, 1967, Wilson issued a diagram of the 3-5-7-11 Hexany, as a result of attempting to discover a set of pitches that had the most condensed set of interrelationships. At first he thought that this structure was just a fluke, but he later realized that other harmonic relationships could be mapped over geometric forms. The Hexany is analogous to the octohedron being formed by a set of tetrads.

² A thing turning into its opposite, from the Greek: *enantios*, opposite + *dromos*, running 1/1



Example 2. Two tones alternate in the upper voice, against a pattern composed of the four remaining tones in the lower voice. All eight of the possible triads in the Hexany are generated by the resulting counterpoint.



Example 3. The harmonic and subharmonic 4:5:6 triads each progress via two intermediate steps to its complement. Two tones are held in common by each successive pair of chords. A musical enantiadromia.² [These two progressions could also be combined end-to-end to produce a seven-chord cycle—Ed.]