MULTIPLE DIVISION OF THE OCTAVE AND THE TONAL RESOURCES OF 19-TONE TEMPERAMENT

BY

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PREFACE

The proper role of the music theorist and the proper relationship of musical theory to practice have been subjects of frequent controversy. Much of the preoccupation of music theory is with the music of the past. Speculation on a possible future course for music involves the risk of serious error. The breaking of ground for new musical patterns of construction seems a particularly speculative preoccupation, and yet a number of theorists have accepted just this as their primary task.

It is the basic presupposition behind this study that inquiry into possible new directions for musical development represents theory in its most creative and constructive role. It would seem reasonable to suppose that in a field as vast and relatively unexplored as the multiple division of the octave there must be misconceptions to be swept away and complications to be simplified so that the creative musician might eventually be able to make a choice based neither on ignorance nor on fear of the unknown.

It was the initial purpose of this study to make a thorough investigation of one of the possible kinds of multiple division, 19-tone equal temperament, only. It became apparent on investigation, however, that the entire field of multiple division had received insufficient comprehensive examination or evaluation since Bosanquet in 1875,

and that as specific a study as was contemplated would be without any link to the body of general knowledge. There is almost nothing in recent literature more specific than the single-chapter references of J. Murray Barbour[1] yet more general than the passionate advocacy of individual systems by such writers as Würschmidt, Wyschnegradsky, Fokker, and Partch.[2]

My purpose is, therefore, to present a general picture of the field of multiple division of the octave as it exists today. In partial retention of the original intent of the study, 19-tone temperament will be emphasized and studied in more detail than the other systems.

WHAT IS 19-TONE TEMPERAMENT?

19-tone temperament, as its name indicates, is the division of the octave into nineteen equal parts instead of the twelve equal parts into which it is divided in common

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[1] J. Murray Barbour, Tuning and Temperament, the most widely recognized work in this country on tuning systems in general.

[2] The works of these writers are listed in the bibliography and are studied in the various chapters of this text. It should be noted that several books have dealt at greater than one-chapter length with multiple division. Augusto Novarro's Sistema Natural de la Música contains many interesting insights into a number of recently explored systems of multiple division, while Partch and Fokker have looked with considerable sympathy into proposed systems of multiple division other than their own. Their works are useful to the student of multiple division as primers, but they explore at best only a few of the problems and possibilities inherent in the field as a whole.

practice. Since the octave is an abstract quality produced by two sounds whose vibration frequencies are to one another as the relationship 2 to 1, it follows that the octave can be divided into as many parts as one wishes, just as it is possible to divide the abstract number 2 into as many fractional parts as one wishes.[3]

The vibration ratio of the smallest interval in 19-tone temperament is approximately equal to the superparticular ratio 28:27, and exactly equal to the repeating decimal fraction $63.\overline{157894736842105 2}$ cents.[4] The interval in 12-tone temperament which has its closest equivalent in 19-tone temperament is the perfect fifth (together with its inversion the perfect fourth), which is approximately 5¾ cents larger than its 19-tone equivalent (which will also be called a "perfect fifth"). To find the discrepancy between corresponding intervals of the two temperaments it is possible to use the circle of fifths, adding 5¾ cents for each fifth used.

There are many ways the intervals might be named and

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[3]These subdivisions of the number 2 may be equal arithmetically, geometrically, or exponentially, or they may be not equal by any method. In 19-tone temperament the subdivisions are geometrically equal, since the systems represents the 19-fold geometric division of the ratio 2:1. The smallest unit is $\sqrt[19]{2}$.

[4]This figure is given by Charles S. Kent in An Introduction to Tuning and Temperament. He gives the value in cents for the smallest interval of all of the commonly proposed temperaments, in every case obtaining a repeated decimal fraction.

the pitches notated. As it would seem particularly to be desired that this study be easily understood by musicians whose entire experience has been with standard diatonic nomenclature, this same nomenclature will be applied to 19-tone temperament throughout this study. Traditional enharmonic equivalences must, in this process, be cancelled out while other enharmonic equivalences are created. C♯ and Db are, in 19-tone temperament, distinct pitches, while E♯ and Fb are one and the same. In Example 1, below, the names of pitches and intervals in the 19-tone system are compared to those of 12-tone temperament.

PRELIMINARY ABBREVIATIONS AND DEFINITIONS

It has proved necessary to use a few uncommon abbreviations or new words for concepts which must be referred to repeatedly and the full description of which in customary language would be most cumbersome.

The smallest interval in any equal-tempered system will be referred to as a <u>unit</u> of that particular system, since the term "semitone" is ambiguous where more than 12 tones are used.

Since it is often necessary to refer to intervals both as units of a temperament and as just ratios, the following procedure has been adopted. Whenever a just ratio is intended, the two numbers are separated by a colon:

EXAMPLE 1

19-TONE				12-TONE	
Name of tone	Interval from C	Cents	Cents	Interval from C	Name of tone
C	perfect prime	00	00	perfect prime	C
C#	aug.prime-dim. 2nd	63			
Db	minor second	126	100	minor second	Db (C#)
D	major second	189	200	major second	D
D#	aug.2nd-dim. 3rd	253			
Eb	minor third	316	300	minor third	Eb (D#)
E	major third	379	400	major third	E
E#-Fb	aug.3rd-dim. 4th	442			
F	perfect fourth	505	500	perfect fourth	F
F#	augmented fourth	568			
Gb	diminished fifth	632	600	aug. 4th; dim. 5th	F#-Gb
G	perfect fifth	695	700	perfect fifth	G
G#	aug.5th-dim. 6th	758			
Ab	minor sixth	821	800	minor sixth	Ab (G#)
A	major sixth	884	900	major sixth	A
A#	aug.6th-dim. 7th	947			
Bb	minor seventh	1011	1000	minor seventh	Bb (A#)
B	major seventh	1074	1100	major seventh	B
B#-Cb	aug.7th-dim. 8th	1137			
c	perfect octave	1200	1200	perfect octave	c

3:2. Whenever a logarithmic fraction is intended, the two numbers are separated by a virgule: 7/12. In the absence of any designation of which interval such a configuration of numbers is the logarithmic fraction, it may be assumed that it is of an octave.

Terms pertaining to specific chapters will be introduced at the beginnings of those chapters, and I have tried to include all unfamiliar concepts and terms in a glossary which appears on page 443.

PART ONE

REASONS FOR THE ADVOCACY OF MULTIPLE DIVISION

The idea of using more than twelve tones in the octave, although many centuries old in the West, and a matter of everyday practice in certain sections of the Orient,[1] nevertheless appears radical to most Western musicians. In spite of the tremendous changes in musical taste and style which have taken place in recent years, very few musicians appear to see any general departure from the limit of twelve tones per octave as likely in the foreseeable future.[2] As believers in a cause recognized, therefore, as radical, the advocates of multiple division, however disparate their views, share a feeling of discontent with the prevailing system, 12-tone equal temperament. The objections these writers have raised to 12-tone temperament can be grouped into one or more of the following three categories:

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[1] Arabian music is generally considered to be based on a scale of 17-tones to the octave, while a 22-tone system is attributed to the Hindus. Neither system involves equal temperament.

[2] "The practice for the past five hundred years has favored almost exclusively systems with only twelve pitches in the octave. There seems no immediate prospect of that practice being discarded in favor of any system of multiple division." Barbour, op. cit., p. 132. Certainly this view is widely held.

1. Acoustic: wherein it is alleged that the intervals created in duodecimal equal division are deficient, usually because of their lack of agreement with small rational-number ratios. The thirds and sixths of our present tuning system are particularly subject to such allegations, but some writers also attack the sevenths, and a few even question the fifths.

2. Historical: wherein it is alleged that earlier musical systems based on other tunings possessed features which would be desirable today, but which have been lost through the adoption of 12-tone equal temperament. In general, the development of musical resources required earlier systems to expand (i.e., encompass a greater number of tones to the octave) or perish. They perished. But many writers would bring them back, expanded through multiple division.

3. Evolutional: wherein it is alleged that while there is not necessarily an inherent defect or inadequacy in 12-tone equal temperament, changes in musical style are working to render it inappropriate for further musical growth. Such a view leads an evolutionally oriented advocate of multiple division to examine historical precedents and acoustic bases for a new system. The emphasis remains on evolution, however, and the third group of writers takes a much less hostile view toward 12-tone temperament than do the other two groups.

A chapter will be devoted to each of the three categories listed above.

CHAPTER 1

THE ACOUSTIC BASIS FOR MULTIPLE DIVISION

Joseph Sauveur, the great deaf-mute mathematician and music theorist active around 1700, proposed that the best tempered system is that in which the octave is not divided in too many parts and in which the pure diatonic intervals are altered as little as possible.[1] In this short statement Sauveur posed several of the questions which have bothered musical theorists throughout the ages. How many tones are "too many"? How small an alteration may be considered "as little as possible"? It appears evident on face value that the more tones one uses in a tempered division of the octave, the more likely one is to find closer and closer equivalents to the pure diatonic intervals. Accepting Sauveur's basic premise for the moment, all temperaments can be seen as representing a balance between economy in the number of tones on the one hand and exactness of correspondence to "pure diatonic" intervals on the other.

Given Sauveur's "pure diatonic" scale as a basis, however, it is self-evident that some temperaments are far

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[1] "Le système tempéré le plus parfait est celuy qui ayant son octave divisée en peu de parties, a des différences les moindres et les moins inégales." Sauveur, Table Generale des Sistèmes Tempérez de Musique, Histoire de l'Académie Royale des Sciences, 1711, avec les Mémoires de Mathématique et de Physique, page 315. The expression "pure diatonic" is taken from the article on Sauveur in Scherchen, The Nature of Music, page 42.

superior to others. 12-tone equal temperament is far superior to 11- or 13-tone temperament in the accurate representation of this scale. If one wishes to use as one's basis a system other than that which Sauveur called the "pure diatonic," it is possible that 11- or 13-tone temperament[2] might serve his purposes better than 12-. Once one abandons a single premise such as Sauveur's the possibilities become infinite. It is therefore necessary to begin by examining the building blocks of musical systems, the intervals, around which theories of consonance and dissonance, of scales, of tonalities, and, finally, of tuning systems, are devised.

THEORIES OF CONSONANCE AND DISSONANCE

Although the terms "consonance" and "dissonance" tend to suggest specific understood phenomena to musicians, and although they are essential elements in almost all theories of music, they have been interpreted in varying ways by different authorities. There are those who define consonance and dissonance in primarily emotional terms, the one as a source of pleasurable sensation, the other as a source of a disquieting sensation. Others prefer to define consonance and dissonance in the context of the dynamics of

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[2] Ernst Krenek has used 13-tone equal temperament for a special musical effect in an electronic musical interlude in his Spiritus Intelligentiae Sanctus. Musical Quarterly, April, 1960, page 219.

a musical phrase, consonance representing a state of "rest" at the end of a phrase, dissonance a state of urgency and a search for "rest." Still others have defined consonance by the relative position of two tones within a scale without regard to specific harmonic considerations.[3] However, the most frequently used, and probably the most universally accepted, definitions of consonance and dissonance rest on one of two assumptions. The first of these is that consonance is produced by agreement between at least some of the component parts of two or more simultaneous musical sounds; this agreement is the result of their fundamentals creating a small-number ratio, such as 2:1, 3:2, etc.[4] The second is that, while indeed certain combinations of sound are consonant and others dissonant regardless of their musical context, it is the conditioning of the ear rather than the inherent qualities of the mathematical ratio that makes them

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[3]Joseph Yasser maintains that the interval between the first and third members of any functionally used scale is a consonance, regardless of the number of tones in the scale. A Theory of Evolving Tonality, page 197.

[4]It need not be dwelt upon that various acousticians explain the agreeable or allegedly agreeable effects of small-number ratios in various ways. Whether it is because of the absence of beats (Helmholtz), the melting together (Verschmelzung) of the individual sounds (Stumpff), or the formation of agreeable and comprehensible rhythmic patterns by the combined sound waves (Opelt), a simple ratio between the concomitant parts is found by all these writers to be the phenomenon which causes a combination of sounds to be consonant. This alone is what matters in considering the implications of the doctrine of natural consonance on systems of tuning.

so.[5] The writers who adhere to the first view might be called members of the "Natural Consonance" school, while those who adopt the second might be considered as believing in "Conditioned Consonance."

Concerning the supporters of conditioned consonance who advocate multiple division, they can hardly base any preference for change on acoustic grounds, since the consonances they recognize are the "consonances" in current use, the 700-cent fifth and the 300- and 400-cent thirds. Believers in conditioned consonance can, of course, support multiple division on the grounds that the ear will eventually become conditioned to the "consonances" of a new system. However, many opponents of proposed systems of multiple division have used, in effect, the argument that the ear has been conditioned to prefer "consonances" as they are.[6] It has been realistically observed that if indeed preference is a matter of conditioning, progress from one tuning system to another must overcome almost insuperable obstacles. The acoustical basis for multiple division

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[5]Harry Partch attributes to Aristoxenos of Tarent the first direct statement of the doctrine that the ear, and not the small number ratios, is the determinant of consonance. Partch argues, however, that "the results by ear and by small-number ratios are of course identical." Genesis of a Music, page 239. Helmholtz is probably the most renowned of the many theorists and acousticians who have advocated natural consonance.

[6]It has been the thirds in particular where believers in conditioned consonance have argued against systems which closely approximate the just ratio 5:4. See below, page 26.

is therefore cited almost exclusively by advocates of natural consonance.

Further complicating the picture is the fact that many writers accept limited portions of a theory of natural consonance while rejecting others. As Partch rightly points out, even those who would discount the idea of the natural validity of the small-number ratios use the most basic small number ratio, 2:1, as the basis for their musical systems.[7] There have been many theorists who have shared the basic view of Pythagoras that octaves and fifths should be represented by small-number ratios but that thirds should not be. The particular qualities of 12-tone temperament have tended to reinforce such a view. There have been many theorists who have maintained that consonance is only possible where the numbers in the ratio were 6 or smaller.[8] Others have accepted as consonant such ratios as 10:9 while refusing to consider the prime number 7 as a factor in musical consonance at all.

It is evident that for many writers who accept small-number ratios in principle, the question "when does a small number become a large number?" must be answered. The

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[7] Op. cit., page 239.

[8] Zarlino was, of course, a most famous exponent of the theory of the senario as the basis for consonance. From the Renaissance to the 19th century, the view of the senario as the basis for consonance seems to have prevailed overwhelmingly over both those who would reduce to 4 and those who would expand to 7 or more the number of natural partials used in generating a musical system.

subsequent parts of this chapter, working upwards from the octave, examine the answers which have been given to these questions.

One other aspect of the general relationship of consonance and dissonance needs to be considered. There has been increasingly strong controversy in recent decades over whether consonance and dissonance should be considered as opposed polar forces at all. The traditional view, enunciated emphatically by Zarlino and maintained for several centuries, accepts the polarity of consonance and dissonance as fundamental to music. Many modern theorists have opposed this view. Arnold Schönberg asserts that so-called dissonant intervals are nothing more than the less simple consonances.[9] The practice of 20th century composers seems to indicate support for Schönberg's view.

Among disciples of multiple division, opinions on the polarity of consonance and dissonance are mixed. One of the major proponents of 19-tone temperament, writing under the pseudonym Ariel, espouses multiple division on the grounds that it is necessary for the restoration of polarity between consonance and dissonance which has become corroded.[10] On the other hand, Ivan Wyschnegradsky, a

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[9] "Dissonanzen sind die fernerliegenden Konsonanzen." Cited in Yasser, A Letter from Arnold Schönberg, AMS Journal, Spring, 1953, p. 53 ff.

[10] Ariel, Das Relativitätsprinzip der Musikalischen Harmonie, p. 148.

leading advocate of 24-tone temperament, considers the abolition of consonance and dissonance as polar opposites to be a basic starting point for his system.[11] It is understandable that a system of multiple division seeking to exploit a "twilight area" between consonance and dissonance or to abolish consonance and dissonance altogether, will differ from a system attempting to reinforce the polarity of the two elements.

Intervals:
THE OCTAVE

"It is universally agreed that the octaves should be exact," wrote the Englishman R.H.M. Bosanquet in 1876 in his Elementary Treatise on Musical Intervals and Temperament.[12] There was much justification for his remark when it was made, and it is quite imaginable that the same statement might be made today, although the word "universally" would have to be changed. The octave is alone among the small-number ratios in retaining its special characteristics in the musical hierarchy in the aesthetics of dodecaphony, and it remains the basis for almost every proposed musical system. The uniqueness of the octave has been described many times, but why it should have such a unique effect

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[11] Wyschnegradsky, Quartertonal Music, its Possibilities and Organic Sources, Pro Musica Quarterly, Oct. 1927, page 30.

[12] p. 2.

appears as unanswerable as why green should look green.[13]

In this century, a few musicians have begun to build systems which do not recognize this hitherto conceded hegemony of the octave. A typical and noteworthy case is Stockhausen's Studie II, where the interval 5:1 is chosen in preference to 2:1 as the basis for an equal temperament.[14] The interval 5:1 is divided into 25 equal parts. Other Stockhausen works have involved other systems in which the octave is not to be found. In some works of the Cologne school the continuum of pitch plays a role secondary to the continuum of saturation (at one end of which is the sine-wave, at the other end, noise) in which no phenomenon similar to the octave is discernable. Such experiments having been started, it is conceivable that the octave may begin to play a diminishing role in the formulation of musical systems.

Ivan Wyschnegradsky, who used musical systems based on the octave, has proposed scales in which pitches do not

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[13] In comparing the audible sound continuum with the visible spectrum one is struck by the fact that while the latter proceeds directly from red to indigo without at any point in the interim suggesting red again, the pitch qualities appear to repeat themselves every octave. C returns, likewise E, G♯, etc. Red never returns. There are in all about 10 recurrences of each pitch class within the limits of the audible range of frequencies.

It is possible that the ratio 2:1 would have the same effect visually that it has aurally. We do not know for sure because the human eye perceives only a color range extending from about 4000 to about 7700 angstrom units. But the tendency of red and blue to unite in violet suggests a possible visual quality to the 2:1 ratio not unlike the effect of the octave.

[14] Die Reihe, Bd. 1, footnote to page 50.

repeat themselves at the octave but rather at the second or third octave.[15] Nicolas Slonimsky includes in his Thesaurus of Scales many which involve repetition at the second or third octave only, or even at a still wider interval. Joseph Yasser at one time suggested an incursion on the hegemony of the octave by treating the perfect fourth or the perfect fifth as a complete identity capable of enclosing a musical system.[16] Such a fourth and fifth he proposed to call a "quartave" and a "quintave" respectively. The tetrachord relationships of the Greek Lesser Perfect System might be considered based on such a quartave principle, and it is possible to view parallel organa and some 20th century fourth doublings as a partial manifestation of a similar principle.

In addition to proposals to eliminate the octave or to replace it as the primary "identity" in music, there have been a few suggestions of recent origin that the octave should not be tuned precisely.

A number of studies have shown that at the highest and lowest ranges of hearing the interval 2:1 must be stretched a substantial amount to produce the psychological effect associated with the octave. The suggestion has recently been made that 2:1 be slightly enlarged throughout

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[15] Manuel d'Harmonie À Quarts de Ton, page 15.

[16] The Highways and Byways of Tonal Evolution, AMS Bulletin, Sept. 1948.

the pitch continuum to produce its most agreeable effect.[17]

It has also been proposed that the octave be made smaller. Adriaan Fokker, one of the great contemporary figures in the field of multiple division, has asserted that the octave should not be regarded as the inviolate basis for a musical system but rather as one of several necessary consonances each of which is equally important. The unit in his temperament represents a mean between rational fractions of the octave, the perfect 5th, perfect 4th, major 3rd, minor 6th, natural 7th, and "super" 2nd. Every one of these intervals is altered so that a minimal total alteration shall be necessary.[18] Example 2 shows the factors in Fokker's determination of the proper size for a unit in 31-tone temperament.

Example 2: Factors in Determining the Unit for 31-Tone System

39.0 cents is 1/18 of 3:2
38.2 cents is 1/13 of 4:3
38.6 cents is 1/10 of 5:4
38.8 cents is 1/21 of 8:5
38.8 cents is 1/25 of 7:4
38.5 cents is 1/6 of 8:7
38.7 cents is 1/31 of 2:1
38.65 cents is the weighted mean.

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[17] Kolinsky has proposed that every octave be enlarged by nearly two cents. AMS Journal, 1951. According to Wilhelm Dupont, a survey by Stumpff and Meyer reported in the Zeitschrift fur Psychologie and Physiologie der Sinnesorgane, Bd. XVIII, heft 5 & 6, discovered that 79% of those sampled preferred octaves larger than the pure 2:1. Geschichte der Musikalischen Temperatur.

[18] Fokker, Le Gamme, La Musique, et le Tempérament égal, 1952, page 247.

In calculating his mean value for a unit of the system, Fokker gives the octave approximately double the weight of each of the other units, and he gives the smaller of the remaining intervals greater weight than the larger ones on the grounds that a slighter change produces more pronounced effects where the interval is smaller. Fokker's octave is 1198.2 cents, just about as shortened as Kolinsky's is lengthened. "Perhaps it might be better to choose that value for the basic interval," suggests Fokker,[19] in a statement which certainly falls short of militancy.

In his charts he also represents the octave as one dimension of a four-dimensional scheme, with unity representing the sum of the squares of the member dimensions.[20]

Despite occasional suggestions to disregard the octave, to enlarge the octave, or to shorten the octave, its role in music remains so assured that it appears to be in little danger of going into eclipse. Among the many proposed changes in musical nomenclature brought into existence by proponents of multiple division and by others, there does not appear to be a single recommendation to reletter the notes so as to deny the existence of an octave identity.

Even the vigorous serial movement, which treats all

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[19] "Peut-être vaudrait-il mieux choisir cette valeur-là pour l'intervalle elementaire du tempérament." Ibid.

[20] Fokker, Les Mathématiques et la Musique, p. 28.

intervals within the octave as equal, recognizes in the octave its traditional value as replica or identity. The recent debate within the ranks of dodecaphonists over whether to permit octave doubling within the "row" suggests that to some, at least, the existence of the octave as an interval outside of the scope of serial treatment, is an enigma, its special treatment creating possibly some discomfort. But the total freedom of melodic octave-displacement within dodecaphony demonstrates further that the octave is not subject to the laws of the other intervals within the system.

In light of the above it appears reasonable to anticipate the continuance of the octave and to stipulate its use in the systems of multiple division which will be explored below. It is also a practical necessity to assume that the addition or subtraction of an octave does not change the essential character of an interval. Otherwise it would be necessary to consider the octave-and-a-fifth (the twelfth) separately from the fifth, etc.

THE 3rd PARTIAL: THE PERFECT FIFTH AND FOURTH

"The strongest consonance in music next to the 2:1," is how Harry Partch characterizes the 3:2 or perfect fifth.[21] This statement by one of the most eminent opponents of present-day tuning procedures accords with the

[21] Op. cit., pp. 304-5.

principles set forth by one of the leading protagonists of a tuning approaching 12-tone equal temperament when it was new and controversial. Said Werckmeister, "The nearer a ratio is to unity, the more perfect is its consonance."[22] The perfect fifth is, after the octave, the nearest possible interval to unity. All of the known tuning systems of the Western world have taken the perfect fifth into consideration and have rendered it with a degree of accuracy which, if questioned by some authorities, has been higher than the accuracy consistently attained by any other interval except for the octave.[23]

As to whether the interval 3:2 is common to all of the world's musical systems, as has occasionally been claimed, Fritz Kuttner asserts that the "fifth" in Chinese music is 20 to 30 cents flat.[24] It is apparently nearly as flat in Siamese music if, as is generally agreed, Siamese

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[22] "Je näher eine Zahl der Unität, je vollkommener dieselbe ist," cited by Dupont, op. cit., page 76. Many historians including Dupont ascribe to Werckmeister an important role in the establishment of 12-tone equal temperament. Barbour doubts, however, that Werckmeister was, in fact, an advocate of equal temperament, claiming instead that he advocated only a "well-tempered" system in which modulation to any key was possible while the keys retained their individual characteristics.

[23] In the meantone tuning system the third is tuned perfectly at the expense of the fifths. This tuning system was, however, of short duration and its adoption was never universal. Its position of favor was probably less influenced by a preference for the third over the fifth as such than by the simple fact that only a quarter as much discrepancy in the fifths was necessary for a perfect third as the other way around.

[24] Communicated by discussion, December 1960.

music approximates 7-tone temperament with its 685.7-cent fifth. Nevertheless, the interval 3:2 has been an essential part of Chinese musical theory for many centuries, and the interval is a strongly felt part of all highly developed systems of the West, Near East, and India. Alain Danielou, a comparative musicologist, has called the 3rd partial the "cyclic" element in music, by which he presumably means that the domain of musical systems is generally defined by cycles of fifths.[25]

To Bosanquet, the fifth is the basic interval of any musical system, and "regular systems are such that all their notes can be arranged in a continuous series of equal fifths."[26]

Recognition of the primacy of the fifths among intervals within the octave extends to those who would temper it considerably (and even, paradoxically, to one who would abandon it). Robert Smith, an 18th century English theorist, in giving tuning instructions, indicates that choosing a temperament for the fifth is a most essential step.[27] Joseph Yasser, a contemporary advocate of 19-tone temperament, considers the cycle of perfect fifths to be a "common

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[25] Danielou, Introduction to the Study of Musical Scales, 1943, pages 230-231.

[26] Op. cit., p. 60.

[27] Smith, Robert, Harmonics or the Philosophy of Musical Sounds, 1759. He would alter the fifth by more than 6 cents.

denominator" for all musical systems. He first produces his 19-tone scale as a series of fifths with a comma of 138 cents.[28] This he does, despite a remarkable decision not to accept 3:2 as the basis for a consonance in the 19-tone system at all.[29]

With the exception of this suggestion from Yasser, surprising in light of his use of the perfect fifth in deriving his system, and with similar exception of a few unique temperaments, generally proposed only for single pieces,[30] the fifth remains an integral part of intonational systems. From the rise of medieval modes through the end of the 19th century this interval has unquestionably played what is correctly called the "dominant" role, harmonically and structurally, in our music. The dominating interval in the "rule of the octave" which so concerned Rameau and his contemporaries, the fifth defines both the authentic and plagal cadences, and its primacy among intervals other than the octave appears to be fundamental to functional harmony.

Because of the mathematical-acoustic basis for the use of the perfect fifth as dominating interval, apologists

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[28] Yasser, A Theory of Evolving Tonality, 1933, page 116.

[29] Ibid., pages 176 and 285. For further discussion see Chapter II, below.

[30] Such as those works of Krenek and Stockhausen which are cited above, pages four and ten respectively.

for the various schools of 20th century composition which have chosen not to use the fifth above other intervals as a source for harmony or tonal progression have been hard put to justify this aspect of their case. Allen Forte, in the introduction to his book, Contemporary Tone Structures, states, "Much of contemporary music is rendered invalid from that point of view (that the pre-eminence of the fifth is naturally ordained)," and he asserts that the overtone series as a natural basis for the dominant-tonic relationship is open to question on several points.[31]

That nature may, however, play a role in the long-manifested human preference for the fifth is evidenced by the simplicity of its ratio and attested by the continuing body of writers who select that interval as an important basis for their musical systems. It would appear that any system of multiple division based on the principle of the consonance of the small-number ratios would have either to include the perfect fifth or to offer a satisfactory explanation for its absence.

The Fifth: Pure or Altered? It remains to be determined whether the perfect fifth shall be pure or altered, and if altered, to what extent. Probaoly the tuning system with the longest history in the West is the so-called Pythagorean system based on absolutely pure fifths. This

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[31]Forte, Contemporary Tone Structures, pp. 10-11.

system, in general use during the Middle Ages, probably employed extensively in antiquity, and used by the majority of string players today,[32] can never be closed perfectly since no number of exact fifths can ever equal an exact number of octaves.[33] Here lies one of the primary defects of Pythagorean tuning, the imperfect relationship between fifth and octave. In the other direction lies its other imperfection, the imperfect relationship between the fifth and other possibly desirable small-number ratios. Just as no power of 3 can ever equal a power of 2, neither of them can ever equal a power of 5, or of 7 etc.

When operative in a 12-tone system, Pythagorean intonation leaves a discrepancy of 23.5 cents, representing the ratio 531441:524288, which is known as the Pythagorean comma. As mentioned above, when applied to a 19-tone system, Pythagorean tuning leaves a discrepancy of about 138 cents, a figure equal to more than 2 units of the tempered 19-tone system. Much less is the discrepancy in a 17-tone system and many theorists believe the Arabic 17-tone system to be Pythagorean.[34] Yasser has offered an

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[32] For a strong statement of its importance by a disciple of its role see Barbour, The Persistence of the Pythagorean System.

[33] This is obvious in the fact that all powers of two must be even numbers, while all powers of three must be odd numbers.

[34] According to Partch, early Arabian musicians such as Zalzal used non-Pythagorean systems but 14th century theorists simplified to Pythagorean tuning which has remained

interesting postulate which derives support from the apparent preference of much of antiquity for Pythagorean tuning. He suggests that it is in the nature of all pentatonic music to eschew the use of the just third, making Pythagorean tuning the "just intonation" for the 5-tone scale.[35]

Among the theorists who have advocated temperaments for the fifth, some have done so reluctantly and sparingly, but others have claimed to prefer the sound of the tempered fifth to that of the pure fifth. Among the temperaments adopted or proposed, our present system involves relatively slight altering of the fifth. Nevertheless, there have been those who have based their search for a better equal temperament on the postulate that the fifth must be even better than the one found in 12-tone equal temperament.[36] A far greater number of advocates of multiple division have been

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since. Helmholtz however prefers to regard the Arabic system as comprised of 9 fifths with 8 tones added a third above all but one of the initial 9. Owing to the close agreement between the major third and the interval comprised of 8 perfect fifths, an agreement which Helmholtz records in the theorem bearing his name, the difference between Helmholtz' explanation and the more usual one involves only a minute gradation in pitch.

[35] Op. cit., page 131.

[36] Janko, a pianist and theorist who is best known for his attempt to popularize a reform in keyboard design, considers the question of multiple division, and makes the initial stipulation that the fifths must be better than those of 12-tone temperament, leading him to a choice between 29-, 41- and 53-tone temperaments, of which he chooses the second. Über mehr als zwölfstufige Temperatur, Beiträge für Akustik und Musikwissenschaft, heft 3, p. 6.

willing to see the fifth altered more than in 12-tone temperament in order to improve other intervals in the temperament. By a shortening of the fifth by 5½ cents, a temperament containing the just major third is achieved, while a fifth which is 7 cents too small produces a system with a just minor third. The specific systems which are created by the various alterations in the size of the fifth will be discussed in later chapters. Relevant to the present discussion is the evaluation of the alterations in the fifth itself, as offered by the various writers.

On the one hand there is Partch who claims that even 2 cents is an excessive alteration.[37] On the other hand there are numerous advocates of multiple division who judge deficiencies of about 7 cents to be within the range of tolerance.[38] None of the systems of multiple division which have obtained broad support possesses fifths with a greater discrepancy than this, suggesting about 7½ cents as an implied limit for the fifth. This is a far smaller figure than that of the discrepancy of other supposed natural consonances in 12-tone temperament.

There are theorists who maintain that the perfect

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[37] "A heard 2-cents discrepancy is a sensible difference in simultaneous soundings." Op. cit., p. 310. For this reason, Partch opposes all equal temperaments.

[38] Ariel asserts that the 7-cent deficiency of the fifth in 19-tone temperament is well within the limit beyond which consonance is destroyed. "Sie bedeutet aber noch lange nicht die Grenze für die Trübung des Wohlklanges." Das Relativitätsprinzip in der Musik, p. 144.

fifth as the principal interval within the octave requires a purer tuning than other consonances. Against this is raised the counterclaim that the more complex and subtle consonances are more easily destroyed than the fifth by slight alterations in tuning. Fifths in practice as well as theory have been the least distorted intervals after the octave.

It should be noted, however, that there have been occasional theorists who have preferred altered to unaltered fifths. Robert Smith, in the 18th century, defending mean-tone principles against the increasing tendency toward 12-tone equal temperament, enunciated the principle that if indeed the consonances of a musical system must be tempered, it is best that they all be tempered moderately rather than that one or two should have to be tempered a great amount in order that other consonances might be almost pure. Evaluating all intervals from this standpoint, which he called equal harmony, Smith attacks the fifths of 12-tone temperament as being "finer than they ought to be."[39] The ideal fifth, according to Smith, would be 5/18 of a comma smaller than 3:2. This is almost exactly six cents. M. W. Drobisch, a 19th century theorist, uses methods similar to Smith's in finding a value for the fifth that will work best

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[39] Smith, Harmonics, p. 166. Sauveur had been concerned with the equalization of the discrepancies of the intervals as well, as can be seen from the quotation in footnote 1 on page 3, but he was content with an alteration of only about 4 cents in the fifth.

with the other consonances of the diatonic system and also selects a flat fifth, 697.2 cents.

In the 20th century, the Danish mathematician and acoustician Thorvald Kornerup has proposed a new system of acoustic valuation aimed at interrelating all of the consonant intervals in a single mathematical progression. As part of his over-all plan, the "ideal" fifth is to be 696.21 cents, nearly 6 cents smaller than 3:2. To support his judgment favoring the smaller fifths, Kornerup relates the 3rd partial to each of the higher partials through number 32. Example 3, below, shows how many fifths must be superimposed to produce each of the partials from 9 to 16, and the size of each of the fifths in order that the partial produced should be pure.[40]

Example 3: Kornerup's Relationships of Fifths to Partials

NUMBER OF FIFTHS	SIZE OF EACH FIFTH	PARTIAL PRODUCED THEREBY
2	701.95 cents	9
4	696.58 cents	10 (or 5)
1/13*	696.05 cents	11
1	701.95 cents	12 (or 6 or 3)
15	696.03	13
10	696.88	14 (or 7)
5	697.65	15
12	700.00	16 (or 8 or 4 or 2 or 1)

* = thirteen fourths rather than fifths: the exact inversion.

[40] Kornerup, Acoustic Valuation of Intervals, 1938, page 13. For a more detailed examination of Kornerup's theories see Chapter 9.

As can be seen, the fifths are all either 701.95, the size of the perfect fifth, or smaller, with most of them in the region 696-697, where Kornerup's "ideal" fifth lies. The table appears to give support to the view that in an equal tempered musical system attempting to give close approximation to the upper partials, the fifth will have to be reduced in size, perhaps substantially. However, Kornerup's methods in arriving at his figures are not above criticism. He appears to have excluded fifths larger than 701.95 quite arbitrarily, even in cases where a shorter cycle of fifths closer to 701.95 would have produced the same partial. For example, in the case of the 13th partial, Kornerup requires no less than 15 fifths, each of which is nearly 6 cents too small. However, if he had substituted a fifth only three cents too large, 705.0 cents, he would have been able to produce the 13th partial with a series of but 8 fifths (705 x 8 = 5640, minus 4800 = 840 cents = 13:8). Kornerup's table remains informative in showing the various amounts of alteration necessary to relate the shortened fifth to other members of the harmonic series, but its validity as a justification for his theory of the foreshortened fifth is severely weakened by the <u>a priori</u> exclusion of the lengthened fifth.

How to evaluate theories giving actual acoustical preference to the foreshortened fifth? Since they all involve relationships between the ratio 3:2 and other numbers, one is tempted to disregard theories of a small fifth

altogether as externally ordained and to question strongly whether any of the authors, given just a fifth, really prefer the foreshortened version. Nevertheless, in view of the unquestionable preference of many contemporary writers for enlarged major thirds over pure 5:4's, the genuine preference of some musicians for an altered fifth is quite possible. In Smith's case, recalling the prevalence in his day, especially in England, of mean-tone tuning involving a distinctly flattened fifth, it is quite possible that conditioned ears or innate conservatism quite convinced him of the acoustical superiority of the small fifth. Certainly if we are to accord a measure of acoustical validity to the treatment of the enlarged major third as a consonance, it would appear only reasonable to accord similar regard to the preference for the foreshortened fifth, especially as the extent of tempering in the latter case is much the smaller, not exceeding the 7.21 cents called for in 19-tone temperament. For the purposes of this paper, all intervals within this range of 3:2 will be treated as representatives of the perfect fifth, and the perfect fifth will be regarded as an essential part of any system of multiple division, with 701.95 cents regarded as the ideal size.

THE 5th PARTIAL: THIRDS AND SIXTHS

With the addition of the 5th partial, four intervals are added to the field of basic consonances: 5:4, the major third; its inversion, 8:5, the minor sixth; 6:5, the minor third, representing the difference between the perfect fifth and the major third; and 5:3, the major sixth, which is the inversion of the minor third. These four intervals, represented in cents, are approximately 386, 814, 316, and 884, respectively. Because every one of these intervals deviates from its equivalent in 12-tone temperament by well over half a syntonic comma, it is on the subject of the fifth partial that the advocates of 12-tone temperament have been historically most on the defensive. Rather than offer the argument that a discrepancy of 13 to 15 cents is negligible, defenders of 12-tone temperament have often contended that a large major third is preferable to a just third. Many string players and singers are taught to exaggerate the size of even the major thirds of 12-tone temperament. As the argument over the proper size of the thirds, and whether they should in fact be considered as representatives of a consonant 5th partial, is basic to nearly all questions related to multiple division, the points made by both sides will be examined in some detail.

Daniélou, who considers the third partial to be "cyclic," calls the fifth partial "modal," describing it

further as the "humanizer" of music.[41] But just as aestheticians disagree on how extensive a role the 5th partial should play in today's music, historians and musicologists differ on the role of the just third in other ages and civilizations. Ratios containing the number 5 exist in the theoretical writings of Ptolemy near the end of the Greek era. It is generally agreed that throughout the early middle ages the fifth partial was not used. However, there is disagreement about the time and extent of its re-introduction into musical theory and practice, and this disagreement closely parallels the disagreement concerning its aesthetic validity.

J. Murray Barbour, a leading expert on tuning systems and a strong partisan for 12-tone temperament, has claimed on several occasions that the first re-introduction of the 5th partial to musical tuning systems occurs in the writings of Bartolomeo Ramos de Pareja in 1482. Thorwald Kornerup and other theorists who have valued the 5th partial more highly than Barbour ascribe to Walter Odington, an English theorist active around 1300, the re-introduction of this partial as a means of explaining the relative consonance of the third. Barbour acknowledges Odington's "discovery" of the interval 5:4 but apparently does not believe that it was used by him or by his contemporaries as part of a

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[41] Op. cit., page 231.

complete musical system.[42]

Building their case around Odington's alleged advocacy of the 5:4 ratio in explaining the third, and around the English practice known as gymel, wherein parallel thirds and sixths were sung as consonances not unlike the later French faux bourdon, the supporters of the 5:4 claim to see a connection between the beginnings of the theory of the just third and the beginnings of the treatment of the third as a consonance. The third in Pythagorean music, not being based on a small number ratio, was a dissonance and was treated as such.[43] If, indeed, the general acceptance of the third as a consonance corresponds in time to the adoption of 5:4 as the correct intonation of the third, important historical support is given to the case for the just ratio. The opposing claim is that the interval 5:4 was introduced after the consonance of the third had begun to be established and that just thirds never did achieve the widespread use that has sometimes been ascribed

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[42] In Tuning and Temperament, Barbour cites Odington's reference to 5:4 and 6:5 as the consonant thirds on p. 3, but chooses Ramos as the founder of just intonation because he was "the first known European writer to break away from the Pythagorean tuning for the tuning of the chromatic monochord," p. 89.

[43] Yasser considers the major third to be a dissonance in all pentatonic music, which, he contends, is based on Pythagorean tuning. A Theory of Evolving Tonality, p. 111. Kurt Blaukopf, a respected musico-sociologist who has been greatly influenced by Yasser's writings, makes the same assertion. Musiksoziologie, p. 62.

to them.[44]

The preference for the just third among theorists and acousticians, especially in the 19th century and shortly before, is quite extensive. Helmholtz sounds the keynote when he says, "The principal fault of our present tempered intonation lies in the thirds, and . . . is the old Pythagorean error of forming the thirds by means of an ascending series of four fifths. The natural relation of the major third to the tonic, both melodically and harmonically, depends on the ratio 5/4 (5:4) of the pitch numbers. Any other third is only a more or less unsatisfactory substitute for the natural major third."[45]

Bosanquet calls the Pythagorean third "disagreeable,"[46] after asserting that "chords formed by the notes ordinarily in use are much inferior in excellence to chords which are in perfect tune."[47]

In this century many of the leading theorists and acousticians have continued to assert the superiority of the natural third. Carl Eitz, at the beginning of the century, asserted that "A good a capella chorus sings, as far as a composition permits, in just intonation, since every

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[44] Op. cit., p. 89.

[45] Helmholtz, Hermann L. F., On the Sensations of Tone as a Physiological Basis for the Theory of Music, translated from the German by Alexander J. Ellis, p. 315.

[46] Bosanquet, op. cit., p. 8.

[47] Ibid., p. 6.

normally gifted singer has an excellent sensitivity for pure octaves, fifths, fourths, thirds, and sixths. . . ."[48] A similar view has more recently been expressed on a number of occasions by Lindsey Norden and by many others.

Even among some writers well disposed toward 12-tone temperament, the 5th partial is recognized as the source of the consonance of the thirds. Rameau in the 18th century is a good example, as he unqualifiedly accepts the senario as the basis for consonance but is not ill-disposed toward 12-tone temperament as a compromise. Hindemith's views on the 5th partial and on 12-tone temperament are similar.

Others, however, have voiced an emphatic preference for the thirds of 12-tone temperament over the thirds of the unaltered 5th partial. In the notes which accompany the first release in Musurgia Records' series on the History of Musical Theory, Fritz A. Kuttner and J. Murray Barbour state,[49] "As will be seen and demonstrated in the recording, there is . . . reason to doubt the musical merit of a superparticular ratio as low as 4:5." In the introduction

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[48] "Ein guter A capella-Chor singt, soweit es die Komposition zulässt, in natürlich-reiner Stimmung, denn jeder normal veranlagte Sänger hat ein feines Empfinden für reine Oktaven, Quinten, Quarten, Terzen und Sexten. . . ." Eitz, Carl, Von den natürlich reine Stimmungsverhältnissen, Melos I, 1920, p. 293.

[49] Kuttner, Fritz, and J. Murray Barbour, The Theory of Classical Greek Music, Introductory Notes, New York, 1958, Musurgia Records, Column 10.

to their next relesse, The Theory and Practice of Just Intonation, the same two authors state that a major defect in just intonation is "boredom with the insipid major and minor thirds that are too perfect for our ears to have much character."[50]

A great number of writers have taken a middle position with respect to the thirds by endorsing the just third for harmony while preferring a larger third for melody. Alfred Jonquière, a notable acoustician whose Grundriss der Musikalischen Akustik of 1898 proved serviceable as a standard work for many years, takes this position on the basis of experiments which showed that musicians prefer the large major third for melody but the small one for harmony.[51] Engbert Brandsma, reporting to the congress of the Internationale Musikgesellschaft in 1909, follows the same path as Jonquière, calling the melodic, Pythagorean intervals "structural" while considering the harmonic, pure intervals a source of "color." Brandsma uses several musical examples to illustrate his point, the first being Agatha's theme from Der Freischutz (Ex. 4), in which the interval 5:4 above the tonic for the E would, in Brandsma's

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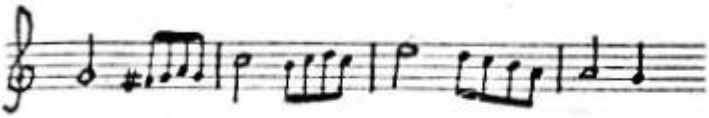
[50] Kuttner and Barbour, The Theory and Practice of Just Intonation, Introductory Notes, column 31.

[51] Jonquière, Grundriss der Musikalischen Akustik, p. 139.

opinion, be "expressionless."[52]

Example 4:



In spite of its apparent harmonic derivation, Brandsma insists similarly that the F# in the opening theme of the finale of Beethoven's violin concerto must be Pythagorean in order to achieve sufficient brilliance. The folksong "Sleep, Baby Sleep," however, is best sung with a just third in order to achieve repose.

Writing in 1955, Thomas P. Frost reacts in much the same way as Jonquière and Brandsma to tapes of Eivind Groven's organ[53] in enlarged just intonation.[54] He concedes that his reaction may be due in part to conditioning: "A musician accustomed to equal temperament will find many of the scale tones, particularly the 3rd and 6th, so far removed from their accustomed pitch as to seem 'sour.' When a scale or solo melody is played, it may appear disconcert-

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[52] Brandsma, Über die Tonverhältnisse in der alten und neuen Musik, Bericht von III Kongress der Internationalen Musikgesellschaft, p. 353 ff.

[53] Eivind Groven is a Norwegian theorist and composer whose 36-tone organ is discussed below in this chapter and in Chapter 7.

[54] Frost, Thomas P., A Matter of Records, Organ Institute Quarterly, Vol 5:3, Summer 1955.

ingly out of tune." Frost adds, however, that "When these same notes are combined into chords, the sounds are magically consonant. The complete and unaccustomed absence of beats between the notes (makes them) all but blend into one tone. The pure richness of the untempered chords is a novel experience which we wish it were possible to share with every reader."

String players are generally instructed to play sharped notes somewhat higher than the enharmonically equivalent flatted notes. This practice tends to produce Pythagorean thirds. Helmholtz, however, informs us that Joachim played just thirds on all occasions,[55] and Dupont does not question this assertion in reporting it in his history of temperament.[56] Arguments persist to this day as to whether singers use just or large thirds when working away from a piano. It appears that more and more authorities are tending to doubt that just thirds enjoy any significant use.[57]

The minor third, 6:5, has stirred up some controversy

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[55] Helmholtz, op. cit., p. 323.

[56] Dupont, op. cit., p. 130.

[57] It is Kuttner's preliminary finding in general research supported by stroboscopic measurements that the intonation of the different musical media, piano, strings, voices, etc. is far less varied than is generally thought. According to Kuttner, a well-tuned piano tends to have slight deviations from equal temperament (perhaps 2 or 3 cents on the average), while voices and strings approach equal temperament nearly as closely. --conversation, December 1960.

of its own. Ranked as a subordinate interval to the major third by Descartes, because it is not built on a replica of the fundamental,[58] the just minor third has been rejected, and with it the minor triad 10:12:15, by some authors who have accepted the harmonic usage of the just major third. Brandsma finds 32:27 better than 6:5 for harmonic as well as melodic use, calling 6:5 a horrible discord (unerträglichen Missklang), while he finds a diminished seventh best constructed as three superimposed 32:27 intervals.[59] Jonquière rejects the interval 6:5 in the construction of minor chords as too like the major (dur-ähnlich).[60]

The interval 6:5, whose first use is attributed by Kornerup to Erastothenes of Cyrene in about 200 B.C.,[61] has received substantial support in its own light. Ariel, in particular, likes this interval and regards it as the fundamental interval for a higher musical order (Grundintervall höherer Ordnung).[62]

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[58] Descartes

[59] Brandsma, op. cit., pp. 359-60.

[60] Jonquière, op. cit., p. 139, p. 145. On page 145, Jonquière points out that the just minor third calls into play a difference tone a perfect fifth below its upper member, a tone totally unsuitable if the minor third is the bottom part of a minor triad. Jonquiere does not choose to discuss the completely unrelated difference tones of the equal-tempered or Pythagorean minor thirds, however.

[61] Kornerup, Musical Acoustics based on a Pure Third System, columns 18-19.

[62] Ariel, op. cit., p. 74. For an elaboration of this point see chapter 10.

Harry Partch does not go so far as Ariel in singling out the interval 6:5, but he calls it "good to the ear,"[63] adding that its Pythagorean equivalent, 32:27, is "beyond the ear's capacity to determine accurately."[64] At first glance this indictment hardly seems relevant, since easiness to determine accurately does not in itself appear to be a satisfactory way to determine the consonance of an interval. But the implications of Partch's allegations go deeper. It is generally accepted that current tuning practice grants far greater leeway to variations in the third than in the octave or fifth.[65] But is not the ear's very willingness to accept a considerable alteration in the third a sign that it has not accepted the 300-cent minor third or the 400- or 408-cent major third as an acoustical standard? And does not the comparison between the relative stability of the small-number consonances (fifths, fourths, octaves) and the relative instability of the artificial consonances (the thirds and sixths of current use) at least suggest that when a musical society tunes its consonant intervals

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[63] Partch, op. cit., p. 241.

[64] Ibid., p. 278.

[65] "Octaves, fifths, and fourths present a certain problem of intonation, since the slightest deviation from the correct pitch in either note is more apparent to the ear than it would be in such intervals as the 6th and 3rd, where the mathematical ratio between the notes is more complex." -- Kent Kennan, discussing violin double-stops in The Technique of Orchestration. Indeed, the mathematical ratio of the thirds and sixths of common practice is "complex!" Small wonder that few musicians would refuse to concede Kennan his point on the flexibility of their intonation.

to small-number ratios it has a more stable basis for its intonation?

THE WEB OF FIFTHS AND THIRDS

With the acceptance of the 3rd and 5th partials as basic to most musical systems, and before proceeding to the consideration of the 7th partial (which remains controversial even among the advocates of multiple division), it is necessary to consider a body of acoustical data and speculation concerning the relationship between the 3rd and 5th partials, and the use of the intervals created by this relationship in the building of musical systems. Jose Würschmidt uses the term "Quinten-Terzengewebe" to describe what he regards as the total musical field for any system.[66] Wesley S. B. Woolhouse, an important pioneer writer in the field of multiple division, writing in 1835, claims that "Every interval may be determined by combinations of the Major 3rd, the Perfect 5th, and the Octave."[67]

Bosanquet asserts, "To provide a material of notes for musical performance it is theoretically requisite in the first place that to every note used we should possess

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[66] Würschmidt, Jose, Die rationellen Tonsysteme in Quinten-Terzengewebe, Zeitschrift für Physik, Bd. 46, Jan. 1928, p. 527.

[67] Woolhouse, W. S. B., Essay on Musical Intervals, Harmonics, and the Temperament of the Musical Scale, p. 73.

octave, fifth, and major third, up and down. Each of these being a note used we may require the same accessories to each, and so on."[68] Bosanquet continues, "If however we provide all the notes necessary for an extended system . . . we have endless series of fifths running up and down, and endless series of thirds running horizontally; and it is possible to show that no two of the notes will ever be exactly the same in pitch. Consequently in practice various approximations are employed, so as to reduce the number of notes required."[69]

As the just fifth and third are fixed intervals, the various relationships within the web of fifths and thirds are immutable. The most important of these relationships with their traditional terms and qualities are shown in Example 5.

Among the theorists who have dealt with multiple division in the past, a majority have built their systems to capitalize on features of the web of fifths and thirds. The "approximations" which Bosanquet notes above have led to two types of systems: equal-tempered systems in which every member can serve as third or fifth above or below another member; and unequal systems where for a greater measure of accuracy or simplicity or both, the complete mutuality of relationship is sacrificed and tones on the

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[68] Bosanquet, op. cit., p. 1.

[69] Ibid., p. 3.

Example 5: Relationships in Web of Fifths and Thirds[70]

Designation	Name	Ratio	Description
A.	Perfect Fifth	3:2	
B.	Major Third	5:4	
C.	Minor Third	6:5	A - B
D.	Major tone	9:8	Two fifths less one octave
E.	Minor tone	10:9	B - D
F.	Diatonic semitone	16:15	(A + B) deducted from one octave
G.	Major limma	27:25	3A - (2B + one octave)
H.	Chromatic semitone	25:24	D - G; E - F; 2B - A
I.	Minor limma	135:128	3A + B - two octaves; D - F
J.	Pythagorean limma	256:243	three octaves - 5A
K.	Major diesis[71]	128:125	one octave - 3B; G - I
L.	Minor diesis	3125:3072	5B - (one octave + A); H - K
M.	Syntonic comma	81:80	4A - (B + two octaves); D - E
N.	Minor comma	2048:2025	three octaves - (4A + 2B); K - M
O.	Pythagorean comma	531441:524288	12A - seven octaves
P.	Schisma	32805:32768	8A B - five octaves; M - N

[70] Most of the materials in this chart are taken from a similar chart by Friedrich Opelt from his Allgemeine Theorie der Musik, p. 21.

[71] The Harvard Dictionary lists the major diesis as 135:128 (four minor thirds minus an octave), and calls the interval shown here, 128:125, a minor diesis.

fringe of the system lack fifth and/or third, above or below.

Würschmidt, in his theory of rational tone-systems, has made one of the most penetrating analyses of the qualities and possibilities of the various equal-tempered systems within the context of the web of fifths and thirds. He is particularly concerned with two kinds of minute intervals which he considers particularly important to tempered systems. The first kind he calls defining (definierend) intervals. These are the very small intervals in the web of 5ths and 3rds which are made to disappear (made equal to zero) in the process of building a specific tempered system. The syntonic comma, for example, is a defining interval in 12-tone temperament, as is the major diesis. The second kind, Würschmidt calls constructing (konstruierend) intervals. These are the intervals each of which is equal to one unit of a tempered system. The constructing intervals in 12-tone temperament are the various kinds of semitones such as 16:15 and 25:24. It is Würschmidt's basic contention that a rational system is one in which the largest defining interval is smaller than the smallest constructing interval.[72] The rational systems, according to Würschmidt

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[72] Würschmidt, Die rationellen Tonsysteme, op. cit., p. 527. Würschmidt considers the syntonic comma, 81:80 and the major diesis, 128:125, to be the defining intervals in 12-tone temperament. The sizes of these intervals are 21.5 and 41.4 cents respectively. As constructing intervals, Würschmidt selects 16:15, 25:24, and 135:128, the smallest of which is 70 cents. 12-tone temperament is therefore

are 5-, 7-, 10-, 12-, 19-, 22-, 31-, 34-, 53-, and 118-tone temperaments.[73]

One of the great advantages of the concept of a web of fifths and thirds is that it provides a method of charting musical systems according to the intervals from which they are derived, thus making visual and easy to perceive the characteristic features of the system.

Bosanquet makes such a graph of the 12-tone system, which he calls the duodene, with the fifths vertical and the thirds horizontal.

Example 6: Bosanquet's Duodene.[74]

B^b	D	$F^\#$
E^b	G	B
A^b	C	E
D^b	F	A

rational. In choosing 135:128 as his third constructing interval Würschmidt is making a most unusual selection. The usual 12-tone system is built as follows, $(25:24)^5 \times (16:15)^4 \times (27:25)^3 = 2$. Würschmidt instead builds his 12-tone system as follows, $(16:15)^7 \times (135:128)^3 \times (25:24)^2 = 2$.

[73] Ibid., p. 556. Würschmidt does pose a limit on the expanse of his system for, if he did not, other small intervals far removed from musical practice would interfere with his calculations. He chooses as limits 11 fifths and 27 thirds in order to eliminate the very small discrepancies from the octave created by the 12th fifth (the Pythagorean comma) and by the 28th third (which exceeds the ninth octave by only a few cents).

[74] Bosanquet, op. cit., p. 2.

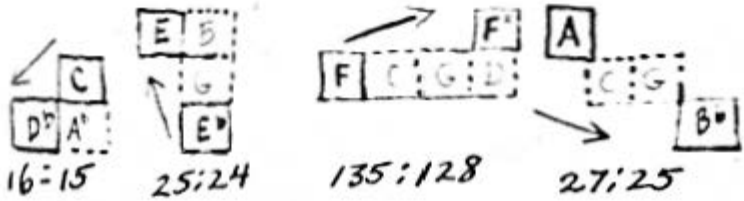
The usual practice in graphing fifths and thirds is to represent the fifths horizontally and the thirds vertically. Ex. 7 shows the same duodene as charted by Adriaan Fokker, who attributes it to the 17th century mathematician, Leonhard Euler.[75]

Example 7: Euler's Genus Diatonico-Chromaticum

A	E	B	F#
F	C	G	D
D♭	A♭	E♭	B♭

Würschmidt, noting that the duodene requires four constructing intervals, chooses a different arrangement of tones. Example 8 shows the four constructing intervals in 12-tone temperament as they appear in Euler's system, while Example 9 shows Würschmidt's solution, eliminating the interval 27:25.

Example 8: The Constructing Intervals in 12-tone Temperament



[75] Fokker, A. D., Just Intonation and the Combination of Harmonic Diatonic Melodic Groups, p. 9.

Example 9: Würschmidt's "normalized" 12-tone system.

	B	F#	C#	G#
C	G	D	A	E
	E♭	B♭	F	

Ariel, who was a contemporary of Würschmidt's and whose work was well known by him, was probably influenced by Würschmidt as well. In 1925, after Würschmidt had already developed his theories of the Quinten-Terzengewebe, but before he codified the 12-tone system as shown above, Ariel published his own views on the 12-tone system, using only the traditional three constructing intervals, 16:15, 25:24, and 27:25. Ariel offers two patterns in interval charts which, when graphed, turn out to be as is shown in Example 10.

Example 10: Two 12-tone systems by Ariel.[76]

F♭				
	A	E	B	
	F	C	G	D
		A♭	E♭	B♭
				D♭

F#			
D	A	E	B
	F	C	G
	D♭	A♭	E♭	B♭

[76]Derived from the charts in Ariel, op. cit., p. 115.

It appears that Kornerup would agree with the second of the Ariel systems, for his own chart includes every one of Ariel's tones except F#, which Kornerup would probably select in the same manner.[77] One other pattern, somewhat similar to Ariel's is offered below in Example 11. It likewise employs only the same three constructing intervals. However, in spite of the strong preference shown by Ariel and Würschmidt for systems involving only three constructing intervals, it is hard to see why any of these systems is preferable to the Bosanquet duodene with its compact pattern involving four different sizes of semitone.

Example 11: An Additional 12-tone System

F#	C#	G#		
	A	E	B	
	F	C	G	D
		a#	E♭	B♭

To Würschmidt, multiple division follows from the logical wish to eliminate the larger of the defining intervals. Since the deisis is the largest of the defining intervals in 12-tone temperament, the next rational system must provide separate tones for pitches separated by that interval. The diesis thus becomes the smallest of the

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[77] Derived from Musical Acoustics Based on a Pure Third System, columns 27-8.

constructing intervals of the next rational system, which is 19-tone temperament. Used instead as a defining interval in 19-tone temperament is the 29.5-cent minor diesis, 3125:3072, consisting of the difference between the chromatic semitone 25:24 and the major diesis 128:125. Both of these latter intervals are constructing intervals, and it follows that since all constructing intervals are equal to one unit of a given system, the difference between any pair of constructing intervals must equal a defining interval. Therefore, in a rational system, the largest constructing interval must always be less than twice the size of the smallest constructing interval. Other of Würschmidt's rational systems will be examined in the chapters on the various specific systems of multiple division.

Just Systems: To theorists preferring exactitude of fifths and thirds to approximation, the problem has been to find ways of closing the open ends of their systems. To provide 12 central tones with Bosanquet's minimal requirement (a fifth and a third both above and below) would require 26 tones to the octave, the outer 14 of which would serve merely as auxiliaries without the acoustical resources to become tonics in their own right. Making potential tonics of all the tones in the traditional twelve major keys would require a minimum of 37 tones per octave, 18 of them lacking at least one auxiliary. Nevertheless, many theorists have tried to come to terms with the infinitude

of possibilities, and just organs and harmonia have been built with almost every possible number of tones to the octave. Barbour examines a number of these instruments[78] beginning on page 108, and Partch studies several in great detail.

Within the web of fifths and thirds, perhaps the most remarkable near-identity is the schisma, which represents the difference between 8 fifths plus a third and five octaves: 32805:32768, or 1.945 cents. Even those theorists who have been implacably opposed to substituting the approximations inherent in equal temperaments for just ratios have tended to accept the principle of an alteration of each fifth by 1/8 schisma (about a quarter of a cent) in order that every eighth fifth might produce a just major third when deducted from five octaves. The treatment of the schisma as an identity can simplify a musical system considerably. Clark Jones rightly remarks that 15 major keys can, with the assistance of a minute temperament of 1/8 schisma, be realized with as few as 24 tones to the octave, whereas 35 would be required if the schisma were nottreated as an identity.[79]

The Norwegian composer Eivind Groven also uses

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[78] In Tuning and Temperament.

[79] Jones, Remarks on Just Intonation and Musical Scales, Acoustical Society Journal, 1947, p. 727.

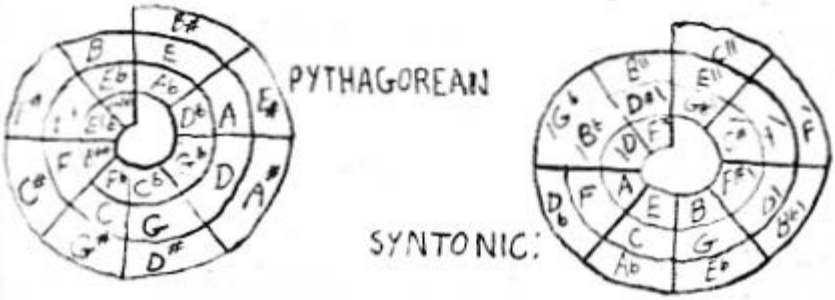
schismatic temperament for his just organ,[80] but feels compelled to add 12 tones to the 24 recommended by Jones, bringing to 36 the number of tones to the octave.

The schisma is the subject of a classical approximation, possibly mislabeled a theorem, Helmholtz' Theorem. This states that the schisma, produced as outlined above, is approximately equal to the error of the fifth in 12-tone temperament. The respective figures in cents are:

Schisma: 1.945 cents
Error of 5th: 1.955 cents.

The use of the schisma as an identity converts the open-ended plane graph into a kind of spiral, or partly detached ring. The perimeter represents a cycle of fifths while the spokes represent a series of thirds. Example 12 illustrates the system of 24-tones advocated by Jones. Two possible nomenclatures are shown.

Example 12: 24-Tone Schismatic Temperament



[80] Groven, My Untempered Organ, Organ Institute Quarterly, Vol 5:3, Summer 1955.

Eivind Groven extends the spiral by 540 degrees, and uses just such a graph to illustrate his article which is cited above.

The Schisma in Pythagorean systems: Since 8 fifths deducted from 5 octaves leaves a major third in schismatic temperaments, a system of fifths yields a just third only if there are at least 8 superimposed fifths. After the first 8 tones, each additional tone of a Pythagorean system will yield the illusion of a just third. Barbour and Kuttner point out that this gives the distant keys of Pythagorean tuning a close resemblance to the near keys in just intonation, while Kornerup, who strongly disapproves of Pythagorean tuning,[81] says of the phenomenon of almost-just thirds over the "back fifths" of the Pythagorean system, "It is the playful way of nature that all exaggeration corrects itself."[82]

Because of the relatively close correspondence between twelve fifths and seven octaves, the tones of an extended cycle of pure (or sub-schismatically tempered) fifths tend to pile up in layers of twelve, each removed from the other by about a comma . . . a Pythagorean comma in the case of pure fifths, and an interval slightly smaller than a

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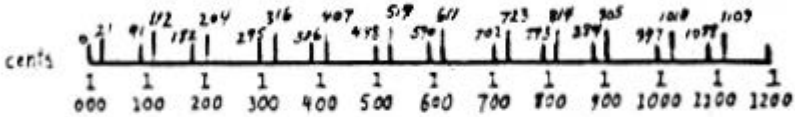
[81] Kornerup likens the octave, fifth, and third, to the three primary colors, maintaining that Pythagoreanism is the musical equivalent of color-blindedness. Musical Acoustics Based on the Pure Third System, column 29.

[82] Kornerup, Acoustic Methods at Work, 1934, p. 19.

syntonic comma in the case of sub-schismatically altered fifths. The 24 tones of Jones' spiral are grouped as follows when converted into a pitch continuum.

Example 13: Pitch Continuum of the 24-Tone Pythagorean Spiral (slightly altered by schismatic temperament)



With each addition of 12 tones another "layer" is added, one comma higher than the preceding one. This fact possibly explains why proponents of this kind of system have tended to adopt systems whose tones numbered multiples of twelve. It will be seen in example 13, above, that the intervals between each of the twelve close groups are of two different sizes, the one being approximately 70 cents, the other, 91 cents. The smaller of these two intervals is approximately three commas in size, while the larger one is about four. In the 24-tone system shown in example 13, there are seven 3-comma gaps, and five 4-comma gaps. As many theorists have pointed out, it is possible to bring the web of fifths and thirds to a very close approximation of a closed system by continuing the spiral until these comma gaps are filled in. A total of 29 additional tones are needed to fill these gaps which brings to 53 the total

number of tones in the system. A 53-tone Pythagorean system is almost identical with 53-tone equal temperament, as 53 fifths (37203.35 cents) is nearly precisely equal to 31 octaves (37200.00 cents). A single fifth in 53-tone temperament is distinguished from the just fifth by only 0.06 of a cent.[83]

The building of 12-tone layers separated by a comma encouraged theorists to use traditional tonal nomenclature, with the addition of a dash over or under the note-name to indicate raising or lowering by a comma. This practice, according to Kornerup, evolved from a system of comma-differentiation by upper and lower case letters used by Hauptmann in 1853 to the fully developed plan of Oettingen in 1866, in which dashes above the note-names signified lowering by a comma. Helmholtz, according to Kornerup, duplicated this system in 1870, but with the dashes used below the letters to lower the pitch. Eitz in 1891 still used the system, but with addition and division signs after the note-names rather than dashes.[84]

Oettingen, Helmholtz and Eitz all used Pythagorean nomenclature, wherein E, representing the just third above

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[83] The size of the fifth in 53-tone temperament as well as much of the data about interval sizes throughout this paper is gathered from an impressive though unpublished collection of charts appended to An Introduction to Tuning and Temperament, by Charles S. Kent.

[84] Kornerup, Musical Acoustics based on a Pure Third System, columns 21-22.

C, appears with the sign of comma modification. The G# 25:16 appears with two comma signs indicating that it is two commas removed from the G# of the Pythagorean series. Kornerup offers a chart showing the nomenclature of his predecessors for four tones as derived from C as center. He also shows his own nomenclature, which is based on a just third system wherein it is the Pythagorean intervals, rather than the syntonic intervals, which are altered by comma signs. Example 14 shows excerpts from Kornerup's chart.

Example 14:

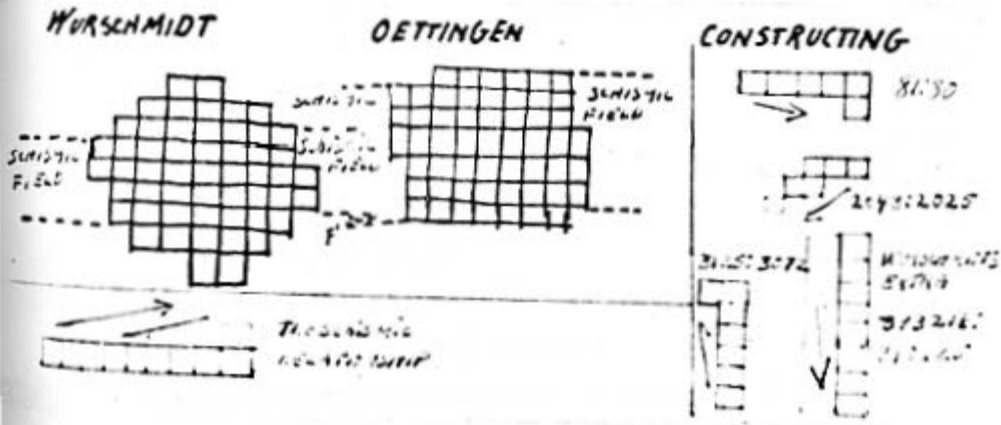
Ratio	Cents	Oettingen	Helmholtz (1870 version)	Eitz	Kornerup	Pythagorean (not in Kornerup's chart)
25:16	-773	$\overline{\overline{g\#}}$	$\underline{\underline{g\#}}$	$g\# \div 2$	$g\#$	$Bbbb$
5:4	386	$\overline{e}$	$\underline{e}$	$e \div 1$	e	Fb
10:9	182	$\overline{d}$	$\underline{d}$	$d \div 1$	d	Ebb
9:8	204	d	d	d	$d+$	D

The debate between adherents of the two nomenclatures has been fierce. Würschmidt, an adherent of syntonic nomenclature, and J. Wallot, a disciple of Oettingen, engaged in a bitter debate in the pages of the Zeitschrift

für Physik[85] in 1921 and 1922.

Würschmidt compares his 53-tone system with Oettingen's in diagram form, attempting further to confound Wallot. Note, however, in Example 15, that Oettingen's is more compact than Würschmidt's and that it makes more extensive use of the schismic relation (shown by the dotted-line extensions in Example 15).

Example 15: Würschmidt's and Oettingen's 53-tone Systems with Defining and Constructing Intervals shown.



It should be noted that a truly Pythagorean 53-tone system would appear on a diagram as a single horizontal row of 53 fifths. Not even Oettingen's nomenclature,

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[85] See particularly Wallot in volume 4, p. 157, and Würschmidt in volume 5, page 111. Würschmidt ascribes the syntonic nomenclature he uses to H. Starke, author of Physikalische Musiklehre (of which Würschmidt cites pages 99 and 100). Würschmidt claims as the chief advantage to syntonic nomenclature that many fewer comma-alterations are needed than in Oettingen's system, a claim which is justified.

complicated by comma-signs, employs the multiple sharps and flats that would be necessary to reflect such a diagram accurately. It would appear that while Würschmidt's nomenclature does commend itself by its relative simplicity, Oettingen's system, as shown in Example 15, above, is the superior one, even in Würschmidt's own terms: It is the more compact; it is the one which uses only three constructing intervals; it makes much greater use of the schisma as an identity.

The two basic nomenclatures of just musical systems and of 53-tone temperament are reflected by two distinct divisions of the 53-tone system to produce the diatonic scale. Joachim Steiner, advocating 53-tone temperament as a basis for measurement refers to the Pythagorean division 9 9 4 9 9 9 4 as string-lydian and to the just division 9 8 5 9 8 9 5 as flute-lydian.[86] The dilemma of the alternate scales and nomenclatures results from the conversion of the syntonic comma from defining to constructing interval. Our note-names are based on the assumption, valid in 12-tone temperament, that two pitches separated by a syntonic comma are in reality one and the same.

The web of 5ths and 3rds does not end with 53 tones. Würschmidt has gone on to diagram 118-tone temperament. The web is, indeed, endless, and it would appear advisable to

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[86] Einfluss der reinen Stimmung auf die Entwicklung der Musik, Bericht von III. Kongress der Internationalen Musikgesellschaft, page 387.

extricate ourselves and move on to the 7th partial.

THE SEVENTH PARTIAL: 7:4, 7:5, 7:6 AND THEIR INVERSIONS

> "In the flora and fauna the figures 1, 2, 3, and 5 are ruling according to the law of precedence that 1, 2, and 3 (as well as multiplies of these figures with each other) are ruling on a lower stage of development; for instance in the flowers of monocotyledonous plants and with zoöphytes; while 5 (and multiples as 10, 15, and so forth) indicate a higher phase of development, -- in the flowers of bicotyledonus plants and with echimoderms (starfish, crinoideans, arctiniae), while other prime numbers indicate teratologies."[87]

The above categorical opposition to the role of the number 7 appears in a theoretical treatise on music. It is not, as one might think, from an opponent of the expansion of musical resources. The statement is by Kornerup, a 20th century advocate of multiple division. It would appear that in whatever tonal worlds may develop, the 7th partial will remain the subject of controversy.

Kornerup, in his other writings, acknowledges the seventh partial. He admits its place among some of the musical systems of the past and present,[88] and even provides an approximation for the interval 7:6 in his Golden

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[87] Kornerup, Musical Acoustics based on a Pure Third System, column 31.

[88] He asserts that the arithmetical division 10:10, 10:9, 10:8, 10:7, 10:6, 10:5 is the basic type for oriental pentatonic music. Acoustics Methods of Work, 1934, p. 11. This work is ten years more recent than the one in which he attacks the 7th partial so vehemently.

System.[89] More implacably opposed to the inclusion of the 7th partial in any kind of musical system is Ariel. He distinguishes between the "natural" number series (which includes all numbers) and the "harmonic" series (which includes no primes or products of primes higher than five).[90] Ariel's opposition touches every aspect of the seventh partial, and is categorical and absolute.

Danielou projects the view that the seventh partial is beyond the scope of normal human perception,[91] and that its introduction would therefore be "dangerous."

In their opposition to the incorporation of the 7th partial into any new musical systems, these men follow a long and distinguished line of theorists whose ranks include Zarlino, Rameau, Oettingen, Riemann, and Hindemith.[92] In his definitive work on the 7th partial,[93] Martin Vogel

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[89]For a thorough discussion of Kornerup's Golden System, see Chapter 9. Kornerup regards 7:6 as smaller than 6:5 by a diesis which, in the golden system, is .618 of a small semitone. The golden approximation of 7:6 is one of the best in Kornerup's system, 265.93 cents, within one cent of the natural ratio (which is 266.87 cents). Kornerup's representation of 7:4 is much less accurate, however.

[90]Ariel, op. cit., p. 47. Ariel goes on to point out (p. 51) "Wenn..zwei Sänger einen Zweiklang singen, dessen Töne genau im Verhältnis 6:7 schwingen, dann ist das weder ein musikalisches Intervall noch die Temperierung eines solchen; es ist überhaupt keine Musik." (If two singers sing at an interval 7:6 apart, the result is neither a musical interval nor the temperament of such an interval; it is indeed not music at all).

[91]Danielou, op. cit., p. 231.

[92]These names are cited by Partch, op. cit., p. 93.

[93]Zahl 7 in der Spekulativen Musiktheorie. Vogel is Privatdozent at the University of Bonn.

leaves little doubt that every conceivable argument which present-day senarists might wish to use has already been offered. However, as Vogel shows, much of the opposition to the theoretical endorsement of the 7th partial has been based primarily on its unavailability in present-day tuning and in its exclusion from the diatonic scale.[94]

There is a large measure of ambivalence toward the seventh partial on the part of many advocates of multiple division. Friedrich Opelt, within a single text,[95] presents a "complete" table of basic intervals from which the seventh partial is completely absent,[96] a table of consonances which includes ratios of the 7th partial,[97] a statement that the seventh partial is disturbing (störend),[98] the statement[99] that all consonances are derived from the single chord 4:5:6:7:8, the recognition of 31-tone temperament as a means of including the seventh partial within a musical system, and finally the decision to exclude the seventh partial from further musical speculation because of the

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[94] Vogel cites the theorist Chladni, who, in referring to Kirnberger's name for the tone created by the 7th partial from C (I), cracked that the natural seventh was named i after the imperative form of the Latin <u>ire</u>, to go, and meant "Go away; we have no place for you here."

[95] <u>Allgemeine Theorie der Musik</u>, L8.

[96] p. 21.

[97] p. 30.

[98] p. 65.

[99] p. 66.

technical difficulties 31-tone temperament would entail.[100]

Euler, the 17th century mathematician on whose theories of the Greek genera a major current school of 31-tone tempered music has been founded, demonstrates much of the ambivalence toward the seventh partial shown more recently by Opelt. Beginning with speculations about mode as based on the senario, Euler rejects acknowledged Greek tunings based on septimal intervals as imperfect and harsh.[101] Speculating about their use nonetheless, Euler becomes increasingly interested in them and, later, endorses the use of the seventh partial. He evolves a mathematical formula for measuring relative degrees of consonance (by adding factors and exponents) whereby the natural minor seventh is the most consonant of the minor sevenths whenever the interval is increased by the interposition of one or more octaves.[102]

Support for the inclusion of the 7th partial in musical theory has as long and distinguished a history as has its opposition. Among the figures who have insisted that a role be found for the number 7 have been Mersenne,

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[100] p. 67.

[101] Euler's Tentamen Novae Theoriae, unpubl. doctoral dissertation by Charles S. Smith, p. 175.

[102] Euler's ingenious though highly questionable system for measuring consonance is treated at some length in Vogel, op. cit., esp. p. 78.

Tartini, Fétis, Serre, Hauptmann,[103] and Kirnberger.[104] The well-known critic, Donald Francis Tovey, is at the end of a long line of musicians of many generations when he complains that music as it is now constituted "contains no room for so simple a thing as the 7th partial."[105]

Many proponents of multiple division have been interested in the possible role of the 7th partial in enlarged musical systems. Yasser, who calls the 7th partial "flavorless" within the diatonic system,[106] incorporates it nonetheless into the body of consonant sonorities of his "supra-diatonic" system.[107]

The interest in the seventh partial on the part of some theorists has been so keen that there have even been proposed "just" 12-tone systems whose non-diatonic members are to be drawn from septimal ratios. One such proposal is by A. M. Awraamoff in the first volume of Melos, 1920. He offers a septimal tuning for the 12 keys of the piano in the name of "freeing" music from the shackles of equal

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[103] Partch, op. cit., p. 93.

[104] Kirnberger's interest in the 7th partial is treated at some length in Vogel, op. cit.

[105] Encyclopaedia Brittanica, 11th Edition, quoted by Fokker, Expériences Musicales, p. 133.

[106] A Theory of Evolving Tonality, footnote to page 193.

[107] For details on Yasser's supra-diatonic system and its use of the 7th partial, see below, Chapter 11.

temperament.[108] Awraamoff's position appears inconsistent, however, for while his theory postulates a just, non-septimally derived diatonic scale with septimal tones between scale degrees, the actual pitches of his system yield no such diatonic scale. The harmonic minor of F is, however, possible in the scale which is shown in Example 16.

Example 16: Awraamoff's Septimal "Just" Scale

Intervals between successive tones:		$\frac{9}{8}$	$\frac{64}{63}$	$\frac{21}{20}$	$\frac{25}{24}$	$\frac{21}{20}$	$\frac{64}{63}$	$\frac{9}{8}$	$\frac{16}{15}$	$\frac{15}{14}$	$\frac{49}{48}$	$\frac{21}{20}$	$\frac{16}{15}$
Pitch names given by Awraamoff:	F	G	Y	A♭	A	B	B♭	C	D♭	Δ	Ɛ	E	F
Intervals upwards from F:	1	$\frac{9}{8}$	$\frac{8}{7}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{21}{16}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{12}{7}$	$\frac{7}{4}$	$\frac{15}{8}$	$\frac{2}{1}$

The weaknesses of this scale are evident. The intervals vary in size from 9:8 to 64:63 and include seven different sizes. Absolutely no scale patterns and very few harmonic patterns are transposable at all, and these only to a few pitches within the system. Awraamoff's concept of tonality is evidently quite static. With the exception of the interval between Ab and A (25:24) and the interval between Δ and Ɛ (49:48), the intervals of Awraamoff's

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[108] Awraamoff, A. W., Jenseits von Temperierung and Tonalität, Melos I, p. 131. Awraamoff's system has the dubious distinction of having been called worse than "the poorest tuning system shown in this book" by Barbour, Tuning and Temperament, p. 24.

scale would fit fairly comfortably into 41-tone equal temperament. Exactly as it stands, however, Awraamoff's entire scale could not be approximated by any temperament within the range of this study.

Adriaan Fokker has recently produced a 12-tone genus which achieves what Awraamoff sought to obtain. Fokker combines four septimal tetrads 4:5:6:7, the first two built harmonically on F and C, the other two built ornamentally downward from F# and B.[109] The sixteen tones thereby obtained include four identities, thereby reducing the number of distinct pitches to twelve. The result involves much less dislocation in the tones of present tuning practice than does Awraamoff's system. The five constructing intervals range in size from 28:27 to 15:14, involving a range of less than 2 to 1 and only slightly greater than the range of constructing intervals in the traditional just 12-tone scale. The entire system fits reasonably well into 31-tone temperament with the single intervals representing 2 or 3 units, generally somewhat extended or shortened. Fokker's 12-tone septimal scale is shown in Example 17.

It should be emphasized that Fokker's septimal 12-tone scale is offered as one of many possibilities, almost as a curiosity, while Awraamoff's curiosity is suggested in all seriousness.

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[109]Fokker, Just Intonation, p. 40.

Example 17: Fokker's Diatonic Genus with 4 Added Sevenths

Intervals between successive tones:	$\frac{15}{14}$ $\frac{21}{20}$ $\frac{28}{27}$ $\frac{15}{14}$ $\frac{10}{9}$ $\frac{135}{128}$ $\frac{16}{15}$ $\frac{15}{14}$ $\frac{28}{27}$ $\frac{21}{20}$ $\frac{15}{14}$ $\frac{16}{15}$
Pitch names suggested by Fokker's signs:	C C♯ D E♭ E F F♯ G G♯ A♭ B♭ C
Intervals upwards from C:	1 $\frac{15}{14}$ $\frac{8}{7}$ $\frac{7}{6}$ $\frac{5}{4}$ $\frac{9}{7}$ $\frac{45}{32}$ $\frac{3}{2}$ $\frac{45}{28}$ $\frac{5}{3}$ $\frac{7}{4}$ $\frac{15}{8}$ 2

A system somewhat similar to Awraamoff's and Fokker's is offered by the Englishman Perrett.[110] His system rightfully belongs among the proposals for multiple division which are considered in succeeding chapters (see Chapter 7). The basis for Perrett's system is the construction of two diatonic scales a septimal semitone (21:20) apart. It is quite understandable that theorists who are interested in the 7th partial are unlikely to be satisfied with any 12-tone system, and Perrett's 14-tone basic scale (he later extends it to 19 tones) is but a first step away from 12-tone thinking under the influence of the 7th partial.

The various intervals created by the 7th partial have the following discrepancies in 12-tone temperament:

7:4	32 cents too large
7:5	18 cents too large
7:6	34 cents too large
9:7	36 cents too small

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[110] Perrett, Some Questions of Musical Theory, 1926-. His scale is in the appendix to Vol. II.

Nonetheless, many theorists have held that the seventh partial explains at least some of the phenomena of the music of the current repertoire. The dominant seventh, in particular, has frequently been attributed to the partials 4, 5, 6, and 7. That many advocates of the expansion of musical resources should argue, as does Fokker, that 4:5:6:7 is the "primary chord" of music,[111] is hardly surprising. Even some of the theorists who have been loath to accept the validity of the 5th partial have accepted the 7th partial as a musical reality. A much-cited survey by Cornu and Mercadier late in the 19th century, which showed a preference for artificial thirds in place of the 5th partial except in the harmonic use of the major third, showed a general preference for the natural seventh in dominant harmony. More than that, Brandsma, who agreed with Cornu and Mercadier about the 5th partial, considers the interval 7:4 to be present in the rich harmonies of Wagner and responsible for their stunning ("glänzende") effect.[112]

Brandsma's thesis that the interval 7:4 pervades Wagner's harmonies leads to interesting speculation about the invertibility of the sonority 4:5:6:7. The smallest

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[111] Fokker, Just Intonation, p. 10.

[112] Brandsma, op. cit., p. 356. Further emphatic support of the role of the 7th partial in dominant harmony is offered by L. Norden in A New Study of Intervals, Organ Institute Quarterly, Vol 5:I, page 31.

rational integers which can be used to indicate the inversion of this sound are 60:70:84:105. Nevertheless, Fokker uses such sonorities, which he calls chords of the sub-seventh, as does Harry Partch.

The speculation arises from the use by Wagner of a vast number of half-diminished seventh chords. The half-diminished seventh, in traditional practice, is the exact inversion of the dominant seventh and is therefore to the chord of the sub-seventh precisely as the dominant seventh is to the chord 4:5:6:7. Brandsma never mentions the half-diminished seventh explicitly. Elsewhere in his article, he states that the diminished seventh should consist of three Pythagorean minor thirds, leaving one to wonder whether he considers the half-diminished seventh to be based on different lower intervals than the diminished seventh. It is possible, but hardly likely, that Brandsma meant the half-diminished chord to consist of two Pythagorean minor thirds with a natural seventh perched on top, 381 cents above the diminished fifth. Owing to his disinclination for the 5th partial, it is possible that the above would be his preference over the more logical $\frac{1}{7}:\frac{1}{6}:\frac{1}{5}:\frac{1}{4}$ or 60:70:84:105.

While the interval 7:4 has been singled out by some writers such as Brandsma, 7:5 and 7:6 have also drawn the particular interest of others. Perrett finds in the tritone 7:5 (583 cents) great beauty which he finds lacking in the

equal-tempered substitute.[113] Perrett cites Tartini in support of his conviction that 7:5 is the proper ratio for the augmented fourth.

7:6 has been offered by Awraamoff, Fokker, and others, as the lower interval of an important septimal minor triad, 6:7:9. This sonority is offered by Awraamoff as the explanation for the supertonic chord in just intonation (supposedly thereby ending the riddle of its apparent dissonance). It is difficult to see the basis for Awraamoff's assertion. The chord 27:32:40, which is the chord of the second degree of the standard just major scale, bears no more of a resemblance of 6:7:9 than it does to 10:12:15, the standard just minor triad. While 32:27 is a comma smaller than 6:5, it is more than a comma larger than 7:6, and the septimal minor triad, like the traditional minor triad, contains a perfect fifth which, if used for supertonic harmony, must throw some other fifth out of tune.

The sonority 6:7:9, like the sonority 4:5:6:7, has been cited considerably more often than its inversion, leading one to question anew whether the sonorities extrapolated from the harmonic series are, in fact, invertible. Partch and Fokker repeatedly insist that they are, but,

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[113] So enamored is Perrett of the sound of 7:5, that he postulates the invention of the enharmonic genus in ancient times to a chance discovery of this interval by Olympus, owing to a slight mistuning of his lyre. Some Questions of Musical Theory, p. 3.

outside of their pages, one is unlikely to find any reference to a septimal major, 14:18:21. In experiments related to the writing of this paper, the septimal minor chord was enthusiastically received while the septimal major was not. Those who are as argumentatively inclined as the many distinguished theorists who have debated in the past the invertibility of the 3rd and 5th partials will have a fine new bone to chew upon should the 7th partial assume a greater role in the music of the future.

The Web of 5ths, 3rds, and 7ths: The addition of the harmonic seventh to the web of thirds and fifths adds a new array of possible intervals which can be used in place of semitone, diesis, and comma. The proliferation of these intervals was regarded as a great threat by Ariel, and their absorption into a musical system does pose a considerable problem. Example 18 shows a few of the most prominent of these new intervals.

The four intervals which are bracketed in Example 18 are the ones Fokker uses as constructing intervals for his 31-tone system. The bottom one is one of his defining intervals, others of which include the syntonic comma, 21.5 cents, and the 8-cent interval, 1029:1024, consisting of three harmonic sevenths and a fifth, minus three octaves.[114]

The two near-identities (225:224 and 1029:1024) are

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[114]Fokker, Les Mathématiques et la Musique, p. 26.

Example 18: Septimal Intervals

Ratio	Size in cents	Description
15:14	118.4	Perfect fifth plus major third, minus natural seventh.
21:20	85	Perfect fifth plus natural seventh, minus octave and third.
28:27	63	Natural seventh minus a Pythagorean major sixth.
36:35	49	Two perfect fifths, minus a major third and natural seventh.
49:48	36	Two natural sevenths, minus an octave and a fifth.
50:49	35	An octave and two major thirds, minus two natural sevenths.
64:63	27	Two octaves, minus two fifths and one natural seventh.
225:224	7.7	Two fifths and two thirds, minus an octave and a natural seventh.

in a somewhat less perfect way to septimal just intonation what the schismatic relationship is to senary just intonation. But the 7th partial adds a third dimension to the web or tone-lattice as Fokker calls it. Because of the added difficulties inherent in this third dimension, nobody has as yet proposed a septimal semi-closed, "rounded" just system comparable with Jones' or Groven's senary just systems. Septimal schismatic temperaments do not, as yet, exist. There are, on the one hand, systems such as Perrett's and Partch's (see Chapter 7) which incorporate

the 7th partial but make no attempt at complete or near-complete mutuality (transposability). On the other hand there is 31-tone temperament with its close approximation for the intervals of the 7th partial, as is shown in Example 19, below.

Example 19:	Interval	Error
Errors of Septimal Intervals in 31-tone Temperament	7:4	1.1 cents
	7:5	2.1 cents
	7:6	4.1 cents
	9:7	9.3 cents

While no semi-closed just system specifically designed to include the seventh partial has been advanced, Bosanquet did not neglect to suggest methods of approximating the seventh partial within his senary systems. The standard minor seventh, 16:9, when flattened by a Pythagorean comma, becomes almost equal to the natural seventh.[115] It may be implied from this that the comma-layer systems of Helmholtz, Bosanquet, Eitz, Groven, and others can offer a close approximation of the 7th partial. However, Bosanquet's approximation is 4 cents in error, and another 2 cents must be added to the error when the more usual syntonic comma is substituted for the Pythagorean. The 6-cent approximation for the natural seventh in many of the systems of fifths and thirds may well be worth noting, although this might still

[115] Bosanquet, op. cit., p. 41.

be an excessive discrepancy for an interval as unfamiliar as the natural seventh.

Meantone temperament offered an exceptionally fine approximation of the natural seventh in the form of its augmented sixth. This is forcefully brought out by such early theorists as Kirnberger, and by Bosanquet[116] and Kornerup.[117]

The availability of the 7th partial as an overtone for brass instruments has been exploited occasionally as by Rossini,[118] Beethoven,[119] and more recently by Britten.[120] Bartok is cited[121] as having advocated its use, arguing that folk-practice and creative necessity justify its incorporation into musical theory.

The possibility of making the intervals of the 7th partial available at any scale degree in any musical context,

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[116] Ibid., p. 42.

[117] Kornerup, Die Vorläufer der gleichschwebenden Temperatur, pp. 1-2.

[118] In a fanfare cited by Gevaert, Histoire et Théorie de La Musique de l'Antiquité, p. 315.

[119] Trio of Eroica Symphony, third movement. It is of course played higher today, but the concert Db's for third horn are produced by the seventh partial and were probably played "low" since the usual practice was to lower more than to raise the natural pitch by hand. Furthermore, these Db's sound very beautiful when played "low."

[120] In the Serenade for Tenor, Horn, and Strings, Prelude and Postlude.

[121] By Fokker in La Gamme...., page 152. Article cited is from the Hungarian publication Uj Idok.

although rejected by some advocates of multiple division, is to others the prime reason for looking beyond the horizon of the 12-tone system.

THE HIGHER PARTIALS

In the expansion of the field of usable elements from the harmonic series, once the 7th partial is accepted, the 8th, 9th, and 10th partials follow without difficulty. The 8th and 10th partials are, of course, replicas of the fundamental and of the 5th partial, while the 9th partial is derived from two superimposed fifths, and is generally conceded to be the basis of the supertonic degree of the just scale.[122] The next controversial point in the expansion of the usable range in the harmonic series is the 11th partial and, as in the case of the 7th partial, the dispute has been fierce, if waged on a more limited scale.

The use of intervals based on ratios involving the number 11 has a long history. Students of various ancient and primitive cultures have taken measurements of pipes and discovered in places as diverse as Greece and Norway evidence of a preference for arithmetical division of the pipe

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[122] Certain theorists, such as Ariel and Kornerup, who tend to deny the musicality of the 7th partial as well, claim the supertonic to be 10:9 above the tonic rather than 9:8. There is still a 9 in the ratio, however, and at some point all "just" diatonic scales must contain two consecutive fifths, creating together the relationship that characterizes the 9th partial.

by twelfths of its length which produces the pentachord $\frac{1}{12}\cdot\frac{1}{11}\cdot\frac{1}{10}\cdot\frac{1}{9}\cdot\frac{1}{8}$. Erik Eggen, the Norwegian musicologist, considers this pentachord, as well as its inversion, 8:9:10:11:12, to be present in the musical scales of many peoples, including the Norwegians themselves.[123] Barbour lists a scale based on the tetrachord $\frac{1}{12}\cdot\frac{1}{11}\cdot\frac{1}{10}\cdot\frac{1}{9}$ as being among the Greek tunings listed by Ptolemy.[124] The interval 12:11 is also cited by Barbour as the middle interval of the tetrachords in Ptolemy's Chromatic Syntonon.[125]

According to Partch, the Arabian lute-player Zalzal recognized the 11th partial as an element in his music, and the Renaissance theorist Vicentino accepted the 11th partial in theory if not in practice.[126] A number of writers have cited the 11th partial as the basis for quarter-tone music (24-tone equal temperament) owing to the close relationship between the intervals 12:11 (150.6 cents), 11:9 (347.4 cents) and 11:8 (551.3 cents) and the corresponding intervals in 24-tone temperament (150, 350, and 550 cents respectively). Partch and Eggen point out

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[123] Eggen, Erik, Zur Entstehung und Entwicklung der Skala, pp. 104-5. Eggen notes a high 4th in Norwegian folk-songs and considers this tone to represent the 11th partial in the diapente 8:9:10:11:12. He elaborates on this in his Skala-Studier.

[124] The Diatonic Heziolon. Tuning and Temperament, p. 21.

[125] Ibid., p. 18.

[126] Op. cit., p. 251.

this relationship, but perhaps more significant is the affirmation of the role of the 11th partial in quarter-tone music by Ivan Wyschnegradsky, one of the most ardent of the quarter-tonists. In his Manuel d'Harmonie À Quarts de Ton, 1933, Wyschnegradsky cites the 11th partial as the best basis for 24-tone temperament.[127] He suggests further, however, that it is also possible to regard the 7th partial as responsible for 19/24 (a discrepancy of 18 cents), the 13th for 17/24, and the 29th through 41st partials as the basis for the complete scale from 21/24 through the next 9/24.

Wyschnegradsky's rather free use of what are probably unperceivable relationships suggests that he is merely looking for an acoustical basis for a pre-existing musical system. Partch, however, creates his musical system out of a determination, a priori, to include the 11th partial. Partch makes some effort to round out his system by adding tones to fill the largest intervals, but he does not try to approximate a closed tempered system, although he takes the trouble to calculate that the harmonic series through 11 can be reasonably approximated by a temperament of 113 tones to the octave.[128] Such a number is too high even for Partch.

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[127] Wyschnegradsky, op. cit., p. 23.

[128] Partch, op. cit., p. 307. For a critical evaluation of 113-tone temperament, see chapter 6.

Lindsay Norden speculates that the 11th partial may well be the next to be brought into the body of music, although he does not consider it to have been of any importance in the art of harmony up till now.[129] Schönberg, on the other hand, regards the 11th partial, as well as the 13th, as basic to the chromatic scale.[130] Considering Schönberg's known views on the necessity of overcoming traditional ways of hearing triadic relationships, the theory enunciated in his article, Problems of Harmony, relating all tones in the 12-tone system to the partials of I, IV, and V, is quite astounding.

Example 20: Schönberg's Acoustical Source

Partial:	1	2	3	4	5	6	7	8	9	10	11	12	13
	C	C	G	C	E	G	Bb	CC	D	E	F#	G	Ab
	F	F	C	F	A	C	Eb	F	G	A	B	C	Db
	G	G	D	G	B	D	F	G	A	B	C#	D	Eb

Schönberg's espousal of partials up to the 13th as being present in 12-tone temperament is attacked quite vehemently by Yasser.[131] Yasser points out that the C# derived as the 11th partial of G represents a pitch almost a semitone from the Db representing the 13th partial of F,

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[129] A New Study of Intervals, p. 38.

[130] Schönberg, Problems of Harmony, Modern Music XI, p. 170.

[131] Yasser, A Letter from Arnold Schönberg, AMS Journal, 1953.

yet both are represented by the same pitch in equal temperament. In spite of discrepancies of nearly 50 cents, Schönberg reaffirms his preference for equal temperament, likening natural intonation to nudity and temperament to presentable dress . . . possibly the most evocative defense of equal temperament that ever has been attempted.[132]

If Schönberg pays a possibly excessive obeisance to the upper partials through 13 in his concept of the 12-tone system, Yasser does the same with respect to the same partials as the basis for the consonant units of his 19-tone system. The errors of the intervals derived from some of these partials sometimes exceed 20 cents and approach the limit of possible error in 19-tone temperament (about 31.6 cents). Examples of such errors which Yasser himself has calculated[133] include 13:8, 22 cents, 11:7, 24 cents, 12:11, 24 cents.

Since the harmonic series is endless, any finite musical system must draw its limits. The Pythagoreans allow no primes higher than 3. Many have been found who draw the line after 5, or 7. Partch, claiming his reasons to be practical and arbitrary rather than based on an immutable theoretical concept, stops after 11. Yasser and Schönberg build on very loose approximations of the

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[132] Yasser, Ibid., p. 55.

[133] Unpublished letter from Joseph Yasser to John Redfield, May 9, 1933. His calculations, in centitones, have here been converted into cents.

partials through 13. One rather exceptional group of English theorists, starting with MacFarren, have held the 17th partial to be an important musical element[134] even while rejecting the 11th and 13th. They may have been influenced largely by the close approximation of the 17th partial in 12-tone equal temperament. Perrett, however, who also espouses the 17th partial[135] while rejecting both the 11th and the 13th, is a sharp critic of 12-tone temperament and an advocate of multiple division. He calculates that with the primes 3, 5, 7, and 17, 171-tone temperament is the nearly perfect scale (see chapter 6).

A possible weakness in any musical theory involving accurate renditions of the upper partials as an objective, lies in newly found data showing the overtones of actual sounding bodies to represent their theoretical pitches with increasing inaccuracy as their distance from the fundamental increases. The noted acoustician, Robert W. Young, has dealt with this phenomenon extensively,[136] and it is possible that further exploration may bring into question many acoustical arguments for just systems involving the higher partials. However, inharmonicity, as the phenomenon

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[134] Shirlaw, The Theory of Harmony. The 19th partial is mentioned by Cuseley according to Shirlaw, as the basis for the true minor third.

[135] Perrett, op. cit., p. 149.

[136] See Young's article in Groves Dictionary of Music and Musicians, Tuning, Piano.

is called, affects octave replicas of lower partials as well, a fact which has not destroyed the role of the octave in music, or even of the quadruple octave. Inharmonicity is cumulative as the distance from the fundamental increases, and therefore is relatively slight in its effect on the superparticular ratios (that is, ratios comprised of numbers only one removed from each other such as 13:12). If 12:11 is deemed an aesthetically desirable phenomenon, the fact of inharmonicity need not affect its achievement. Psychologically, however, the knowledge of inharmonicity is likely to prove a handicap to favorable consideration of the upper partials; one can already hear the question "Why consider them when they are bound to be out-of-tune anyhow?"

Despite the unquestionably satisfying melodic character of the series 8:9:10:11:12, the addition of the 11th partial to musical systems involves such complications that venturesome pioneers such as Partch who really use it will probably long continue to be outnumbered by those who cite it to justify such diverse phenomena as 12- (Schönberg), 19- (Yasser), and 24- (Wyschnegradsky) tone temperaments whose substance, in most cases, has little to do with the 11th partial. Nevertheless, so broad are the possibilities of multiple division that, as Partch has shown by example, it affords ample room to those who would explore partials as high as 11, and possibly even higher.

THE ACOUSTICAL POSITION OF 12-TONE TEMPERAMENT

Every interval in 12-tone temperament has, as the foregoing shows, been attacked at one time or another. The octave has been attacked for being too perfect; the fifth both for being too perfect and not perfect enough. The major third has been attacked in theory for being too large, but in practice it has often been made still larger. Finally, 12-tone temperament has been attacked for making no provision for intervals involving the numbers 7 and 11.

12-tone temperament was more a discovery than an invention, since its properties were inherent in the condition of "12-ness" long before Werckmeister proposed the "well-tempered" scale. Werckmeister may, in fact, not be the decisive advocate of 12-tone equal temperament at all[137] but, whatever the case, its qualities reflect a fundamental approach of his, which is to regard as most important among the ratios those which are closest to unity.[138] Whatever can justly be said against 12-tone temperament it represents with greatest fidelity those intervals whose ratios

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[137] Dupont and most of the traditional sources cite Werckmeister as the principal exponent of 12-tone temperament, but Barbour holds that Werckmeister never recommended precisely equal temperament at all, but rather a rough approximation. The question is not essential here, because, in the totality of the field of possible tuning systems, Werckmeister's and 12-tone equal temperament cannot be far apart. And 12-tone temperament does realize Werckmeister's premise.

[138] "Je näher eine Zahl der Unität je vollkommener dieselbe ist." Quoted by Dupont.

most closely approach unity. Example 21 shows how the discrepancies in the representation by 12-tone temperament of small-number intervals increase as the highest prime number in the ratio increases.

Example 21: Discrepancies in 12-tone Temperament

Interval class:	Average discrepancy in cents	Average discrepancy in % of possible error
Prime class 2: 2:1	0	0%
Prime class 3: 3:2 (inversions not shown)	2	4%
Prime class 5: 5:4, 6:5, 9:5	14	28%
Prime class 7: 7:4, 7:5, 7:6, 9:7	30	60%
Prime class 11: 11:8, 11:7, 11:6, 11:9, 11:10	40	80%

The closeness of 3:2 to 7/12 is quite remarkable, especially when one considers that in order to find an equal temperament with an approximation of the fifth which is closer, one must proceed as high as 29 tones to the octave. Therefore, if one holds, with J. Murray Barbour and Fritz A. Kuttner that[139] "superparticular ratios beyond the limit 7:8 cannot have musical value . . . there is even reason to doubt the musical merit of a superparticular ratio as low as 5:4," then 12-tone temperament seems more than an adequate system but rather almost a perfect one. It is to those who regard a close approximation to the 5th

[139] Notes to The Theory of Classical Greek Music, op. cit., column 10.

partial or any kind of approximation of the 7th partial as desirable that 12-tone temperament must appear inadequate. For such musicians as desire closer approximations for the 5th and 7th partials or higher, multiple division of the octave is justified on acoustical grounds.

THE ACOUSTICAL POSITION OF 19-TONE TEMPERAMENT

A shift from 12- to 19-tone temperament would involve the sacrifice of some of the purity of the fifth in return for thirds which are much closer to just thirds than their counterparts in 12-tone temperament. The intervals of the 7th partial are represented more faithfully in 19- than in 12-tone temperament when measured in cents, but only very slightly so when measured on the possibly more realistic basis of percentage of the maximum possible error. Example 22 shows the discrepancies of the groups of basic intervals in 19-tone temperament and compares them to the corresponding errors in 12-tone temperament.

There is improvement over 12-tone temperament for all interval classes except the first, not only in the total size of the discrepancy but also in the percentage of possible error. The improvement is insufficient to provide a satisfactory basis for the 7th or 11th partials. It is in the region of the 5th partial that 19-tone temperament scores its most important gains. At the expense of the loss

Example 22: Discrepancies in 19-Tone Temperament

Interval class	Average discrepancy in cents		Discrepancy in % of maximum possible error	
		Improvement		Improvement
Prime class 3: 3:2	7	-5	24%	-20%
Prime class 5: 5:4, 6:5, 9:5	5	9	14%	14%
Prime class 7: 7:6, 7:5, 7:4, 9:7	15	15	48%	12%
Prime class 11: 11:6, 11:7, 11:8, 11:9, 11:10	23	17	74%	6%

of the great accuracy of its fifths, 19-tone temperament eliminates the rather large errors in the 12-tone thirds and can easily, therefore, be considered an acoustical improvement . . . or setback depending on one's point of view.[140]

[140] For a more detailed treatment of this subject see Chapter 12.

CHAPTER 2

HISTORICAL PRECEDENTS FOR MULTIPLE DIVISION

"Greek music of the best period . . . was a thing of beauty unsurpassed and unattained in any subsequent age."[1] Opinions of this kind lead their holders to seek out ways of reestablishing the beauties of bygone eras. The alleged beauty may be of the music of ancient Greece, of the Renaissance, of the Baroque, or of a non-Western culture. In general the pattern of the arguments is the same. The older system gave way to newer systems not owing to any inferiority of its intervals or sonorities. Quite the contrary, the older system gave way to newer, coarser systems because the older system was not flexible enough to satisfy the expanding demand for harmonic and modulational resources. Thus, the argument continues, a better intonational system was lost because musicians of an earlier epoch were incapable of adding the number of extra tones to the octave which might have been necessary to preserve the advantages of their intonational systems while expanding harmonic resources. Today, however, with our vastly increased technical resources, it would be possible to recreate the beauties of earlier intonational systems without sacrificing the present modulational possibilities demanded by musicians. By this line of reasoning, multiple

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[1]Perrett, op. cit., p. 7.

division becomes a potential time-machine, bringing back the lost intonational subtleties of the past to be enjoyed with or without the full sophistication of the present. Chief among the systems of the past that recent advocates of multiple division have singled out to imitate have been the harmoniae of the ancient Greeks, the just intonation of the Renaissance, and the meantone systems of the late Renaissance and the Baroque.

THE GREEKS

Musical theory today, in studying the theory of the Greeks, seems to be interested primarily in the Greater-Perfect System and the network of modes which theorists of the Christian epoch tried unsuccessfully to imitate. As far as intonation is concerned, the Greek mathematician and theorist Pythagoras is recognized for his system of tuning by perfect fifths, a practice still widely adhered to today. Pythagorean intonation is, as Partch points out, a very direct ancestor of 12-tone equal temperament, a temperament in which the fifths are nearly pure at the expense of the other small-number intervals.[2] But while large numbers of theorists have concerned themselves with the diatonic modes and with Pythagorean intonation in Greek music, a smaller group has concerned itself with a much thornier area in

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[2]Partch, op. cit., p. 280.

Greek musical theory: the genera, enharmonic and chromatic as well as diatonic; and the tuning systems enumerated by Ptolemy as part of the Greek practice.

The neatness of Pythagorean systematization is weakened when applied to the chromatic genus, and Pythagorean tuning breaks down completely when an attempt is made to apply it to the enharmonic genus. In standard explanations the chromatic tetrachord is diagnosed as "semitone, semitone, sesquitone."[3] But in Pythagorean tuning the size of the "semitone" is not precisely half the size of the tone; it is smaller. It is with the second semitone that the difficulties of the Pythagorean system commence. If both semitones are 90 cents (the recognized size of the Pythagorean semitone) then the remaining interval in the tetrachord must be 316 cents, the interval subtended by no less than 9 fifths. This is far less economical than the diatonic tetrachord which requires but 5 fifths. The seven-tone diatonic scale is produced from a series of six contiguous fifths. The seven-tone chromatic scale requires twelve fifths and is not a self-contained completed system when produced by fifths, as is the diatonic scale.

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[3]In order to conform to the pattern established elsewhere in this paper, I am counting from the bottom up and hope that this will not cause too much confusion in this chapter since, with the music of the Greeks, it is more customary to count from the top down.

The Pythagorean's problems are compounded when he tries to explain the enharmonic genus as based on fifths alone.[4] The standard explanation of the enharmonic genus is "quarter-tone, quarter-tone, di-tone." How large is the quarter-tone to be? Assuming, for a beginning, that the large interval is to be the usual size of the Pythagorean third, each of the quarter-tones would be 45 cents. This interval, however, does not exist within the Pythagorean series, although a fair approximation is offered by the double-comma (about 47 cents) subtended by 24 fifths. An approximation is, however, against the precise spirit of the Pythagoreans. If the large interval is to be the size of the third produced when 8 fifths are deducted from 5 octaves, 384 cents, then each quarter-tone must be 57 cents in size. This interval is also not produced within the Pythagorean system. Therefore the enharmonic genus cannot consist of equal quarter-tones produced by the Pythagorean process.

Evidence seems to suggest that two radically conflicting views on musical intonation existed side by side for quite some time; the one was Pythagorean and excluded all genera other than the diatonic; the other was based on

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[4]Perrett states, "The problem which the Pythagoreans failed to solve was how to divide a major semitone melodically and harmonically." He adds, "The correct solution gives us an 'open sesame' back to the world of genuine and beautiful Greek music." op. cit., p. 11.

various combinations of small-number ratios and included all three genera as well as hybrids of them.[5] Besides Pythagoras, Aristoxenos of Tarent is considered by many as a disciple of simplification, although some of his writings and those of his disciples leave his position somewhat ambiguous. Aristoxenos is known for suggesting the possibility of 12-tone temperament, which makes him possibly its first advocate. He is also supposedly known for his diatribes against the harmonists, who advocated music based on small-number ratios; this is one of our ways of learning that the harmonists did exist, and that they did have some influence.[6]

The harmonists seem to have had a martyr in Timaeus, whose devotion to the enharmonic genus and to small-number ratios was sufficient to have induced him to defy the populace of his community so that he was banished.[7] Katheleen Schlesinger has attempted to reconstruct his scale. The martyred Timaeus stands as a kind of culture-hero to those who feel cut off by the simplifications of 12-tone temperament much as Timaeus was cut off by the Pythagoreans.

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[5]Kathleen Schlesinger, a major musicologists who devoted much of her work to a study of Greek music, holds this view, as do many exponents of multiple division.

[6]Schlesinger and Perrett emphasize the apparent antipathy between Aristoxenos and the harmonists.

[7]Kinkeldey, Otto, The Harmonic Sense, M.T.N.A. Proceedings, 1927.

Positive evidence of a considerable impact by the proponents of small-number ratios exists in the writings of a number of theorists and historians, among them Archytus of Tarent, Didymus, Plutarch, and Ptolemy, whose catalogue of tunings has received wide circulation through its publication in Barbour's Tuning and Temperament. Ptolemy lived several centuries after the crest of Greek civilization and after the supposed decline of the non-diatonic genera. For this reason his catalogues are sometimes discounted as mere speculation based on the reading of what even in his day was ancient theory. Nevertheless, as reliable an authority as the late Otto Gombosi takes the view that Ptolemy's figures were based on actual practice.[8]

Probably the strongest evidence of the use of small-number intervals in Greek music comes from surviving instruments. In her major work, The Greek Aulos, Schlesinger represents measurements of many such instruments as proving conclusively that the wind instruments of ancient Greece were attuned to small number ratios. Gombosi also indicates quite a number of scales based on septimal intervals.

Along with questions of the extensiveness of the use and influence of small-number ratios and the non-diatonic genera, there is the question of their artistic quality as

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[8] Gombosi, Otto, Tonarten und Stimmungen der Antiken Musik, 1939, p. 103. "Man darf nicht vergessen dass sie nicht von Ptolemaios geprägt worden sind, sondern aus der Praxis stammen."

opposed to that of the music of the Pythagoreans. Perrett suggests that "Perhaps . . . Pythagoras and Aristoxenos between them were the death of music as an art."[9] He admits that the contrary view--that they helped clarify and ennoble the art--is more prevalent. Nevertheless, no less an authority than Gevaert, perhaps the first of the great modern musicologists to have concentrated his attentions on Greek music, reports favorably on the enharmonic. He states quite emphatically that the best sources considered the enharmonic to be the genus par excellence, the most distinguished, the best ordered, and the most exact.[10] It is opinions such as this that interest many proponents of multiple division.

The 16th and 17th centuries produced a number of theorists who sought to reproduce the genera of the Greeks through multiple division on an experimental basis. Some of these theorists have, in turn, had a profound influence on contemporary advocates of multiple division. Vicentino, one of the first builders of a multiple-division instrument, is said to have attempted in theory a reconciliation of Greek ideas with 16th century practice.[11] Giovanni Battista

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[9]Perrett, op. cit., p. 9.

[10]Gevaert, Francois Aug., Histoire et Théorie de La Musique de L'Antiquité, 1875, p. 296. "Le jugement que les Grecs portent sur le genre enharmonique a de quoi nous surprendre davantage. Selon les écrivains les plus compétents 'c'est le genre par excellence, le plus distingué le mieux ordonné, le plus exact.' "

[11]Cited in Barbour, Tuning and Temperament, p. 117.

Doni in the 17th century built an elaborate organ with three keyboards, each tuned to a different mode. By a combination of tones from the different keyboards the genera were also obtainable.[12] Perrett ascribes the origin of the enharmonic genus to just such a combination of diatonic modes, although he proceeds quite differently from Doni. Where Doni separates the "tonics" of his modes by major thirds, Perrett separates them by the interval 21:20, derived from the interval 7:5 (a fifth lower) which Perrett considers an exceptionally fine consonance. Perrett, like Doni, constructs his modes in just intonation, and a curious feature in the interval construction is that he chooses, like Doni, to use different modes. In Perrett's scheme the modes are symmetrically complementary, the one being the inversion of the other.

Leonhard Euler was another 17th century theorist deeply interested in the Greek genera. Euler's ideas have been influential among an important group of 20th century theorists interested in multiple division. He reduced the various Greek scales to mathematical formulae involving combinations of thirds and fifths and sometimes sevenths. It is this systematic attempt to codify the practice of the Greeks which Fokker seizes upon as the basis for his 31-tone system. "Euler has given us our scale," asserts Fokker on several occasions. According to Barbour, Euler

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[12] Ibid., p. 111.

was also one of the first to suggest an instrument based on tiers separated in pitch by a comma.

Euler's greatest influence on advocates of multiple division derives from his method of constructing (or postulating the reconstruction of) genera from groups of intervals. The diatonic genus is, according to Euler, produced by the formula 3^3x5. The 3^3 means that from a starting pitch a cycle of three fifths is marked off (as for example from F -- to C, G, and D). The "5" indicates that for each of the notes produced a major third above is added (in the case of the pitches named above the added tones would be A, E, B, and F#). Euler's diatonic genus contains 8 pitches and yields not one but two diatonic scales (they are not identical).

Kirnberger, the great 18th century theorist, spoke out for the inclusion of the enharmonic intervals. His statement on the subject was made by Bosanquet the frontispiece for his Treatise. In Bosanquet's translation, it reads as follows: "Greater certainly would be the gain of Song if we really had the enharmonic intervals in our system. For then singers would accustom themselves, from their youth up, to sing correctly the smallest harmonic intervals, and the ear of the listener to appreciate them; and thereby would it be possible, in many cases, to make the expression of the passions very much stronger."[13]

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[13]Bosanquet, op. cit., Frontispiece.

The Greeks have supplied the imaginations of musicians with interest in the septimal intervals, the very small intervals of the enharmonic genus[14] and scales of unequal intervals produced by harmonic and arithmetic series. But they have provided even more as a mystique: an unknown music associated with one of the greatest cultures history has known. Those who interest themselves in this mystique are drawn quickly beyond the limiting sphere of the systems of tone relations of the present.

JUST INTONATION

"Chords formed by the notes ordinarily in use are much inferior in excellence to chords which are in perfect tune,"[15] states Bosanquet. The tuning known as just intonation permits the principal chords to be in "perfect tune." When the dominant is exactly a perfect fifth above the tonic and the subdominant a perfect fifth below, and the major triads of the tonic, dominant, and subdominant, contain both pure thirds and pure fifths, the scale thereby

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[14]Typical of the interest in the Greeks by advocates of multiple division is an article by Edoardo Cavallini, Il Pluricromatismo nell' evoluzione musicale, in the Rivista Musicale Italiana 1946, p. 130. Cavallini emphasizes the importance of the Greek enharmonic to contemporary multiple division, and reconstructs the extant fragment from the Orestes according to enharmonic theory with rather pleasing results.

[15]Bosanquet, op. cit., p. 6.

formed is said to be in just intonation. Technically, however, any scale based entirely on unaltered rational-number intervals is in just intonation.

Designed to make more perfect the consonances of the Renaissance, this tuning, involving two different sizes of whole-tone, had a short life-span owing to its inflexibility. Since the early death of just intonation, however, attempts at its resuscitation have filled volumes. Still more volumes have been filled in refutation.

A number of controversies regarding just intonation still smolder. They concern its history, its merits, and its present use, if any. How extensive was the use of just intonation? How early in Western history can it be found, and how long did it last? Was it generally discarded because it failed to do what it was supposed to do or because it failed to do what it was not supposed to do? And, finally, do choruses and brass instruments use just intonation when they are able to do so, as some writers allege? Although these questions are quite distinct from one another, they tend to be treated as one, because just intonation has managed to lend itself extraordinarily well to opinion-forming. Those who favor just intonation argue that just intonation had an exalted and extended history, that its replacement by meantone and then equal temperament was due to changes in the musical requirements rather than to any inherent weakness in just intonation. They further

argue that it is ideal, at least for harmony, and that it is still used for some music by the best choruses. Their opponents argue that the history of just intonation was, at best, short and undistinguished, that just harmonies sound insipid, that just intonation was condemned by its own inherent weaknesses, and that choruses foolish enough to try to use just intonation will go radically flat.

It is possible to make all kinds of hypotheses about the use of just intonation at the end of the Middle Ages and the beginning of the Renaissance. It is generally acknowledged that the Englishman Odington[16] claimed that the proper tuning for the thirds should be 5:4 and 6:5 in about the year 1300. Whether this represented theoretical wishful thinking or actual musical practice can only be guessed at. It is also quite possible that the 5:4 third represented a local practice. England may have used a just third before the continent; musicologists generally acknowledge that England, through the practice known as gymel, used the third as a consonance earlier than the continent.[17]

With Bartolomeo Ramos de Pareja in the 15th century

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[16]Kornerup, who likes just intonation, tends to emphasize Odington's role, while Barbour, who does not, tends to minimize it.

[17]Besides being possibly the first, England was probably the last country in the West to use the just third. Organs in meantone temperament were still extant and in use in England in the late 19th century. (Bosanquet)

begins a long series of theorists who tuned their monochords to just intonation. Barbour notes that Ramos cited expedient simplicity rather than musical necessity as the reason for including the intervals of the 5th partial in his tuning. Barbour also points out that Ramos' tuning is more weighted in favor of the perfect fifth over the third than are the later, more standard, just tunings. From this Barbour deduces that at the time of Ramos just intonation had behind it neither the support of widespread practice nor the emphatic endorsement of theorists on the basis of its superior sound.

As to the early end to the practice of just intonation, if such practice there was at all, Barbour points to the difficulty of singing the more chromatic works of Marenzio, Cipriano de Rore, and Gesualdo without instrumental accompaniment. Barbour points out in this connection that a capella did not mean "unaccompanied" in its earlier usage and that choruses must have adopted the intonation systems of whatever instruments were used as accompaniment.

Whether musicians unhampered by fixed intonations could distinguish just intonation at all is another subject for contention. Recent experimentation has indicated a wide span of preferences among musicians concerning the thirds to be employed. According to the preliminary findings in a study by Lewis Richardson at Indiana University,[18]

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[18] Conversation with Mr. Richardson. His findings will become part of an unpublished dissertation at Indiana University.

some excellent musicians demonstrated great inconsistency in the thirds they "preferred." Many theorists have postulated different thirds for melody and harmony and it is conceivable that some of Richardson's subjects may have been thinking "melodically" and "harmonically" on different occasions when they were tested. Fokker has recognized the difficulty contemporary musicians have at forming the intervals of just intonation, and has written a graded series of exercises aimed at developing these skills.[19]

Proponents of just intonation assert that the current general inability to measure thirds accurately is the direct result of atrophy caused by the long disuse of just intonation. At the same time other proponents of just intonation (in some cases the same ones)[20] insist that choruses do, indeed, use just intonation. There appears to be an inconsistency here.

A similar contradiction can be noted on the other side, however. Barbour and Kuttner deny that unaccompanied choruses use just intonation[21] and then proceed to attribute the tendency of unaccompanied choruses to fall in pitch to

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[19]Fokker, Just Intonation is the work in question. The exercises also attempt to train the musician to hear and use the seventh partial.

[20]Norden argues that choruses use just intonation and that those who don't prefer just intervals to tempered ones through the 7th partial demonstrate insufficient ear-training. Organ Institute Quarterly, 5:1, p. 31 ff.

[21]Introductory notes to Musurgia Records A-2, op. cit., columns 29-30.

the use of just intonation.

The argument that just intonation causes flatting is one of the classical and oft-repeated points in the case against the use of pure fifths and thirds. It is based on the double employment of the supertonic. In chords in which the supertonic is employed with subdominant function, the supertonic is derived by thirds from the subdominant, and has the value of 10:9 above the tonic (4:3 - 6:5 = 10:9). In chords of dominant function, however, the supertonic is derived by fifths above the dominant and has the value of 9:8 above the tonic (3:2 x 3:2 ÷ 2:1 = 9:8). Since the normal course of harmonic progression is from subdominant to dominant rather than the reverse, harmonic music is full of instances in which the supertonic is first sung (or played) as a member of subdominant harmony and held over in an upper voice to become a member of dominant harmony. Assuming that the pitch of the supertonic is held at a constant level, the pitch of the tonic must move downward as the interval between the two increases from 10:9 to 9:8. Thus, for every progression from ii to V in just intonation, where the supertonic is sustained at a constant pitch, the tonic falls by the interval 81:80 or 21.5 cents. In one of the great tours de force of this branch of musical theory, Gustav Engel has analyzed the entire score of Don Giovanni for progressions in which the double employment of held-over tones would force the music upwards or downwards. Engel

concludes that were Don Giovanni to be performed in its entirety in just intonation, the pitch would slip by about an augmented fourth in the course of the evening.[22] The many audiences which have heard choruses slip a whole tone in less than five minutes can hardly be alarmed at Engel's estimate.

In refutation Partch suggests that the supertonic need not be held constant. According to Partch, if the pitch of the supertonic is raised a comma during the progression ii-V, the result is not disturbing and the pitch of the original tonic is thereby retained. In any case, the tendency of choruses to drop in pitch may demonstrate that left to their own devices singers will tend to adopt some kind of just intonation.

Despite the vehemence with which Barbour and Kuttner attack just intonation, and especially its thirds, they leave the door open to those who would re-establish just intonation through multiple division when they conclude, "Of course it is the severe limitation of just intonation to 12 notes in the octave, with enharmonically equivalent notes forbidden, that has caused its outright rejection by musicians."[23] The record which accompanies these remarks

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[22] Engel, Gustav, Eine mathematisch-harmonische Analyse des Don Giovanni von Mozart, VMW, III, 1887, p. 491 ff.

[23] Op. cit., column 31.

amply demonstrates that more than twelve tones would indeed have to be available if just intonation were to be applied to music having any substantial harmonic range.

It is from just such a premise that Bosanquet and Fokker began their speculations on multiple division. It is also from this premise that the vast number of unequal scales such as Helmholtz' (24-tone), Eitz' (50-tone) and Groven's (36-tone) have arisen. Partch also makes just intonation his starting point, although insisting on the extension of the principle to include the 7th and 11th partials. Würschmidt and Ariel used just intonation as the theoretical basis for 19-tone temperament much as Zarlino, Mersenne and Rameau used just intonation as the basis for musical systems for which they too advocated temperament.[24] The coincidence of the just minor third with its equivalent interval in 19-tone temperament (the discrepancy is a small fraction of one cent) is one of the strongest arguments used for that particular system, especially by Ariel.

Technically, just intonation supplies the proponent of multiple division with a model whose consonant thirds differ substantially from those of 12-tone temperament. It also interests him through its greater variety of basic intervals, which require smaller subdivisions of the octave

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[24]The temperament was meantone in the former case, a sub-commatic temperament not unlike equal temperament in the second, and equal temperament in the third.

than 1/12. It provides him with such relatively minute intervals as 25:24 the difference between major and minor thirds (which Ariel considers to be the basic unit for 19-tone temperament) and 81:80, the difference between the major and minor whole-tones (which is widely held to offer the basic unit for 53-tone temperament).

But, as with Greek tunings, just intonation is equally important for the intangibles it brings to the modern proponent of multiple division. There is the mystique of small rational numbers, and the ideal of perfect consonance. And there is, as with Greek tunings, the association with a golden age in the history of the world--in this case with the Renaissance. The attempt to achieve just intonation in more complex music than was obtainable in the Renaissance has stimulated many projects in multiple division and will no doubt continue to do so.

MEANTONE AND OTHER SUB-COMMA TEMPERAMENTS

Used literally, the term "meantone temperament" applies only to Pietro Aron's ¼-comma temperament of the fifth, wherein the major thirds are exact and the major seconds (tones) are the mean (geometric) between 9:8 and 10:9.[4] In the more generic sense in which "meantone temperament" is sometimes used, it has come to mean any temperament of the fifth, for the purpose of improving the thirds, wherein

¢ 29:29 = 192.56¢ or $\sqrt[2]{\frac{9}{8}\cdot\frac{10}{9}} = 193.16$

the fifth is smaller than in 12-tone equal temperament. In this sense there are nearly as many varieties of meantone temperament as there are 16th, 17th and 18th century theorists combined. The generic meaning will be used here.

Meantone temperament figured heavily in the theoretical writings of the 16th through 18th centuries. Its importance arose out of the realization that pure fifths and pure thirds could not exist side by side in a musical system containing a small number of tones. Theorists, therefore, returned for convenience to the Pythagorean practice of producing the third by combining four fifths and deducting two octaves. However, whereas the Pythagoreans used perfect fifths, allowing the thirds to be a comma larger than the just thirds, the proponents of meantone temperament reduced the size of the fifths in order to improve the thirds. In Aron's ¼-comma temperament, the most significant and the only literally correct "meantone temperament," the fifths are altered about 5½ cents in order that the thirds can be pure.

The shift in emphasis from the perfect fifth to the major third as the basic unalterable interval within the octave is partly explained by the development of the third as a consonance in musical usage. But there is a logical reason for preferring meantone temperament to Pythagorean intonation. Since the third is composed of four fifths (less two octaves), a minute alteration of a single fifth

will have a fourfold effect on the size of the third. By making each fifth smaller by ¼ comma, the third is improved one whole comma, which is enough to make it perfect. Therefore, whereas a system containing pure fifths will have thirds in error by a whole comma, a system with perfect thirds will have fifths in error by only ¼ comma. If thirds and fifths are considered equally important in a musical system, then clearly it is better to choose the system involving ¼-comma of error than the system involving a discrepancy of an entire comma.

Other so-called meantone temperaments have arisen from attempts to compromise the differences between fifth and third, making neither perfect while holding to a minimum the disagreeable discrepancies in both. Still others have developed from the shortening of both the fifth and the major third in order to improve other intervals in the temperament, most notably the minor third. Meantone temperament is looked upon with disfavor by those who consider the pure fifth to be more important than the pure third or who look upon the pure thirds as inferior to altered thirds. Nevertheless, it represents an important phase in the history of intonation and one which saw the birth and development of harmonic practice.

There are, as will be shown, two important reasons why there is a close kinship between meantone temperament and multiple division. In the first place there is a

theoretical relationship between any given meantone temperament and an equal temperament involving multiple division. This is because meantone temperament is a perfect but incomplete system. The sharps on the one hand and the flats on the other are never joined together, each meantone system having its own particular size of "wolf" fifth where the two ends "meet." But it is possible to continue the series of fifths in theory until, with the aid of only negligible additional tempering, they do meet. This produces an equal temperament which may be thought of as the fulfillment or basis or background for the particular kind of meantone temperament. Theorists of the meantone period, such as Sauveur, actually did use multiple division as a theoretical basis for meantone temperament. Conversely, today, this relationship is used to justify multiple division as a means of restoring the advantages of meantone temperament.

The second close relationship between meantone temperament and multiple division is in the practical realm. Many of the proponents of meantone temperament actually conceived it for instruments possessing more than twelve keys. Renaissance instruments were built and amply described which made multiple division, particularly those systems involving 19- and 31-tones, a reality. As Charles Kent asserts,[25]

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[25]Kent, Charles, An Introduction to Tuning and Temperament, p. 46. This hitherto unpublished document is particularly valuable for the beginner due to the thoroughness of its information and the lucidity of its presentation.

referring to Aron's ¼-comma temperament, "If one were to design an instrument capable of playing the complete meantone system . . . (involving both the sharps and the flats in common use) . . . it would require 19 keys to the octave." It might be added that if one wished to make of ¼-comma temperament a closed system, he would require 31 tones to the octave.

Meantone temperament gradually gave way to 12-tone equal temperament (which is, itself, a form of meantone temperament when the term is used in its broadest sense). 12-tone equal temperament is the result of the alteration of the fifth by approximately 1/11 comma, which temperament leaves the third 7/11 comma too large, and the minor third 8/11 comma too small. There appears to have been a substantial period between the maximum influence of Aron's ¼-comma temperament and the triumph of 12-tone equal temperament when a compromise temperament of 1/6 comma, sometimes called Silbermann's temperament, enjoyed widespread use.[26]

Of importance to the present study are evidences of close historical ties between meantone temperament and theories of multiple division. Equally important is the question whether the deficiencies in meantone temperament which caused its downfall were of the kind that multiple

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[26] Sauveur undoubtedly meant this temperament when he says that the 55-tone temperament is in general use by musicians as the intervals are essentially the same. See Scherchen, The Nature of Music, p. 42.

division could have prevented. If it can be established that such close ties as have been suggested did exist, and if it can be further established that meantone temperament fell because of its "wolves"[27] and not because of its small fifths and thirds, then a strong historical case can be made for multiple division as a justifiable extension of meantone temperament.

Of particular relevance is the work of the deaf-mute theorist, astronomer and mathematician, Joseph Sauveur. Although himself unable to hear, he worked with the constant corroboration of musicians, and obtained such insight into music that Hermann Scherchen was led to place him with Bach and Beethoven among the great innovators of music.[28] Sauveur's writings on music appear in several articles in the Mémoires of the Royal Academy of Sciences of Paris and, except for the brief summary by Scherchen, have not been translated into English. Scherchen alludes to Sauveur as a proponent of 43-tone temperament and as an opponent of 12-tone temperament. The latter is quite true, as the following statement, translated by Scherchen, bears out.[29] "This

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[27] That is, the bad fifths between the tones at the ends of the system. These could be corrected by multiple division, while the small thirds and fifths are basic to the meantone system and are not corrected by the addition of more tones.

[28] The Nature of Music. In this book, Scherchen honors Sauveur, Bach, and Beethoven with an essay apiece, using their transcending greatness and their physical disabilities as a central theme.

[29] The English is not by Scherchen but by William Mann; Scherchen's translation was to German. The Nature of Music, p. 43.

system (12-tone temperament) is preferred by our less competent instrumentalists; it allows one to transpose easily without changing of interval. But more sensitive musicians have rejected it, because its intervals deviate from the diatonic norm."[30]

It is also true that Sauveur proposes 43-tone temperament, but there is no evidence that he ever envisaged music with 43 actual tones to the octave. Sauveur evidently meant 43-tone temperament to be the theoretical basis for a meantone temperament involving the flatting of the fifth by 1/5 comma. The crucial statement by Sauveur which Scherchen passes over without comment is that "55-tone temperament is in universal use by musicians." It is obvious that taken in the literal sense this statement is absurd. 55 tones to the octave is a radical projection today and a 55-tone octave has never enjoyed "universal use." What Sauveur evidently had in mind is 1/6 comma temperament which, in the early 18th century, is known to have enjoyed widespread use. The fifth in 55-tone temperament is altered by almost precisely 1/6 comma, rendering 55-tone temperament the theoretical completion of the

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[30] "Ce systême à son usage chés les Joüeurs d'Instrumens les moins habiles, a cause de sa simplicité et de la facilité pouvant transposer les notes ut re mi fa sol la si sur telle touche qu'ils veulent, sans aucun changement dans les intervalles: mais les différences des intervalles de ce systême avec ceux du systême Diatonique juste étant trop grandes, les habiles Joüeurs d'Instrumens l'ont rejetté." Mémoires, 1701, p. 214.

meantone system involving fifths altered by 1/6 comma. By the same logic, the 43-tone temperament advocated by Sauveur can be construed as the theoretical completion of a meantone system involving fifths altered by 1/5 comma. Consideration of Sauveur's reasons for advocating this temperament bear out the assumption that this is what he intended. He lists 9 reasons:

1. It preserves the just minor second of the diatonic system.[31]

2. It preserves a major second and chromatic semitone that represent arithmetical means between the just alternatives.[32]

3. The deviations of the other intervals are equal, except that of the minor third which is double; equal discrepancies bother the ear less than do unequal ones.[33]

4. If the unit of 43-tone temperament is itself divided into 7 parts, all of the discrepancies fall very close to units of this smaller subdivision.

5. The most perfect interval, the 4th (5th), is better represented than the thirds.

6. 43-tone temperament is a mean between the other legitimate systems. Its whole-tone is an

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[31] This and the succeeding 8 points are taken from Table Generale des Sistêmes Temperez de Musique, Mémoires, 1711, pp. 516-17.

[32] This seeming error is clarified by Sauveur's sixth point.

[33] This point, somewhat alien to our age, is made repeatedly in the 18th century. Our 12-tone system has most unequal discrepancies as between 5th and 3rd, while the fifth and major third of 19-tone temperament are in error by almost precisely the same amount.

arithmetical mean between two minor and three major tones, precisely the proportion which appears in the octave.

7. The sum of all deviations of the intervals, taken positively, is least of all the systems.

8. Eptamérides (the smaller subdivision mentioned above in no. 4) are equal to the 301st part of the octave and represent ordinary logarithms.

9. I have tried all of the temperaments on my clavecin, and my colleagues have preferred the sound of 43-tone temperament.

Sauveur's third point, that he prefers equal deviations to unequal ones, is probably at the root of his preference for 1/5 comma temperament. The perfect fifth and the major third, representing the intervals from the fundamental to the upper consonant partials, are the important ones to most of the theorists. It is self-evident that 1/4-comma temperament favors the third completely while Pythagorean tuning favors the fifth. The arithmetic mean between them would be 1/8 of a comma, but the arithmetic mean is hardly valid here, since discrepancies of the third would then be four times as great as discrepancies of the fifth. Sauveur's 1/5 comma temperament is the equivalent of a geometric mean between the Aron meantone temperament on the one hand and Pythagorean tuning on the other. The third as well as the fifth is in error by 1/5 comma.[34]

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[34] According to Kent's figures in op. cit., the fifth in 43-tone temperament is 4.28 cents flat while the third is 4.39 cents sharp. The discrepancy between the two deviations is a reminder that 43-tone temperament is only an approximation (although a good one) for 1/5 comma temperament, for in the latter system the deviations would be exactly equal.

An aspect of Sauveur's case which emphasizes that his interest in 43-tone temperament was as a basis for a musical system rather than as a complete system is in his emphasis in the advantages of 43-tone temperament for measurement. 1/301 has been found by a number of theorists to be a useful unit of measurement because the logarithm of 2 (the octave) is 30 103. 1/301, called an éptaméride by Sauveur, is called a savart by others who use it. Use of savarts makes possible the measurement of intervals by ordinary logarithms of their ratios, without transposing by a complicated constant as in the case of cents or milioctaves. Theorists as far removed from the advocacy of 43-tone temperament as Würschmidt and Danielou have made use of the savart for purposes of measurement. But this has little relevance to the musical merits of 43 tones to the octave. Neither do Sauveur's other points, which stress the diatonic intervals but in no way pertain to a system containing many extra tones.

As Sauveur's 1/5-comma and Silbermann's 1/6-comma temperaments have their close equivalents in 43- and 55-tone temperaments, so Aron's ¼-comma temperament has its close equivalent in 31-tone equal temperament. The Dutch mathematician Huygens pointed out this equivalence in the 17th century. A practical connection between the two was established, at least for experimental purposes, by Vicentino, late Renaissance composer, theorist and musician extra-

ordinary. His archicembalo was tuned, he says, to the prevailing intonation of his day, which was ¼-comma temperament.[35] His instrument possessed 31 keys to the octave and, if it was attuned to the prevailing intonation of his day, was nearly equal-tempered. This is shown by example 23, below, in which the 17 pitches shown by Kent for the meantone system are contrasted with the equivalent pitches for 31-tone equal temperament. As will be observed, there is no discrepancy as large as a schisma. The two systems are nearly identical, so that one might say with full justification that ¼-comma temperament, which enjoyed widespread use, is the direct ancestor of 31-tone temperament, which has its very strong advocates today.

During the earliest period of meantone tuning, some theorists proposed to alter the fifth by more than ¼ comma in the interest of rendering the minor third and major sixth closer to the ratio 6:5. The minor third is created in these systems by three superimposed fifths. Where the fifth is pure the minor third, like its complement within the fifth the major third, is out of tune by a comma. For the minor third to be in perfect tune, the fifth must be altered by 1/3 comma, which will also be the error of the major third. Two important Renaissance theorists, Zarlino and Salinas, proposed such a temperament. It should be stressed that both theorists also proposed other temperaments and that they did not single out 1/3-comma temperament for special favor.

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[35] Barbour, op. cit., p. 117.

Example 23: Meantone and 31-tone Temperaments Compared[36]

31-tone temperament	Meantone tuning		Discrepancy
0.00	A	0.00	
38.71			
77.42	A#	76.05	1.37
116.13	Bb	117.11	0.98
154.84			
193.55	B	193.16	0.39
232.26			
270.97			
309.68	C	310.27	0.59
348.39			
387.10	C#	386.31	0.79
425.81	Db	427.37	1.56
464.52			
503.23	D	503.42	0.19
541.94			
580.65	D#	579.47	1.18
619.35	Eb	620.53	1.18
658.06			
696.77	E	696.58	0.19
735.48			
774.19			
812.90	F	813.69	0.79
851.61			
890.32	F#	889.74	0.42
929.03	Gb	930.79	1.76
967.74			
1006.45	G	1006.84	0.39
1045.16			
1083.87	G#	1082.89	0.98
1122.58	Ab	1123.95	1.37
1161.29			
1200.00	A	1200.00	

1/3-comma temperament can well be considered a direct forbear of 19-tone equal temperament. According to Barbour, Salinas never realized the possibility of closing his system through the use of 19 tones to the octave. Zarlino, who

[36]All figures are from Kent, op. cit.

proposed 1/3-comma temperament, but only as one of many and without the enthusiasm he reserved for ¼- and 2/7-comma temperaments, had a 19-tone instrument constructed, very probably the first of its species, but there is no evidence that he tuned it to 1/3-comma temperament at any time. Nevertheless, the correspondence between 1/3-comma temperament and 19-tone equal temperament is extraordinarily close, as Example 24 shows, and 1/3-comma temperament can be considered a direct ancestor of the later system.

Example 24: 1/3-Comma and 19-Tone Temperament Compared

19-tone temperament	1/3 comma temperament		Discrepancy
00.00	A	00.00	0.0
63.16	A#	63.52	0.36 cent
126.32	Bb	126.06	0.26 cent
189.47	B	189.57	0.10 cent
252.63			
315.79	C	315.64	0.15 cent
378.95	C#	379.15	0.20 cent
442.11	Db	441.69	0.42 cent
505.26	D	505.21	0.05 cent
568.42	D#	568.73	0.31 cent
631.58	Eb	631.27	0.31 cent
694.74	E	694.79	0.05 cent
757.89			
821.05	F	820.85	0.20 cent
884.21	F#	884.36	0.15 cent
947.37	Gb	946.90	0.47 cent
1010.53	G	1010.43	0.10 cent
1073.68	G#	1073.94	0.26 cent
1136.84	Ab	1136.48	0.36 cent

A number of compromises between 1/3- and ¼-comma temperaments were proposed. Zarlino's name is generally associated with 2/7-comma temperament, while the 18th century

English theorist Robert Smith endorsed 5/18-comma temperament. 50-tone equal division might be used to complete either of the above systems, as the fifth is very close to 696 cents in both. Smith's is the nearer one to 50-tone temperament, having been calculated with the 50-division in mind.

Smith is also on record as opposing 12-tone equal temperament which, during his lifetime, had begun to dislodge meantone temperaments from their position of supremacy. "The harmony in this system of 12 hemitones is extremely coarse and disagreeable."[37] Smith's and Sauveur's opposition at least suggest that the approaching triumph of 12-tone equal temperament was viewed with alarm by some musicians who fully understood its implications.

A number of modern advocates of multiple division have taken particuler interest in the theory and practice of meantone temperament. Fokker regards meantone temperament as having been a highly satisfactory system of intonation whose undoing was caused by the inconvenience of requiring separate keys for G# and Ab.[38] He is of the opinion that the music of Sweelinck sounds best when performed in meantone temperament.[39] Of the shift from

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[37]Smith, Harmonics, p. 166. (1759)

[38]Fokker, La Gamme... p. 152.

[39]Fokker, Ibid., p. 161.

meantone to equal temperament, Fokker writes, "There was a growing tendency to sacrifice the refinement of the variety of semitones in order to make them all equal and thereby to gain the complete freedom of modulation."[40]

Fokker acknowledges his debt to Huyghens who offered 31-tone temperament as a means of completing Aron's meantone temperament.[41] In Fokker's final summation, historical and acoustical considerations (the latter relating primarily to the 7th partial) share equally in his choice of 31-tone temperament. "Tricesimoprimal equal temperament has two faces. One is turned to the past. It recalls into being the all-but forgotten beauties of the music of the great old masters by restoring the temperament they breathed in. The other face is turned to the future, with its possibility of coping with new harmonies and with autochthonous music outside the compass of traditional secular Western music."[42]

Kornerup is another advocate of multiple division who is well aware of its close relationship with meantone temperaments. He considers Aron's ¼-comma temperament to be the forerunner of 31-tone temperament and Salinas' 1/3-comma temperament the forerunner of 19-tone temperament.[43]

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[40] Fokker, Equal Temperament with 31 Notes, Organ Institute Quarterly, Vol. 5:4, p. 41.

[41] Fokker, Un Orgue a Demitons Majeurs et Mineurs, Recherches Musicales, p. 182.

[42] Fokker, Equal Temperament with 31 Notes, p. 44.

[43] Kornerup, Die Vorläufer der Gleichschwebenden Temperaturen mit 19 oder 31 Tönen in der Oktave, p. 6. Kornerup attributes both meantone temperaments to Arnold Schlick, rather than to Aron or Salinas.

These two systems of multiple division are the ones in which Kornerup has the greatest interest.

Meantone temperament left behind specimens of instruments with "split-keys" and other devices for realizing more than 12 tones to the octave. Lacking, perhaps, the intangible prestige of the supposed Greek tunings or the illusion of perfection in just intonation, meantone temperament offers a background of practical multiple division as well as a theory suggesting greater multiple division. It is hardly surprising that advocates of multiple division such as Fokker, Kornerup and Bosanquet, having determined to their satisfaction that meantone temperament was not dropped because of its basic intervals,[44] cite meantone temperament as precedent for their proposals. While the split keys on the instruments possessing 13 to 16 tones per octave could push the "wolves" away from the door, they can be eliminated entirely by a completed system of multiple division.

Periods other than the Greek, Renaissance, and Baroque, have seen isolated phenomena suggestive of multiple division. The famous Montpelier codex of the 13th century

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[44]Bosanquet devotes several pages to an extended discussion of Bach's apparent reasons for preferring equal temperament, and concludes that Bach's only objection to meantone temperament was to the "wolf" which can be removed through multiple division. Except for the "wolf", says Bosanquet, "no claim to equal temperament (over meantone) can be made in Bach's name." op. cit., p. 36.

possessed marks generally thought to indicate mictrotonal alterations in pitch. Joseph Gmelch carefully shows why the signs ⊢ , ⊣ , ⊢ , ⊤ and ⊥ cannot indicate rhythmic or dynamic alterations but must apply to pitches.[45] At least one quarter-tone composer, Alois Haba, refers to the Montpelier codex as an historical precedent for multiple division.[46] Sachs has written of a most unique medieval instrument with septimal tuning[47] but as far as is known this instrument has not yet been cited as a precedent for multiple division.

A BRIEF NOTE ON ORIENTAL AND FOLK MUSIC

Closely linked with the intonational systems of the past as models for multiple division are the present systems of other cultures. Such leading advocates of multiple division as Fokker, Bosanquet, Kornerup, Yasser, Partch, and Wyschnegradsky have shown considerable interest in the musical systems of the Orient. The presence within the world of intonational systems other than our own offers a chance to test other possibilities and a challenge to

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[45] Gmelch, Joseph, Die Vierteltonstufen im Messtonale von Montpellier, 1911, pp. 9-19.

[46] Haba, Kwart-tonen-Problemen, De Muziek, 1927, p. 109.

[47] Sachs, Curt, A Strange Medieval Scale, A.M.S. Journal, Fall, 1949, p. 169. According to Sachs, two instances show that "in the time around 1100, the wide-meshed 8:7 scale was used in addition to the customary . . . scales."

enlarge our own system so as to incorporate the best of other systems. No less a musician than Bartok is cited[48] as urging finer divisions than those provided by 12-tone temperament in order to realize the autochthonous folk-music of Eastern Europe. Fokker attributes Haba's preoccupation with multiple division to a similar interest in Eastern European music.[49] Fokker himself is greatly interested in the salendro scale of Indonesia, which he considers to be based on the trisecting of the perfect fifth.[50] Such a trisection is possible in 31-tone temperament where the fifth is 18 units in size. Wyschnegradsky also cites the near-equal 5-tone scale as a feature of his system (24-tone temperament), and he mentions Arabic music as a basis for his system.[51] Bosanquet concerns himself at length with Indian music. While many musicians argue that large areas of the Orient possess music with less subtle regard for nuances of pitch then Western music, some Eastern cultures, particularly those in the near-East and India, have paid considerable attention to precision in the intonation of

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[48] By Fokker in La Gamme... p. 152. Bartok is here cited as advocating in particular the seventh partial. Bartok uses tones lying outside the 12-tone system in his Violin Concerto.

[49] Ibid., p. 152.

[50] Just Intonation, p. 34.

[51] Wyschnegradsky, La Musique à Quarts de Ton, Revue Musicale, Jan. 1937, p. 26.

their intervals.[52] The fact that Arabian and Indian musical systems employ more than twelve tones to the octave serves as an example to musicians of the West that sensitivity to 12 distinct pitches in the octave can hardly be the limit of human capacity.

CONCLUSION

Multiple division has firm roots in the actual music of the distant and near past, and in other musical cultures of the present. Some of the proposed systems of multiple division, such as 31-tone equal temperament, involve intonations which (with negligible deviations) were actually used in music. Others, such as 19-tone temperament, involve intonations which were seriously proposed by musical scholars to meet problems of an earlier age but were never extensively used. The case for multiple division is unquestionably enhanced by the extent to which it can draw on historical precedent.

But the presence of various tunings in the music of the past is not in itself proof that all or any of them should be revived today. In the final analysis one's own reading of the implications of history depends upon his views of the acoustical properties of 12-tone temperament.

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[52] Prof. Paul Matthen of Indiana University tells of a visiting Arabian singer who expressed astonishment at how many different musical intervals were notated and regarded in Western music simply as a major third.

Is it a better tuning than those which preceded it, or rather only a more convenient one? For those who have accepted the latter alternative, history has supplied a stimulus for further study, and for the proposal of systems of multiple division.

CHAPTER 3

MULTIPLE DIVISION AS THE RESULT OF EVOLUTION

In the words of Roger Sessions, "It is hardly necessary to point out on these pages that change is inevitable at any period in the development of an art."[1] For about two centuries, the changes which have taken place have not involved the essential manner of tuning, but there have been a considerable number of theorists in recent years who believe that a stage in musical evolution has been reached where changes in the manner of tuning would be most desirable.

The view that the process of evolution calls for new concepts in the field of intonation does not necessarily carry any implicit disapproval of 12-tone temperament as an acoustical entity. The advocate of multiple division through evolution may be very favorably disposed toward 12-tone temperament and may prefer it to earlier historical systems of intonation. He is likely, in fact, to regard 12-tone temperament as a step in a long chain of progress; a chain whose next link must soon be found.[2]

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[1] Musical Quarterly, April 1960, p. 159.

[2] A typical representative of this view is Kornerup, who regards musical evolution as a series of victories for the harmonists (Harmoniker) over the canonists (Kanoniker), that is to say of the progressives over the rule-bound conservatives. The original advocates of the 12-tone system of tuning Kornerup places among the harmonists. Today's advocates of 12-tone temperament who oppose an extension in musical resources to 19- or 31-tone systems, Kornerup considers canonists. Akustische Gesetze, p. 35.

Advocacy of multiple division as evolutionally decreed implies either a critical view toward the materials and practices of contemporary music or else the view that contemporary practices are transitory or transitional. This might be best brought in relief by a citation of the opposite, conservative viewpoint. "An 'exhaustion' of the resources of the 12-tone system in the relevant future is not only unforeseeable, but unthinkable. . . . In its vastness of structural means, its flexibility, and its precision, the 12-tone system cedes nothing to any musical system of the past or present that has engaged the mind of musical man."[3] The above statement, by Milton Babbitt, does not necessarily represent the thinking of all contemporary dodecaphonists, however.[4]

Besides a limited estimate of the growth possibilities of present musical practice, the advocate of multiple division through evolution tends nonetheless to regard organic growth as a necessity. He tends to see as important the inter-performability of new and old music in the same milieu, preferably on the same instruments. Particularly stressed is the viewpoint that a new system can be defined only through practice, rather than through theory.

Finally, many of the advocates of multiple division

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[3]Babbitt, Musical Quarterly, April 1960, p. 259.

[4]This will be developed in some detail later in this chapter.

through evolution are concerned with signs they believe already to be present which indicate leanings toward a new musical system involving more tones to the octave.

INTONATION SYSTEMS BY HISTORICAL DIALECTIC

In his article in 1930, Tonality and Atonality as Synthesized by Supra-Tonality, Yasser states directly a dialectic concept which is implied throughout his writings. In the very title of the article, Yasser conveys his view that tonality and atonality represent thesis and antithesis within the tuning system of 12-tone temperament. From their clash must come a synthesis wherein the 12-tone scale, previously atonal owing to its lack of a modal physiognomy (through the fact that its intervals are all equal), becomes a tonal entity as part of a larger unit, 19-tone temperament.

Hermann Pfrogner also sees in tonality and atonality the coexistence of a kind of thesis and antithesis, but he finds harmony rather than disharmony in the presence of opposites. Where Yasser sees a new system as resulting from and resolving the conflict, Pfrogner sees the conflict as self-sustaining and creating a balance of nature within the 12-tone system. Any changes which might occur in the tuning system must come from the tonal aspects of the system alone, for it is in its tonal sense that the 12-tone system contains the enharmonic equivalents which may force

expansion.[5]

Yasser discusses the evolution of our own and other musical systems, attempting to demonstrate a similar dialectic process to that which he holds will resolve the conflict between tonality and atonality. He considers the Siamese scale to have evolved from a simple 5- to a more complex 7-tone system.[6] The necessity for 12 tones in Western music he considers to have arisen from the gradually evolving tendency to use all seven of the tones of the preceding ("infra-diatonic") system as scale members. Yasser believes the 14th century Ars Nova to have been the point in musical history where the "infra-diatonic" yielded to the diatonic. Jean de Muris, in his attacks on the music of the Ars Nova, betrayed himself to be an infra-diatonicist, according to Yasser.[7]

Melchior Sachs, in asserting that our 12-tone system is the result of evolution by accretion from a simpler 7-tone system, points out that our means of notation is a carry-over from an earlier stage in the evolu-

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[5]Pfrogner, Die Zwölfordnung der Töne, pp. 252-3.

[6]A Theory of Evolving Tonality, p. 43.

[7]Tonality and Atonality... op. cit., p. 347. Modern scholarship seems to indicate that Yasser picked the wrong man in Jean de Muris, but his observations would be equally valid if assigned to the real opponents of Ars Nova.

tionary process.[8] Sach's assumption, like Yasser's after him, is: as twelve was evolved from simpler systems, so can more complex systems evolve from twelve.

IMPLIED RESERVATIONS ABOUT CONTEMPORARY PRACTICE

Since the evolution of whole musical systems has always been demonstrably slow, there would be little reason to advocate further evolutionary transformation at this time were all theorists as satisfied as Babbitt (above) with the state of musical materials at the present time. It is hardly necessary to be a specialist in multiple division, or even in theory and composition, to be aware that this is not the case. Indirectly, of course, any attack on present practices is an invitation for further search for alternate techniques. In addition, however, there is often a direct correspondence between the advocacy of multiple division and a negative view toward major aspects of contemporary practice.

Ariel, to cite an extreme case, advocates 19-tone temperament to regenerate a music which has become enfeebled.[9] A thread of very severe criticism of contemporary music runs through much of the literature of multiple

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[8] Report of the 4th Congress, International Music Society, 1911, p. 279. For further information on the work of Prof. Sachs see Chapter 8.

[9] Das Relativitätsprinzip, p. 164.

division.[10] Perrett cites with evident approval a violent attack on much contemporary music by G. Dyson.[11] Partch writes with strong disfavor on current trends away from "corporeal" music. The article by Scherchen in praise of Sauveur is full of predictions of the disintegration of 12-tone temperament:[12] "The science of acoustics will have to bear Sauveur's experiments in mind, as the disruption of our equal-tempered scale, and the musical system on which it is based, becomes complete."

The criticism of contemporary practice involves, either together or separately, both tonal harmonic practices and atonal, serial procedures. Tasser accepts the atonalists' view that the addition of chromatic tones eventually "disintegrated tonality."[13] Elsewhere, however, he states that atonality was not a "better path."[14]

Almost one hundred years ago, Helmholtz foresaw the domination of music by dissonance. He considered this an

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[10] The exception, interestingly enough, is the literature endorsing exact quarter-tones, wherein a majority of writers seem to be quite comfortable in their surroundings, and quite convinced that their work is but an extension of the work already begun in the 20th century. See Chapter 4.

[11] Some Questions of Musical Theory, page 11. Citation is from Dyson, G. The New Music, 1923, p. 75, to the effect that much new music is for the "tone-deaf."

[12] The Nature of Music, p. 42.

[13] A Revised Concept of Tonality, M.T.N.A. Journal, 1935, p. 100.

[14] A Theory of Evolving Tonality, p. 237.

unfortunate development and placed the blame on the equal-tempered scale. His statement deserves to be quoted at some length, for it represents very probably the beginning of a pattern of dissent which has since characterized a large body of the advocates of multiple division. By an extension of Helmholtz' reasoning, opposition to extreme dissonance becomes opposition to the current tuning system which Helmholtz considers to be the cause for the trend toward dissonance.

> Finally, we cannot, I think, fail to recognize the influence of tempered intonation upon the style of composition. The first effect of this influence was favorable. It allowed composers as well as players to move freely and easily into all keys, and thus opened up a new wealth of modulation. On the other hand, we likewise cannot fail to recognize that the alteration of intonation also compelled composers to have recourse to some such wealth of modulation. For when the intonation of consonant chords ceased to be perfect, and the differences between their various inversions and positions were, as a consequence, nearly obliterated, it was necessary to use more powerful means, to have recourse to a frequent employment of harsh dissonances, and to endeavor by less usual modulations to replace the characteristic expression, which the harmonies proper and the key itself had ceased to possess. Hence in many modern compositions dissonant chords of the dominant Seventh form the majority, and consonant chords the minority, yet no one can doubt that this is the reverse of what ought to be the case; and continual bold modulational leaps threaten entirely to destroy the feeling for tonality. These are unpleasant symptoms for the further development of art. The mechanism of instruments and attention to their convenience, threaten to lord it over the natural requirements of the ear, and to destroy once more the principle upon which modern musical art is founded, the steady predominance of the tonic tone and tonic chord.[15]

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[15] <u>On the Sensations of Tone</u>, p.

What Helmholtz regarded as a denial of primacy to consonance has developed toward the denial of harmony as an element of musical construction. Richard Franko Goldman, writing on Stravinsky,[16] considers this denial "basic in the post-Webern world," adding, "It is certainly possible that harmony is dead, that the composers of the 18th and 19th centuries have explored all the possibilities of what we have come to call traditional harmony, and that the composer today can add nothing to what they have left us." Elsewhere in the same article, Goldman indicates that he is far from certain that this is so, and he does not suggest, as many of the advocates of multiple division do, a specific means of finding new facets or possibilities for what we have come to call traditional harmony.

While one branch of multiple division literature is devoted largely to the further development of traditional harmonic values, another is opposed to these values. Yasser believes, for example, that each musical system formed by evolution must have its own specially fitted harmonic language wherein consonances and dissonances are transfigured from less advanced systems.[17] Perrett, whose approach is more historical than evolutional, decries the inherent poverty of materials in the restriction to two

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[16] Musical Quarterly, April 1960, p. 263.

[17] For a detailed treatment of this subject see chapter 11.

modes, major and minor, the purity of one of which he also questions.

Of atonality Perrett applauds the logic where there is equal temperament, but he considers it "unnatural," likening it to the completely egalitarian society "where no one's anybody."[18] Elsewhere, Perrett takes a more unequivocal position against atonality and a more conciliatory stand with respect to traditional harmonic concepts: "the future of music does not lie in atonality, but on the direct contrary, in extending the principle of tonality to and throughout the chromatic scale."[19] Perrett's prediction has not been realized in its first thirty-five years but there continue to be musicians who, for the long run, would agree with Perrett.

A somewhat atypical view of the disintegration of tonality, a view which traces its lineage to Helmholtz and leads to an acoustically oriented advocacy of multiple division, is Ariel's. He feels that major and minor have been weakened primarily because of the acoustical deficiencies in their representation by equal temperament. The difference in nature between the two kinds of third is about 70 cents, Ariel maintains, while the smallest unit in 12-tone temperament is 100 cents. A tonal system with an interval close to 70 cents (19-tone temperament contains

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[18] Perrett, op. cit., p. 29.

[19] Ibid., p. 110.

the interval 63.16 cents) would, according to Ariel, strengthen anew the dualism between major and minor.[20]

While opposition to present practice tends to be associated with the belief that something new will have to be found to replace it, sometimes the latter view appears coupled with a profound respect for current practice. Stravinsky, despite his current absorption in the technique of dodecaphony, is willing to concede the possibility that musical evolution may take another direction.[21]

The view that contemporary music is in a transitional state awaiting the development of a set of values which will effectively replace those of traditional harmony has been held by many distinguished commentators on new music for quite some time. René Lenormand, in the conclusion to his Study of Twentieth Century Harmony in 1914 states, "We should see new formulas arise of which we cannot at present foresee the character."[22]

Some musicians, such as Babbitt, have come to believe that 12-tone serial technique has fulfilled such predictions as Lenormand's. But as long as even such present

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[20] Ariel, op. cit., p. 164. For a detailed account, see chapter 10.

[21] Stravinsky and Craft, Conversations with Igor Stravinsky, p. 151. Our musical language, says Stravinsky, "is serial at present, and though our contemporary development of it could be tangential to an evolution we do not yet see, for us this doesn't matter. Its resources have enlarged the present language (of music). . . ."

supporters of dodecaphony as Stravinsky appear hesitant to make such claims, it is unlikely that its numerous critics will accept Babbitt's view. The adamant refusal of the majority of composers of electronic music to affix pitch boundaries along traditional lines may be a sign that the days of 12-tone temperament are, indeed, numbered.

GRADUAL DEVELOPMENT AND INTERPERFORMABILITY

Advocates of multiple division by evolution tend to stress the role of practice over theory, and of general and widespread practice over unique practice. Augusto Novarro, in his interesting study of various possible musical systems, pointedly reminds his readers that 12-tone temperament itself was imposed neither by a famous physicist nor by an eminent musician but rather by the public, by which Novarro means the Italian lute-players and the Spanish vihuelists.[23]

Yasser offers a rather precise and detailed theoretical system quite in advance of any popular clamor. Nevertheless he supports views similar to Novarro's on the primacy of practice over theory. It is the artist and not the scientist, maintains Yasser, who will first determine the melodic intonation of the 19-tone scale.[24] Opposition to

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[23] Novarro, op. cit., p. 249. "El temperamento de doce sonidos actualmente en uso en occidente, no ha sido impuesto por ningún físico famoso o músico eminente, lo impuso el pueblo: los laudistas italianos, y más aun los vihuelistas españoles."

[24] A Theory of Evolving Tonality, p. 222.

the mechanical expansion of the resources of music has characterized the arguments of Würschmidt[25] and other proponents of 19-tone subdivision against the quarter-tone advocates. 24-tone temperament is attacked as offering new tones through theoretical decree (based on convenience) rather than artistic necessity.

Proponents of 24-tone temperament, however, argue in the same vein, claiming that the truly organic growth is that which retains the present system lock, stock, and barrel. Needless to say, all 12-tone music is performable on equal-tempered 24-tone instruments, and an unquestionable advantage of quarter-tone music over other species of multiple division is the possibility of mixing the new with the old and thereby introducing new musical phenomena in digestible proportions.

The performability of 12-tone music on 24-tone instruments is incontestable. Similar claims have been made for 19-tone tempered instruments. According to the claims of some of the advocates of 19-tone temperament as a product of evolution, it is comprised of the 12-tone scale, plus 7 auxiliary tones based on the diatonic scale. Can both the diatonic and 12-tone chromatic systems as we know them be successfully played on 19-tone instruments? Melchior

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[25] Würschmidt, Viertel- und Sechsteltonmusik, Neue Musik-Zeitung, Vol. 42, 1921, p. 183.

Sachs says yes,[26] while Yasser appears rather uncertain. In A Theory of Evolving Tonality, Yasser states that 19-tone temperament is satisfactory for diatonic melodies, although he concedes the discrepancies to be "somewhat greater" than in 12-tone temperament.[27] In a letter to John Redfield, Yasser concedes that 19-tone temperament possesses "little if any harmonic advantage over 12- for the performance of diatonic music," at any rate not enough to justify the shift to 19-tone temperament for this purpose alone.[28] According to Yasser as well as Sachs, then, diatonic music is performable on 19-tone instruments.

What about chromatic music? Chromatic materials are included in Sachs' outright assertion that 12-tone music is playable on 19-tone instruments. Rudimentary experimentation shows Sachs to be on rather shaky ground. The differences between the two systems of intonation are quite great as regards chromatic music. Assuming C to be the point of reference, the F#'s of 12- and 19-tone temperaments are more than 30 cents apart. Fx involves a discrepancy between the two systems of close to 70 cents. The two systems can hardly be considered equivalent as regards chromatic music. It is

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[26] In his report to the 4th Congress of the I.M.S., op. cit., p. 69.

[27] A Theory of Evolving Tonality, p. 131.

[28] The letter is dated May 1, 1933 and is, of course, unpublished.

Yasser's contention that genuinely chromatic[29] music is wrong on 12-tone instruments and correct on 19-tone instruments. In view of the fact that composers were consciously working with the 12-tone scale, this is a questionable assertion, but it is more tenable than the argument of Sachs that the two systems are interchangeable in the rendition of fully chromatic music. Yasser supports his argument by referring to a number of features of music using the entire chromatic gamut (as a basic scale) as anticipations of 19-tone temperament.

SIGNS OF CHANGE

It is inherent in the evolutionary process that the seeds of the new are, at some stage, present in the old. Yasser believes that a number of contemporary practices show an instinctive leaning toward a new intonational system. Among the signs to which Yasser points is the extensive use of octave displacement. It is his view that in 19-tone temperament what is now called the chromatic scale will have great melodic value as a scale. It is not effective as a melodic scale today, according to Yasser, because it is "out of tune." Composers, instinctively sensing this, avoid the direct use of the chromatic scale but, desiring its benefits nonetheless, they disguise it

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[29] That is to say music in which all twelve tones function as part of the basic scale.

through the use of 7ths and 9ths in place of seconds, thereby hiding the unsatisfactory intonation.[30]

A rather revealing corroboration of this argument comes from Bosanquet, more than a half century earlier. In this instance Bosanquet is not speaking of multiple division. "In chords formed of a succession of minor thirds, major sixths frequently occur. Care must be taken to dispose them so as to make this interval correct. If a deviation is necessary, it is better, if possible, to extend the interval by an octave; the resulting major thirteenth (3:10) is not very sensitive."[31] It is the relative lack of sensitivity of the octave-displaced intervals which Yasser believes makes the phenomenon of octave displacement so attractive to composers.

Polytonality is another contemporary phenomenon cited by Yasser as evidence of the anticipation of a new system. He demonstrates how the diatonic scale and diatonic harmony can be produced by the simultaneous use of two pentatonic scales with simple quartal harmony. Postulating a similar synthesis at a more advanced state of musical development, Yasser asserts that there is no such thing as polytonality; that polytonal combinations represent single harmonic units in a more elaborate system.[32] Although this

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[30] A.M.S. Journal, 1953, p. 59.

[31] Bosanquet, op. cit., p. 72.

[32] A Theory of Evolving Tonality, p. 250.

view may be questioned on the basis of composers' awareness of the phenomenon of polytonality when they use it, Yasser is probably supported by the very difficulty a musician has in attempting to extract two or more separate chords from a single polytonal combination in listening. Nor is a polytonal sonority necessarily intended to be heard as two or more separate chords.

Yasser is especially interested in the harmony of Scriabin.[33] Scriabin's so-called mystical chord is treated by Yasser and others[34] as a combination of the following members of the harmonic series: 1, 5, 7, 9, 11 and 13. These are the very partials which Yasser considers to comprise the basic consonant hexad in 19-tone temperament. Yasser postulates from the above that Scriabin must have envisioned something like the 19-tone system in evolving his "mystical chord" and his harmonic thinking in general. This view has been experimentally corroborated by Tillman H. Schafer, a physicist who has long held a strong interest

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[33] It should be born in mind that for a period preceding the writing of Yasser's book Scriabin was regarded with considerable awe by a rising generation of composers and that his influence was once regarded as far greater than it is today.

[34] A Theory of Evolving Tonality, p. 232. Ernst Bindel has more recently published a similar analysis of the Scriabin chord: C F# Bb e a^{b} d'
1 11 7 5 13 9. If this is, indeed, the Scriabin chord, it is dreadfully out of tune in both 12- and 19-tone temperaments. In fairness to Yasser, however, it should be stated that much the worse would be the 12-tone tempered rendition.

in Yasser's theories.[35] After participating in the construction of a 19-tone instrument, Schafer reported to Yasser that "A friend played Scriabin's 5th Sonata (on the 19-tone instrument) and evidence that he sensed the plane was everywhere."[36]

A phenomenon which occurs from time to time in the standard contemporary repertoire and can easily be interpreted as presaging an era of multiple division is the use of tones outside of the 12-tone realm for special effect. Among the works in the standard repertory which employ unusual subdivisions are Benjamin Britten's highly successful Serenade for Tenor, Horn, and String Orchestra which uses the natural 7th and 11th partials of the horn in solo passages, Aaron Copland's piano trio Vitebsk which uses approximate quarter tones in the two stringed instruments, and Bartok's Violin Concerto. Ernest Bloch and Charles Ives are among the other recognized composers who have made occasional use of microtones.

James Mursell writes of equal temperament that it has frozen the development of music, consigning innovation

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[35]His honors thesis is entitled The Music of Tomorrow: The Supra-Diatonic Scale, a New Concept of Timbre, The Electronic Musical Instrument (1941). His instrument-building was in collaboration with James Piehl.

[36]Letter from Schafer to Yasser, unpublished, dated February 14, 1947. Schafer offered a similar report to the Acoustical Society of America at about the same time.

to the speed of flow of a glacier.[37] On one level, Mursell appears wrong. Has there ever been a period in the history of music in which styles have changed as rapidly or as completely as in the last 70 years? On another level, however, the rapid changes of style can be construed as efforts to surmount an obstacle which blocks the entry into a more fertile domain. Change and progress need not be synonymous, and Mursell is clearly referring to the latter when he speaks of "innovation."

Advocates of multiple division through evolution believe that our general habituation to 12-tone equal temperament is the greatest obstacle to further progress in music and that a kind of "supermusic" is possible which awaits only the arrival of composers and performers equal to its superior challenge.

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[37] Mursell, James L., Psychology and the Problem of the Scale, Musical Quarterly, Vol. XXXII, p. 565. Cited by Blaukopf, Musiksoziologie, p. 117.

PART TWO

THE WRITINGS OF THE ADVOCATES OF MULTIPLE DIVISION

The advocates of multiple division share only the view that the present tuning system is or soon will be inadequate. As the reasons for their holding this view vary, so do the remedies which they propose. Many of those whose only interest in a new system is evolutional have proposed to build systems of multiple division in which the 12-tone scale remains unchanged; it is merely subdivided. This is the basis for so-called quarter-tone music. Others have proposed temperaments based on a more perfect fifth than is available in the present equal-tempered system. Others still have proposed temperaments based on less perfect fifths but more perfect thirds, resembling the meantone systems of the past. A few have contemplated the idea of temperaments that make no attempt to duplicate the fifth or the third. Still others have despaired of meeting their acoustic demands by a performable equal temperament and have proposed unequal systems.

The following chapters are arranged according to the categories outlined above. 19-tone temperament is reserved for the end of this section in order that it may be examined in greater detail. Chapters are devoted to the early writers on 19-tone temperament and to each of its three leading modern exponents, Kornerup, Ariel, and Yasser.

CHAPTER 4

MULTIPLE DIVISION BASED ON 12-TONE TEMPERAMENT

The number of musicians who have worked with quarter-tones in their compositions is already large. The line extends from the first decade of this century (and possibly earlier) to the present,[1] and includes representatives from many countries. The ready availability of instruments tempered to 12 tones per octave has probably contributed far more than any acoustical theories (although they exist) or historical precedent (although one is said to exist) to the practical fact of 24-tone equal temperament. By taking two instruments attuned to 12-tone temperament and setting them 50 cents apart, 24-tone temperament is obtained. Two pianos, so tuned, have provided the quarter-tonists with a standard medium. Four pianos, so tuned, have provided a quarter-tone "orchestra."[2]

At least three of the quarter-tone composers have published their theoretical views at some length. The earliest of these was the German Willi Möllendorff, whose

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[1] Dupont acknowledges Richard H. Stein as having been the first to compose with quarter-tones in 1906. Op. cit., p. 115. This is probably earlier than the first quarter-tone works of Haba, Hans Barth, or Jörg Meger who, together, are credited by Novarro with originating 24-tone temperament. Op. cit., p. 248. More likely correct is José Asuar's assertion that Stein was the first to publish quarter-tone music (2 pieces for Cello and Piano, 1906). There was undoubtedly earlier experimentation.

[2] Wyschnegradsky calls his work, Ainsi Parlait Zarathustra, a symphony for an orchestra of four pianos.

pamphlet, Musik mit Vierteltönen, appeared in 1917. A decade later Alois Haba, probably the best known of the composers whose primary output has involved multiple division, published his Neue Harmonielehre. The Neue Harmonielehre was selected by Gustav Reese for inclusion in his list of the 80 most important treatises of all time about music not available in English.[3] In 1933 Ivan Wyschnegradsky, a quarter-tone composer whose works were to receive a fair measure of local success some years later, published a Manuel d'Harmonie à Quarts de Ton.

Möllendorff, Haba and Wyschnegradsky share a common interest in the problem of bridging the chasm between 12- and 24-tone music, but the attitude of each of the three men is quite different from those of the others toward this, as it is toward the goals of quarter-tone music in general.

Möllendorff's chief concern throughout his pamphlet, Musik mit Vierteltönen, seems to be the defense of quarter-tone music against charges that it is too radical. He is by present standards an out-and-out conservative. He is for retaining the traditional scales and modes, with the use of quarter-tone intervals for occasional enrichment of sonority but primarily for the enlargement of the possibilities for modulation. He divides his 24-tone system into two distinct 12-tone systems pitched a quarter-tone apart.

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[3]Reese, Fourscore Classics of Music Literature, 1957.

A large portion of this pamphlet is devoted to simple modulational formulae for moving from a given key in one 12-tone system to each of the 12 possible keys in the other system. Besides showing the formulae as musical examples he cites three general rules for such modulations, which he claims always to follow in his own practice:

1. In passing between two keys which belong to opposing 12-tone systems, use only familiar chords, each complete in a single 12-tone system.
2. Since such a procedure prevents the use of a common tone in passing from one system to the other, follow the traditional rule that in the absence of a common tone there must be contrary motion.
3. Each individual voice must move by a single quarter-tone over the break between the two systems rather than by a larger interval.

Möllendorff devotes a major portion of his pamphlet to the refutation of those arguments which he considers to be those most frequently raised against quarter-tone music. These arguments remind us that in 1917 the main current of the 20th century was still far from dictating or determining the aesthetic center of gravity. Many of the arguments against quarter-tone music in this section appear to be remarkably similar to the arguments against main-stream 20th century music in general, arguments long since abandoned. First Möllendorff cites the objection that music with quarter-tones will make people even more nervous than they already are ("die Menschen noch nervöser gemacht werden als

sie jetzt schon sind"[4]). He devotes considerable space to denying the contention. The second objection which he cites is that quarter-tone music will destroy the tempered system. This obvious inaccuracy he deals with summarily, as he does with the equally fallacious third objection, that because of the availability of quarter-tones all of the present literature will have to be re-composed (umkomponiert). The fourth argument which Möllendorff disposes of is that the introduction of quarter-tones signifies a retrogression to the primitive, even barbaric, condition of a lower cultural level. A fifth and final negative argument is cited, the only one among the five that is still heard today. This argument is that the quarter-tone would be much too small an interval to permit two neighboring tones to be properly distinguished from one another.

The objections which Möllendorff cites are far more of interest today than his routine replies, which might equally well have been written in defense of Le Sacre du Printemps. Indeed, as will emerge in the study of Haba and Busoni, the early development of interest in multiple division seems aesthetically to have been regarded as a legitimate wing of musical modernism. If this be true, the knot was probably untied just after the end of World War I when a new 20th century musical elite came to power and

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[4]Op. cit., p. 40.

defined limits of acceptability within which the multiple-division advocates could never thereafter penetrate.

Möllendorff's pamphlet is, as the foregoing might suggest, dated. Its arguments appear naive today and its musical goals quite modest. Of some interest is Möllendorff's own harmonium (which he offers for sale, citing proudly its Deutsches Reichspatent) with a keyboard design which inexplicably calls for 36 rather than 24 keys to the octave.

Alois Haba is undoubtedly the most world-renowned of the composers regularly employing multiple division. He is probably also the most agressively anti-traditional of the multiple-division composers in his attitude toward the music of the past and the composer's duty to turn away from it.[5] Most of the Neue Harmonielehre consists of a simple catalogue of possible scales and sonorities in each of several systems of equal temperament. It is in keeping with the philosophy which Haba outlines in his extended introduction that this is so. He explains that it would be wrong to single out progressions to demonstrate to the student, because the student must learn above all to create his own progressions: progressions which have never been used before.[6] Haba also

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[5]Haba's attitude combined with his renown may have contributed to the public image of the advocate of multiple division as an extreme radical.

[6]Neue Harmonielehre, p. 112.

denies the existence of any such thing as inherent harmonic tendency. Any such tendency, he asserts, exists only in the creator's mind.[7]

The impression of Haba as a kind of musical anarchist is reinforced by a reading of Haba's own view of the history of theory. Franz Skuhersky is lavishly praised by Haba for his doctrine that any triad may be used at any time on any degree of any kind of scale.[8] Karl Stecker is cited with respect for having extended this same principle to tetrads, and Vitezslav Novak is commended for having brought pentads and hexads into the same promiscuity of relationship. To all of this Haba adds his own extention: any combination of two or more tones can be used in conjunction with any other combination of tones regardless of the systems to which each set might belong.[9]

The Neue Harmonielehre begins with a thorough catalogue of possible scales and sonorities in traditional 12-tone temperament. Then follows a section on 24-tone temperament and a section of 36-tone temperament. Finally, as a crowning system, he offers 72-tone temperament which, among other things, combines all of the possibilities of each of the other 3 systems in the book. He does not make

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[7] Ibid., p. 129.

[8] Ibid., p. viii. Haba calls Skuhersky's the first truly original theory since Rameau.

[9] Ibid., p. vi.

any attempt to show the peculiar acoustic advantages of 72-tone temperament, which are worth noting (see below, page 162). He is unconcerned about reforms in notational procedures, and uses eleven different kinds of sharp signs to provide for every possible tone between the whole-tones in 72-tone temperament.

Haba's compositions have been performed in many places, often with considerable success.[10] His works include pieces for many different media and include at least one opera, Die Mutter, which appears to have been produced with considerable excitement in Munich in the late 1920's. The orchestration calls for the complete body of quarter-tone instruments available to Haba. A regular though diminished string section is supplemented by 2 quarter-tone clarinets,[11] 2 quarter-tone trumpets,[12] 2 slide trombones, a quarter-tone piano, a quarter-tone harmonium, and 2 harps tuned a quarter-tone apart. The piano, clarinets and trumpets were specially built for the occasion.[13]

Haba appears to have regarded 24-tone temperament as a basis for notation and a starting point for his system

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[10] Ibid., p. 246.

[11] According to Sigmund Klein, Quarter-Tone Data, in the Pro-Musica Quarterly of March 1925, p. 22, quarter-tone clarinets were first built for Richard H. Stein.

[12] According to Ibid., a quarter-tone trumpet was built in the 19th century in Odessa. The addition of a fourth valve connecting with a short pipe-extention is all that is needed to convert a trumpet to quarter-tone use, albeit with rather inexact intonation.

[13] Haba, Meine Vierteltonoper Die Mutter, Anbruch, Monatsschrift für Moderne Musik.

rather than as a rigid system of intonation to be adhered to at all times. In the introductory notes to his Music for Unaccompanied Violin, opus 96, Haba states that he conceives the quarter-tone as representing approximately even subdivisions of the semitones as they are normally played on the violin. This means, acknowledges Haba, that the chromatic and diatonic semitones are different in size, and the player is instructed to bear this in mind in estimating the proper intonation of the quarter-tones.[14] In the introductory notes to the companion work, Fantasy for Solo Violin, Haba also stresses the necessity for "plasticity of intonation."

Fokker interprets Haba's flexible view of the intonation of 24-tone temperament to mean that Haba is more interested in microtones in general than in the specific system of microtones he employs. It is Fokker's opinion that with the opportunity to use and hear 31-tone instruments Haba would have accepted that system.[15] This may well be wishful thinking on Fokker's part. Haba through his life's work became quite habituated to thinking along the lines of the 24-tone system, which he recognized to be a dual, conditionally related unit (as opposed to the 12-tone system which he calls an acoustically related unit).[16]

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[14] Haba, Musique pour Violon Seul, p. 5, Phantaisie pour Violon Seul, p. 3.

[15] Fokker, La Gamme... p. 152.

[16] Haba, Phantaisie pour Violon Seul, p. 5.

Ivan Wyschnegradsky belongs to a younger generation than Haba and some of his writings show the moderating influence of the neo-classical revival which temporarily halted the contempt for the practices of the past which influenced Haba so strongly. Wyschnegradsky's writings show two contradictory strains, the one exulting in the overthrow of traditional harmonic concepts in the vein of Haba, the other highly conciliatory and rational in its attempt to link the old and the new. His quarter-tone music, to judge from its critics,[17] would appear to reflect both strains, sometimes highly reminiscent of Wagner and Scriabin, sometimes extremely dissonant and anti-tonal. Periodically Wyschnegradsky returns to a concept which he calls pansonority, a concept which has as its basic premise the destruction of consonance and dissonance as polar forces.[18] However, in his other, more valuable works, Wyschnegradsky seems almost to be a different author.

Of all Wyschnegradsky's writings, the one which sheds the most valuable light on his musical practices is probably his Manuel d'harmonie à quarts de ton, published in 1933. It is a moderate work, in some respects reminiscent of

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[17] The impression is confirmed by the opinions of both the "pros" such as Boris de Schloezer, and the "antis" such as Léonid Sabanéev (Musical Times, June 1, 1929, p. 503).

[18] Wyschnegradsky, Quartertonal Music, Its Possibilities and Organic Sources, Pro Musical Quarterly, Oct. 1927, especially pp. 30 and 31.

Möllendorff. Wyschnegradsky begins, as did Möllendorff, by dividing his 24 tones into two groups of 12. One group he calls the old scale, the other, the new scale. He then considers interval nomenclature at some length, preserving the names of the intervals of the 12-tone system as they stand and adding the following new names:

Seconds, Thirds, Sixths, and Sevenths

neutre.....between major and minor
plus que majeure....¼-tone larger than major
moins que mineure...¼-tone smaller than minor
suraugmentée.....¼-tone larger than augmented
sous-diminuée.....¼-tone smaller than diminished.

Perfect Intervals

majeure.....½-tone larger than perfect
mineure.....½-tone smaller than perfect.

Additional Accidental Signs

raise by ¼-tone #
raise by 3/4-tone ##

lower by ¼-tone ♮
lower by 3/4 tone ♮♮

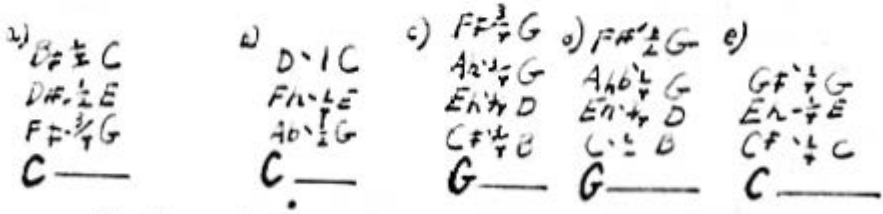
Next, Wyschnegradsky carefully considers the component parts of the 24-tone system. Eight intervals form complete 24-tone circles. This is exactly twice the number of intervals which form complete 12-tone circles in the 12-tone system. In the 24-tone system these intervals are 1/24, 5/24, 7/24, 11/24, and their respective inversions. In addition, four intervals form 8-tone circles within the system. These are 3/24, 9/24, and their inversions. Wyschnegradsky implies that the field for modulation is

greatly enriched by the availability of these additional long cycles.

The above serving to introduce the properties of the system, Wyschnegradsky begins the first of his two major units: quarter-tones as accidentals, altered chords, and the basis for modulation within a more traditional harmonic context. It is this section which bears some resemblance to Möllendorff. Wyschnegradsky, however, has codified his ideas far more carefully, and has certain tested and favored progressions to offer to the reader.

For music fundamentally in the "old" 12-tone scale, non-harmonic ornaments from the "new" scale should be no more than 3/4 tone removed from their resolutions.[19] Example 25 shows several of Wyschnegradsky's favorite simultaneous ornaments resolving over pedal-tones.

Example 25: Quarter-tone Ornaments by Wyschnegradsky



Wyschnegradsky is conscious of the enharmonic ambiguities inherent in some of his ornaments. Vertically, the upper tones of these sonorities are traditional chords,

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[19] Manuel d'Harmonie à Quarts de Ton, p. 6.

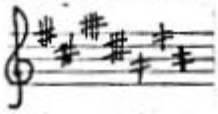
major, minor, etc., strongly suggesting spellings consistent with their sound. Horizontally, through their resolution in contrary motion, these groups of ornaments appear to require an altogether different spelling. The situation is, of course, not unprecedented, existing in such traditional functions as the augmented sixth chords which sound like dominant sevenths.

Next, Wyschnegradsky examines chord alterations by quarter-tones. He shows all (eleven) forms of an altered triad wherein one third is allowed to remain either major or minor and not altered. The so-called neutral triad, which consists of an equally bisected perfect fifth, he considers doubly altered since neither third is left unchanged.

Among his preferences in chords and scales, Wyschnegradsky lists the "Neapolitan" with root raised a quarter-tone, and the so-called Gypsy scale with its two augmented seconds decreased through the raising of their lower members by a quarter-tone. In the many chords and scales, an implied underlying principle seems to be present. Where traditional 12-tone practice admitted two sonorities whose only difference lay in the modification by a semitone of one the pitches, the use of the quarter-tone between these two pitches is acceptable practice.

This principle is carried over into tetrads. Wyschnegradsky is fond of altering the fifths of seventh

chords by a quarter-tone, placing them between the diminished and perfect categories.[20]

Next, Wyschnegradsky turns to modulation. As an introduction to his treatment of modulation between "old" and "new" tonics, Wyschnegradsky shows various key signatures for "new" major and minor scales. E ♯ major has the following signature:  .[21]

Wyschnegradsky's modulating procedure is rather like Möllendorff's in that he recommends the avoidance of any motion in any voice by large intervals between two tones in opposite systems. At the moment of modulation, only the single move to the neighboring quarter-tone is recommended. He endorses sequential formulae involving quarter-tones as an excellent means of achieving such modulation. He adds the recommendation that as the modulation approaches, the quarter-tone barrier be softened by the use of ornaments from the "new" scale. The use of seventh chords and non-harmonic tones at the point of crossing is also suggested.

The second part of the Manuel d'Harmonie à Quarts de Ton deals with "artificial quarter-tone scales." While pointing out that such scales need not use the octave as their basic unit, Wyschnegradsky chooses to restrict himself to those which do. Proceeding as did the Greeks,

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[20] Ibid., p. 12.

[21] Ibid., p. 13.

Wyschnegradsky builds his scales from identical disjunct tetrachords. He permits his tetrachords to cover 9/24, 10/24, or 11/24. For a detailed example of what he considers to be a particularly successful quarter-tone scale, Wyschnegradsky singles out the scale he calls the neutral mode, C, D, E♮, F, G, A♮, B♮, c.[22] As can be seen, the principle of this scale is that all degrees which have different tones in major and minor are represented by the quarter-tones lying between them. The key signature for this scale is♮♮♮.

Next, Wyschnegradsky determines the most closely related transposition of the same mode. He analyzes his scale in terms of the intervals of which it is made, producing a table of all consecutive degrees, all alternate degrees, and every interval created by each third degree. The table reveals 6 instances of the neutral third, 5 of the perfect fourth, 4 of the neutral second, 3 of the major second, 2 of the major fourth, and 1 of the minor third. By this method Wyschnegradsky concludes that the circle of neutral thirds is the primary basis for modulation when using the neutral scale, just as the perfect fourth is the basis for the simplest modulation in the major or minor modes. In the course of modulating by neutral thirds, the modification of the key signature is rather interesting. As stated above, the key signature for C neutral contains

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[22] Ibid., p. 18.

3 ♮'s. The most closely related key is E♮ neutral. The scale of E♮ neutral is E♮, F ♯, G, A♮, B♮, C, d, e♮. No ♮'s have been added or removed, but rather a ♯ has been added. Continuing upward, the next neutral key is G, whose scale includes the F ♯ and modifies the signature of the preceding key by omitting the A♮. Thus in proceeding upward from key to key around the circle of neutral thirds, the signatures alternately add ♯'s and omit ♮'s.

Wyschnegradsky next considers scales possessing more than 7 tones. Particular favor is accorded a thirteen-tone scale involving two disjunct heptachords (separated by a semitone), each of which consists of five semitones and a quarter-tone. The basic interval for modulation is the major fourth (the interval by which the two heptachords of the scale are separated), 11/24, which, like the neutral third, results in a complete 24-key cycle. In his "symphony," Ainsi Parlait Zarathustra, Wyschnegradsky utilizes this 13-tone scale extensively. On the occasion of one of its performances, Olivier Messaien payed special tribute to this scale in his review.[23]

A contrasting artificial scale mentioned by Wyschnegradsky is the almost equi-distant pentatonic scale. Wyschnegradsky incorrectly lists the intervals as two whole

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[23]Messaien in La Monde Musical, read as a reprint with date and page unspecified.

tones and three intervals of 5/4 tones.[24] The total falls a quarter-tone short of an octave. Presumably the scale is achieved by a single whole-tone and four intervals of 5/4 tone.

Wyschnegradsky next turns to what he calls the free use of quarter-tones. One such use is in "expressive unison." In an expressive unison, two tones are sounded simultaneously which are separated by a quarter-tone. A similar procedure is mentioned for an "expressive octave." That Wyschnegradsky thought well of this device is confirmed by the score of Ainsi Parlait Zarathustra where tremoli involving two or more simultaneous "expressive unisons" occur quite frequently.[25] In light of the frequent use of major sevenths and minor ninths in recent music of 12 tones per octave, it is interesting to speculate on whether those intervals are also used or intended as "expressive octaves." If so, the smaller deviation of the quarter-tone may be a superior means to an effect desired by many composers besides Wyschnegradsky.

Wyschnegradsky endorses parallel sonorities involving composites of quarter-tone relationships, and these also figure prominently in Ainsi Parlait Zarathustra. Parallel neutral thirds and parallel large seconds (5/24), often in

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[24] Op. cit., p. 19.

[25] Ainsi Parlait Zarathustra, publ. by L'Oiseau Lyre.

rather long chains, appear with great frequency in the score. Wyschnegradsky shows several examples of polytonality presenting simultaneously a major key using "old" tones, a minor key using "new" tones, and a neutral key using both. He also offers examples of what he calls polyatonality consisting of 12-tone patterns functioning against one-another.[26] Finally, he shows some examples of widely spaced sonorities in which all 24 tones appear.

His main text completed, the author devotes an appendix to consideration of the acoustical basis for quarter-tone music. He begins by stating that the 11th partial is the best basis for the system, a statement born out by the close equivalence between 11:8 (551.3 cents) and 11/24 (550.0 cents). If he had stopped here he would be on firm ground.[27] However, he turns to other possible acoustical sources. Regarding C as the generating tone, he decides to consider B♭b as the 7th partial, F♯ as the 11th partial, A♮ as the 13th partial, and the scale from B♮ to E♯ as every partial from the 29th to the 41st. Eb he attributes to the 57th partial while other tones still remain without partials of their own. Presumably they are derived from transpositions of the series already shown.

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[26] Op. cit., p. 22.

[27] It should be noted that the particular preference of both Wyschnegradsky and Messaien for the quarter-tone scale whose basic interval is 11/24 jibes with the excellent acoustical basis for this interval.

The above explanation is hardly satisfactory. The 7th partial is more than 18 cents removed from B♭; that this seems excessive is emphasized by the fact that 25 cents is the limit of possible error in the quarter-tone scale. Most of the higher partials are approximated far more closely, but it is difficult to take seriously any arguments that there can be a genuine acoustical basis in partials as high as 41 and 57.

In a later article,[28] Wyschnegradsky attempts to trace the sources of quarter-tone music to the Greek enharmonic tetrachord and to the music of the Arabs. He is careful to differentiate current practice from that of the ancients who used quarter-tones only as a means of altering fundamental seven-tone scales. He claims that quarter-tones can permeate the sound ideal of classical music through tendency tones. When D♯ is about to resolve to E, Wyschnegradsky prefers the use of D♯♯. To use only a few quarter-tones, however, is to commit an affront to the sense of style. He recommends the achievement of equilibrium through the use of approximately the same number of tones from each 12-tone system. This equilibrium is, in fact, encouraged by the performing medium of two (or four) pianos, since the equalization of the importance of the roles of the various performers serves to establish the

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[28] La Musique à quarts de ton et sa réalisation pratique, La Revue Musicale, Jan. 1937, pp. 26-33.

desired tonal equilibrium.

Wyschnegradsky's compositions have had a modest measure of success, with perhaps the most favorable critical reaction following a concert in Brussels in 1947. Of the many laudatory reviews of that performance, probably the most lavish praise came from Louis Verschreager, who remarked[29] that Ainsi Parlait Zarathustra offered a "prodigious revelation of a new musical domain." The dissonances he considered hardly severe for an ear accustomed to Stravinsky and Schönberg.

The greatest ferment of activity by composers of quarter-tone music took place in the 1920's and early 30's. Among its practitioners in Russia were Arthur Lourie, who, as an early commissar of music, gave multiple-division composers a strong but short-lived boost, and Georgi Rimsky-Korsakoff, son of the more famous Rimsky-Korsakoff. In Czechoslovakia, besides Haba, Fr. Wiesmeyer composed quarter-tone works, as did a whole galaxy of composers in Germany. Jörg Mager was, among them, perhaps the most important for his pioneering work at instrument construction. According to Klein, in 1925, he was at work trying to build an instrument capable of playing music written for

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[29] La Cité Nouvelle, Brussels, 18 Feb., 1947. The work was also performed in Paris on November 10, 1945, at which time one of the four pianists was Pierre Boulez.

any tuning.[30] The quarter-tonists were represented in Italy by Silvestro Baglioni, in Spain by Dominguez de Burrueta and Panach Ramos, and in France by Marina Scriabin and Yvette Grimaud, in addition to Wyschnegradsky, an import from Russia. Latin America was represented above all by Julián Carillo of Mexico, who made a quasi-religious crusade over his campaign for multiple division which he labeled 'El Sonido Trece' (the 13th sound).[31]

In the United States, Hans Barth and Mildred Cooper were among the leading exponents of quarter-tone music, but it should be mentioned that Charles Ives was continually interested in microtones, and essayed a few works with quarter-tones. He wrote a short article in Pro Musica Quarterly in which he says, "The assimilation of quarter-tones with what we have now into some reasonable and satisfactory basic plan will be, it seems to me, along

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[30] Klein, Sigmund, Quarter-Tone Data, Pro Musical Quarterly, March 1925. The names listed on this page are taken either from this article or from Asuar, Jose, De los Microtonos y su Aplicacion como Sistemas Temperados, Revista Musical Chilena, Oct-Nov. 1957, p. 59. Several names are also taken from Kallenbach-Greller, Lotte, Die Historischen Grundlagen der Vierteltöne, Archiv für Musikwissenschaft, Sept. 1927, p. 475. Miss Kallenbach-Greller attributes the first quarter-tone compositions in the 19th century to Joseph Petzval, beginning in 1862!

[31] "We are about to see the accomplishment of the greatest transcendence that has taken place since before the time of Christ; ... the psychological transformation produced by Christian doctrine was surely no greater than the formidable revolution which 'The 13th Sound' will produce in the art of tones." --- El Sonido 13, Feb. 1924, p. 6.

harmonic lines with the melodic coming as a kind of collateral . . . in a sense opposite to the way our present system has developed."[32]

Ives predicts that it will take generations or even centuries before quarter-tones are absorbed into the instinct of the inner-ear, but that it is still worthwhile to attempt to use them. He feels that chords with four distinct tones are better than triads, and that pentads are best of all, on the grounds that one member of a quarter-tone triad will sound out-of-tune but a fourth tone can correct the impression. He cites the ability of quarter-tones to "relieve the monotony of literal repetition" and to make a rhythmic contribution to structure in this way. He notes a problem which has concerned advocates of other multiple divisions (see supplement, page 398, below), namely, that it is difficult to distinguish inversions of quarter-tone sonorities. He blames his own ear,[33] but the problem may well lie in the sounds themselves.

The general interest in multiple division and in quarter-tone composition seems to have declined since the early 1930's. Nevertheless, there remain an unknown number

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[32] Ives, Charles, Some Quarter-Tone Impressions, Pro Musica Quarterly, March 1929, p. 28. Leonid Sabaneev takes just the opposite viewpoint, advising quarter-tone composers to concentrate on melody, if necessary to the total exclusion of harmony. Musical Times, June 1, 1929, p. 504.

[33] Op. cit., p. 33.

of composers who continue, in obscurity, to compose quarter-tone music, generally for two pianos. One such man is Mordecai Sandberg of New York. His musical credentials, which include a doctorate from Berlin,[34] are probably better than those of many well-known composers, but he has chosen to amass a huge body of quarter-tone works in almost total anonymity.

There is a small literature of published quarter-tone music, and a substantial literature of unpublished quarter-tone music in manuscript. More detailed studies of this branch of multiple division would well prove fruitful at this time, before much of the unpublished music becomes increasingly unavailable for research.

THE BASIS FOR 24-TONE TEMPERAMENT

There has been much severe criticism of the particular choice of 24 tones to the octave from the advocates of other schools of multiple division. Whether polemic, as when coming from Ariel,[35] or closely reasoned and moderate, as when coming from Novarro[36] or Handschin,[37]

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[34] Knowledge of his very existence, as well as the brief biographical data included, were learned quite by chance in a discussion of other matters with Fritz Kuttner.

[35] Ariel, op. cit., p. 135.

[36] Novarro, op. cit., pp. 153-4.

[37] Handschin, Akustisches aus Russland, Gedenkboek aangeboden aan Dr. D. F. Scheurleer, p. 146.

the criticism tends to follow one general line. According to these and many other writers, the basic mistake of the advocates of 24-tone temperament is that they use 12-tone temperament as their starting point . . . as their image of a perfect system. But 12-tone temperament is itself a compromise and possesses serious weaknesses which are endured for the sake of simplicity. What, it is asked, does 24-tone temperament add to 12-tone temperament in return for the extensive loss in simplicity? Unquestionably, 24-tone temperament offers a good approximation of the 11th partial. Partch and Novarro, among others, concede this. But to what avail is a good 11th partial when the 5th and 7th are lacking?

I have already discussed Wyschnegradsky's claim that 24-tone temperament improves upon the 7th partial offered by 12-tone temperament. In terms of percent of possible error, the 7th partial of 24-tone temperament is worse than the poorly approximated 7th partial of 12-tone temperament. As for the 5th partial, a moderately weak point in 12-tone temperament, it is, of course, unimproved by subdivision into quarter-tones. But, whereas the error of 13½ cents represents only 27 percent of the possible error in 12-tone temperament, the figure becomes a far less tolerable 54 percent in 24-tone temperament. It is quite understandable that the leading proponents of 24-tone temperament wrote favorably of abolishing all concepts of consonance as such.

To the obvious truth advanced by proponents of quarter-tones that all of the advantages of 12-tone temperament are retained in 24-tone temperament, the opponents insist that this is not enough. As the number of tones in a tuning or temperament is increased, imply the critics, one is entitled to increasing accuracy in the approximation of the intervals which form the acoustical basis for the musical system.

It is this altogether reasonable doctrine, that takes the factor of performance and listening difficulty into account, which renders so complex the matter of choosing a most suitable multiple division. Were it not for the reasonableness of this argument, all intonation problems could be solved by taking a basic unit so small that the deviation of all desired intervals would be minute. In 600-tone temperament no interval can be incorrect by more than a cent.

If the advocates of 24-tone temperament are somewhat at a loss in the debate over an acoustical basis, they do have serveral strong historical precedents to draw upon. There is, to begin with, the Greek enharmonic cited by Wyschnegradsky. Barbour cites Kircher's Musurgia Universalis, page 208, as evidence that Aristoxenos has long been looked upon as a precursor of 24-tone temperament.[38] Most of

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[38] Tuning and Temperament, p. 117.

Ptolemy's enharmonic tunings do not divide the semitone equally, however, and the Greek enharmonic can be used to justify other multiple divisions as well as 24-tone temperament. However, another tuning of the tetrachord, listed by Ptolemy and based on "ornamental" division $\frac{1}{12}:\frac{1}{11}:\frac{1}{10}:\frac{1}{9}$, might be regarded as precedent specifically for 24-tone temperament, largely because of the prominence of 11 in the ratios. Erik Eggen, after citing the prevalence of this tetrachord in much extant folk-music, points to the ratios involving 11 and concludes that quarter-tones are thereby established as rivals to whole- and half-tone systems.[39]

The primary defense for 24-tone temperament rests neither on acoustical nor on historical arguments, however, but rather on evolutional grounds, wherein 12-tone temperament is accepted as a fixed quantity and the next musical system is postulated as developing directly out of it. There is expediency involved in such a view, and this is somewhat confirmed by evidence of a non-concern for exactness in intonation which has been attributed to the chief advocates of 24-tone temperament. Fokker, in citing Haba as regarding 24-tone temperament as a loose approximation, approves. Fritz Kuttner, citing Wyschnegradsky's piano tuning as a haphazard affair frequently involving errors

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[39] Eggen, Skala-Studier.

as great as 20 cents,[40] considers such inaccuracy to be a poor reflection on quarter-tone music.

But if expediency is the reason for the choice of 24-tone temperament, this is not necessarily to be condemned. Expediency has been a factor in the development of musical systems in the past--certainly in the development of our own 12-tone system. Perhaps the most justifiable charge which can be brought against the pioneers in 24-tone equal division is simply that they have been insufficiently critical of the past and the present.

MORE THAN 24 TONES

Other writers and composers have sought to subdivide 12-tone temperament into still finer units. Among these are Busoni, who advocated 36-tone temperament but is not known to have composed in this system; Barbieri, who has composed music in 48-tone temperament which is not, however, generally available; Novarro, who has advocated a music which, in effect, uses 60-tone temperament, although actually employing only some of the tones; and Carillo, who has composed music supposedly in 96-tone temperament (a "Preludio a Cristobal Colón," which has been published[41]

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[40] Conversation with Mr. Kuttner. These estimates are said to represent actual measurements, and my own experience with unusual tunings would lead me to concur that such errors are possible.

[41] Published by the New Music Society, 1934.

and recorded).

Busoni presents his proposal for 36-tone equal temperament in his short book, Sketch of a New Esthetic of Music. Busoni's approach is entirely evolutional. Nature's gradations of pitch are infinite, he asserts, and it is therefore incumbent upon the present generation of musicians to move a little nearer to infinitude.[42] To this end, Busoni suggests that besides two-fold division of the whole-tone, three-fold division might be used. While other writers have started with this assumption and advocated 17- or 19-tone temperament, Busoni makes it clear that he is referring to exact divisions of the tone as it now exists. Because of his basic proposal, commentators have erroneously ascribed the advocacy of 18-tone equal temperament to Busoni. He leaves little room for doubt about his intentions, however. He acknowledges that a system of third-tones without semitones would represent a net loss as against the present tuning system, owing to the disappearance of the perfect fifth and the minor third. He therefore proposes to keep both, by creating two systems of third-tones, each separated from the other by a semitone (or three systems of semitones, each separated by a third-tone). This is 36-tone temperament. Busoni would make no use, for the present, of the smallest unit, the sixth-tone,

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[42] Sketch of a New Esthetic of Music, pp. 30-31.

as such. However, he states his belief that the sixth-tone will eventually become integrated into musical speech also.[43]

It is fascinating to speculate on what Busoni's views on multiple division would be if he were alive today. Would he have turned toward 17- or 19-tone temperaments which may well achieve his limited objectives more satisfactorily than 36-tone temperament? It is quite possible that the advent of the dodecaphonic school would have satisfied Busoni's aesthetic wishes within 12-tone temperament. He quotes Nietsche approvingly: "I could imagine a music whose rarest charm should consist in its complete divorce from the Good and the Bad." This divorce has been achieved without the intercession of multiple division of the octave.

Barbieri's 48-tone temperament is passed over for lack of available materials to discuss. Novarro's implied 60-tone temperament will be discussed in Chapter 6 under the heading 15-tone temperament.

72-tone temperament is a system selected by both Haba and Novarro for special attention. Haba selects it much as Busoni selects 36-tone temperament, as a means of fusing two simpler systems. Where Busoni uses 36-tone temperament to fuse 12- and 18-tone temperaments into a single system, Haba does the same with 72-tone temperament, combining 12, 18, 24, and 36-tone temperaments. The

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[43] Ibid., pp. 31-2.

guiding principle is that all of these numbers are factors of 72. Haba revels in the vastness of the resources of 72-tone temperament. Indeed, the combinations are so numerous that his ideal of completely different harmonic progressions for each composer might have a chance at realization.

Novarro, who carefully examines the merits of every possible equal temperament by checking the correspondence of intervals within each system to small-number ratios, has a great deal to say for 72-tone temperament. Indeed, it is far more than just another multiple of 12. Among the intervals Novarro considers adequately represented in 72-tone temperament are the following, plus their inversions.[44]

21:20 as 5/72	8:7 as 14/72	5:4 as 23/72
16:15 as 7/72	7:6 as 16/72	9:7 as 26/72
10: 9 as 11/72	6:5 as 19/72	7:5 as 35/72
9: 8 as 12/72	4:3 as 30/72	

Pleased with the correspondence of the ratios based on prime numbers 7 or smaller, Novarro also assigns values within the temperament to intervals involving the 11th, 13th, and (in one case the) 17th partials.

11:9 as 21/72	11:7 as 48/72	13:7 as 64/72
11:8 as 33/72	13:8 as 50/72	17:9 as 66/72
13:9 as 39/72	11:6 as 63/72	

Except for 16:15, and the ratios involving the number 13, where the discrepancies are somewhat larger, all

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[44]Novarro, op. cit., p. 156.

of the intervals cited by Novarro are represented within 4 cents by 72-tone temperament, and most of them are realized far more closely than that. 72-tone temperament is one of those favored by the algorithmic process of Viggo Brun (for which see Chapter 13).

A brief analysis of the scale-degree numbers on Novarro's chart is revealing. The number of units in the intervals 8:7, 7:6, 9:7, 11:7, and 13:7 is always a multiple of two, informing us that the 7th partial is as faithfully represented in (72 ÷ 2 =) 36-tone temperament as in 72-tone temperament (an advantage about which Busoni says nothing, although Partch comments upon the accuracy of the 7th partial in the 36-tone system). The intervals whose ratios involve the number 11, 11:9, 11:8, 11:7, and 11:6, are all represented by scale degrees which are multiples of 3. This shows that the intervals involving the 11th partial are as well represented by (72 ÷ 3 =) 24-tone temperament as by 72-tone temperament. It will be recalled how excellent the representation of the 11th partial is in 24-tone temperament.

A similar perusal of the simple intervals involving the perfect fifth (4:3 and 9:8) shows that the scale degrees are multiples of 6. This indicates that the fifths of 72-tone temperament are still the fifths of (72 ÷ 6 =) 12-tone temperament, a fact which need not be discouraging, since the fifth is well represented in the simpler system. It should be noted that where the error of the fifth in 12-tone

temperament is less than 4 percent of the possible error, it is nearly 24 percent in 72-tone temperament. This amount is still moderate, however, by the standards of the thirds in 12-tone temperament.

The advantages which accrue from the combining of 12-, 24-, and 36-tone temperaments are summarized above. But 72-tone temperament has an important acoustical advantage which none of the other systems considered so far has. This concerns the representation of the 5th partial. It will be noted on Novarro's chart (above, page 163) that for the intervals 21:20, 16:15, 10:9, 6:5, and 5:4, the numbers of the scale degrees are all prime. This indicates that the (moderately) close approximation of the 5th partial available in 72-tone temperament is not available in any of the simpler systems based on factors of 72.

In order to appreciate completely the acoustical position of 72-tone temperament, it is advisable to consider an additive principle in the creation of compound temperaments out of simpler temperaments. This principle is implied by a number of authors and applied by others, even when not stated directly. When an equal temperament contains "a plus b" tones to the octave, where "a" and "b" do not contain common factors, then the temperament will represent a weighted mean between the characteristics of the equal temperament possessing "a" tones and the equal temperament possessing "b" tones. The temperament "a" or "b"

containing the larger number of tones will have the greater influence, in proportion to its greater size, on the characteristics of the temperament containing "a plus b" tones.

This principle is to be observed in the selection, by such theorists as Yasser and Kornerup, of a Fibonacci series of "good" temperaments. In obtaining 31 by the addition of 12 and 19, it is implicit that 31-tone temperament will represent a mean between the features of 12- and 19-tone temperaments, somewhat weighted in favor of the characteristics of 19-. The flat fifths (1/3 comma) of 19-tone temperament and the very slightly flat fifths of 12-tone temperament combine to produce a fifth flat by ¼ comma. The large major thirds of 12- and the small major thirds of 19-tone temperament combine to produce an almost perfect major third in 31-tone temperament (386.1 cents).

With this in mind, it is revealing to note that 72-tone temperament, more than a simple product of 12-tone temperament in six-fold subdivision, is also a mean between two highly regarded equal temperaments of opposite characteristics, 31- and 41-tone temperaments (whose fine qualities will be discussed at length in Chapter 5). 31-tone temperament possesses excellent representations of the 5th and 7th partials, while 41-tone temperament is especially noted for its excellent 3rd and 11th partials. 72-tone temperament also represents a departure from 53-tone

temperament, considered ideal by many music theorists, in the direction of 19-tone temperament, itself widely regarded with favor.

Although probably unrealistic for practical purposes because of the cumbersome number of tones required, 72-tone temperament is an excellent theoretical entity, worth bearing in mind as a possible meeting place between those who would base multiple division on 12-tone temperament and those who would look for other temperaments with superior acoustical properties.

A conceivable disadvantage of 72-tone temperament, as with all temperaments involving simple multiples of 12, is that the cycle of fifths does not embrace the entire system. Whether this constitutes a distinct liability or not would depend on the expectations of the musician employing the system. However, it should be noted that some musicians have postulated that the fifth should be a cyclic interval in any system, and Bosanquet eliminates systems based on multiples of 12 from his consideration for that reason. By Bosanquet's definition of regular systems (see below, chapter 5, page 170) neither 24-, 36-, nor 72-tone temperament is applicable.

The 96-fold division of the octave remains to all intents and purposes unexplored. Julian Carillo, in his Preludio a Cristobal Colón, claims to have written "the first composition in the world" in 16th tones. The 96-

division deals only with the unorthodox notation. In actuality the music is almost entirely limited in complexity to quarter-tones, with a few brief passages on one instrument in eighths of a tone. The work is for soprano (vocalizing), flute, violin, harp, quarter-tone fretted guitar, and octavina, a stringed instrument which is always played pizzicato and which, alone among the instruments, is briefly assigned eighths of a tone.

Above the figure 96 no further subdivision of 12-tone temperament has been proposed.

CHAPTER 5

EQUAL TEMPERAMENTS BASED ON FIFTHS AND THIRDS

Section 1: Positive Systems

This chapter introduces a domain of rather complex theories and counter-theories. Bosanquet's General Theory of the Octave,[1] offers a valuable procedure for relating and measuring the various systems encountered in this phase of the study. His theory possesses both the advantages and defects inherent in the use of such familiar landmarks as our 12-tone tempered system and the diatonic scale as a basis for comparison.

Bosanquet's contribution to the field of multiple division of the octave is almost too immense to evaluate. His writings have influenced as diverse a group as Fokker, Kornerup, and Partch, and his own efforts at instrument-construction provided a major breakthrough and an excellent example for those who followed him.[2] His General Theory of the Octave supplies a precise language for dealing with

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[1] An Elementary Treatise on Musical Intervals and Temperament, 1876, p. 60.

[2] The instrument for which Bosanquet is justly famous is a harmonium normally tuned to 53-tone equal temperament, and possessing a "generalized keyboard" whereby for any acoustic grouping or relationship of sounds, the spatial relationship between the keys to be depressed remains exactly constant, regardless of transposition.

multiple divisions based on fifths and thirds. His interests covered the whole broad range, acoustical, historical, and anthropological, of subjects related to the multiple division of the octave.

BOSANQUET'S GENERAL THEORY OF THE OCTAVE

Bosanquet's theory begins with several useful definitions which will be cited and then observed throughout this and subsequent chapters.

> Regular Systems are such that all their notes can be arranged in a continuous series of equal fifths.[3]
>
> Regular Cyclical Systems are not only regular, but return to the same pitch after a certain number of fifths. Every such system divides the octave into a certain number of equal intervals.
>
> Error is deviation from a perfect concord.[4]
>
> Departure is deviation from an equal-tempered interval. Unless the equal-temperament is otherwise specified, departure is measured from the intervals of 12-tone equal temperament.
>
> Intervals taken upwards are called positive, taken downwards, negative.
>
> Fifths are called positive if they have

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[3]Nothing is said, under regular systems, about the necessity of the fifths meeting. Both Pythagorean intonation and meantone tuning are regular systems by Bosanquet's definition. Equal-tempered systems are not only regular, but cyclical as well.

[4]Perfect concord meaning a concord based on a small-number ratio.

positive departures.[5]

Systems are said to be positive or negative according as their fifths are positive or negative.[6]

Regular cyclical systems are said to be of the rth order, positive or negative, when twelve of the approximate fifths of the system exceed or fall short of the octave by r units of the system.[7]

The above list of definitions gives Bosanquet a yardstick, 12-tone equal temperament, which may influence his judgment, but which does not constitute a commitment to that effect. He now proceeds to formulate theorems pertaining to the nature of regular systems.[8]

Th. I. In any regular system, five 7-fifths semitones and seven 5-fifths semitones make up an exact octave.

By his interest in the two kinds of semitones, Bosanquet is concerning himself with the diatonic scale.

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[5]Note that departure rather than error is the basis for classification of the fifths. This is a most important aspect of Bosanquet's theory, as it establishes 12-tone temperament as the norm for comparison. The perfect fifth has, of course, a positive departure.

[6]This establishes 12-tone temperament as the mean between "positive" and "negative" systems.

[7]The concept of an order, r, may require explanatory illustration. Note that a unit of a system is its smallest interval. Let us take 19-tone temperament as an example. An approximate fifth in 19-tone temperament is 11 units of the system. Twelve such fifths total (12 x 11 =) 132 units. Seven octaves equal (19 x 7 =) 133 units. Therefore, in 19-tone temperament, "twelve of the approximate fifths of the system . . . fall short of the octave" by 1 unit of the system. In 19-tone temperament, r = -1.

[8]Op. cit., p. 61.

Theorem I asserts that for any regular system the practice of determining two kinds of semitones from superimposed fifths is valid. This theorem is easily proved[9] by the identity $7 \times 5 = 5 \times 7$.

> Th. II. In any regular system the difference between the 7-fifths semitone and the 5-fifths semitone is the departure of 12 fifths, having regard to sign.

This is self-evident, and is based on the simple arithmetical fact that 5 fifths plus 7 fifths equals 12 fifths and the assumption that the difference in the semitones will be created by the departure in the fifths since where there is no departure in the fifths (in 12-tone temperament) the two kinds of semitone are identical.

The next five theorems concern Bosanquet's theory of r, and are presented here without individual commentary. The concept of r further relates multiple division to 12-tone temperament, since for each value of r there is an endless series of regular cyclical systems, separated from one another by the addition or deletion of 12 tones to the

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[9] In 12-tone temperament, 12 semitones equals an octave. In any other system the semitones are altered according to a multiple of the departure of the fifth. As the size of the fifth is constant, the departures of seven 5-fifth semitones and of five 7-fifth semitones will each equal 35 times the departure of a single fifth. But because the 5-fifth semitone is decreased as the fifth is increased, while the 7-fifth semitone is increased with the fifth, the two 35-fifth departures will cancel out one-another, leaving only the 12 equal-tempered semitones which continue to make an octave.

octave.[10] The temperaments which have been considered in the preceding chapter would all be considered by Bosanquet to have r equal to 0. Temperaments of a given order of r can differ from one another quite extensively although they will possess some common characteristics as opposed to 12-tone temperament. Within a given order of r, the more tones there may be to the octave, the greater is the influence of 12-tone temperament on the intervals of the series (12-tone temperament is the limiting case of any order of r where n approaches ∞).

Th. III. In a regular cyclical system of order plus or minus r, the difference between the 7-fifths semitone and the 5-fifths semitone is plus or minus r units of the system.

Th. IV. In any regular cyclical system, if the octave be divided into n equal intervals, and r be the order of the system, the departure of each fifth of the system is r/n semitones of 12-tone temperament.

Th. V. If in a system of the rth order the octave be divided into n equal intervals, $r + 7n$ is a multiple of 12, and $\frac{r + 7n}{12}$ is the number of units in the fifth of the system.

Th. VI. If a system divide the octave into n equal intervals, the total departure of all the n fifths of the system equals r semitones of 12-tone temperament, where r is the order of the system.

Th. VII. If n be the number of divisions in the octave in a system of the rth order, then $n + 7r$ will be divisible by 12, and $\frac{n + 7r}{12}$ will be the number of units in the 7-fifths semitone.

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[10] When $r = -1$, for example, we have the endless series, 7-, 19-, 31-, 43-, 55-, 67-, 79-, etc.

Having explored the properties of the fifth, octave, and semitones, of the systems of the various orders, Bosanquet proceeds to theorems relating to other intervals.

> Th. VIII. Negative systems form their major thirds by four fifths up. Corollary: The departure of a third of a negative cyclical system $\underline{n}$ of order $-\underline{r}$ is $\frac{-4r}{n}$.

Barbour insists[11] that this formula is not valid where $\underline{n}$ is greater than 12. If Barbour is correct, all negative systems including such attractive and popular ones as 19- and 31-tone temperaments must be eliminated on the spot. Since the thirds of 12-tone temperament are produced in this manner and are substantially larger than 5:4, the negative systems which have been seriously proposed unfailingly improve the quality of the thirds.

> Th. IX. Positive systems form approximately perfect thirds by eight fifths down. Corollary: the departure of such third is $\frac{-8r}{n}$.

The application of this theorem, which makes use of the schismatic relationship, has a weakness. Since the third is produced by eight fifths, relatively slight deviations in the fifth are magnified 8 times in their effect on the thirds. 29-tone temperament affords a useful illustration, being a positive system of the order r = 1. In this tempera-

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[11] Tuning and Temperament, p. 130.

ment, the error of the fifth is only 1½ cents. The error of the third, however, consists of the schisma plus eight times the error of the fifth, a total of 14 cents, enough to render 29-tone temperament useless as a system with fifth and third.

> Th. X. Helmholtz' Theorem. The third thus formed with perfect fifths has an error nearly equal in amount to the error of the equal-tempered 5th.

This theorem involves only approximation and therefore cannot be proved. The two ratios which are semi-equated arise from different series, the one involving only 3 and 2, the other involving 5 as well, and their near-identity is fortuitious.

> Th. XI. In positive systems an approximate harmonic seventh can be obtained by fourteen fifths down.

> Th. XII. In negative systems an approximate harmonic seventh can be obtained by ten fifths up.

Owing to the number of fifths involved in each seventh, small variations in the fifths can effect huge displacements in the sevenths. Applied to 31- and 41-tone temperaments, theorems XII and XI, respectively, produce excellent results. 36-tone temperament, wherein r = 0, possesses an excellent harmonic seventh which cannot be obtained by any combinations of fifths at all. It is recalled from the previous chapter that the fifths in 36-tone temperament are not cyclic over the whole system, and the two members of the interval 7:4

belong to different cycles of fifths in 36-tone temperament which is, when all is said, an irregular system by the General Theory of Bosanquet.

WHERE R EQUALS 1

Bosanquet's classification of regular cyclical systems according to a value or r, positive or negative, affords a particularly useful perspective for examining the various possible temperaments. The minimal variation from 12-tone temperament is provided by temperaments in which r is equal to plus or minus 1. In view of the general concern with pure fifths and thirds in general, and the preservation of the acoustical advantages of 12-tone temperament in particular, it should come as no surprise that the vast majority of authors who have recommended multiple division have advocated systems in which r is no greater than plus or minus 1.

The order, r equals 1, includes the temperaments of 5-, 17-, 29-, 41-, 53-, and 65 tones per octave. In this order, the fifths are always larger than 700 cents, with 700 cents representing the limiting case as n approaches infinity. The operation of this principle is observed in the fifths of the six systems listed above, which are, respectively, 720 cents, 705.9 cents, 703.5 cents, 702.4 cents, 701.9 cents, and 701.5 cents. The great advantage of the systems of this order is their fifths, which are particularly

excellent in the last four of the cited temperaments. The thirds, however, are often weak. Since the fifths are larger than in 12-tone temperament, the thirds formed by 4 fifths are even larger than the large thirds of present use, and are consequently impracticable. Following the procedure Bosanquet recommends (in Th. IX.), the thirds formed by 8 fifths vary considerably from system to system. In 5- and 17-tone temperaments the major third is, to all intents and purposes, a non-existent interval. In 29-tone temperament it is still 14 cents too small, an error prompting even Novarro, who experimented with the temperament at length, to discard it.[12]

In 41- and 53-tone temperaments, the third makes rapid improvement, being 6 cents too small in the former, and slightly more than 1 cent too small in the latter. When the temperament is increased to 65- per octave, the third becomes nearly two cents too large. Several theorists, among them Ariel and Würschmidt, but earlier Bosanquet,[13] have noted the opposite deviations of the thirds in 53- and 65-tone temperaments. Since both have excellent fifths, the temperament involving their sum of tones, 118-, must be nearly perfect from the standpoint of fifths and thirds.

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[12] Novarro, op. cit., pp. 162-4.

[13] Bosanquet mentions the performability of 118-tone temperament (not quite all of the tones) on his harmonium in his article, Temperament, or the Division of the Octave, from the Proceedings of the Musical Association of London, 1874-5.

All of the temperaments of the order $r = 1$, between 5 and 53-tones per octave, have been used or have been recommended. 5-tone temperament is the approximate tuning of the Indonesian salendro scale. 17-tone temperament has generated interest from time to time, and is often considered to be the properly constructed Arabian scale.[14] There have been at least two 20th century advocates of 17-tone temperament, Edmond Malherbe and Vladimir Karapetyan. Malherbe's views are published,[15] and he is seen to be interested in a 17-tone system for "third-tones" which he considers to have more validity here than in an 18- or 36-tone system. He may not even belong properly with the advocates of 17-tone temperament, for he asserts a leaning toward the Pythagorean intonation of his 17-tone system.[16] Perhaps his most interesting contribution is his suggestion for the notation of simultaneous sharps, flats, and/or naturals on the same line of the staff. This perplexing problem of contemporary music, aggravated by multiple division, he solves with the following, self-explanatory symbols.[17] ♯♭ ♯♯ ♮♭

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[14] Barbour takes this view in Tuning and Temperament, p. 114.

[15] Le Menéstrel, 19 July, 1929, p. 329.

[16] Ibid., p. 330.

[17] Ibid., p. 337.

Vladimir Karapetyan, a professor of engineering, argues for the use of 17-tone temperament on the grounds that it is a simpler expansion from 12-tone temperament than 19- and that it possesses better fifths than 19-tone temperament.[18] Barbour imagines that the larger-than-Pythagorean thirds and the small semitones might make 17-tone temperament excellent for melodic, though not for harmonic, purposes.[19]

29-tone temperament is the first of the multiple divisions with better fifths than 12-tone temperament. Novarro credits its first advocacy to Cheve. Novarro fretted a guitar to 29-tone temperament and conducted a series of experiments but discarded it, as had von Jankó and others, because of its weak thirds.

Von Jankó, famous primarily for his attempt to popularize a keyboard based on Bosanquet's multiple-division keyboard for general use in the 12-tone system, wrote one article on multiple division[20] in which 41-tone temperament is singled out for favorable attention. A number of major theorists, including Fokker, have given 41-tone temperament respectful attention as well. Like 17- and 29-tone tempera-

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[18] Letter from V. Karapetyan to J. Yasser, Aug. 22, 1935, unpubl.

[19] Op. cit., p. 114.

[20] "Über mehr als zwölfstufige gleichschwebende Temperaturen," Beiträge zur Akustik und Musikwissenschaft, 1901, p. 6.

ments, it contains a major third which is too small, but the error is, in 41-tone temperament, reduced to six cents. Besides superb fifths (less than half a cent too large), 41-tone temperament contains a natural seventh which is less than 3 cents too small, an excellent 9th partial, of course, and an 11th partial which is about as inexact as the major thirds. It is the first of the equal-tempered systems where the errors of all partials through 11 are sufficiently small to render the system discussable as a realization of all of these partials. Prof. Viggo Brun shows, by a process of algorithms,[21] how 41-tone temperament belongs properly to the mathematical series of temperaments which represent optimally the partials through 11.

Fokker considers 41-tone temperament quite seriously as a function of the partials 3, 5, and 7, without including 11. He goes to considerable effort to illustrate this system with a diagram of the three-dimensional tone-lattice as he conceives it.[22] He also charts the deviations of the various intervals based on small-number ratios in 41-tone temperament, measured in terms of units of the system. Example 26, an excerpt from Fokker's chart, shows the size of each of the intervals involving numbers 11 and smaller

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[21]Unpublished letter to the author, but note his publications in bibliography.

[22]Rekenkundige Bespiegeling der Muziek, section beginning p. 183. Similar charts from this book on 53- and 31-tone temperaments are reproduced, below.

and the errors of their approximations in 41-tone temperament. 1.000 equals 1/41.

Example 26: Small-Number Intervals in 41-Tone Temperament

	2	3	4	5	6	7	8	9	10	11
2	0	23.98								
3	.02	0	17.02	30.22						
4		-.02	0	13.20	-	33.10				
5		.22	-.20	0	10.78	19.90	27.80	34.77		
6				.22	0	9.12				35.85
7			-.10	.10	-.12	0	7.90	14.87	21.10	26.74
8				.20		.10	0	6.97		18.84
9				.23		.13	.03	0	6.23	11.87
10						-.10		-.23	0	5.64
11					.15	.26	.16	.13	.36	0

Fokker's final conclusion on 41-tone temperament is that it offers no acoustical improvement over 31-tone temperament to justify the complications involved in using 10 additional tones. He bases this conclusion on the figures of Ligtenberg, who used the sum of the squares of the deviations of fifth, third and seventh in arriving at his recommendations.

That 41-tone temperament offers no general improvement over 31-tone temperament is essentially true, since the 6-cent discrepancy of the thirds in 41-tone temperament

replaces the 6-cent discrepancy of the fifths in 31-tone temperament. Nevertheless, to theorists such as von Jankó, who postulate a near-perfect fifth as the sine qua non for a temperament, 41-tone temperament does represent a distinct gain over 31-.

53-TONE TEMPERAMENT

With 53-tone temperament we arrive at what many theorists including Bosanquet have regarded as almost the utopian condition for music. Discovery of 53-tone temperament is generally credited to Mercator, the scientist famous for the map projection in which East is always East, and West is West, although Greenland becomes many times the size of Australia. (1)

Yasser, however, cites the Chinese King Fang as the discoverer of the 53-tone system.[23] It is unlikely that either King Fang or Mercator thought of his discoveries in practical musical terms. They are both responsible for the discovery of the remarkable near-coincidence between 53 fifths and 31 octaves. The actual ratio for the interval separating the two is $3^{53}:2^{84}$ or:

$$\frac{19,383,245,667,680,019,896,796,723}{19,342,813,113,834,066,795,298,816} \quad !!$$

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[23] A Theory of Evolving Tonality, p. 31. Partch adds: "no royalty implied."

(1) This is a gross error. Nicholas Mercator and Gerhardus Mercator were two different people.

In recent years, Esther Tipple has attempted to revive interest in 53-tone temperament with a number of tables, charts, a slide-rule, and an instrument. The name of Mercator is again associated with the venture, as she has established the Mercator Foundation in Georgia, under whose auspices her work has been done.[24] Although she has suggested scales with various numbers of tones from within the system of 53-tone temperament, no known compositions have come from the Mercator foundation. Miss Tipple's 19-tone scale from within 53-tone temperament is noted below in chapter 12. p 731

Since the time of its mention by Mercator, 53-tone temperament has been considered by many theorists, usually favorably. Ariel includes it in his series of "good" systems, coming after 34-tone temperament. Würschmidt considers 53-tone temperament among the finest of the rational tone systems, devoting considerable space ot its analysis.[25] Ellis had earlier called 53-tone temperament among the three best[26] (together with 12- and 31-. Fokker was to select the same three temperaments). In 1912, the advocacy of 53-tone temperament appears in its most

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[24]Tipple, Music Logarithmic Spiral and other works, see bibliography. The address of the Mercator Foundation is given as Black's Bluff Road, Rome, Georgia.

[25]Würschmidt, Die rationellen Tonsysteme... op. cit., p. 550 ff.

[26]Appendix to Helmholtz, op. cit., p. 435.

suspenseful and mysterious form: an anonymous interview granted to G. Bender by a disciple of "Futurism" in Le Guide du Concert, who stresses that in 53-tone temperament Fxx (quadruple sharp) = Dbbbb. This is true.

It is to Bosanquet, however, that the greatest credit for the modern interest in 53-tone temperament belongs. He points out that not only is the fifth excellent in 53-tone temperament (its discrepancy is less than .1 cent) but the third is less than 2 cents too small, an error smaller than that of the fifth in 12-tone temperament. The single unit of 53-tone temperament is only slightly larger than the syntonic comma. Bosanquet points out that the syntonic comma equals 1/55.8 octave.[27] The discrepancy between 1/53 and 81:80 is slightly larger than one cent. The Pythagorean third is nearly 0.3 cents flat, and the error of the just major third is the sum of the two figures, 1.4 cents.

Bosanquet attempted to make practical reality out of 53-tone temperament with an instrument which may well prove to be the direct ancestor of any multiple division instrument in general use in the future.[28] That the 53-tone system is closely linked by Bosanquet to the various multiple just systems based on perfect fifths is evident

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[27] Elementary Treatise... op. cit., p. 13.

[28] Bosanquet's generalized keyboard is employed with slight modifications by Fokker for his 31-tone organ which has had considerable influence in recent years. A diagram of the generalized keyboards is included in the appendix of this study.

not only from his own text but also from the accompanying advertisement of the manufacturer of his instrument. The buyer who wishes an organ with multiple division but cannot afford the 53 tones or 84 keys demanded by Bosanquet may buy an organ with 24, 36, or 48 keys to the octave. The discrepancy between the tones of the systems with 24, 36, and 48 perfect fifths on the one hand and 53-tone temperament on the other is all but negligible. 53-tone temperament provides an approximation that will be in error by about 1 cent for every 12 fifths traversed in the Pythagorean systems. The discrepancy is larger, of course, if the fifths of the just system are tempered by 1/8 of a schisma to produce just thirds. It probably may be surmised that the organs in Mr. Jennings' advertisement were designed with perfect, rather than octa-schismatically altered fifths in mind.

The traditional diatonic scale can be played in two different ways in 53-tone temperament, as Steiner has pointed out.[29] With Pythagorean intonation, the whole-tones are produced by intervals of 9/53 and the semitones by intervals of 4/53. The discrepancy is negligible. A version of just intonation with an average deviation of less than a cent is obtainable with 9/53 for the major tone, 8/53 for the minor tone, and 5/53 for the diatonic semitone. For

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[29] Bericht von III Kongress, I.M.G., op. cit., p. 394.

musical requirements closely related to the diatonic scale, 53-tone temperament is little short of perfect. It may, however, be much too complex for so simple a use.

Among the theorists writing on 53-tone temperament, Wesley Woolhouse, in 1835, some 40 years before Bosanquet, complains about the inaccuracy of the thirds. Woolhouse, the initial proponent of 19-tone temperament, would hardly seem to be the man to be disturbed by an error of only 1½ cents. He does not state the size of the thirds or their error and it is possible that he calculated them incorrectly.

Partch, however, objects strongly, and with considerable justification, to the representations of the 7th and 11th partials in 53-tone temperament. The former is almost 5 cents too sharp, and the latter close to 7 cents too flat, the discrepancies being quite substantial in view of the fineness of the temperament.

The 5-cent error in the 7th partial notwithstanding, Fokker accepts this interval as an essential feature of 53-tone temperament, building a three-dimensional tone-lattice as is his custom with other multiple temperaments. His diagram is reproduced several pages hence in Example 28.

Neither Fokker, Öttingen nor Würschmidt, all of whom offer specific "just" intonations for 53 tones per octave, present any comparison between their proposed systems and 53-tone equal temperament. Example 27 shows the approximate tunings, in cents, of the 53-tone scales of these three as

deduced from their diagrams, and compares them to Pythagorean and equal-tempered 53-tone systems. The intervals are measured from the middle-most tone in each diagram and the discrepancies likewise. The average deviation shown at the bottom involves all of the intervals as measured from a central tone, and is not the same thing as Barbour's mean deviation, which is concerned with the constructing intervals (units of the system).

Example 27: Comparison of 53-Tone Systems

	53-t temperament	Ottingen	Wirschmidt	Fokker	Pythagorean
0	00.00	00.00	00.00	00.00	00.00
1	22.62	21.51		13.8	23.46
2	45.28	41.06		43.4	46.92
3	67.92	70.68		70.68	66.77
4	90.57	92.18		84.4	90.23
5	113.21	111.74		111.74	113.69
6	135.85	133.24		133.24	137.15
7	158.49	162.85		155.1	156.99
8	181.13	182.40		182.40	180.45
9	203.77	203.91		203.91	203.91
10	226.42	223.46		231.2	227.37
11	249.06	244.97		239.55	250.82
12	271.70	274.58		266.85	270.68
13	294.34	294.14		288.4	294.14
14	316.98	315.64		315.64	317.60
15	339.62	335.20	345.25	337.55	341.06
16	362.26	364.81	356.70	359.05	360.90
17	384.91	386.31		386.31	384.36
18	407.55	407.82		400.1	407.82
19	430.19	427.47		427.47	431.28
20	452.83	456.99		457.0	451.13
21	475.47	478.50		470.75	474.59
22	498.11	498.05		498.05	498.05
23	520.75	519.56		519.56	521.51
24	543.40	539.11		541.3	544.97
25	566.04	568.72		555.2	564.81
26	588.68	590.22		582.5	588.27

Example 27 (Continued)

	53-t temperament	Ottingen	Würschmidt	Fokker	Pythagorean
27	611.32	609.78		617.5	611.73
28	633.96	631.28		631.28	635.19
29	656.60	660.89		653.2	655.03
30	679.25	680.44		674.65	678.49
31	701.89	701.95	701.95	601.95	701.95
32	724.53	721.50		729.25	725.41
33	747.17	743.01		737.6	748.87
34	769.81	772.53		777.53	768.72
35	792.45	792.18		786.4	792.18
36	815.09	813.69		813.69	815.64
37	837.74	835.19	843.30	(Six figures see below)	839.10
38	860.38	864.80	854.75	857.1	858.95
39	883.02	884.36		884.36	882.41
40	905.66	905.86		898.1	905.87
41	928.30	925.42		933.15	929.33
42	950.94	955.03		941.5	949.17
43	973.58	976.54		968.8	972.63
44	996.23	996.09		996.09	996.09
45	1018.87	1017.60		1017.60	1019.55
46	1041.51	1037.15		1039.5	1043.01
47	1064.15	1066.76	all others	1053.2	1064.86
48	1086.79	1088.26	same as	1088.26	1086.32
49	1109.43	1107.82	Ottingen.	1115.6	1109.78
50	1132.08	1129.32		1129.32	1133.24
51	1154.72	1158.94		1151.2	1153.08
52	1177.36	1178.49		1172.7	1176.54
53	1200.00	1200.00		1200.00	1200.00
Average deviation:		2.395	3.044	5.629	0.91
Average number of intervals used:		3.577	3.462	2.941	13.5

The six different pitches for Fokker's 37th tone are:

835.6
835.2
843.3
827.5
848.7
822.0

Example 27 reveals a number of characteristics of 53-tone temperament worth noting. As with 12-tone temperament it is the perfect fifth which is the dominating interval.[30] The very existence of more than one interpretation of just intonation shows that with 53 tones to the octave, as with 12, Barbour's statement is applicable: "There is no such thing as just intonation; but, rather, many different just intonations; . . ."[31] Barbour's statement continues, ". . . of these, the best is that which comes closest to the Pythagorean tuning." This is even more true of 53-tone temperament than of 12-, as the low figure under Average Deviation shows.

In general, 53-tone temperament would seem to be as good a system as most theorists have regarded it. The average deviations range from less than a cent, when compared to Pythagorean tuning, to around 2½ to 3 cents for the systems using fifths and thirds, to over 5½ cents for Fokker's system employing the natural seventh as well. By comparison the average deviations for various systems in 12-tone temperament are: Ptolemy's just intonation, 12 cents; Aron's meantone temperament, 13 cents; Pythagorean tuning,

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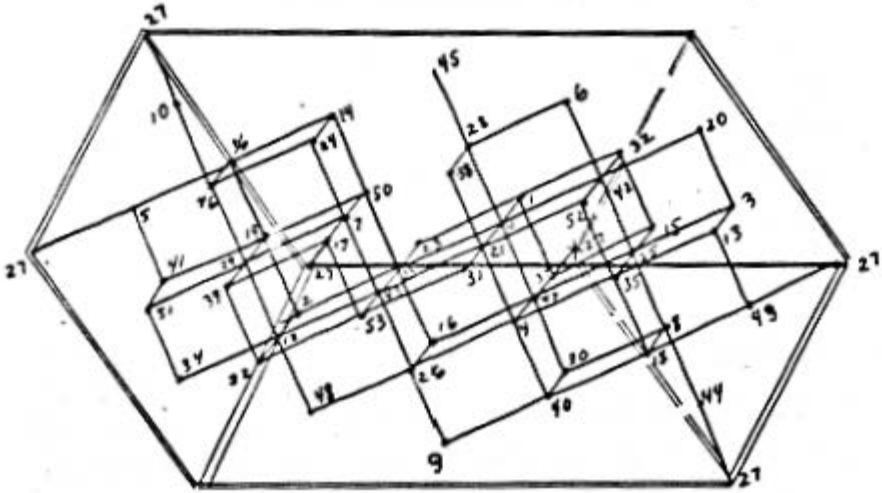
[30] Ariel, who likes to consider each of the "good" systems as related to the interval which is best represented by its temperament, acknowledges the hegemony of the fifth in 12- and 53-tone temperaments but not, of course, in 19- or 31-.

[31] Op. cit., p. 105.

6 cents. The extent of improvement in 53-tone temperament is about proportional to the increase in the number of tones. The improvement is even better in the case of the Pythagorean system and in the case of systems involving septimal intervals. Septimally derived intervals in 12-tone temperament are all but unthinkable.

Of the four systems shown, Fokker's is the most interesting. Example 28 is a diagram of the system upon which the figures in Example 27 are based. I have transposed the numbers shown below by adding 10 to each of them, in order to make the tone shown by Fokker as 43 the generating tone of the system, since it is the most centrally located of the tones in his diagram.

Example 28: Fokker's 53-Tone System



In Example 27, there is shown for each system the average number of steps which must be taken to reach each member of the system from the central or generating tone. It can be presumed that some advantage would accrue to those systems which required fewer separate steps in relating one tone to another. It is here that Fokker's tri-dimensional system shows to its greatest advantage, for it involves an average of fewer than three separate steps to reach each tone from the generating tone even though it is not precisely at the center, whereas the bi-dimensional systems require more than 3. It is also here that the weakness of the uni-dimensional Pythagorean system is most apparent, with 13.5 steps separating the average member from the central tone. Würschmidt's system, slightly inferior to Öttingen's in intonation, is the more compact of the two, as can be seen by Example 27.

Fokker's system is suspended between six different representations of the 27th tone of his system (the 37th on our chart). It is symptomatic of the weakness of his system from the standpoint of intonation, that the six representatives of this one tone vary in pitch by 26.7 cents, more than the smallest unit of 53-tone temperament! The smallest interval between consecutive tones in Fokker's system is only slightly larger than 8 cents, while the largest is about 35 cents. It may well be justifiable to conclude that a tri-dimensional tone-lattice does not work well for a system in

which one basic interval is very much better represented than another. The weaknesses of Fokker's system reflect the weaknesses of the 7th partial in 53-tone temperament.

In concluding our consideration of 53-tone temperament, it would seem reasonable to suggest that it is a superb temperament for fifths, a good temperament for fifths and thirds, and a questionable temperament once higher prime partials are brought into consideration. 53-tone being the most complicated equal temperament to be considered in any detail in these pages, this would appear the most suitable time to point out that no matter how minute the subdivision of the octave, just intonation always remains a step ahead of equal temperament and they never can be brought together. 53-tone temperament also demonstrates that as long as there are two or more theorists considering a given musical system, there will always be contention over the just intonation.

WHERE R EQUALS 2

Four systems in which r = plus 2 are of sufficient value to merit some attention. Three of these have been proposed, 22-, 34-, and 118-tone temperaments, of which the latter will be considered in Chapter 6 in comparison with other temperaments involving a very great number of tones. In addition to the other temperaments already named, 46-tone

temperament has acoustical features possibly worthy of consideration. Example 29 shows the size of the intervals most closely approximating 3:2, 5:4, and 7:4 in each of the three above-named systems, together with the discrepancies involved in each case.

Example 29: 22- 34- and 46-Tone Systems Compared

Interval	22-Tone	34-Tone	46-Tone
FIFTH	709.08	705.88	704.34
ERROR OF FIFTH	7.13	3.93	2.39
THIRD	381.8	388.24	391.3
ERROR OF THIRD	4.4	1.93	5.0
SEVENTH	981.8	952.9	965.2
ERROR OF SEVENTH	13.0	15.9	3.6

These systems compare not altogether unfavorably with those temperaments previously examined. The double order is reflected in the high degree of sharpness of the fifths, especially in 22-tone temperament.

Since the errors of the third and fifth in 22-tone temperament are in different directions, the error of the minor third is equal to their sum, almost 12 cents. This is probably excessive, but the system is otherwise within the realm of consideration, with its flaws about commensurate with those of 19-tone temperament. It is possible that the positive fifths, untried historically, might be more

bothersome than the negative fifths of 19-tone temperament, which are also about 7 cents in error. The major third in 22-tone temperament is better than the major third in 19- (by a rather small margin), but the minor third is much worse. Neither temperament offers a close approximation to the natural 7th, although that of 22-tone temperament, erring on the same side of the seventh in 12-tone temperament but less than half as much, would sound relatively pure to our ears. 22-tone temperament possesses a familiar landmark, the tritone of 12-tone temperament. Opelt and Würschmidt report with some favor on this system, as does Novarro, who attributes its introduction to the East Indians. The 22 srutis of Indian music are not generally considered to represent an equal-tempered system, however.

34-tone temperament can be compared profitably with its negative equivalent, 31-tone temperament. The fifths are slightly better and the major thirds slightly less good. The minor thirds are better owing to the fact that both fifth and third are positive in error. Where 34-tone temperament is weakest in comparison to 31- is in its representation of the harmonic seventh. It is for this element that 31-tone temperament must be preferred to 34- for possible general use.

46-tone temperament improves the 7th partial of 34-tone temperament to a point where it might be useful, but the third is weakened considerably in the process. The

great improvement of the fifths and thirds in 53-tone temperament (over 46-), achieved at only slight cost in the 7ths, renders the slightly more complex temperament the more likely to be useful.

Section 2: Negative Systems

Theorists have been reluctant to abandon the practice of building thirds by the superposition of four fifths. As the major third in 12-tone temperament is more than 13 cents too large, many theorists have built musical systems by reducing the size of the equal-tempered fifth.

WHERE R EQUALS -1

By far the most common order of these systems is that in which twelve fifths fall short of seven octaves by one unit. The limit of the fifth in this order, as in all orders based on 12-tone temperament, is 700 cents. This limit is approached from below, so that the fifth is always too small. This is a congenital weakness in the negative orders, one which has caused Drobisch to leave them out of his series of ideal temperaments[32] and Jankó to reject them out of hand. Even Bosanquet gives the negative systems a subordinate role.[33]

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[32] Drobisch, op. cit., p. 88.

[33] His harmonium is constructed primarily for positive systems, although it can be adapted for use with negative systems.

The series in which $\underline{r} = -1$ begins with 7-, 19-, 31-, 43-, and 55-tone temperaments. All five of these systems have played a role in either musical practice or musical theory. The fifths in each of the five systems, in ascending order of $\underline{n}$, are 685.7 cents, 694.7 cents, 696.8 cents, 697.7 cents, and 698.2 cents. The gradual improvement in fifths compensates in large measure for the added complications as $\underline{n}$ increases, but the improvement is too gradual to justify an unambiguous preference for those systems in this order where $\underline{n}$ is largest. The major thirds in the order approach a limit of 400 cents from below, crossing the desired 386.3 about where $\underline{n} = 31$. The thirds for the five systems listed above are, in ascending order of $\underline{n}$, 342.9, 378.9, 387.1, 390.7 and 392.7.

The harmonic seventh, as formed by ten ascending fifths according to Bosanquet's theorem (no. XI., see above), varies from 857.14 cents (more than 100 cents too small) in 7-tone temperament, to a limit of 1000 cents, very much too large. It is 21 cents too small in 19-tone temperament and 8 cents too large in 43-tone temperament. In 31-tone temperament, however, the harmonic seventh is approximated within 2 cents. It is the combination of excellent major thirds and natural sevenths which has made 31-tone temperament a particular favorite of many theorists.

The five principal temperaments of the order $\underline{r} = -1$ will be taken up one by one, with 31-tone temperament

reserved for the conclusion of this chapter, and 19-tone temperament reserved for a more detailed examination in subsequent chapters.

7-tone temperament is of significance more because of practice than theory. Siamese music is based on the approximation of this temperament. As will be noted subsequently, Joseph Yasser, one of the chief exponents of multiple division, sees in the Siamese musical system a direct ancestor of diatonic musical practice, frozen from further development by the arrival of equal temperament. Siamese musical practice is pentatonic, but the closed and complete system is essentially equal-tempered heptatonic.

If, as Fritz Kuttner has stated,[34] the fifths in Chinese music are consistently very flat, it is conceivable that some kinship may exist between Chinese musical practice and an approximation of 7-tone equal temperament. Such a kinship is merely speculative and is contradicted by other intonational practices in Chinese music; nevertheless, it would seem a better explanation for the phenomenon of the flat fifths than the dodecaphonic concept of the lü or the Pythagorean explanation usually given for the pentatonic scale.

7-tone temperament is of little value in Western music today, except for experimental purposes, since it would represent a retrogression from current practice both

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[34]Conversation, December 1960.

in complexity and in acoustical relationships. It is interesting to hear diatonic melodies played in 7-tone temperament, and very difficult not to conclude emphatically that the diatonic unequal scale is an aesthetic improvement. It is Yasser's contention that the abolition of 12-tone equal temperament in favor of a more complex system would provide more appropriate contours for 12-tone melodies much as 12-tone temperament provides a more suitable vehicle than 7-tone temperament for heptatonic melodies.

43-tone temperament is mentioned in most studies as the system proposed by Sauveur. It seems more likely, however, that Sauveur used equal temperaments only as a means of simplifying meantone systems for theoretical purposes and that he envisaged no music using 43 tones to the octave. Nevertheless, Sauveur's advocacy represents an endorsement of the acoustical attributes of 43-tone equal temperament since they are no different from those of the 1/5-comma temperament system Sauveur was actually recommending. It is in this temperament that the fifths and thirds yield equally to the demands of the other, rendering the temperament ideal for those who wish to preserve the relationship whereby the third is produced by four fifths without favoring either interval at the expense of the other.

Although cited in the standard works on multiple division, 43-tone temperament as a musical reality has yet to be advocated by anyone, unless Sauveur is considered to

have advocated it. This may well be because of the compromises which characterize the system. The fine thirds and sevenths of 31-tone temperament are given up, as well as its relative simplicity, for what is in reality only a slight improvement in the fifths. This has hardly seemed worthwhile to most theorists in view of the almost perfect fifths in 53-tone temperament at the expense of only slight further complication. A 43-tone system without equal temperament has been advocated and used extensively by Harry Partch (see below, Chapter 7).

55-tone temperament is the complete system, part of which is represented in Silbermann's 1/6-comma temperament, much favored by many instrument makers and tuners around 1700 and, according to Sauveur, "in universal use by musicians"[35] at that time. According to Novarro, 55-tone temperament as such has had an advocate in Esteve. Its fifths represent a slight improvement over those in 43-tone temperament, provided at the expense of the thirds and of what might still remain of simplicity. It seems far more useful as the basis for a 12-tone meantone temperament than as a system to be used in its totality. As a meantone temperament it possesses the great advantage of thirds which are produced by four fifths. As a system used in its totality it possesses no such advantage, since how the thirds may be derived in an equal-tempered system is purely academic; the

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[35]Scherchen, The Nature of Music, p. 42.

only meaningful question concerns their quality. The chief intervals in 55-tone temperament compare most unfavorably with those in the positive system which is its most obvious rival, 53-tone temperament. Fifths, thirds, and sevenths are all distinctly purer in the latter, rendering the acoustical defense of 55-tone temperament an extremely difficult matter.

WHERE R EQUALS -2: 50-TONE TEMPERAMENT

Before proceeding to 31-tone temperament, I am considering 50-tone temperament, the only temperament of order r = -2 to have been seriously proposed, because its problems very closely resemble those of 55-tone and other first-order negative systems. Formed by the adding together of the numbers 19 and 31, the number 50 designates an equal temperament possessing the basic strengths and weaknesses of those two temperaments which are yet to be considered, with the significant added weakness created by the increased complexity of the system. Its fifths are almost 6 cents flat; this represents a deviation of over 48 percent of the possible error where the intervals are so small. The thirds are only 2.3 cents too small, but this is a greater error than that of the thirds in 53-tone temperament, a temperament whose principal virtue lies not in its thirds but in its fifths. 50-tone temperament represents the completion of Zarlino's

2/7-comma temperament and Robert Smith's 5/18-comma temperament. Its advocacy as a system of multiple division is generally attributed to Henfling, a contemporary of Sauveur, probably as erroneously as in the case of Sauveur and 43-tone temperament. It is probable that Henfling, like Sauveur, meant his system to be a basis for measurement (50 is a convenient number for that purpose) and the superstructure for a simple meantone system. Robert Smith advocated it in this way as later did Woolhouse.

Like 43- and 55-tone temperaments, 50-tone temperament seems most useful for the simple manner in which its thirds are derived from its fifths, a usefulness which is essential to the construction of a suitable meantone temperament using a small number of tones, but of no acoustical value in the complete cyclical system. It is 53-tone temperament, which forms its thirds and fifths independently of one another, and which cannot be used therefore as the basis of a twelve-tone meantone temperament, which otherwise completely dwarfs its near rivals in acoustical powers.

31-TONE EQUAL TEMPERAMENT

It is the negative systems which dominate the field of equal temperaments between 12 and 41. Two systems with order $\underline{r} = -1$ lie within this area, 19- and 31-tone temperaments. These two systems have been advocated jointly by

Kornerup; Yasser has advocated the former for the present and the latter for the future. Würschmidt, Opelt and many of the other writers who have spoken with great favor of the one have done so equally of the other. Other writers, however, have made a more positive choice between the two systems. As will be demonstrated, most of the acoustical advantages lie with 31-tone temperament, while 19-tone temperament possesses those advantages which accrue from greater simplicity.

Holland and 31-tone temperament have maintained an affinity which stretches from the time of its discovery to the present. It was the Dutch mathematician, Christian Huygens, who demonstrated in 1732 that 31-tone temperament is a closed system representing the completion of Aron's meantone temperament. It was another Dutch scientist, Adriaan Fokker, retired professor and curator of the Teyler Institute in Haarlem, who built there a pipe organ tuned to 31-tone temperament which has become the central point in a recent surge of interest in that tuning system.

During the Renaissance, when Aron's ¼-comma temperament was in widespread use, a number of instruments were built using 31 tones to the octave. Best known of these was Vicentino's archicembalo, which appears to have had a companion instrument, an archiorgano. There is some confusion over Vicentino's exact intentions for the intonation of his instrument or instruments, a confusion which persists

in the face of brave new attempts to pinpoint the pitches on the basis of Vicentino's own ambiguous instructions.[36] It is fairly well established, however, that at least the first orders of keys, and possibly the entire instrument, were tuned to ¼-comma temperament, rendering the arciorgano almost an equal-tempered instrument.

Several other 31-tone instruments were built shortly after Vicentino's. The patterns of flat and sharp tones on these instruments were complex and showed no regard for facility of modulation or transposition, a most understandable circumstance considering when they were built. One instrument, however, Fabio Colonna's "Sambuca Lincea," is said[37] to have had the first "generalized" keyboard, wherein each order resembled the one before it. According to Barbour, Colonna's orders are separated by 1/5 of a tone, and each order contains 7 tones to the octave. It would appear that Colonna's keyboard was only partly generalized, since 31 divided by 7 does not leave an interval that would be the basis for any musical system of the Renaissance or early Baroque.

31-tone temperament shares with other negative temperaments the quality of having excessively small fifths.

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[36] For the most recent and detailed examination of the tuning of Vicentino's instruments see Kaufmann, Henry W., Vicentino's Arciorgano; an Annotated Translation, Journal of Music Theory, April 1961, p. 32.

[37] By Barbour in Tuning and Temperament, p. 119.

The deficiency is nearly 5.2 cents, representing 27 percent of the largest error possible. Against this defect must be balanced a major third and a natural seventh of exceptional quality. The thirds are 0.8 cent sharp while the sevenths are 1.05 cents too small. Theorists have generally written favorably about this temperament,[38] but often with reservations about its fifths.[39] Some writers have found more cause for objection in the minor thirds (major sixths) than in the fifths.[40] The discrepancy for this interval in 31-tone temperament is 6 cents, the sum of the discrepancies of the fifth and the major third. Writers who have most emphatically endorsed 31-tone temperament while rejecting 19-, such as Fokker, have tended not to consider the minor third as an interval of the same rank of importance as the major third.

Woolhouse, on the other hand, is concerned with the minor third, as were Smith and Harrison before him. Since

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[38]Probably the most enthusiastic statement about 31-tone temperament was by B. S. Wedell, a Copenhagen actuary who occasionally wrote articles on music for Danish-language periodicals. Kornerup quotes Wedell as writing of 31-tone temperament that it is "an Eldorado for music theorists." Kornerup, Acoustic Valuation of Intervals, p. 7.

[39]Novarro, op. cit., p. 165-6, suggests 31-tone temperament for those types of instruments only, where beats, such as will be created by out-of-tune fifths, are not heard too pronouncedly.

[40]Robert Smith in the 18th century and Woolhouse in the 19th adopt this view. See Smith, op. cit., preface, and also page 121; and Woolhouse, op. cit., p. 49, where in spite of his reservations he says of 31-tone temperament: ". . . a very good scale . . . has been much approved of by many musicians."

the minor third is created by deducting the major third from the perfect fifth, the error in the minor third of any given equal-tempered system will be a function of the errors of the perfect fifth and major third. Where the error of one of these two determining intervals is positive and the other negative, as in 31-tone temperament, the error for the minor third will be equal to the sum of the errors for the perfect fifth and major third. Where, however, the errors of both of the determining intervals carry the same sign (as in 19-tone temperament, where both the perfect fifth and major third are negative), the error for the minor third will be equal to the difference between the errors of the major third and perfect fifth. In 19-tone temperament the errors for the major third and perfect fifth, although each quite substantial, cancel one another almost completely, leaving the minor third extremely close to perfection. This near-perfection is of significance to Woolhouse, but is of no importance to Fokker, since the minor third is to him not a generating interval.

If 12- and 53-tone temperaments are essentially Pythagorean, in that they feature particularly satisfactory fifths, 31-tone temperament is not. The major third is the most satisfactory interval, and of the just intonations offered for 31-tone systems, those that offer the closest approximation to 31-tone equal temperament are the ones which use the greatest number of thirds and the fewest

fifths. Among the proponents of 31-tone temperament, at least three, Würschmidt, Ariel, and Fokker, have proposed systems of just intonation involving 31 tones to the octave, as well as methods for defining or deriving the temperament. Würschmidt and Ariel derive the system through thirds and fifths alone, whereas Fokker includes sevenths in a three-dimensional tone-lattice. Würschmidt and Fokker both define the temperament by the intervals which are made to equal zero. For Würschmidt, using two determinants, there are two such intervals. For Fokker, using three, there are three.

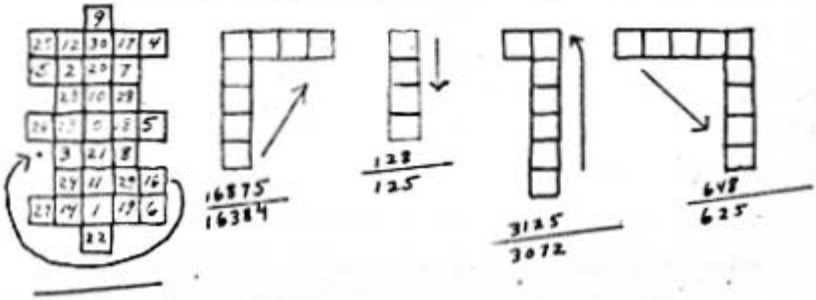
According to Würschmidt, one of the two intervals which are made equal to zero to establish 31-tone temperament is the syntonic comma, 81:80. A third is thereby made equal to four fifths minus two octaves. Since this is also true in 12-tone and other temperaments, it does not by itself define the system. Würschmidt's other identity in 31-tone temperament is between eight thirds, and two octaves and a fifth. The interval which is made thereby to become zero is 393216:390625, about 6 cents in size.

Würschmidt makes two diagrams for 31-tone temperament. The first, symmetrical, requires four different constructing intervals. These are 128:125, the difference between three thirds and an octave (this occurs 18 times in the system); 3125:3072, three thirds minus an octave and a fifth (10 times); 16875:16384, three fifths and four thirds minus three octaves (2 times); and the somewhat larger interval 648:625,

representing four fifths minus four thirds and an octave or else four minor thirds minus an octave (1 time).

Würschmidt acknowledges that by upsetting the symmetry through the alteration of a single tone (tone number 16 in Example 30) the fourth and largest constructing interval is eliminated, rendering the system presumably stronger. Example 30 shows Würschmidt's original, symmetrical diagram, with an arrow showing the alternate location of tone 16. The tones are numbered in ascending scalewise order, and are placed by fifths and thirds. The four constructing intervals are shown by the accompanying figures. The alteration shown by the arrow eliminates the single 648:625 from the system and removes one of the 10 appearances of the smallest unit, 3125:3072, replacing these with a nineteenth instance of the diesis, 128:125, and a third instance of 16875:16384.

Example 30: Würschmidt's 31-Tone System

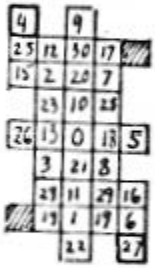


Ariel, in contrast to Würschmidt, makes no use of the interval 16875:16384, but rather uses Würschmidt's three other intervals as follows: 128:125 sixteen times, 3125:3072 twelve times, and 648:625 three times. Revising Würschmidt's diagram the minimum number of times necessary to produce Ariel's system of intervals, it reads as follows in Example 31.

Example 31: Ariel's 31-Tone System

dark border = added by Ariel to Würschmidt's system.

completely dark = deleted by Ariel from Würschmidt's system.



Example 32 shows the properties of the principal 31-tone systems which have been proposed. As the average deviations of the three foregoing systems are within 0.1 of a cent of being identical, only one of them Würschmidt's first system, is included in Example 32. Using Barbour's system of calculating the mean deviation of the constructing intervals, however, the difference between the three methods proposed above is more substantial, Würschmidt's second system involving the smallest mean deviation, 5.2 cents, and Ariel's the greatest, 6.4 cents. These figures compare

Example 32: Chart of Intervals in 31-Tone Just Systems

Unit of System	Equal Temperament	Würschmidt	Fokker	Alternate Septimal Tunings	
0	00.00	00.00	00.00		
1	38.71	41.06	27.3	48.7	35.65
2	77.42	70.68	92.17	70.68	84.44
3	116.13	111.74	119.5		
4	154.84	162.85	155.1		
5	193.55	203.91	203.91		
6	232.26	244.97	231.2		
7	270.97	274.58	266.85		
8	309.68	315.64	323.4	301.9	315.64
9	348.39	345.25	359.0		
10	387.10	386.31	386.31		
11	425.81	427.37	435.1		
12	464.52	456.99	470.75		
13	503.23	498.05	498.05		
14	541.94	539.11	(six figures see below)		
15	580.65	568.72	590.22		
16	619.35	609.78*	617.5	Würschmidt:	
17	658.06	660.89	653.2	*or 631.28	
18	696.77	701.95	701.95		
19	735.48	743.01	729.25		
20	774.19	772.63	764.9	786.4	772.63
21	812.90	813.69	821.4		
22	851.61	854.75	857.1		
23	890.32	884.36	884.36		
24	929.03	925.42	933.15		
25	967.74	955.03	968.8		
26	1006.45	996.09	996.09	1017.60	1003.8
27	1045.16	1037.15	1061.0	1039.5	1052.6
28	1083.87	1088.26	1088.26		
29	1122.58	1129.32	1115.6		
30	1161.29	1158.94	1172.7	all others the same as Fokker's	
31	1200.00	1200.00	1200.0		
Average deviation:		5.18	6.75	6.05	5.17 cents
Average number of steps		2.9	2.55	2.55	2.4
Mean deviation scalewise		5.2	10.4	9.7	7.9

The six different pitches for Fokker's 14th tone are:

525.25
533.7
541.4
545.85
554.6
and 563.95 cents. For calculating the average deviation, the best, 541.4 is used for all three septimal systems shown.

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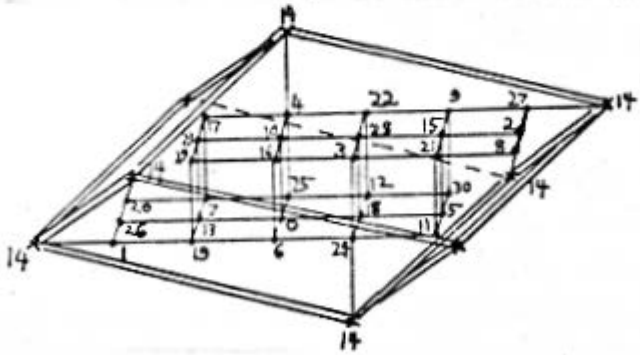
favorably with the mean deviations for just intonation and meantone temperament in a 12-tone system, even allowing for the difference in size of the basic intervals of the two systems.

The diagram of Fokker's tone-lattice is shown below in Example 33.

Example 33: Fokker's 31-Tone Tone-Lattice

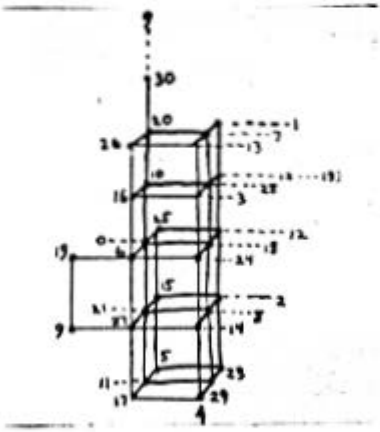


The numbers Fokker actually used are altered both here and in Example 32 by the deduction of the constant, 2, from every tone.

As in 53-tone temperament, Fokker pays a price for the addition of a third dimension. The intervals of his just system are less accurately portrayed by the equal temperament than are those of Würschmidt's and Ariel's systems. In the Fokker diagram the fifths are horizontal, the thirds vertical, and the sevenths diagonal in order to present an illusion of depth. Why, one might ask, does Fokker use the fifth far more than either of the other intervals when it is the fifth that is the weakest of the three basic intervals in the 31-tone system? The reason does not lie in a Pythagorean aesthetic, since Fokker never, in his theoretical writings, singles out the fifth for special emphasis among the basic intervals. The explanation appears, rather, to lie within the dynamics of the interrelationships of the three basic intervals in the 31-tone system. The diagonal is limited to two spaces in the diagram, because three sevenths would interfere with the formation of fifths and can therefore never be used. Since the seventh is itself created by two thirds plus two fifths, one of these latter intervals must be used only once. Since the third is the better of the two intervals in 31-tone temperament, one might expect Fokker to use only one fifth. Indeed, it is almost possible to reverse the fifths and thirds in Fokker's diagram and still have a complete system. But again Fokker would have been confronted with a limit caused by another identity in the 31-tone system: three thirds plus

two sevenths forms a fifth. A system based on one fifth, two sevenths, and two thirds would produce only 3 x 3 x 2 = 18 tones, not nearly enough to complete the system. The remaining tones can be added by further use of thirds and sevenths, if desired, but only in a most un-compact manner. Such a system is shown, below, as Example 34. Its average and mean deviations are markedly smaller than those of Fokker's system,[41] but the absence of compactness and symmetry are serious counterbalancing liabilities.

Example 34: A Three-Dimensional 31-Tone System Built Primarily on Thirds



In the accompanying system, the largest deviation from tone zero to any other tone, as opposed to equal temperament, is approximately 7 cents to tone nine. An alternate tone 9, through which this discrepancy is greatly reduced, is shown by the dotted line.

Fokker's basic system is built on three fifths (a fourth fifth cannot be used regularly as four fifths form an identity with one third), two sevenths, and one third.

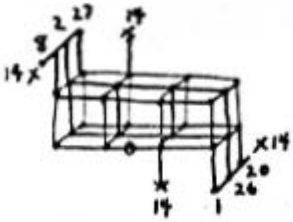
[41] The mean deviation of the system shown in Example 34 is 4.35 cents as compared to 10.4 cents in Fokker's system.

In the diagram, Example 33, this forms a basic parallelopiped containing 4 x 3 x 2 = 24 tones. The two systems listed as "alternate septimal" systems in Example 32, above, use Fokker's basic parallelopiped to account for what might be considered their 24 central tones. The two alternate systems propose alternative methods to Fokker's Pythagorean extension in order to obtain the remaining seven tones (six, not counting the 14th tone which Fokker offers in six different ways) in a manner which will minimize the discrepancies as against equal temperament.

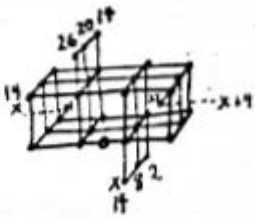
The first alternative system alters each of the six fixed outer tones of Fokker's system by a syntonic comma, resulting in the replacement of Fokker's two sets of three extra fifths (each placed a seventh apart from one another in each set) with two sets of extra thirds (similarly placed with respect to one another). The extra tones in all systems are numbered 1, 2, 8, 20, 26, and 27. The second alternate system draws each of the six outer tones from whatever source it can in order to produce a series of intervals with the lowest possible deviations from equal temperament. The figures at the bottom of Example 32, above, show how great an effect the change of these six tones has on the statistics of deviation. Example 35, below, shows the tone-lattice of each of the two alternate systems which appear in Example 32.

Example 35: Two Alternate Septimal Systems

First Alternate System:



Second Alternate System:



In the systems with many major thirds, the diesis, 128:125, forms the backbone of the scale structure.[42] This interval is completely absent from Fokker's system and the systems shown in Example 35. Because this diesis differs in size from the unit 1/31 by only three cents, those systems which use it show up particularly well with respect to mean deviation which is calculated on the basis of the constructing intervals.

The mean deviation of the single units of the system is a questionable basis for measuring the appropriateness of a multiple-division system such as 31-tone temperament;

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[42] Würschmidt uses this diesis 18 times in his symmetrical system and 19 times in his improved system. The system in Example 34 uses this diesis 11 times.

it is highly unlikely that the single unit will be used harmonically, or even with much frequence melodically. By the method of averaging the deviation from equal temperament of all of the intervals of the system from one central or generating tone, a set of figures more representative of the harmonic and melodic accuracy of the tempered system emerges. By this method of calculation the difference in quality between Fokker's and Würschmidt's systems becomes much less, and with the second of the suggested modifications, Fokker's system attains an average deviation as small as Würschmidt's.

Despite the close approximation to 31-tone temperament by the systems based on large numbers of superimposed thirds, Fokker's system, especially with the second of the modifications here proposed, seems superior to any other just 31-tone system. The additional fifths render a service proportionate to the intonational problems they create. The fifth, as the most perfect consonance after the octave, would appear to be an interval required plentifully in any system of multiple division based on an acoustical concept of consonance. The presence of sufficient fifths is shared by Fokker's and Würschmidt's systems, but not by the system shown in Example 34. The sevenths which distinguish Fokker's system from Würschmidt's offer a new dimension and direction for consonance in the 31-tone system at almost no cost in

intonational accuracy. With the recommended modifications,[43] Fokker's system would appear to possess sufficient resemblance to the tempered system (which he actually uses in practice) to be of thorough serviceability.[44]

CURRENT PRACTICE IN 31-TONE TEMPERAMENT

Since the construction in the middle 1940's of an organ in 31-tone equal temperament at Teylers Stichting in Haarlem under Professor Fokker's leadership, a growing literature of tricesimoprimal music has developed. Beginning as a strictly local concern, music for the 31-tone organ has increasingly attracted composers from other European countries.[45] Other instruments have been designed recently for 31-tone temperament. Among them is a 31-tone

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[43]The modifications might be objected to on the grounds that Fokker's original system meets at 6 of its ends while the modified systems meet only at 4. One's viewpoint depends, of course, on which features of a tone-system he deems important. To me the modification is thoroughly justified for the more equal intervals which result.

[44]One might ask why, since all the above roads lead to equal temperament, it should make any difference how the system is derived. Practical questions involving intonation are involved, since it can be assumed that singers and players of variable-pitch instruments such as violin and trombone would attempt to come as close as possible to the just intonation. Also, in judging the tempered system it is necessary to know for what purposes it is supposed to be suitable. Finally, it is reasonable to assume that chord- and scale-building within a tempered system will depend on the composer's views, conscious or unconscious, as to the theoretical basis of the system.

[45]Letter from Prof. Fokker, March 9, 1960. The English composer Alan Ridout is cited as an example.

trumpet, an adaptation by Martin Vogel, the author of the definitive work on the seventh partial. Prof. Fokker, in reporting on this instrument in his letter does not describe its mechanism. Presumably a complete re-ordering of valves is required, possibly involving 1, 2, 4, and 8 units of the system respectively.

The first compositions for the 31-tone organ reflect Fokker's preoccupation with Euler's genera even when they are not by Fokker. A competition was held in 1947 for which it was specified that the composers use a set of these genera. The winning work, Mart. J. Lursen's Modi Antichi Musiche Nuove, has been published. Lursen's work reflects the conservative approach to multiple division of the Fokker school. In each of the pieces, only from 4 to 8 of the available pitches are used per octave. The ten genera employed represent all of the threefold combinations of the 3rd, 5th, and 7th partials: 3^3, $3^2 \times 5$, 3×5^2, 5^3, $3^2 \times 7$, $3 \times 5 \times 7$, $5^2 \times 7$, 3×7^2, 5×7^2, and 7^3. Modi Antichi Musiche Nuove consists of 20 very short pieces, among which are included small dances such as a gavotte and a sarabande, several pieces titled after the technical means employed, such as "Bimodal Perpetual Inverted Canon," and a few miscellaneously titled pieces such as "Prelude," "Rondino" and "Inventionetta." Reaching out beyond the limited timbres offered by the 31-tone organ, Lursen specifies varied instrumental groups as ideal for each of the small pieces, should

they ever develop the capacity to play them.

A recording of tricesimoprimal music has been released commercially in Europe (Phillips 400090, 45 r/p/m), while additional recordings exist in University libraries.[46] It is hoped that some of these recordings will become more available in this country.

With generally favorable acoustical credentials and with a distinguished precursor in meantone temperament, 31-tone temperament offers one of the most promising realms within the field of multiple division. Under Professor Fokker's nurturing the beginnings of a literature of music have emerged. While it could hardly be considered likely, it is within the scope of the possible that a sudden great increase in the use of this temperament might occur in the near future.

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[46] A recording of tricesimoprimal works by Jan van Dijk, Henk Badings, and Arie de Klein is in the library of the University of Basel, Switzerland. ---Letter from Prof. Fokker.

CHAPTER 6

UNUSUAL SUGGESTIONS FOR EQUAL TEMPERAMENTS

In this short chapter appear briefly temperaments with many tones in the octave which have been regarded as ideal but impractical, and temperaments with relatively few tones belonging to unusual orders of r and containing serious defects in their thirds or fifths.

"Ideal" Systems

Example 36 shows the basic acoustical features of seven of the leading systems of multiple division involving more tones to the octave than even their firmest supporters have deemed practical.

It is obvious from a quick examination of this chart that no truly ideal temperament in which the partials through 13 are accurately represented has been found. It is patently absurd to use the same kind of standards as are used in 12-tone temperament when measuring the discrepancies of systems having over a hundred tones per octave. With 100 tones to the octave, the individual pitches are only 12 cents apart, bringing any imaginable pitch within 6 cents of a member of the system. One would expect "ideal" temperaments such as these to have a particularly high degree of correspondence between tempered and just ratios to compensate, at least in some measure, for their

EXAMPLE 36: "IDEAL" SYSTEMS

Number of Tones:	74	87	113	118	171	347	612
Cents per unit:	16.22	13.79	10.62	10.17	7.02	3.46	1.96
Size of the fifth	697.30	703.45	700.88	701.69	701.75	702.01	701.96
Error of the fifth	4.64	1.50	1.07	0.26	0.20	0.06	0.005
% of possible error	57	22	20	05	06	04	00
Size of the third	389.2	386.2	382.3	386.4	386.0	387.3	386.3
Error of the third	3.1	0.1	4.0	0.1	0.3	1.0	0.0
% of possible error	38	01	75	02	08	59	00
Size of the seventh	973.0	965.5	966.4	966.1	968.4	968.3	968.6
Error of the seventh	4.2	3.3	2.4	2.7	0.4	0.5	0.2
% of possible error	52	48	45	53	11	29	20
Size of 11:8	551.35	551.7	552.2	549.2	554.4	549.8	551.0
Error of 11:8	00.0	0.4	0.9	2.1	3.1	1.3	0.3
% of possible error	00	06	17	41	88	75	31
Size of 13:8	843.2	841.4	838.9	844.1	842.1	840.3	841.2
Error of 13:8	2.7	0.9	1.6	3.6	1.6	0.2	0.7
% of possible error	33	13	30	70	46	12	71

impracticability. Otherwise, why propose them in the first place?

74-tone temperament can be disposed of quickly, since it was proposed not as an ideal temperament but rather as part of a mathematical formula for obtaining satisfactory temperaments.[1] It represents the approximate fulfillment of a system of 2/9-comma temperament, a temperament which did figure in some late Renaissance theory. It combines the assets and liabilities of 31- and 43-tone temperaments, which are too much alike to be worth combining, since the resulting temperament is too similar to both to be worth the many additional tones involved. It can well be dismissed from further consideration, with only the regret over losing so fine a representation of the 11th partial.

87-tone temperament is the only temperament between 28 (!) and 559 obtained by Barbour using ordinary ternary continued fractions.[2] Since ternary continued fractions relate only to the octave, fifth, and major third, it is not surprising to find the error in the 7th partial rather large. The 5th and 11th partials are excellent, and the 13th partial is quite adequately approximated. However, in addition to the 7th partial the 3rd partial is not as well represented as one would wish in as complex a system as

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[1]By Drobisch as reported in Tuning and Temperament, p. 129.

[2]Ibid., p. 130.

this one.

113-tone temperament is suggested by Partch as the simplest temperament in which all of the partials through 11 are adequately represented.[3] There appears to have been an error in his calculations, because there are thirds that he would never tolerate in a musical system, even one involving far fewer tones than 113-tone temperament. It is true that many scale units of Partch's system (see chapter 7, below) are quite close to intervals representing multiples of 10.62 cents (the unit in 113-tone temperament), but many others are not.

118-tone temperament has enjoyed the most widespread approval of all the "ideal" temperaments among theorists. Bosanquet, Würschmidt, Ariel and Barbour are among the writers who have reported favorably on this temperament. Both the fifth and the third are obtained within small fractions of a cent, rendering the discrepancies for these two intervals small, even for a system involving such minute units. Far less fortunate in 118-tone temperament are the upper prime partials, each missing its mark by more than the comparable interval in Partch's 113-tone system. The ideal temperament is indeed an elusive matter.

171-tone temperament is Perrett's choice among the

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[3] Genesis of a Music, p. 307.

equal temperaments.[4] He is concerned with the partials through 7, and, indeed, for the 3rd, 5th, and 7th partials, this temperament is the first to approach "ideal" standards. 0.4 cents is the maximum deviation for any of the partials below 11, an impressive attainment even for a temperament with as many tones as this. With the 11th partial the system becomes as useless as 12-tone temperament for the same partial; the desired tone lies almost midway between two members of the tempered system.

347-tone temperament bears the seal of approval of Von Janko and von Hornbostel. With this many tones in the system, a single cent's discrepancy must be considered serious, and the 3rd partial is out of tune by that much. It is hard to understand why these men selected 347 in preference to, say, 171. It is likely that the discovery of 347 was by chance and that before Perrett nobody had explored the advantages of 171-tone temperament.

The last of the charted temperaments, 612, is endorsed by such diverse writers as Barbour, Bosanquet, and von Janko. The fifths and thirds are obtained with a

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[4]Perrett, op. cit., p. 149. Perrett admits the weakness of the 11th and 13th partials in 171-tone temperament, but regards them as unimportant, using instead the 17th partial. "If the 11th and 13th harmonics can be dispensed with, 171 may claim to be the perfect scale," he concludes.

[5]Op. cit., from p. 6. von Janko considers 41- and 53-tone temperaments to be the best until 347. He then also mentions 400, 453, 506, 559, and 612-tone temperaments, the numbers each removed from one another by 53 integers.

precision that would be remarkable in any equal temperament. Even the seventh partial is closer in the more meaningful category (% of possible error) than in any of the "ideal" systems except for 171-tone temperament. It may be of some consolation in an imperfect world to consider that if some kind of living creature in a distant solar system should develop ears and a mind of incomprehensible powers, nature has, in 612-tone temperament, an almost perfect system for him. Almost perfect, that is, if he is content to forego the 11th, and especially the 13th, partials.

SYSTEMS WITHOUT FIFTHS OR THIRDS OR BOTH

From time to time, systems lying outside the field of consonant thirds and fifths have been proposed, usually for an isolated, specific use. With the availability of electronically produced music, not dependent on a fixed tuning system, the practice of adopting unique tuning systems for individual pieces may well increase. In his recent work, Spiritus intelligentiae sanctus, Ernst Krenek makes use of 13-tone equal temperament in an electronic interlude.[6] In this interlude he makes use of conjunct heptachords in a serial manner. It would be no less logical to construct an electronic interlude out of 11-tone temperament, 23-tone temperament or any other system. As long as

[6]Musical Quarterly, April 1960, p. 248.

consonance is not an objective, such irregular systems commend themselves as very possibly superior to systems with good fifths or thirds.

Augusto Novarro has fretted guitars in 15-tone temperament, thereby obtaining considerable improvement in the minor thirds and natural sevenths as against those of 12-tone temperament. He uses these guitars in conjunction with other instruments, traditionally tuned, in order that he may also have access to well-tuned fifths. To close the combined system would require 60-tone temperament, but Novarro envisages no such closing of the system; rather the simple coexistence of 12- and 15-tone temperaments. One would imagine that the three pitches common to both systems would have special importance to Novarro, and that the 400-cent interval between them would be of particular significance in music so conceived.

Busoni's supposed 18-tone system has been discussed above. It is in reality a 36-tone system. It can, however, be thought of as combining with 12-tone temperament in an incomplete 36-tone system much like Novarro's incomplete 60-tone system above. Busoni might have tried to maintain the 18-tone scale as a distinct entity, in which case he would have had a fifth-less scale.

The list of proposed equal temperaments without third or fifth is, indeed, quite small. It would seem that once the basic consonances of the harmonic series are eliminated, one system would do quite as well as another.

CHAPTER 7

MULTIPLE DIVISION WITHOUT EQUAL TEMPERAMENT

Most of the writers advocating multiple division have been interested in equal temperament. They have sought out multiple division as a means of attaining acoustical niceties lost through 12-tone equal temperament while retaining the advantages of equal-temperament in general. But, as the previous chapters have shown, there is no such thing as a perfect equal temperament. However much the acoustical consonance of the intervals may be increased by the judicious choice of an equal temperament involving multiple division, it is possible, obviously, to raise higher standards of accuracy, or to insist on the inclusion of higher and higher partials, or both. There have been a number of writers who have preferred the perfection of some interval combinations in their systems to the transposability of all. Within the field of multiple division they occupy a position akin to that held by the advocates of just intonation within the diatonic and 12-tone systems.

The category "multiple division without equal temperament" can include such historical events as the many meantone systems involving split keys. Within the last century these have not been revived, however, and experiments in multiple division without equal temperament

have taken one of two directions. The first has been the building of systems of fifths, either pure or altered by only 1/8 of a schisma (less than ¼ of a cent). These systems have been mentioned, above, in Chapter 1, and again in Chapter 5 in the discussion of 53-tone temperament, which, because of its near-perfect fifths, represents a kind of completion or fulfillment of all of these systems. The number of authors who have proposed systems of this kind, especially in the 19th century, is great, and many instruments embodying perfect or near-perfect fifths in 24- or 36-fold profusion have been built. Debate over their relative merits has filled the pages of many a 19th century German periodical. The goal of most of the exponents of systems such as these seems to have been the perfect intonation of all fifths (and frequently of the thirds as well) that might be called for in the music in general use. For such purposes, a basic 12-tone Pythagorean system, with one or two alternate tones for each member, a comma higher or lower, would be sufficient.

Among the exponents of 24-tone systems of this kind are Helmholtz and more recently Clark Jones, both of whom are aware of the advantages of forming pure thirds by eight sub-schismatically altered fifths.

36-tone systems of this kind number among their supporters at least one important contemporary figure, Eivind Groven of Norway, who has made some tape recordings

from an organ built to contain 36 sub-schismatically altered fifths. Among earlier users of 36-tone systems of fifths are Puhlmann, whom Groven credits as the first, and Eitz, who also proposed a similar system with, oddly, 52 tones to the octave.

Other, similar systems have been proposed as follows: by Brandsma, 48 tones; by Würschmidt (before his interest in 19-tone temperament developed), 50 tones; by Eisleben, 55 tones. The list is undoubtedly much longer, but the systems do not differ markedly from one another. A greater range of difference is found in the instruments designed to realize these systems.

The second basic category of multiple division without equal temperament includes those systems based in part on intervals not included in a series of fifths. Two leading exponents of such systems in this century have been Wilfrid Perrett in England and Harry Partch in the United States, both of whom have built instruments to bring their systems into practical reality and have published their theoretical doctrines.

PERRETT

Wilfrid Perrett began publishing his Some Questions of Musical Theory in 1926. A series of thin volumes followed one another at varying intervals of time. The volumes seem invariably to have been bound separately, and

as there were only 150 copies printed, copies of some of the volumes are quite rare. Perrett appears to have been a highly learned man; he also published works on phonetics, spelling reform, and the antecedents of Shakespeare's King Lear.[1] His erudition is apparent in a vast number of learned citations, including a quotation from Plutarch in Greek. It is, in fact, the interpretation of Plutarch which is at the heart of Perrett's hypothesis: that the Greek enharmonic genus was formed from the combination of two diatonic scales related by a septimal ratio.

Perrett quotes Plutarch as follows:

φαίνεται δ' Ὄλυμπος αὐξήσας μουσικήν, τῷ ἀγένητόν τι καὶ ἀγνοούμενον ὑπὸ τῶν ἔμπροσθεν εἰσαγαγεῖν, καὶ ἀρχηγὸς γενέσθαι τῆς Ἑλληνικῆς καὶ καλῆς Μουσικῆς

Admitting that this passage can be interpreted in different ways, Perrett states his view that Plutarch means that Olympus created a scale none of whose tones were the

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[1]The titles of Perrett's other works in question: Some Questions of Phonetic Theory, Peetickay: an Essay towards the Abolition of Spelling, and The Story of King Lear from Geoffrey of Monmouth to Shakespeare.

same as those of the national scale, but whose contours were the same, that is to say Dorian in mode, since Olympus used the new scale to compose in that mode. Since the term enharmonic means "fitted in," Perrett concludes that the enharmonic genus represented the "fitting together" of the two Dorian scales. Having proceeded this far on an interpretation of a statement by an historian and an etymological deduction, Perrett approaches the most difficult part acoustico-aesthetically. He notes that if the paramese used by Olympus had been tuned 7 cents lower than normal (by "normal" Perrett is referring to Ptolemy's diatonic genus), it would have completed an interval of 7:5 with another member of the system. This interval Perrett considers extremely beautiful and notes that Tartini considered 7:5 to be the best augmented 4th.

Having found the slightly lower paramese by accident, postulates Perrett, Olympus used it to generate several of the principal tones of the second scale of which, however, it was not a member. Perhaps this can be made clearer by reconstructing Olympus' system as suggested by Perrett.

The original system, based on the Dorian scale of E (but calculated in cents by Perrett from the generating tone A) is: E 720; F 814; G 1018; A 000; B 204; c 316; d 520. In the original system, the note B is accidentally tuned seven cents flat, i.e., lowered to 197. This creates the pleasing interval 7:5 between F and B (7:5 = 583 cents).

From the generating tone B 197, a perfect fifth upward is measured, yielding a septimal Gb (to be called ^{7}Gb) 899. From this ^{7}Gb a just major third and then another perfect fifth are measured, finally yielding ^{7}F 787. This septimal F is the starting tone for the Dorian Octave of the new scale (it is interesting to see how Perrett's theory parallels the traditional theory of alternate tunings on E and F). As this tone is 85 cents higher than E, the two scales are essentially identical in form and separated from one another by 85 cents (representing the ratio 21:20). If the two scales were completely identical in form, Perrett's 14-tone system would be as follows:

^{7}F 787;	^{7}Gb 899;	^{7}Ab 1103;	^{7}Bb 85;	^{7}C 289;	^{7}Db 401;	^{7}Eb 605;	^{7}F 787
E 702;	F 814;	G 1018;	A 00;	B 204;	C 316;	D 520;	E 702.
16:15	9:8	10:9	9:8	16:15	9:8	10:9	

Perrett makes one alteration in the new scale, however. Where two whole-tones are placed consecutively, between ^{7}Db and ^{7}F, Perrett reverses the order of major and minor tones by lowering the pitch of ^{7}Eb a syntonic comma from 605 to 583 cents. This gives him a 14-tone system consisting of the following pitches (in cents):

0
85
204
289
318
401
520
583
702
787
814
899
1018
1103
1200

Basic to Perrett's approach is the desire to avoid uniformity. The alteration of ^{7}Eb adds to the number of combinations available but limits the number of exact transpositions which are possible. As Perrett points out, only two major scales, (C and ^{7}Db) are available in his system, and their forms are not identical. Perrett is delighted to find present in his system such Ptolemiean tetrachords as 28:27, 15:14, 6:5, and 21:20, 10:9, 8:7, as well as a number of enharmonic tetrachords. Using russet-brown keys for his two new tones, and a special system of notation,[2] Perrett notes several progressions involving Ptolemiean tetrachords.

In his 14-tone system, Perrett has five gaps which are wider than 85 cents. Four are as large as 119 cents while the other is smaller, 97 cents, the result of the

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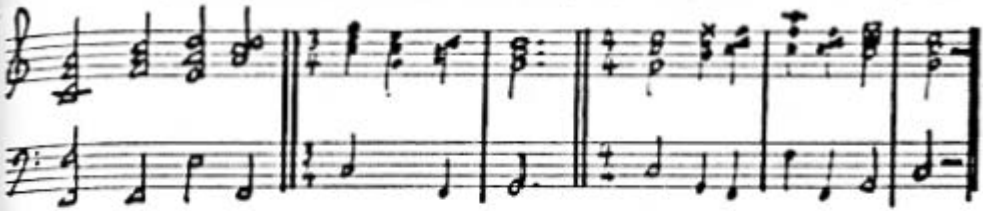
[2]Triangular note-heads signify black keys higher in pitch than the standard white ones, while x-shaped note-heads are played on the russet-brown keys below F and C in pitch.

Example 37: Progressions Cited by Perrett[3]



rearrangement of the modes. Perrett adds five additional tones to his system to fill these gaps. These tones are added at a distance of 70 cents (25:24, the difference between the major and minor third) above or below one of the already existing tones. The new tones are E^{b} 632, 70 cents below E, Ab and Bb, similarly located with respect to A and B and a septimal D and a septimal G, each located 70 cents above the neighboring septimal flatted tones. As a result of these additions, many of Perrett's configurations are transposable to many additional keys.

Perrett calls his enlarged scale a tierce-tone scale; it possesses 19 tones to the octave, and its intervals range in size from 27 to 85 cents. With this scale, Perrett is delighted to discover that he can harmonize the enharmonic tetrachord of Archytas.[24]

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[3] Some Questions of Musical Theory, pp. 20-21.

[4] Ibid., appendix to Chapter 4.

Although some transposition is possible in Perrett's system, it is not his goal to make transposition easy. He considers it to have been a great virtue of the Greeks that they associated specific intervals with specific pitches. Such an association obviously cannot be made in a transposable system. In order to enjoy free harmonization of all of the tones in his system and to use them all as tonics, Perrett claims to require 62 tones, using occasional schismatic compensation.[5] This would still not achieve a closed system; for this an equal temperament of 171 tones to the octave would be necessary.

The creator of an instrument, the Olympion, named after the supposed inventor of the enharmonic genus, and the author of a highly imaginative if occasionally extravagant treatise, Perrett belongs among the best of the speculative theorists of our day. His inquiries were motivated by a conception of the beautiful which he always kept foremost in mind, and a reverence for his subject.

PARTCH

Perrett's obscure book did not go entirely unnoticed. Among its most avid readers was Harry Partch, who had already embarked on the development of a musical system which resembled Perrett's in many respects but which was far more

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[5] Ibid., pp. 132-3.

ambitious.

Partch's attainments as composer and instrument builder have dwarfed his work as a theorist. Nevertheless, in Genesis of a Music, Partch puts forth as forcefully as ever has been done the case for just intonation in principle, multiple division in general, and his own system of 43 tones to the octave in particular.

The basis for Partch's musical system is a concept which he calls monophony. This concept allows the derivation by simple ratios of all tones from a single source. From one central tone is built what Partch calls an otonality, which is the reduction within the octave of the first 11 members of the harmonic series whose fundamental is the central tone. From this same tone is also built a utonality, consisting of the exact inversion of the otonality; the utonality is given a standing within the total system exactly equal to that of the otonality. Discounting octave replicas there are 6 tones in each otonality or utonality, representing every odd number from 1 to 11. Arranged in ascending order, the tones of an otonality, reduced to within a single octave, proceed by the sequence 6:7:8:9:10:11:12, with 8 representing the generating tone. The utonality consists of these same figures as denominators with a fixed numerator.

The entire system of 43 tones is built from the simple phenomena of otonality and utonality in two stages.

The first stage is the completion of a system of monophonic ratios wherein each of the six basic values relates to each of the other five both as numerator and as denominator. In the process, each tone of the initial otonality serves as unity for a utonality, while each tone in the original utonality serves as unity for an otonality. Example 38 shows the complete repertoire of monophonic ratios arranged in what Partch calls the tonality diamond. The ratios are shown in their most basic form. In actual practice octave displacements are used to narrow the range.

Example 38: Partch's Tonality Diamond

Otonalities....read down
Utonalities....read across.

$$\begin{array}{cccccc}
\frac{1}{1} & \frac{1}{3} & \frac{1}{5} & \frac{1}{7} & \frac{1}{9} & \frac{1}{11} \\
\frac{3}{1} & \frac{3}{3} & \frac{3}{5} & \frac{3}{7} & \frac{3}{9} & \frac{3}{11} \\
\frac{5}{1} & \frac{5}{3} & \frac{5}{5} & \frac{5}{7} & \frac{5}{9} & \frac{5}{11} \\
\frac{7}{1} & \frac{7}{3} & \frac{7}{5} & \frac{7}{7} & \frac{7}{9} & \frac{7}{11} \\
\frac{9}{1} & \frac{9}{3} & \frac{9}{5} & \frac{9}{7} & \frac{9}{9} & \frac{9}{11} \\
\frac{11}{1} & \frac{11}{3} & \frac{11}{5} & \frac{11}{7} & \frac{11}{9} & \frac{11}{11}
\end{array}$$

It will be observed that there are 36 ratios (6 x 6) in the above diamond. Of these, the six comprising the diagonal from the top left turn out to be the same, 1:1.

Two others prove equal to 3:1 and two to 1:3. All of the remaining intervals intervals are distinct, however, so that the number of distinct monophonic ratios proves to be 29. These ratios, when arranged within the octave, give Partch his basic scale of 29 tones. These tones are marked with an asterisk (*) in Example 39, below.

Somewhat concerned over the discrepancy in size between the various intervals which separate the monophonic ratios when they are arranged as a scale, Partch adds 14 additional tones to fill in the larger gaps. In the process he adds to the number of available transpositions for his basic otonality and utonality as well as to the number of available scale patterns. The range of intervals in Partch's original system of monophonic ratios is between 14.4 cents (121:120) and 150.6 cents. 14.4 cents remains the smallest interval in the complete system, while the largest interval is 38.9 cents (45:44).

It will be seen from Example 39 that the discrepancies between Partch's intervals and those of 43-tone equal temperament are considerable. From the 9th to the 34th tone in Partch's system is 737.6 cents, nearly 40 cents larger than the tempered interval involving as many degrees. Transposability is a commodity available only in limited quantities within the Partch system.

Nevertheless, those musical features such as o- and utonalities, which Partch deems most essential to his musical

Example 39: Partch's System

	Ratio	Cents	tempered equivalent		Ratio	cents	tempered equivalent
0	1:1	* 0.0	0.0	22	10:7	*617.5	614.0
1	81:80	21.5	27.9	23	16:11	*648.7	641.9
2	33:32	53.2	55.8	24	40:27	680.5	669.8
3	21:20	84.5	83.7	25	3:2	*702.0	697.7
4	16:15	111.7	111.6	26	32:21	729.2	725.6
5	12:11	*150.6	139.5	27	14:9	*764.9	753.5
6	11:10	*165.0	167.4	28	11:7	*782.5	781.4
7	10:9	*184.4	195.3	29	8:5	*813.7	809.3
8	9:8	*203.9	223.3	30	18:11	*852.6	837.2
9	8:7	*231.2	251.2	31	5:3	*884.4	865.1
10	7:6	*266.9	279.1	32	27:16	905.9	893.0
11	32:27	294.1	307.0	33	12:7	*933.1	920.9
12	6:5	*315.6	334.9	34	7:4	*968.8	948.8
13	11:9	*347.4	362.8	35	16:9	*996.1	976.7
14	11:4	*386.3	390.7	36	9:5	*1017.6	1004.7
15	14:11	*417.5	418.6	37	20:11	*1035.0	1032.6
16	9:7	*435.1	446.5	38	11:6	*1049.4	1060.5
17	21:16	470.8	474.4	39	15:8	1088.3	1088.4
18	4:3	*498.0	502.3	40	40:21	1115.5	1116.3
19	27:20	519.5	530.2	41	64:33	1116.8	1144.2
20	11:8	*551.3	558.1	42	160:81	1178.5	1172.1
21	7:5	*582.5	586.0	43	2:1	*1200.0	1200.0

demands, are transposable into a number of keys. The fourteen tones which Partch adds to those representing the original 29 monophonic ratios provide for a considerable increase in the number of musical phenomena which are transposable and in the number of keys into which already transposable phenomena can be placed. An example of the former is the Ptolemeian diatonic scale, performable in one key only using the original monophonic ratios (that key which uses 4:3 as the tonic). It is, in the complete system, available in at least five keys. The tonics of the major form of this scale include 4:3, 1, 16:9, 8:5, and 6:5.

Partch tackles directly one of the most often advanced arguments against systems of just intonation: that based on double employment. To those who complain that the supertonic of the diatonic scale represents a tone a comma higher when it is employed in dominant harmony than when employed in subdominant harmony, Partch replies that there is nothing wrong at all in this situation. The shift of a comma, he claims, is far less disturbing than the poor intonation of one or both of the chords caused by an alternative, just or tempered. From this view it is apparent that Partch is, after all, a harmonist, disturbed by harmonic imperfections of minute proportions, but undismayed by the necessity of melodic adjustments many times larger. This is, of course, a thoroughly legitimate viewpoint which can

never be proved right or wrong. It is somewhat paradoxical, however, to find it presented as one of the foundations of a musical ideology whose other central points include a great emphasis on the solo human voice, the wellspring of melody. The two polar elements in Partch tend to serve as checks and balances on one another.

To perform music based on the acoustic system outlined above, Partch has adapted and built instruments of many kinds over a long period of time. Although his first instrument was an adapted viola, lavishly fretted and held as though a cello, most of his stringed instruments have been plucked rather than bowed instruments. Two huge kitharas, especially designed for the playing of hexads (presumably tuned to o- and utonalities) and simultaneous parallel glissandi; two monochord-type instruments, one of them double, each consisting of a large number of strings of equal lengths with movable bridges; and three adapted guitars, round out of the string section of Partch's basic orchestra. The wind instruments consist of two adapted harmoniums. One uses many extra keys to provide the full 43 tones to the octave. The other uses only the standard keyboard, but still provides 43 pitches to the octave, the compensation being made by deducting radically from the range of the instrument. As a result, the space usually associated with the octave represents only about a minor third.[6] These two instruments are

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[6]Partch quite characteristically permits the octave coupler to remain intact, giving him parallel minor thirds as a matter of simple course.

called chromelodeons by Partch because of the brilliant colors of the keys, colors associated with the musical ratios involved in the respective pitches. The chromelodeons are tuned by ear on the basis of beats and the absence of beats. They then serve as the basis for the tuning of all the other instruments. A number of scores by Partch make use of the clarinet; the player is expected to supplement the tones available on the instrument by alternations in air pressure and the position of his lips.

The percussion instruments are of various materials, with wood tending to predominate. Marimbas, of all sizes, shapes, and tone-qualities, emit perhaps the most characteristic timbres of the Partch orchestra.

Partch's instruments have gradually gained a substantial reputation, but his equally impressive corpus of music remains all but unknown, despite the availability of recordings of complete or substantially excerpted versions of four of his major works. The recordings which have been released, in order of their publication, are Plectra and Percussion Dances, Oedipus, The Bewitched, and U.S. Highball. The order of composition is almost reversed; U.S. Highball dates from the early 1940's; Oedipus, the Dances, and The Bewitched, in that order, were composed in the 1950's.

U.S. Highball is one of a group of five works entitled The Wayward. In it are discernable most of the characteristics of Partch's music. The two predominant elements are:

energetic and somewhat asymmetric rhythms, and a mood nocturnally wistful in an almost pictorial sense. U.S. Highball is a kind of tone-poem with sung words, representing a ride on freight trains from California to Chicago. Imitations of railroad sounds and the sounds of windblown mountain nights are blended harmoniously with melismatic, distorted calls of station names in a haunting evocation. The score has a plaintive character whose impact can be very strong. The enharmonic intervals of Partch's system tend to be used in this as in his other works in very conjunct melodic patterns of intense emotional expression. The more relaxed sections tend to be diatonic, as do the relatively few disjunct melodic figures.

Oedipus is the most contrapuntal of Partch's offerings to date. The plaintive note, as might be imagined, predominates, and rises to an apotheosis in an antiphony between Oedipus, a deep bass, and the chorus of women. The choral music is mostly diatonic, while Oedipus sings a mixture of diatonic and microtonic intervals.

The Plectra and Percussion Dances include Castor and Pollux, which is purely instrumental. Here the rhythmic element prevails, although the sliding sonorities of the kitharas which begin both of the two parallel sections suggest that other dominant strain of Partch. A typically direct melody of a very simple, tuneful character develops out of a music in which textural and rhythmical elements

had previously prevailed. In Ring Around the Moon, the second of the dances, a more whimsical side of Partch is revealed in full. This lighter aspect has prevailed in the works offered since, especially in The Bewitched. In these latest works, a level of technical mastery of the novel instrumental media is achieved which makes "experimental" seem an inappropriate label for the music.

The Bewitched has been particularly successful in broadening the base of Partch's following among musicians.[7] Like Oedipus, a work of large proportions, The Bewitched seems to be more influenced than Partch's earlier works by external developments in Western music. The use of four standard instruments (cello, clarinet, bass clarinet, and piccolo) undoubtedly contributes to the more standard effect. As always in Partch the idiomatic features of the specific instruments tend to determine the style. In most cases reorchestrating any section of a Partch score would be entirely out of the question.

As this is written the world premier of Partch's latest work, Revelation in the Courthouse Park, is

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[7]Suzanne Bloch and Peggy Glanville-Hicks, two musicians of considerable reputation, wrote letters to the New York Times when it failed to publish a music review of The Bewitched after its New York performance in 1959. "Several years ago I reacted to Mr. Partch's music with great antagonism. This time I found myself deeply impressed," says Miss Bloch. "His insight and action are as basic to contemporary music's real crisis as the activities of most of his contemporary 'innovators' are irrelevant," says Miss Glanville-Hicks, a composer not yet noted for the use of microtones.

being awaited. It will be interesting to see whether the recent preoccupation of the composer with satire and whimsy has been continued (as is suggested by the title) and whether the apparent tendency toward reconciliation with other musical systems in our society has been continued.

In an age that has accepted as its own the complex statement in a simple tonal system, Partch has insisted tenaciously on the simple statement in a complex tonal system. Partch has explored the path in multiple division that lies at the opposite end from Haba's. With his 43 tones, superceded in strength of impact by his novel timbres, Partch has attempted to show that multiple division can by its real complexity liberate music from the necessity of contrived complexity.

It is quite likely that in the era of the assembly line, where the finished product is the result of the work of many hands, Partch's individual hegemony over his entire domain from theory, from instrument construction, to composition and performance, may be an obstacle to his recognition. Partch seems to leave his fellow composers (and judges) no room to share and feel important in his movement.

If Partch has, by turning away from traditional instruments in favor of his own, renounced the practical possibility of being a leader of a specific musical style, he remains America's leading figure in the field of multiple division, and his music could still serve as a stimulant

toward the expansion of microtonal music in general. In his book Partch argues repeatedly for a pluralistic approach to musical systems,[8] hailing as pioneers even those advocates of unusual theoretical systems with which he is in disagreement.

As a specific system, Partch's monophony may well be too closely tailored to his individualistic impulses to outlast the lifetime of its creator as the basis for musical composition. In a broader sense Partch's chances for success seem much greater. He has taught by example the duty of the student of music to "question the corpus of knowledge, traditions, and usages that gave us a piano . . . the music on its rack . . . and the philosophies that are responsible for these things."[9] His work is of great importance to all who contemplate a revision in our tuning procedures.

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[8]"In a healthy culture differing musical philosophies would be co-existent, not mutually exclusive." Op. cit., p. xi.

[9]Op. cit., pp. ix-x.

CHAPTER 8

THE ANTECEDENTS AND EARLY ADVOCACY OF 19-TONE TEMPERAMENT

The advocacy of 19-tone equal temperament has been for the most part a 20th century phenomenon. Three writers in this century have presented extended treatises favoring its adoption--two of them in book form and one in a series of related pamphlets. Each of these writers will be studied in some detail in a separate chapter.

19-tone temperament is referred to without advocacy by at least one theorist in the 18th century. It is modestly advocated by several writers in the 19th century of which the first is evidently the Englishman, Wesley Woolhouse, and is the subject of an enthusiastic report by the German, Melchiorre Sachs, to the 4th Congress of the Internationalen Musikgesellschaft in London, in 1911. Finally, it is written of with favor by a number of authors in scattered periodicals leading to its espousal in the 1920's and 1930's by Kornerup, Ariel, and Yasser.

In two important respects 19-tone temperament has its roots in the Renaissance, although it appears never to have been directly advocated before the 19th century. Theoretically, it resembles very closely one of the tempered tunings which gained the qualified endorsement of several of the leading Renaissance theorists. Practically, 19-tone temperament can claim an antecedent in the several instru-

ments using 19 tones to the octave which were built during the Renaissance. We do not know precisely how these instruments were tuned, but the evidence suggests that they were tuned neither to 19-tone temperament nor to the meantone temperament which most closely resembles it (1/3-comma temperament). Nevertheless, the existence of these instruments has been widely known, and the possibility that they were tuned in such a way as to anticipate 19-tone temperament has been recognized by several of the advocates of 19-tone temperament. As these instruments were easily retunable cembali, it is possible that they were experimentally tuned and retuned to many possible temperaments including 1/3-comma.

SALINAS AND 1/3-COMMA TEMPERAMENT

In his famous treatise De Musica, the blind Spaniard Salinas, 1513-1590, takes up at some length the subject of temperament which he considers a necessity. He lists three temperaments which he deems to be worthy of consideration; the first alters the fifths by 1/3 of a comma, the second by 2/7 of a comma and the third by ½ of a comma.[1] In each case he carefully lists the various tunings of the tetrachords--diatonic, chromatic, and enharmonic--which he considers renderable in the temperament. In each case he

[1] De Musica, Liber III

also diagrams the temperament, showing the position of each tone with respect to its position in the diatonic genus of Ptolemy and with respect to the others. Salinas discusses the three alternatives as though they were equally known (or unknown) to musicians of his time, but where scholars have found earlier instances of the advocacy of ¼- and 2/7-comma temperament, they have apparently found no earlier reference to 1/3-comma temperament.[2] If Salinas was the inventor of 1/3-comma temperament, he bears his achievement modestly. He treats it first among the three possible temperaments, at somewhat shorter length than the other two, and he acknowledges the superiority of the major thirds and the fifths in the other two temperaments. Nevertheless, Salinas did not take a stand <u>against</u> 1/3-comma temperament, and as long as he appears to have been the first major writer to acknowledge it as a reputable temperament, his role as an unwitting grandfather of 19-tone temperament appears secure. The two temperaments are almost identical, and it appears that Salinas was interested in a 19-tone application of the temperament to the three genera.[3]

Barbour points out that Salinas never stressed the

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[2]Barbour refers to 1/3-comma temperament as the "invention of Francisco Salinas." <u>Tuning and Temperament</u>, p. 33. It appears likely that it was of earlier invention. Kornerup attributes it to Arnold Schlick, substantially earlier, claiming 1/3-comma temperament to be included in the supplement of his <u>Speigel der Orgelmacher und Organisten</u>. Attribution of a number of meantone temperaments to Schlick is traditional and is strongly disputed by Barbour, who claims Schlick's tunings to have been irregular.

[3]Barbour, <u>op. cit.</u>, p. 34.

advantage that 1/3-comma temperament possessed in being a closed system (through 19-tone equal temperament). This is quite understandable, since Salinas predated the general concurrence that there is any advantage a priori to a closed system as such. Salinas could not have failed to notice the tendency of 1/3-comma temperament to produce approximately equal intervals, for he meticulously diagrammed each of his temperaments, and 1/3-comma temperament produces intervals which appear exactly equal on his chart.[4] It is unfortunate that Salinas did not record his reaction to what he must have noticed on his diagrams.

The impression left by Salinas and other writers who included mention of 1/3-comma temperament is that it was an accepted, though junior, partner of ¼- and 2/7-comma temperaments which, because of their better thirds and fifths, had more prestige and more widespread use.

It was Zarlino who is generally credited with ordering the construction of the first known instrument with 19 tones to the octave. Another more famous 19-tone instrument was the "clavicembalo universale," mentioned by Praetorius in his highly renowned work Syntagma Musicum. Praetorius reports that the instrument was built by Elsass and was in the service of the duke of Prague. Elsass has accordingly been given occasional credit for building the first 19-tone instrument and even for originating 19-tone

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[4] De Musica, op. cit.

temperament,[5] although it is highly unlikely that his instrument was tuned to a very close approximation of 19-tone temperament.

What music could be performed on the clavicembalum universale? According to Praetorius, works in the enharmonic genus and works in the chromatic genus such as Marenzio's madrigals could be performed on such an instrument. Hauptmann has effectively disputed this assertion, pointing out that in such chromatic madrigals as Quel Suo Antico Stylo Marenzio uses C# and Db in different voices at the same time as apparent identities.[6] A 19-tone instrument would be just what is not needed to perform such works of Marenzio, which clamor instead for 12-tone temperament.

Praetorius' apparent misunderstanding is itself revealing. That he could have made such a mistake in failing to see the semitonal implications of Marenzio's notation suggests how far 12-tone equal temperament remained from the consciousness of the average musician in 1600. To such a musician C# and Db were different tones and could only be realized together if instruments other than the conventional keyboard instruments played them.

The increase in the use of chromatic resources, and the rise of interest in modulation forced musicians gradually to a choice between additional keys to the octave on

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[5]Kornerup, in Acoustic Valuation of Intervals, refers to 19-tone temperament as Elsass' 19-tone temperament, p. 11.

[6]Hauptmann, M., Temperatur, Jahrbücher für Musikalische Wissenschaft, I, 1863. The reference cited is from p. 40.

the one hand, and the gradual erosion of the differences in the semitone on the other. It was the latter course which was chosen, at the expense of the purity of the thirds. Throughout the 17th and 18th centuries, the fifths and thirds grew generally larger as meantone temperament of ¼-comma was superceded by 1/6-comma temperament and finally by equal temperament. This brought music farther and farther away from the intonations of 19-tone temperament, and from that acoustic condition in which tones numbering more than twelve could make a substantial contribution to the mobility and consonance of the system. As the thirds and fifths grew larger and the semitones became more and more equal, instances of "split keys" became rarer and rarer. It is conceivable that there was a point around Salinas' and Zarlino's time when music might have evolved as easily toward 19-tone temperament as toward 12-. With the enlargement of the fifth and third in the 17th century, an unchangeable course toward 12-tone temperament was established. It is hardly surprising that in this atmosphere little or nothing about 1/3-comma or 19-tone temperament was said.

It is quite appropriate that England, the country which relinquished meantone temperament most belatedly and most reluctantly,[7] produced whatever interest in 19-tone

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[7]According to Bosanquet, at least two organs in major English cathedrals were still tuned to meantone temperaments in 1875. While Bach in Germany was advocating and using an almost equal temperament of 12 tones, Händel, in England, had been using an organ with 16 keys to realize a broad range in what may well have been a temperament of more than 1/6 of a comma.

temperament existed before the middle of the 19th century. After the continent had long since established ¼-comma temperament as an outer limit for the reduction of the fifth, and had passed through 1/6-comma temperament toward equal temperament, Robert Smith in England, in 1759, published his treatise Harmonics or The Philosophy of Musical Sounds, advocating the temperament of the fifth by 5/18-comma. On page 158, Smith includes 19-tone temperament in a table also listing 12-, 31-, 50-, and meantone temperaments. 19-tone temperament is dismissed by Smith because of the discrepancy between the degree of accuracy obtained by the major sixth (almost exact) and that of the fifth and major third (about 1/3-comma error). It is Smith's thesis that insofar as possible the errors involved in a temperament should be equally shared by the three principal consonances. While excluding Smith from any list of advocates of 19-tone temperament, it is worthwhile noting that his inclusion of the major sixth among the basic consonances shows a point of view in common with almost all of the later supporters of this octave division, one of whose peculiar features is the almost perfect representation of that interval.

Another Englishman, Wesley S. B. Woolhouse, may well have been the first advocate of 19-tone temperament as such. Presumably a mathematician, he was Head Assistant on the Nautical Almanac. His Essay on Musical Intervals, Harmonics,

and the Temperament of the Musical Scale, 1835, contains a direct espousal of 19-tone temperament as a practical musical system.

In his over-all view of temperament, Woolhouse is essentially a disciple of Smith. Regarding the perfect fifth, the major third, and the minor third as the three basic consonant intervals of music (not counting the octave), Woolhouse seeks to find that temperament in which the total dissonance involved in these three intervals will be a minimum. Although this represents a somewhat different emphasis from that of Smith, whose primary concern was in equality of error, it produced a similar result, because of Woolhouse's method, which was to calculate dissonance by the square of the error of each interval. The use of squares renders preferable that system in which the discrepancies tend to be equal, because a large discrepancy in any single interval would produce an exceptionally large square.[8]

By a complicated algebraic process Woolhouse relates the errors of each of the three intervals to the alteration

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[8]Consider, for example, two systems involving two intervals, the errors for one system being each 3 units in size, the errors for the other system being unequal, 1 and 5 units respectively. Calculating without squares the two systems are of apparently equal error, the sum of each being 6 units. But calculated by the squares of the errors the system with unequal errors is much the worse, containing a total of 26 square units of error while the system of equal errors contains only a total of 18 square units.

of the 9:8 whole-tone in each of the systems. He then finds the ratio of tone to semitone which will produce the minimum sum of the squares of the errors, and finds it to be close to 8:5, whereupon he suggests that in the ideal musical system the tone would have 8 units and the semitone 5. This would give 50 units to the octave, the very temperament advocated by Smith.

Woolhouse, however, is conciliatory in nature, finding much to praise in both 12- and 31-tone temperaments, and especially in 53-tone temperament. After giving serious concern to nearly all of the temperaments which have since been advocated, Woolhouse selects 19-tone temperament as the most useful. "The scale of 19 sounds in the octave, which gives great accuracy to the harmony, though not to such theoretical minuteness as the scale of 53 sounds, would, on account of its simplicity and easy adaptation to the construction of the instrument, be a useful improvement in the correctness of the harmony, without infringing on its practicability as regards the performer. It is to be hoped that some persevering experimental musician will construct an instrument on this scale and give it a fair trial."[9]

Such is Woolhouse's view, and the points made by later advocates have tended to be elaborations and substantiations of Woolhouse's position, with only occasional and scattered arguments of a really different nature.

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[9]Woolhouse, op. cit., p. 54.

From an English Nautical Almanac assistant, 19-tone temperament turned next to a German finance minister for support. In 1852 Friedrich Opelt published his Allgemeine Theorie der Musik. Beginning with a theory of consonance based on the simplicity of the polyrhythm set up by two or more fundamental waves, Opelt proceeds to measure various musical systems by the standards of proximity of pitches to consonances.

An interest in the 7th partial causes Opelt to consider seriously the merits of 31-tone temperament, but he rejects 31-tone temperament as too difficult for use.[10] He therefore chooses not to use the 7th partial in constructing his musical systems, and asserts that only 12- and 19-tone temperaments can provide representation for the remaining natural consonances.

As between 12- and 19-tone temperaments, Opelt considers 19- in many respects to be the better of the two. The chief advantage of 19- which he stresses is the absence of any error greater than 1/3-comma in the representation of the basic consonances. His favoring of 19- over 12-tone temperament is supplemented by a consideration of the likelihood of a change from the one to the other. He concedes that such a change would be difficult, but he does not consider such an alteration in keyboard instruments to be

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[10]Opelt, op. cit., p. 66.

impossible (unausführbar) or even improbable (unwahrscheinlich).[11]

Between the dates of Woolhouse's and Opelt's publications, a harmonium with 19 tones to the octave was constructed by P. S. Munck, a professor at Lund, Sweden.[12] This instrument may well have been the first effort at a consciously equal-tempered instrument with 19 sounds to the octave. The work of Prof. Munck, whose instrument is presently at the Stockholm Museum, represents the beginning of very fruitful labor on multiple division by inhabitants of the Scandinavian countries. Scandinavians have remained in the forefront of the advocacy of multiple division to this day.

Unfortunately we do not have any of the music Prof. Munck played on his harmonium. The first known descriptive information about the performance of music written for a 19-tone instrument comes from the report of Prof. Melchior Sachs to the 4th Congress of the International Music Society in London in 1911. The actual music is again missing (although a manuscript may well exist somewhere), but we have enough descriptive material to obtain a fairly good idea as to what it was like. It probably involved traditional harmony with an abundance of 7th chords employing the

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[11] Ibid., p. 53.

[12] Kornerup, Musical Acoustics based on a Pure Third System, Munck is said by Kornerup to have built his 19-tone harmonium about the year 1845.

unique enharmonic relationships of 19-tone temperament, wherein the "common tone" of 12-tone temperament is replaced by "resolution" in 19-. Sachs' emphasis is on the playability of 12-tone music on 19-tone instruments. This playability he supposedly demonstrated at the Congress in London. It is easy to disagree with his conclusions, however, especially in chromatic music, where the different systems may involve discrepancies approaching and even exceeding half a semitone.

Sachs' presentation to the congress included a theory of the development of musical systems which closely anticipates Yasser's. Sachs sees the development of music as having progressed from 3 to 5 to 7 to 12 tones per octave, with 19 as the next system. That Sachs believes in the same additive concept of tonal evolution as Yasser is evident from the following statement: "As the 12-tone system arose out of the combining of the 5- and 7-tone systems, so, in the further course of development, the 19-tone system, which must also be tempered, will come out of the union of the 7- and 12-tone systems."[13]

To Sachs the advent of 19-tone temperament will result in a greater degree of purity in the triads, but of much

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[13] Report of the 4th Congress, International Music Society, 1911, p. 279. "Wie aus der Verbindung des Fünf- und Siebentonsystems das Zwölftonsystem entstand, so entsteht im weiteren Verlauf der Entwicklung aus der Verbindung des Sieben- Zwölf- und Neunzehntonsystem, welches auch ein temperiertes sein muss."

greater importance to him is the increase in the richness of musical expressive means (Bereicherung der musikalischen Ausdrucksmittel) that can be expected through 19-tone temperament. As an example, Sachs cites the addition, through 19-tone temperament, of a third form of the deceptive cadence. In the key of C, he points out, one can, in the 12-tone system, resolve the dominant chords to either A minor or Ab major and produce what is called a deceptive cadence. In 19-tone music a third deceptive resolution to G# is possible. It seems unlikely that anyone today would consider this of great importance, but Sachs' arguments make sense for their time, and point out a possible use for multiple division which has not gone altogether unnoticed by its later advocates. It might be possible with 19-tone temperament, or with another multiple division, to expand functional harmony, beginning at the point where it transcended its own limits within a 12-tone framework and disintegrated. Such a theory of the disintegration of functional harmony is implied by Yasser, although he does not state it directly.

Finally, Sachs should be noted for several fascinating suggestions for the reform of musical notation. Briefly, they include the use of multicolored staff lines to simplify pitch differentiation and obviate the need for accidentals even in a 19-tone system, and a graphic notation of pitches by horizontal strokes whose length gives exact duration and

whose thickness gives an exact dynamic indication which would, among other things, simplify the designation of principal voices.

During the years immediately following Sachs' report to the International Music Society a flurry of activity in support of 19-tone temperament was begun. In Denmark, P. S. Wedell built an instrument capable of both 19- and 31-tone temperament, and later published, with Bertelson, a series of articles showing the purported advantages of 19- and 31-tone temperaments over 12-tone temperament by the sum of the squares of the errors in the consonances.[14] The approximate errors of the fifth, major third and minor third in 12-, 19-, and 31-tone temperaments are, respectively, 2, 14, 16; 7, 7, 0; and 5, 1, 6 cents. The sums of the squares of these intervals are 456, 98, and 62 square cents. Using this method, the jump from 12- to 19-tone temperament would be emphatically justified by the improvements of the consonances.

In 1919 and 1921 Prof. Valentin Kowalenkow published articles (in German in the Russian publication Iswestija Akademitschenkowo) which went almost unnoticed outside of Russia, but which formed the basis for an article which Jacques Handschin, the noted musicologist, published several

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[14] According to Kornerup, Wedell and Bertelson calculated by smallest squares in 1915 and published their findings in Gottfried Skjerne's monthly Danish-language periodical, Musik, in 1917 and 1919.

years later in a book in honor of Dr. D. F. Scheuerleer.[15] Handschin shows Kowalenkow's chart of the intervals of what he considers to be the just enharmonic scale (21 tones, representing the seven lettered degrees each three times, once each with natural, sharp, and flat). The intervals are given in decimal ratios, and are shown together with the equivalent intervals in 12- and 19-tone temperament. The error in each of these temperaments is approximated in commas and half-commas. The table and approximations are shown, below, in Example 40.

The emerging pattern shows that 19-tone temperament is nearly perfect for those interval relations built of minor thirds, better than 12-tone temperament for those built of major thirds, and not as satisfactory for those relations built on perfect fifths.

Wedell and Bertelson prefer 19- to 12-tone temperament for its consonances. Kowalenkow prefers it on the basis of a table of all of the intervals. The case would be all but closed were it not for one basic weakness in both arguments. Neither provides for the possibility that the intervals he considers are of unequal importance. It is the retrogression in the quality of the perfect fifth, deemed the most important of the intervals within the octave, that imposes the gravest doubts on the acoustical

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[15] Handschin, Jacques, Akustisches aus Russland, published in the Gedenkboek aangeboden aan Dr. D. F. Scheuerleer, p. 143.

Example 40: Kowalenkow's Chart of 12- and 19-Tone Temperaments

Distance	12-tone	Tone	Pure	19-tone	Distance
----	1.000	C	1.000	1.000	----
over 1 comma	1.059	C#	1.042	1.037	under ½ comma
over 1 comma	"	Db	1.08	1.075	under ½ comma
under ½ comma	1.122	D	1.125	1.116	over ½ comma
over 1 comma	1.189	D#	1.172	1.157	1 comma
over ½ comma	"	Eb	1.200	1.200	under ½ comma
over ½ comma	1.260	E	1.250	1.244	under ½ comma
2 commas	1.335	E#	1.302	1.291	over ½ comma
over 1 comma	1.260	Fb	1.28	"	over ½ comma
under ½ comma	1.335	F	1.333	1.339	under ½ comma
over 1 comma	1.414	F#	1.389	1.388	under ½ comma
over 1 comma	"	Gb	1.44	1.440	under ½ comma
under ½ comma	1.498	G	1.500	1.494	under ½ comma
over 1 comma	1.587	G#	1.5625	1.549	over ½ comma
over ½ comma	"	Ab	1.600	1.607	under ½ comma
over ½ comma	1.682	A	1.667	1.666	under ½ comma
over 2 commas	1.782	A#	1.736	1.728	under ½ comma
over ½ comma	"	Bb	1.8	1.792	under ½ comma
over ½ comma	1.888	B	1.875	1.859	over ½ comma
over 1 comma	"	Cb	1.92	1.928	under ½ comma
2 commas	2.000	B#	1.953	"	1 comma

quality of the temperament. Handschin turns his attention directly to this problem, accepting in principle the priority of the fifth but not to the point where it precludes the consideration of other factors in the selection or rejection of 19-tone temperament.

In comparing 19- with 24-tone temperament, which was already quite well known at the time of the publication of his article, Handschin takes a very well-defined position: 24-tone temperament would add only to the chromatic element in music, while 19- would also result in an increased proximity to just intonation.

Handschin partially disassociates himself from the views expressed in his article in a preface which establishes his role as a transcriber rather than a convinced partisan.

The articles of Jose Würschmidt show a gradual development from an interest in multiple division in general to the advocacy of 19-tone temperament in particular. Citing Busoni, Würschmidt agrees that 12-tone temperament is near the end of its usefulness. But he disagrees with Busoni about 36-tone temperament and with the advocates of 24-tone temperament on the grounds that their proposals involve no organic growth for music, but rather, pure mechanical growth.[16] He follows his criticism of 24- and 36-tone temperaments

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[16] Würschmidt, "Viertel- und Sechsteltonmusik," Neue Musik-Zeitung, 42: p. 183.

with an article extolling 19-tone temperament.[17] In comparing the acoustical values of 12- and 19-tone temperaments, he declares for the latter despite a chart which places the former in a far better light than did those of Wedell or Kowalenkow. Where Wedell restricted himself to three consonances, and where Kowalenkow considered all of the intervals in the enharmonic scale of 21 tones, Würschmidt presents the major scale as his unit, comparing the intervals from the tonic in 12- and 19-tone temperament. He uses the just scale of Ptolemy as the basis for comparison. As three of the intervals, 3:2, 4:3, and 9:8, are derived from fifths alone, 19-tone temperament does not fare well in this comparison. Würschmidt's chart, shown below as Example 41, uses the ordinary logarithms of the interval ratios.

Example 41: Würschmidt's Chart of 12- and 19- Systems

Pitch	Just	12-tone	deviation	19-tone	deviation
C	0 000	0 000	000	0 000	0
D	5 115	5 017	-98	4 753	-362
E	9 691	10 034	+343	9 506	-185
F	12 494	12 543	+49	12 675	+181
G	17 609	17 560	-49	17 428	-181
A	22 185	22 577	+392	22 181	-4
B	27 300	27 594	+294	26 934	-366

[17] Würschmidt, Die 19-stugige Skala, Neue Musik-Zeitung, vol. 42: p. 215.

It would appear that with the simpler scale possessing the advantage, 4 intervals to 2, and possessing the smaller total discrepancy when compared with the just diatonic scale, Würschmidt would favor that system. But Würschmidt sees no advantage in simplicity in this case. Having already stated his conviction that the resources of 12-tone temperament are near exhaustion, he also asserts that he can see no impossible obstacles to surmount in the shift to the more complex temperament. 19-tone temperament will be no problem at all for strings, he asserts, while for the piano it need cause no major changes. The white keys can be kept as they are; the existing black keys can be subdivided and two new ones added.[18] Where others might read the statistics in his chart as an endorsement of 12- over 19-tone temperament, Würschmidt sees in it support for 19-tone temperament, on the grounds of its near equality to 12-tone temperament on an acoustical basis. This near-equality, combined with the assumption of the exhaustion of the resources of 12-tone temperament (which is fundamental to Würschmidt's viewpoint) produce as strong a leaning toward 19-tone temperament as that of those authors whose acoustical surveys found 19-tone temperament well ahead.

Würschmidt's case for 19-tone temperament rests, then, on its being the most useful temperament containing more than 12 tones and not on its being intrinsically better than

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[18] Ibid., p. 216.

12-. Würschmidt's theory of rational tone systems (discussed above in Chapter 1) shows why he considers this to be so and also suggests a possible subsequent course of evolution to 22-tone temperament, which Opelt had also declared possible, but which later advocates of 19-tone temperament were strongly to oppose.

The relative weakness, in 19-tone temperament, of the representation of the diatonic scale, which Würschmidt inadvertantly exposes without being particularly concerned, appears to have caused considerable concern to the later advocates of 19-tone temperament. None of the three major advocates of 19-tone temperament who follow uses the diatonic scale of Ptolemy as a musically basic phenomenon in the 19-tone system. Kornerup and Ariel both replace it with the diatonic of Didymus, thereby improving the rendition of the supertonic in 19-tone temperament considerably.[19] Yasser goes much farther; he eliminates the diatonic scale as a point of reference for 19-tone temperament in favor of a "supra-diatonic" scale with 12 members. As Kowalenkow's chart (Example 40) demonstrates, the more non-diatonic tones used which are related to the diatonic members by thirds, the better 19-tone temperaments fares in comparison to 12-.

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[19] The supertonic is 9:8 in the tetrachord of Ptolemy, and 10:9 in the tetrachord of Didymus. The respective cent values are 204 and 182. The supertonic in 12-tone temperament is 200 cents; in 19-tone temperament, 189 cents. By this comparison it is evident that the shift of this one tone from the tetrachord of Ptolemy to that of Didymus is sufficient to shift the balance of accuracy for the whole scale from 12-tone to 19-tone temperament.

With the above, our consideration of the theory of 19-tone temperament and its advocacy up to the time of Kornerup, Ariel, and Yasser is completed. The next three chapters will be concerned with the writings of this triumvirate, with whose work the cause of 19-tone temperament is closely associated.

CHAPTER 9

THORVALD KORNERUP

The 20th century has produced three writers who have written extensively in advocacy of 19-tone temperament. Two of these men have used the medium of books to present their views. Kornerup's advocacy, however, has taken the form of a series of pamphlets. There are thirteen, dating from 1922 to 1938, and they show a gradual evolution in Kornerup's emphasis and interest, balanced against a consistent viewpoint throughout. He is at all times a thoroughly convinced advocate of 19- and 31-tone temperament. Here is a list of his pamphlets from 1922 to 1938, all of them published in his native city of Copenhagen.

1. Musical Acoustics based on the Pure Third System, 1922
2. Akustische Gesetze für die Akkordbildiung, 1930
3. Das Tonsystem des Italieners Zarlino, 1930
4. Die Vorläufer der gleichschwebended Temperaturen (zu 19- and 31 Töne), 1930
5. Die Hochteilung der Oktave, als Ablösung des syntonischen Quint-Terzen Systems, 1930
6. Die akustische Atomtheorie ausgewandt auf das pythagoräische Tonsystem, 1931
7. Von der Ur-form 5-töniger Skalen zu den goldenen Tönen, 1931
8. Indisches Tonsystem aufgeklärt durch tertiäre Teilung, 1931
9. Acoustic Methods of Work, 1934
10. Acoustique theorique, 1934
11. Das goldene Tonsystem, 1934
12. Equilibre entre les sons perfectionnes (dores) et les sons naturels, 1936
13. Acoustic Valuation of Intervals, 1938

Kornerup's militant writings are not confined to

music. They include proposals for political and calendar reform.[1] Throughout his works, Kornerup writes as though he were a debater, scholarly and thorough with all data tending to support his conclusions, oblivious to all contrary data.

In his early writings, Kornerup is concerned with refuting the Pythagoreans. The small fifths which characterize 19- and 31-tone temperaments are desirable, to Kornerup, because they make possible relatively pure thirds. In the writings of what might be called his middle period Kornerup is interested in ancient and oriental musical systems as models for a 19- or 31-tone system. Finally, in his later works, he is preoccupied by an almost mystical conception of a musical application for the golden mean. This is his most original and challenging proposal, and it leads, as do his other doctrines, to the advocacy of 19- and 31-tone temperaments. We shall look at representative works of each period.

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[1]In the political sphere, Kornerup's proposal is for a compromise between two methods of counting ballots for proportional representation. He presents a mathematical mean between two generally considered plans, one of which is alleged excessively to favor the large political parties, the other the small ones. Kornerup's proposals for calendar reform involve the equalization of the months with leftover "monthless" days.

PURE THIRDS ALTER A SECOND

The basic proposal of Kornerup's first pamphlet, Musical Acoustics Based on a Pure Third System, is that the supertonic generally attributed to the just scale, 9:8, is fundamentally wrong and should be replaced by 10:9. In showing how this view evolves from the more traditional concept of the just major scale, Kornerup first reproduces diagrams of the traditional scale based on fifths and thirds.

A	E	B	
F	C	G	D

He then cites what he considers to have been an improvement in this pattern by Shohe Tanaka in 1890. In this diagram, Tanaka staggers the thirds, giving to major and minor thirds equal importance.

D	A	E	B
	F	C	G

Kornerup points out that with Tanaka's revision the horizontal aspect of the system is symmetrical. However, he believes Tanaka's system to be deficient in one respect: the tonic is not at the center of the symmetry. In order to make C the center of a symmetrical pattern similar to

Tanaka's it is only necessary to substitute the D lying a fifth below the tone A in the system for the D lying a fifth above the tone G.

A	E	B	
F	C	G	D

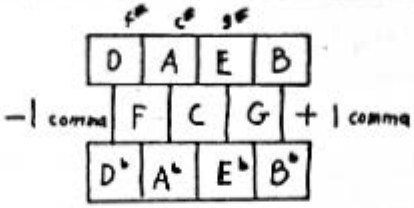
The D in the above diagram has a value of 10:9 as computed from C. Once Kornerup has replaced D 9:8 with D 10:9, the rest of his argument for 19-tone temperament falls into place easily for it is with the supertonic 9:8 that 19-tone temperament suffers most in comparison to 12-tone temperament.

Example 42: The Just Major Scale Compared to 12- and 19-Tone Temperaments

Tone	JUST	12-tone	19-tone	error:12	error:19
C	00	00	00	00	00
D 9:8	204	200	189	04	15
D 10:9	182	"	"	18	7
E 5:4	386	400	379	14	7
F 4:3	498	500	505	2	7
G 3:2	702	700	695	2	7
A 5:3	844	900	884	16	0
B 15:8	1088	1100	1074	12	14
Total error with D as 9:8				50	50
Total error with D as 10:9				64	42

Kornerup extends the system beyond diatonic confines by adding another group of tones below the row F-C-G in the diagrams, making C the central point of a symmetry which

affects the vertical as well as the horizontal patterns. Within the bottom two rows, C is the center of a scale which Kornerup calls the doric minor.



It is part of Kornerup's opposition to everything Pythagorean that he considers the above values and those related to them by thirds to be the tones <u>not</u> commatically altered. Tones to the right of those shown above Kornerup considers to be raised by a comma (or more), while tones to the left are commatically lowered.

The result of Kornerup's theory of the Pure Thirds is the relative de-emphasis on the 3rd partial, and with it the minimization of the importance of the one acoustical area in which 19-tone equal temperament is inferior to 12-.

HISTORICAL JUSTIFICATIONS

Having established an acoustical basis for his recommended reform of musical temperament, Kornerup turns his primary attention to historical precedents for the

flat fifths he is recommending. He devotes special attention to three of the meantone temperaments, those involving the alteration of the fifth by ¼, 2/7, and 1/3 comma respectively. He attributes the 2/7 comma temperament to Zarlino (an attribution which is generally supported) and the other and the other two to Arnold Schlick (both of which are generally contradicted).[2] He cites the early construction of 19-tone instruments, without conceding that they might have been tuned to systems hardly even approximating 19-tone equal temperament. His principal and most valid point concerning the tuning systems of late Renaissance theory is that they countenanced a considerable reduction in the size of the perfect fifth for the purpose of improving the thirds. His pamphlets abound with charts showing how the fifths of his own proposed systems compare closely with those advocated by the disciples of meantone temperaments.

THE GOLDEN SYSTEM

In his early writings Kornerup underlines his view that the acoustical bases for music are threefold, the three bases being the 2nd, 3rd, and 5th partials.[3] He is aware,

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[2]Barbour ascribes ¼-comma temperament to Pietro Aron, 1/3-comma temperament to Francisco Salinas.

[3]Kornerup likens these three values to the three primary colors, declaring that theorists such as the Pythagoreans who fail to recognize one of these values are the musical equivalent of color-blind. Musical Acoustics based on a Pure Third System, column 29.

of course, that the series created by the powers of the three numbers must proceed toward infinity without ever meeting, and that any relationship between values based on one series and values based on another can only be approximate. But this troubles him because he is anxious to have a universal basis for measurement that will provide answers to questions of right and wrong in tuning systems. He finds such a universal basis in the age-old mathematical concept of the golden mean.

The golden ratio, which Kornerup also refers to as Kepler's "Proportio Divina" is that proportion between two values, "a" and "b" where "b" is greater than "a," which is capable of being repeated exactly in the proportion between the larger term, "b," and the sum of the two terms, "a + b." In an algebraic equation, the precondition for the golden ratio is as follows:

$$\frac{a}{b} = \frac{b}{a + b}$$

The value, $\frac{a}{b}$ is generally referred to as ω, and the equation can, after several simple substitutions, be expressed as follows:

$$\omega^2 + \omega - 1 = 0.$$

Solving for ω, we obtain the value 0.618034. It is Kornerup's basic postulate to let the chromatic and diatonic semitones differ from one another by the ratio ω. It is relatively easy, algebraically to solve for a and b

(chromatic and diatonic semitones respectively) such that $\frac{a}{b} = .618034$, while $a^5 \cdot b^7 = 2$ (five chromatic semitones plus seven diatonic semitones equals an octave). The resulting values, in cents, for a and b are 73.5 cents and 118.9 cents respectively. These values are fairly close to just values for these intervals which are, respectively, 25:24 = 70.7 cents, and 16:15 = 111.7 cents.

Because of the special nature of the golden ratio, the tone, which represents the sum of the two kinds of semitones, will represent the value "a + b," which will be a denominator in a fraction equal to ω when the diatonic semitone is the numerator. This tone, "a + b," is 192.4 cents, which is approximately the mean between 9:8 and 10:9. The ratio, ω, also represents the relationship between the major second and the minor third, "a + 2b." This minor third is 311.4 cents in size. The ratio, ω, also represents the relationship between the minor third and the perfect fourth, 2a + 3b. The golden fourth measures 503.8 cents. The entire hierarchy of intervals from the chromatic semitone to the perfect fourth can be expressed as follows:

$$\frac{\text{chromatic semitone}}{\text{diatonic semitone}} = \frac{\text{diatonic semitone}}{\text{tone}} = \frac{\text{tone}}{\text{minor 3rd}} = \frac{\text{minor third}}{\text{perfect fourth}} = 0.618034$$

If the values represented by the five intervals are added together, the sum will be exactly equal to an octave.

If just the last term (perfect fourth) is omitted, the result is the golden fifth, 696.2 cents. If the last two terms are omitted, the result is the golden major third, 384.9 cents. This is how Kornerup brings the disparate values for third, fifth, and octave into a single universal basis for measurement. The golden third, fifth, and octave cannot relate to one another by ω, but rather as sums of values each related to one another by ω.

Kornerup carries the golden ratio through intervals smaller than the chromatic semitone (although the smaller values are not included when members of the series are added together to form the major third, perfect fifth, or octave). Proceeding from large to small, the first two values after the chromatic semitone, which Kornerup calls golden molecules, are 45.4 cents (the golden diesis) and 28.1 cents (the golden comma). At the commatic level the golden interval seems excessively distorted to be of any acoustical value (the syntonic comma is 21.5 cents, nearly 25% smaller), but the larger intervals are approximated sufficiently well to render the golden system at least discussable.

Kornerup claims for his system of intervals at a fixed ratio to one-another that it is capable of serving as a basis for acoustical valuation of musical systems.[4] This

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[4]The intensity with which Kornerup expresses his advocacy of the golden tone, and the extent of his claims for it, can only be shown with a direct quotation: "The relations of the golden division are like a <u>kingly</u> thought

seems a somewhat extravagant assertion in view of the fact that nature has supplied its own basis for acoustical valuation in the intervals of small-number ratios. Barbour, who considers the golden system on "Ignis Fatuus" points to the rather substantial error in the golden fifth and third.[5] The fifth is nearly 5.8 cents in error; an error greater than that of ¼-comma meantone temperament. The third is 1.4 cents in error, but it is an error on the side of flatness which is an aggravating factor to Barbour and others who prefer a large third.

The golden system is probably more realistically appraised when it is considered not as a universal basis for measurement but as simply another proposed musical system, sharing with equal temperaments the imperfection of all intervals other than the octave. As do equal temperaments, the golden system offers compensating advantages for its acoustical inaccuracies. These advantages must be weighed against the extent and importance of the inaccuracies. The advantages of the golden system are less tangible than those of an equal-tempered system. Not even Kornerup has advocated the use of pure golden intonation, since it would take a proliferation of tones to render such a system transposable. Rather, it is asserted

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which shows Beauty and Justice as two forms of the same, a latent natural fundamental law, at the same time both aesthetic and ethic." Acoustic Valuation of Intervals, p. 9.

[5] Tuning and Temperament, page 128.

for the golden system, the use of a constant ratio between the various sizes of component intervals provides an almost mystical harmoniousness. The parts are alleged to harmonize with one another. The comma and diesis are no longer just left-over parts. They belong to a series which also contains two kinds of semitones, the tone, the minor third and the perfect fourth of the system. Supplementing the sense of unity in a system where the great and the small are part of a single series is the mystique of the ratio of the golden mean itself. The object of great respect and even awe of brilliant men for many centuries, the golden mean is said to be embodied in nature in the construction of certain leaves[6] and in much of the world's greatest architecture as well.[7]

Whether or not the reader finds the mystique of the golden mean and of the single hierarchy of intervals appealing as a musical element, he may be impelled to ask what use it could possibly have if Kornerup's initial claims for it are rejected. Not even Kornerup has proposed its use in practice. What value might it have if it is acoustically unsuited to be the basis for valuation?

There is a close organic relationship between the

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[6]Kornerup gives a moderately extensive bibliography of non-musical writings about the golden mean in Acoustic Valuation of Intervals, page 21.

[7]Kornerup's source is Macody Lund, Kathedralen i Nidaros, Oslo, 1919.

golden system and a certain series of related temperaments, the Fibonacci series[8] ... 12, 19, 31, 50 ... . It can be proved that the limit as $n \to \infty$ of $\frac{n}{n - 1}$ in any Fibonacci series is none other than , 0.618034. It can likewise be shown that the golden intervals represent the exact tuning of that system which is the limit of the series ... 12, 19, 31, 50 ... . This having been established, the golden system, as advocated by Kornerup, becomes, essentially, a means of advocating the expansion of tonal resources by means of this particular series. The higher one proceeds within this series, the closer the ratios between the component parts approach the limit ω . Although there are discrepancies between the intervals of any finite tempered system and those of the golden system, such discrepancies become increasingly small as the number of tones employed increases. The table below shows the discrepancies between certain intervals in 31-tone temperament and the equivalent intervals in the golden system.[9]

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[8] A Fibonacci series is a series of numbers in which any given number is the sum of the two preceding numbers, or $n = (n - 2) + (n - 1)$.

[9] For a similar comparison between golden intervals and 19-tone temperament, see below, chapter 12, page 314, for a chart, followed by a general discussion.

	31-TT	Golden
Diesis	38.71	45.4
Chromatic semitone	77.42	73.5
Diatonic semitone	116.13	118.9
Tone	193.55	192.4
Minor Third	309.68	311.4
Major Third	387.68	384.9
Perfect 5th	696.8	696.2

569.8 Fourth

Except for the smallest unit of the system, 31-tone temperament represents a rather faithful rendering of golden tuning. Therefore, in the most practical sense, the case for the golden cut is a part of the case for 31-tone (and to a lesser extent for 19-tone) temperament. Whatever attractive mystique may rest in the principle of the golden mean can, with very slight discrepancies, be attributed to the equal temperaments of the aforementioned Fibonacci series,[10] without altering the acoustical advantages or disadvantages of the temperaments within the series.

It is unnecessary to concern ourselves here over the many other uses Kornerup makes of the golden series as the basis for musical measurement.[11] Whether or not it is regarded as deciding, the value of the golden cut as an

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[10] As chapter 12 will show, some weaknesses are present in the application of the Golden system to 19-tone temperament which do not apply to 31- or 50-tone temperament.

[11] Kornerup finds values for the higher partials on the basis of combinations of golden intervals. His seventh partial is some 6 cents too flat, and is the sum of the golden tone, minor third, and perfect fourth, minus the golden diesis. Kornerup also uses the golden system as a starting point in some Socio-musicology, declaring, for example, in his Indisches Tonsystem..., page 7, that the majority of Indian intervals are very close to golden intervals.

argument in favor of 19-, 31- or 50-tone temperament, supplementing but not replacing acoustical considerations, deserves to be taken into consideration. Because of this, Kornerup's theory of the golden cut must be regarded as an important contribution to the theory of multiple division ever though his basic premise for its use is extravagant to the point of invalidity.

A final note should be added about the broadness of scope of Kornerup's work in acoustics and multiple division. In his pages appears the most detailed chronology of the advocacy of 19-tone temperament to have been published;[12] one of the most thorough treatments of the subject of the different possible ways of dividing a string into musical intervals;[13] and a most sensible proposal for the universal naming of tones.[14] His pamphlets contain many charts, often as illuminating concerning his viewpoint as many pages of text. Several such charts are appended here.[15]

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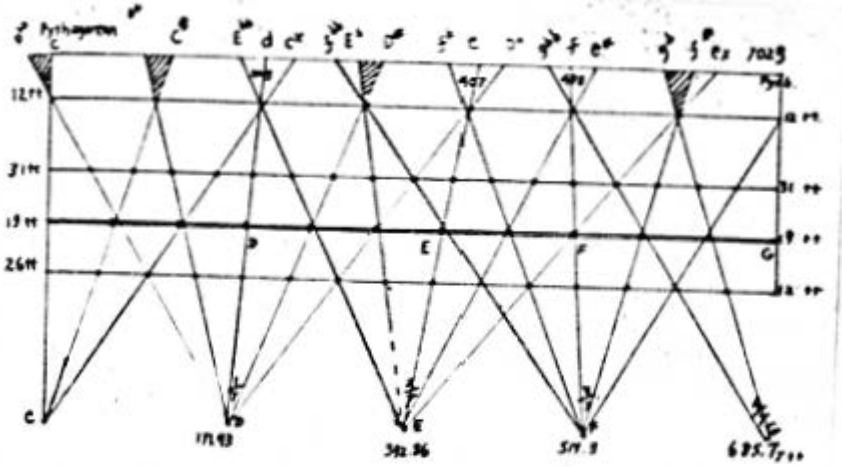
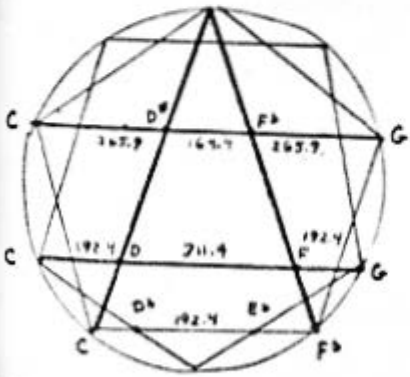
[12]Such chronologies appear in several of his pamphlets. Of the important early writers only Woolhouse is omitted. There is, however, a tendency to attribute advocacy of 19-tone temperament where no such advocacy is implied as in the case of Elsass, the builder of the early 17th century Clavicembalo Universale.

[13]Kornerup's treatment of this is particularly useful and thorough. It is one of the chief subjects of Acoustic Methods of Work.

[14]Kornerup proposes the use of the German suffixes is and es to denote sharps and flats respectively, but the elimination of the letter h from the roster of tone designations, and the use of b to denote b natural.

[15]From Die Vorlaufer..., page 5.

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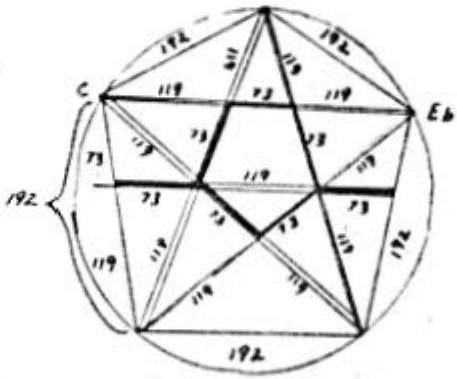
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CHAPTER 10

"ARIEL"

In 1925, a new publishing house appeared in Leipzig called the Neunzehn Stufen Verlag. Citing as its purpose nothing less than the reform of music, the house issued one book and the unfulfilled promise of two additional works and then faded into immediate and lasting oblivion. The one published volume is Das Relativitätsprinzip der Musikalischen Harmonie by an anonymous author writing under the pseudonym "Ariel." A detailed description of a 19-tone instrument with three rows of keys was to follow, along with a proposed system of notation, in a second volume. Volume III was to be a harmony text for 19-tone music based on the so-called principle of relativity. It is to be presumed that financial difficulties postponed the publication of these successor volumes and that political upheavals destroyed all traces of the company and its benefactors.[1]

THE 59 BASIC INTERVALS

Das Relativitätsprinzip is, from the start, a violently partisan work. Ariel insists that all genuine

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[1]The real name of Ariel appears in none of the catalogues where the book is listed. The copyright was issued to the Neunzehn Stufen Verlag, with a Mme. Boas de Jouvenal signing the necessary papers. Whether she was the author or simply an agent is unknown, but the latter would appear the more likely assumption.

music, if its tone relationships are to be logical, must arise out of certain pure intervals. Ariel quite arbitrarily selects 59 such "pure" intervals, and they range from such fundamental intervals as 3:2 to such complex intervals as 20736:15625. Ariel then shows by an appended chart how much more logically the 59 intervals group together in 19 separate units than in 12. One can question several aspects of Ariel's reasoning. Why should 20736:15625 be considered as important as 3:2 in establishing the validity of a temperament? Why, indeed, should 20736:15625 be considered at all?

Reduced to simple intervals, 20736:15625 is the interval created when two major thirds are deducted from four minor thirds. A process as complicated as this would hardly be warranted in the quest for "basic" intervals. What is basic about four superimposed minor thirds? The minor third, of course, possesses exceptional accuracy in 19-tone temperament. In the absence of a meaningful explanation to the contrary, Ariel would seem to be guilty of circular reasoning. He justifies 19-tone temperament on the basis of a group of intervals which seem themselves to have been chosen because they are themselves based on combinations of those intervals which are known to be effectively duplicated by 19-tone temperament. Because Ariel's 59 intervals appear doubtful and arbitrary, they can probably be dismissed from consideration as a serious basis for 19-tone temperament.

Example 44: Ariel's 59 Intervals

1:1	6:5	64:45	27:16
128:125	3125:2592	36:25	128:75
648:625	768:625	625:432	216:125
25:24	100:81	4608:3125	125:72
16:15	5:4	40:27	27648:15625
27:25	32:25	3:2	16:9
625:576	162:125	15625:10368	9:5
3456:3125	125:96	192:125	3125:1728
10:9	20736:15625	125:81	1152:625
9:8	4:3	25:16	50:27
15625:13824	27:20	8:5	15:8
144:125	3125:2304	81:50	48:25
125:108	864:625	625:384	625:324
75:64	25:18	5184:3125	125:64
32:27	45:32	5:3	2:1

THE FOUR REQUIREMENTS OF A TONAL SYSTEM

Ariel moves to a more tenable position when he considers the basic requirements of a system of temperament. He maintains that one must make four demands on any tempered system:[2]

> 1: That the smallest intervals where there is the possibility for direct progressions (direkte Tonbewegungen) be available.
> 2: That the consonances should be as pure as possible.
> 3: That the proportions between the sizes of the basic scalar intervals should resemble those of the natural intervals from which they are taken.

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[2] Das Relativitätsprinzip, page 134. "Man kann für die Anforderungen an ein temperiertes System vier Thesen aufstellen: I. Die kleinsten Intervalle, in denen direkte Tonbewegungen noch möglich sind, müssen vorhanden sein. II. Die Konsonanzen müssen möglichst rein sen. III. Die Tonfolgen müssen in den Verhältnissen ihrer natürlichen Abstufungen zum Ausdruck kommen. IV. Konzonanzen und Dissonanzen müssen als solche Gegensätzlich wirken. Natürliche Dissonanzen dürfen nicht zu Scheinkonzonanzen wirken."

> 4. That consonances and dissonances should function as such against one another. Intervals which are by nature dissonant should not be tempered so as to appear consonant.

On all four counts he finds 19-tone temperament to be far superior to 12.

Smallest Interval: That 19-tone temperament should be preferable to 12- on the first count, that of supplying the smallest possible interval, is self-evident. But Ariel makes a rather important point about the special significance of that particular smallest interval. The natural chromatic semitone, he asserts, is 25:24. This interval he calls the "Terzendifferenz," as it represents the difference between the major third, 5:4, and the minor third 6:5. It is Ariel's contention that this Terzendifferenz is of great importance in music; this appears quite sound in view of the role of major and minor in our musical heritage. But the smallest interval in 12-tone temperament is 100 cents, while the natural Terzendifferenz is 70 cents. For this reason, Ariel strongly rejects 12-tone temperament, even for the performance of the current repertory. He turns, instead, to 19-tone temperament, whose smallest interval, 63 cents, is a much closer approximation of the Terzendifferenz.

The Purity of the Consonances: According to Ariel, the following are the consonant intervals of music: 10:9, 6:5, 5:4, 4:3, 25:18, 36:25, 3:2, 8:5, 5:3, and 9:5. Of these ten consonances within the octave, six are traditionally

regarded as consonances (the minor and major thirds and sixths and the perfect fourth and fifth). In accepting the major second and minor seventh as consonances, Ariel would appear to be justified by much 19th as well as 20th century practice, wherein the minor seventh and/or the major ninth is often added to triadic sonorities with little sense of dissonance. What may appear subject to question about Ariel's use of the major second as a consonance is his choice of 10:9 over the more traditional 9:8. In making this choice he takes the same position as Kornerup, but for his own very different reasons.

He is quite emphatic in his choice. 9:8, he insists, is a dissonance. It is produced by the artificial process of superimposing two fifths.[3] The inversion, 16:9, involves relatively high numbers and is also a dissonance. On the other hand, 10:9 is the inversion of the ratio 9:5, representing the combination of the perfect fifth and minor third. In performing the consonant dominant seventh chord, asserts Ariel, singers attempt to make both the fifth G-d and the minor third, d-f pure. This produces the consonant minor seventh, which must be 9:5. The consonant dominant seventh chord, as the basic generator of consonant harmony, is a

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[3]Ariel seems in this respect to be implying that the combination of like intervals tends to produce dissonance. This is Zarlino's basic view. In the case of the tritone, however, he appears to contradict this view. Perhaps the one consistent way in which Ariel's view might be interpreted is this: the combining of like consonances in itself does not produce a consonance.

theoretical phenomenon with a rather respectable history, and Ariel is in good company in asserting that this consonant seventh chord is comprised of the relationships 20:25:30:36. It is the first and last members, 20:36 which relate to one another as 5:9, and invert to 10:9. Since Ariel is determined to reject the natural harmonic seventh as the basis for any music,[4] there may indeed be acoustical validity to his choice of 9:5, the simplest remaining ratio, as his consonant seventh.

Ariel's "consonant" tritone 25:18, and its inversion, the so-called "Doppelterz" 36:25, are, like the minor seventh, drawn from the consonant seventh chord 20:25:30:36. The Doppelterz is therefore declared consonant in spite of its being the sum of like intervals. It may be somewhat difficult to accept the tritone as a consonance. It is generally referred to as a dissonance, and Hindemith gives the tritone a special classification as the most dissonant interval.[5] However, it should be remembered that the tritone (like the perfect fourth as well as the diminished fifth) was treated as a semi-consonance, in the Renaissance, usable between the upper voices as a consonance. It is only

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[4]Ariel is most adamant in rejecting the 7th partial outright. He argues against its incorporation into music in several ways, concluding with the statement on page 51 that when two singers sing an interval of 7:6, the result is neither a musical interval, nor the temperament of such an interval. It is, "überhaupt keine Musik."

[5]Hindemith, The Craft of Musical Composition.

since the era of 12-tone equal temperament, with an interval 31 cents removed from Ariel's Doppelterz, that the harmonic use of the tritone has come to imply the ultimate in instability and dissonance.

Having established the above intervals as consonant, Ariel compares their representation in 19- and 12-tone temperaments.

3:2 and 4:3: Ariel admits to the superiority of the 12-tone fifths. He insists, however, that the 7-cent error of the fifth in 19-tone temperament is far from transgressing the limits of agreeable sound,[6] and warns against basing a judgment on the fifths alone.

5:4 and 8:5: Ariel complains of the bad thirds in 12-tone temperament, and points out that the major thirds in 19-tone temperament are almost twice as pure. He makes no distinction between the positive error of the 12-tone major third and the negative error (considered by Barbour and others to be far more objectionable) of the 19-tone third.

6:5 and 5:3: The error of this interval in 19-tone temperament is less than 0.1 cents. To Ariel this is the most important of intervals. He refers to it as the child of an eternal father (the perfect fifth) and an eternal mother (the major third). As the offspring of so propitious a union, as the Urkind, the minor third is the foundation

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[6] Op. cit., page 144 "... Sie bedeutet aber noch lange nicht die Grenze für die Trübung des Wohlklanges."

for a higher order of music.[7] Without the knowledge of this fundamental significance of the minor third, maintains Ariel, harmony, and especially modern harmony, is not to be understood. The error in 12-tone temperament is over 15-cents.

10:9 and 9:5: If one is able to agree with Ariel that these are the proper ratios for the major second and minor seventh, and that they are consonances, then 19-tone temperament has a clear edge over 12- in their representation. The approximate margin is 7 cents to 18.

25:18 and 36:25: As in the case above, whatever controversy might exist as between the two systems lies in whether or not these intervals are the correct ratios for tritone and diminished fifth. The error in their representation is a minute fraction of a cent in 19-tone temperament, and about 31 cents in 12-tone temperament. No alternative ratios involving smaller numbers are generally offered for 12-tone temperament, since the interval is generally conceded to be a dissonance in that system. Fokker, on behalf of 31-tone temperament, argues quite persuasively that the consonant tritone is 7:5 and that all other approximations for this interval are dissonant.

For the ten intervals which Ariel has selected as

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[7] Ibid., page 74, "Jener Ausgleich, der aus Gegenbewegung von Quinte und grosse Terz hervorgeht ist die kleine Terz. Diese, gleichsam das Kind von Ur-Vater (Quinte) und Ur-Mutter (gr. Terz) ist Grundintervall höherer Ordnung."

his consonances, the average error in 19-tone temperament is 4.2 cents, in 12, 16.2. Granted the premise of his intervals, Ariel's case for the superiority of the consonances of 19-tone temperament is quite solid indeed.

The Mutual Relationships of the Small Intervals: According to Ariel, there are five essential small intervals in our musical system, the chromatic and diatonic semitones, the whole-tone, the diminished third (to which he is evidently referring by the name of its enharmonic equivalent in 19-tone temperament, the augmented second), and the minor third. In 12-tone temperament the two types of semitone are enharmonic equivalents, as are the augmented second and the minor third. This is, according to Ariel, a serious distortion of the logic of their basic relationship to one another. Ariel considers the mutual relationships singly and in groups. The chromatic and diatonic semitones relate to one another and to the whole-tone[8] approximately as 37:63:100. The twelve tone scale can do no better at reproducing this formula than 50:50:100. Markedly better is the 19-tone approximation, 33:67:100. Ariel shows, similarly, that the 3:4:5 relationship of 19-tone temperament is a rather close approximation for the series 10:9, 144:125, 6:5, while 12-tone temperament is powerless to represent such gradations. In the end, it is Ariel's preference for two

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[8] Ibid., page 147. Compare these figures with the golden ratio of Kornerup which is approximately 38:62:100.

kinds of semi-tone and for a distinct interval larger than the whole-tone but smaller than the minor third which leads him strongly again to endorse 19-tone temperament.

The Polarity of Consonance and Dissonance: Ariel's fourth demand on a tempered system is that it differentiate clearly between its consonances and its dissonances. In this respect, Ariel objects to 12-tone temperament on the grounds that it possesses only one set of dissonances (the minor second and major seventh) which isn't so clearly removed from consonant intervals that it cannot be falsely mistaken for a set of consonances. In contrast, there are four such sets of dissonances in 19-tone temperament. There are, to begin with, two sets (instead of one) of semitones and sevenths. The two sets are 1/19 - 18/19, and 2/19 - 17/19. In addition there are two sets, 7/19 - 12/19, and 4/19 - 15/19, which Ariel considers to be unambiguously dissonant in 19-tone temperament,[9] and which lack any counterpart in 12-tone temperament.

Ariel illustrates the advantage of unambiguous dissonance by showing that the augmented triad and diminished seventh are dissonances in 19-tone temperament, while they are ambiguous through the lack of any completely dissonant intervals in 12-tone temperament. The diminished seventh

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[9] As just representations for 4/19 and 7/19, Ariel gives 144:125 and 32:25 respectively. The deviation of these intervals from their tempered equivalents is somewhat greater than is the deviation of the consonant intervals.

in 19-tone temperament must contain an interval of 4/19 or 15/19, while the augmented triad must have either 7/19 or 12/19.

There appear to be a number of weak points in Ariel's theory of consonance and dissonance employed as polar opposites. It is difficult to see why 25:16 should be regarded as a dissonance while the more complex formulation 36:25 is considered consonant. At best it is hard to imagine why two such similar intervals as 25:16 and 25:18 should have effects which function as polar opposites.

Ariel's theory of consonance and dissonance falls completely if his fundamental rejection of the 7th partial is denied. He has established as consonant a number of intervals containing integers higher than 7. This has not been done by the traditional supporters of the senario (and nothing more). The decision to include as consonances intervals involving integers larger than 6 makes somewhat questionable on logical grounds the rejection of 7 as a basis for consonance.

Despite the theoretical weaknesses of Ariel's hierarchy of consonances and dissonances, I must add that my own subjective appraisal of the characteristics of the intervals in 19-tone temperament concurs with Ariel's theory. This may be less the result of valid acoustical principles than a matter of familiarity. Ariel has named as dissonances 4/19 and 7/19, which are the two basic sounds in 19-tone

temperament that possess no equivalent in present-day music. They may simply sound "dissonant" by reason of their lack of familiarity.

THE INTERRELATION AND EVOLUTION OF TONAL SYSTEMS

In common with the other major 19-tone advocates, Ariel finds "good" tonal systems to be related. Where Kornerup and Yasser find the ultimate relationship between desirable systems to lie in a single Fibonacci series, Ariel is concerned with other aspects of continuity between the different systems. His approach is close to Opelt's (with whose work he is familiar) and even closer to Würschmidt's. Like Würschmidt, Ariel sees the problem as one of finding three suitable constructing intervals, based on pure fifths and thirds, which will not differ too greatly in size. Ariel is not as systematic as Würschmidt,[10] relying more on instinct and on trial and error. His results are quite similar to those of Würschmidt, and remarkably similar to those obtained by Viggo Brun using a carefully worked out mathematical formula.[11]

Reduced to its fundamentals, Ariel's view of the relatedness of desirable musical systems is as follows. Tempered

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[10] Würschmidt's views are dealt with at considerable length in Chapter 1, and again briefly in chapter 8. Opelt is considered in Chapter 8.

[11] Viggo Brun's mathematical formula, first presented in 1919, is still virtually unknown. It is taken up in detail in Chapter 13.

systems are necessary to music, but they are of value only to the extent that they succeed in the approximation of just intervals. It must therefore be possible to formulate any complete musical (tempered) system in terms of the just intervals on which it is built. The basic just intervals remain the same in every "good" system. The systems with many tones result simply from the subdivision of the intervals of the systems with fewer tones.

The seven-tone system, for example, has three constructing intervals, 9:8, 10:9, and 16:15. These intervals are used three times, twice, and twice, respectively. This is, as all will recognize, the just diatonic scale, which Ariel recognizes as the basis for 7-tone temperament. This same just diatonic scale is present in all "good" systems involving additional tones; the accuracy of its representation increases in the temperaments involving more tones. The 12-tone system is formed by retaining the smallest of the intervals of the 7-tone system, 16:15, and subdividing (factoring the ratios of) the other intervals. As there can only be three constructing intervals (a basic premise of Ariel's which might be subject to some criticism), an effort must be made to use the held-over interval (16:15 in this case) as a factor, and to find common factors for the remaining intervals. In the conversion from the 7- to the 12-tone system, 9:8 is subdivided into 25:24 and 27:25, the former member of which is also used, together with 16:15,

as factors of 10:9. The complete 12-tone system is thereby found to contain five intervals 25:24, four intervals 16:15, and three intervals 27:25. By similar processes, further subdivisions yield 19-, 31-, 34-, 53-, 65-, 118-, 171-, 289-, 323-, 441-, 612-, 730-, and 1053-tone temperaments.[12]

The full details of each of the simpler systems are in Das Relativitätsprinzip.[13] Here it would seem relevant only to note Ariel's just 19-tone scale and how it is derived from the just 12-tone scale. As might be expected, the Terzendifferenz, 25:24, is the most important interval in the system. The five intervals, 25:24, which appear in the 12-tone system, are held over. In addition, 25:24 is used as a factor in the subdivision of each of the other intervals. This brings to twelve the number of intervals of that size employed in the 19-tone scale. The remaining seven intervals represent the left-over factors of 16:15 and of 27:25, and are 128:125 and 648:625 respectively. These two new intervals represent on the one hand the

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[12] It is fascinating to compare Ariel's seemingly empirical deductions with Brun's which develop a priori from a mathematical formula. Not only are the systems similar with respect to the number of tones used (with a few exceptions), but also with respect to the constructing intervals in a number of cases. For example, the syntonic comma first becomes a constructing interval for Ariel in 34-tone temperament. Note, in Chapter 13, that it is the graduation of the syntonic comma to an important position in Brun's equation that makes 34- a member of his series.

[13] Beginning page 160.

difference between the octave and three major thirds, and on the other, the difference between the octave and four minor thirds. With the other constructing interval representing the difference between the two kinds of third, it can well be seen that the thirds play the primary role in the construction of the 19-tone scale.

Ariel's "system of systems" is quite impressive, especially as it contains all of the most sought-after of the equal tempered systems. Ariel's is one of the few systematic combinations which link the 53-tone system with the 19- and 31-tone systems. Ariel's hierarchy of systems possesses the considerable advantage of having one universal just intonation which is equally adapted to all of the member systems.

There are, however, several grounds on which Ariel's sequence of systems is subject to question. It is difficult to see why three should have to be the limit to the number of constructing intervals. It is quite possible that a more stable system might be possible with two or four such intervals. Ariel's total reliance on empirical methods and his refusal to draw precise limits can be questioned. He rejects Opelt's proposal for 22-tone temperament, for example, on the grounds that the three constructing intervals must vary excessively in size,[14] the largest being 25:24,

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[14] Würschmidt disputes Ariel on this point, offering an alternative construction which does not use the syntonic comma as a constructing interval. The constructing intervals are, instead, $(25:24)^9$, $(250:243)^3$ and $(648:625)^{10}$.

and the smallest the syntonic comma, 81:80. But Ariel never states precisely what the proper limits for size variation should be. Finally, in Ariel's series of systems, the absence of the 7th partial as a factor stands out as a possible fault as much as in his considerations of the necessary properties for the individual system.

CULT OF NUMBERS

In his attempt to leave no stone unturned under which might be found an argument for the 19-tone scale, Ariel indulges freely in a kind of astrology of numbers. He points out that the Arabian shiek, Mudschial Eddin ibn Arabi (1172-1240), considered 19 to be the number which explained the development of the world.[15] Its double, 38, is a much-mentioned mystical number (mehrfach erwähnte mystische Zahl), according to Ariel. There is also some significance to its square, 361, and the proximity of this square to 360, the number of degrees in a circle and the product of 2^3, 3^2 and 5. Ariel reminds us further that the Asian Go-board is 19x19! Although he disclaims any interest in the Occult as a basis for 19-tone temperament, his citation of traditional mystical and occult sources to support his numerical theories is both interesting and somewhat disquieting. It is a measure of his fanaticism.

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[15] Op. cit., page 43, "die Zahl der Weltentwicklung."

SUMMARY

Behind a mask of fiery polemics and mystical fervor, Ariel presents a number of ideas of considerable importance on the interrelation of musical systems, the universal validity of a single just intonation, the function and adequacy of consonance and dissonance in different tempered systems, and the proper demands a musician can make upon a tempered system. Many of his ideas seem quite arbitrarily established, such as his choice of consonances and dissonances in 19-tone temperament. But, as with his choice of consonances in 19-tone temperament, many of his ideas stand up surprisingly well under subjective evaluation. If Das Relativitätsprinzip has not proved the revolutionary manifesto its author intended it to be, it is still an interesting document.

CHAPTER 11

JOSEPH YASSER

Joseph Yasser's major work, A Theory of Evolving Tonality, is the only one of the large writings advocating 19-tone temperament to have been published in the United States. It is by far the most substantial work on the subject to have been published in the English language. For these reasons it is far better known in this country than the other important writings on the subject, and many of its readers may be under the illusion that the arguments Yasser offers are the standard or even the only ones used in support of 19-tone temperament. Actually, even within its highly specialized area, Yasser's work represents a highly individual viewpoint which is independent and perhaps almost oblivious of the principal lines of theoretical development. Where Kornerup and Ariel approach 19-tone temperament on an acoustic-harmonic basis, Yasser is far more concerned with the socio-melodic aspects. Where the search for a consistent acoustical system with modulational possibilities combined with the purest possible intonation for chords leads Kornerup to his "Golden System" and Ariel to his universal just intonation, concern about the modality of scales leads Yasser to what he calls the Supra Diatonic Scale. To Yasser the ear perceives intervals not through absolute acoustic value but through their position in the

scale.[1] If Ariel, with his emphasis upon harmony, might be loosely considered a follower of Rameau, Yasser might similarly be considered to be in the camp of Fetis.

TONALITY, MODALITY, AND THE PRINCIPLE OF DIATONICITY

In Yasser's view, the real basis for the perception of tonality in music is in an abstract principle which we might call diatonicity. Diatonicity results when the actual scale used in a piece of music contains inequalities in its intervals. These inequalities result from the absence from the scale of certain tones which are nonetheless present in the complete system from which the scale is taken. "Diatonic," as we know it, refers to a seven-tone, unequal scale within a closed twelve-tone system. Yasser holds that the abstract principle of diatonicity is essential to the spontaneity and coherence of musical expression, but that the specific manifestion, the seven-tone scale in a twelve-tone field, is only one of many possible systems embodying the principle of diatonicity. If the resources of a specified diatonic system are exhausted, a new system may be evolved which supplies new tonal resources. Such a new system should, according to Yasser, continue to embody the abstract principle of diatonicity.

Yasser is particularly concerned with the course the

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[1] Theory of an Evolving Tonality, page 56.

evolution of such a new system should take. Yasser examines musical systems more primitive than ours and traces the probable evolution of our diatonic system from them. He concludes that musical systems evolve by a procedure wherein the entire gamut of tones of the earlier system becomes the functional scale of the more advanced system. This later system in turn acquires new auxiliary tones in the number and approximate pattern of the earlier functional scale.

This evolutionary process Yasser demonstrates most thoroughly by examining the relationship between the common-practice diatonic system and the pentatonic system which is alleged to be its predecessor. He points out that in most pentatonic systems there were generally two additional tones, "pien-tones" in the usage of the Chinese, which Yasser refers to as "auxiliary tones" in the complete system, making the pentatonic scale part of a 7-tone system.[2] The evolutionary process develops from a natural tendency to use the auxiliary tones of a musical system more and more freely until they begin to function as scale tones. At some point in the process the need for new auxiliary tones and for a modal shape for the increased scale (to be created by making unequal its intervals) makes itself felt.

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[2]Yasser points out that just as Western music has developed 12-tone equal temperament to facilitate the modulation of its 7-tone scale, Siamese pentatonic music is drawn from a 7-tone field very closely approximating equal temperament.

In Yasser's view the 14th Century Ars Nova was the period in which Western practice reached this point in departing from the pentatonic scale.[3] Today, according to Yasser, a similar process on a more advanced level is called for.

In the diatonic system are five auxiliary tones, equal in number and approximate contour to those of the older pentatonic scale. In like manner, Yasser postulates, the next system will contain seven auxiliary tones equal in approximate contour to the diatonic scale. The twelve tones of our present system will become the functional scale of the next system, bringing the total number of tones in the new system to 19. This system, which Yasser refers to as "12 plus 7," he calls the Supra-Diatonic system.

Yasser postulates universality to the method of growth illustrated above. According to his theory of evolution, tone systems must expand from member to member of the Fibonacci series (2, 5, 7,) 12, 19, 31, 50, etc. This series is identical with Kornerup's although the premises on which the two men derive their series are markedly different.[4]

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[3] Tonality and Atonality as Synthesized by Supra-Tonality, in Modern Music, supplement to issue of Nov.-Dec., 1930, page 347.

[4] Kornerup sees desirable acoustic principles in the Fibonacci principle itself. He makes the Golden Cut, which is the limit of the ratio of $\frac{n}{n+1}$ in the Fibonacci series, the basis for acoustic valuation of all intervals. Kornerup sees in the series 12, 19, 31, 50, acoustic superiority on the basis of characteristics inherent in the series as a

THE VARIABILITY OF JUST INTONATION

Having established primarily that a 19-tone system will arrive by the evolution of the scale, Yasser is nonetheless concerned about the acoustical and harmonic basis for such a system. He acknowledges that there should be a just intonation for the supra-diatonic scale, just as it is generally recognized that a just intonation exists for the diatonic scale. He asserts, however, that such an intonation will have harmonic value only; that the artist and not the scientist will determine the melodic intonation of 19-tone music.

To Yasser, the problem of determining the proper just intonation for the 19-tone system is far more difficult than it was for Ariel who simply subdivided the larger intervals of the just 12-tone system. Yasser is bound by his view that the origin of music is in its scale and by his sociologically oriented reasoning method whereby future development is postulated on the social events of the past. He holds that there is no absolute just intonation, even though he readily concedes that the harmonic series is involved in all just intonations.[5] "The harmonic series,"

Fibonacci series. Yasser, who considers psychological conditioning far more important than acoustics, postulates the system on the grounds of empirically demonstrated tendencies in social evolution.

[5] "Just intonation is subject to the acoustical laws of the natural harmonic series but in no specified form." A Theory of Evolving Tonality, page 177.

explains Yasser, "affects the scale through the medium of consonant chords."[6] However, the determinant of consonance in the chords is, once again, the scale. "Musical intervals are perceived by the human ear, not through their absolute acoustic value (or, at least, not always so) but through their relative position in one scale or another."[7] And so, in the quest for the consonant chords which will determine the just intonation of the 19-tone system, we are brought back to the scale.

THE DISSONANT SECOND AND THE CONSONANT THIRD

Yasser's formula for the determination of consonance and dissonance from the scale is the essence of simplicity. In any of Yasser's systems, dissonance is the property of consecutive members of the functional scale, while consonance is the property of alternate members of the scale. As the number of tones in a functional scale increases, it can be seen that the size of both the consonant and the dissonant intervals decreases, with the dissonant intervals always the smaller ones. Since Yasser chooses to name the intervals in each system ("second," "third," "fourth," etc.) according to the number of functional scale degrees involved, his seconds are always dissonant and his thirds are always

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[6] Ibid., page 179.

[7] Ibid., pages 56-57.

consonant though they vary in size from system to system. The "Infra-thirds" of pentatonic music are the size we consider to be a perfect fourth. The "Supra-thirds" of Yasser's 19-tone system are 3/19 and 4/19, respectively. All, according to Yasser, are consonances.[8]

The basic consonant chord of any tonal system is produced by superimposing as many "thirds" as will "fit" into an octave without leaving the inversion of a "second" as a remainder. In our diatonic music this is the triad, since the seventh chord leaves a dissonant remainder. For the same reason, the consonant chord in pentatonic music is the simple diad. A third tone can not but leave a dissonant "Infra-second" as a remainder. But in 19-tone music, employing 12 tones in the functional scale, it is possible to form a hexad out of superimposed "Supra-thirds," such that no consecutive scale members are present in the sonority. This hexad is, to Yasser, the basic consonance of his Supra-Diatonic System.

All intervals produced by the superposition of supra-

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[8]Yasser expatiates at considerable length upon the consonant and dissonant intervals in each of the three systems he is considering. He is careful to show, for example, that Pythagorean intonation is the just intonation of Pentatonic music (this corresponds with its almost universal usage prior to the early Renaissance) because the intervals 5:4 and 6:5 are not the pure infra-thirds but are smaller intervals in the system, while 4:3 is the perfect infra-third. The thirds of the diatonic system are, of course, precisely what we know today as thirds, are consonant, and require the use of the 5th partial. The supra-thirds will be smaller and will be based on higher partials ...see below.

thirds Yasser labels consonant, and all intervals not produced by this process he calls dissonant. This process leads to most interesting and questionable results. The consonances are 3/19, 4/19, 6/19, 7/19, 9/19, and their inversions. Dissonant, besides the supra-seconds 1/19 and 2/19 (and their inversions), are 5/19 and 8/19 and their inversions, including 11/19, the perfect fifth! The two fundamental consonances of the system according to Ariel are thus treated as dissonances by Yasser. In a single stroke Yasser thereby eliminates the interval which has played the greatest role in the evolution of musical systems (not counting the octave), and the interval in which the small-number ratio and its equivalent in 19-tone temperament are in closest correspondence.

Having declared 11/19 and 5/19 to be dissonances, Yasser concludes that the small-number ratios 3:2 and 6:5 cannot function as consonances and therefore should be eliminated from supra-diatonic music altogether. The 3rd partial, a factor common to both of the rejected intervals, is therefore eliminated from Yasser's theoretical basis for the system. Yasser goes even further, suggesting that in constructing supra-diatonic instruments the builders consider eliminating the 3rd partial as far as possible from the <u>timbre</u> of the tones (as present-day instrument makers

are sometimes said to do with the 7th partial).[9]

Yasser's consonant hexads consist of the six lowest partials which are possible without any octave replicas or any 3rd partials. These are the 1st, 5th, 7th, 9th (1), 11th, and 13th partials. Condensed into the octave 8-16, the hexad contains the ratios 8:9:10:11:13:14:(16). The twelve-tone supra-diatonic scale consists of two such hexads, the roots of which are 11/19 apart. Again Yasser rejects the interval 3:2, although he admits that it would be the best interval to use in separating the hexads if the goal were simplicity of ratios for the combined system. The interval 11/19 must, according to Yasser, be treated as a dissonance. Therefore, Yasser separates the hexad roots by the interval 182:121, larger than 3:2 by almost five cents.[10] The two hexads result in a twelve-tone scale with very complex numerical relationships, and considerable discrepancies from the nearest possible intervals in 19-tone temperament.[11]

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[9] Op. cit., page 285. Tillman H. Schafer and James Piehl, in constructing a 19-tone instrument actually followed this mandate of Yasser's, Schafer reporting in a letter to Yasser an instrument whose sounds did not include the 3rd or 6th partials to any significant extent.

[10] That this is, Yasser's logic notwithstanding, a most difficult doctrine to accept, hardly needs to be stated here. By Yasser's own frequently stated viewpoint, the ear does not hesitate to render "corrections" for intervals whose deviation greatly exceeds 5 cents. Furthermore, Yasser's just dissonant interval, 182:121, is farther from the tempered interval 11/19 than either is from the perfect fifth, 3:2. Thus, to equate 11/19 with 182:121 while ignoring the intermediate interval 3:2 represents a theoretical juggling act of no mean proportions.

EQUAL TEMPERAMENT

Since Yasser uses his elaborate and questionable scheme for just intonation merely as a basis for recommending the practice of 19-tone equal temperament, it is fortunately possible to consider his ends without accepting his means. Yasser is particularly interested in the phenomenon of equal temperament and its role in the evolution of musical systems. His conclusions are indicative of his preference for empirical and sociological data as the basis for aesthetic doctrine.

Toward 12-tone temperament Yasser is, unlike Ariel, hardly an implacable enemy. Its widespread use alone affords grounds for accepting its validity. The phenomenon of equal temperament in widespread musical practice shows, according to Yasser, that "The human ear adjusts itself with comparative ease to artificial intonations even if their deviations from Just Intonation are very perceptible."[12] Yasser adds that "Equal temperament in no way impairs the inherent qualities of a scale . . . providing its (the scale's) most characteristic feature--the chain of specifically alternating, if somewhat acoustically changed, whole steps and half steps--is preserved."[13] Yasser goes a step farther, to the

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[11] This aspect of Yasser's theory is considered and evaluated at some length in Chapter 12, beginning with the chart on page 314.

[12] Op. cit., page 55.

[13] Ibid., page 57.

assertion that deviations from just intonation might in fact be a positive artistic factor in music.

However, equal temperament is of relative rather than absolute advantage. It is valuable in that it facilitates modulation within a system containing more tones than are used in the scale. It is, however, an unfortunate hindrance in that it interferes with evolution. This it does, because the next scale, containing the same number of tones as the previous complete system, will arrange these tones on an unequal basis. As illustration, Yasser suggests that it would be most difficult for the Siamese, with their 7-tone equal temperament, to imagine the intonation of our diatonic scale, so different is the tuning. Similarly, he concludes, it is difficult for us to comprehend the most desirable intonation for the 12-tone scale. Our atonal music represents only a shadow of the best possible music using the 12-tone scale, much as Siamese experiments involving the use of all 7-tones of their system could hardly begin to realize the aesthetic possibilities of the Western 7-tone diatonic scale. In this argument, Yasser returns to his much-repeated premise that the most important ingredient of a musical scale is a modal physiognomy, i.e., a pattern of larger and smaller intervals the differentiation of which provides the structural basis for the comprehension of the music and for the development of subtlety of expression.

As mentioned above in Chapter 3, Yasser devotes

considerable attention to what he regards as evidence from musical practice that the next stage in the evolution of tonal systems is near at hand. He cites octave displacements, whole-tone music, the harmonic concepts of Scriabine, and the phenomena of polytonality and dodecaphony as evidence in support of his assertion (for details see above, pages 129-133). Nearly 30 years later, events have not borne out Yasser's premise that such practices as cited above would lead to a major change in our concepts of intonation. However, it is worth noting, that except for dodecaphony (which was least stressed among the above as indicative of a shift toward a 19-tone system) the practices Yasser cites have all but proved untenable in the long run with 12-tone temperament.

THE INFLUENCE OF YASSER'S THEORY

Yasser's theory of the evolution of tonal systems has influenced several fields of musical endeavor, through both his own further activities and those of others who have accepted all or part of his doctrine.

Yasser himself has worked at considerable length on the analysis of what he calls infra-diatonic music, especially of the Gregorian chant. His views on the suitable harmonization for Gregorian or Chinese music, based entirely on his theory of Infra-diatonic harmony, correspond remarkably closely to one's impressions of such Oriental polyphony as exists and to the practice of Organum.

Several writers in the field which might broadly be called Ethno-musicology have been attracted to Yasser's views as an approximate basis for studying the history of differences between one musical society and another. Curt Blaukopf, in particular,[14] has cited Yasser's theory as a valuable breakthrough toward the understanding of the bases for the various musical systems of the different peoples of the world.

Within the more specific field of 19-tone temperament, Yasser's work led directly to the experiments of Tillman Schafer and James Piehl, and to the recognition of multiple division as a major challenge by such leading acousticians as Robert W. Young.[15] Schafer's thesis[16] combined a summary of Yasser's writings with research on methods for constructing electronic instruments embodying 19-tone temperament. Schafer and Piehl have built several 19-tone instruments together, several of which still survive in Denver. Schafer presented a report to the

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[14] Blaukopf is the author of Musiksoziologie, 1947, a work of some considerable prestige in the field of Ethno-Musicology. His acknowledgment of Yasser's influence demonstrates that Yasser's writings have attracted attention in Europe as well as the United States.

[15] "Some Problems of Postwar Musical Acoustics," in the Acoustical Society Journal of October 1944. Young lists the ten most challenging problems for musical acoustics and lists the testing of Yasser's theories as one of them.

[16] The Music of Tomorrow: The Supra-Diatonic Scale, A New Concept of Timbre, The Electronic Musical Instrument, an unpublished thesis presented to the Division of Mathematics and Natural Science, June 1941, Mills College.

Acoustical Society of America in 1947 in which recordings of music by Scriabin and others were played. Schafer reports in a letter to Yasser the favorable impression made by a Scriabin sonata when played in 19-tone temperament.[17]

Yasser appears to have been the principal influence on McClure in England, who built a 19-tone harmonium. According to Schafer, who maintained a correspondence with McClure (now deceased), McClure was not altogether satisfied with the results and finally decided that the fifths and thirds were too small.[18] According to Schafer, McClure preferred ¼-comma meantone temperament as a compromise. One can infer that McClure, if he were alive today, might be a disciple of Fokker's 31-tone school.

Yasser has maintained a record of his correspondences which has enabled some contact to be retained among American theorists interested in the possibilities of 19-tone temperament. The group has been small but distinguished, representing most of the great regions of the country, with scientists and musicians about evenly divided. If 19-tone temperament should develop into a substantial musical movement in this country, a major share of the credit will certainly belong to the preliminary work of Yasser.

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[17]Letter from Schafer to Joseph Yasser.

[18]Letter from Schafer to the author.

CHAPTER 12

AN ACOUSTICAL COMPARISON OF 19-TONE SYSTEMS

On the previous pages we have seen 19-tone temperament advocated by, among others, Opelt, Würschmidt, Ariel, Kornerup and Yasser. Each of the aforementioned has a different just scale in mind which he considers fittingly represented by 19-tone temperament. On the following page is Example 45, showing what each of these men considers the proper just intonation of the 19-tone scale. In addition is Perrett's 19-tone system, which he never intended to have represented by an equal temperament, but which is there for comparison. Several other possible ways of obtaining 19 tones in the octave are also shown.

In the case of Yasser's system, only 12 pitches are shown, since he calibrates his just intonation entirely on the 12-tone supra-diatonic scale which he singles out for use from within 19-tone temperament.

As Example 45 demonstrates, the systems of Opelt, Ariel and Würschmidt differ only in a few details from one another (Kowalenkow's system, incidentally, is identical with Opelt's). All are based on combinations of fifths and thirds, and it is only in the outer members of the system that there is disagreement among the three authors. Fokker, not a specific advocate of 19-tone temperament, offers a 19-tone system rather similar to one of Würschmidt's, as

		Wührschmidt				Temperament						New System		
	19-tones	Ariel	Opelt	1	2	Fokker	¼-comma	31-tone	Yasser	Kornerup	Perrett	Plain	Septimal	
D	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
D♯	63.2	70.7	70.8	70.7	70.7	70.7	76.0	77.4		73.5	85	70.7	71	68.0
E♭	126.3	111.7	133.2	111.7	133.2	111.7	117.1	116.1	92 W	118.9	134	133.2	119	135.9
E	189.5	182.3	203.9	203.9	203.9	182.3	193.2	193.5	166 E	192.4	204	182.3	204	181.1
E♯ or F♭	252.7	253.1	274.6	274.6	274.6	274.6	269.2	271.0		265.9	289	253.1	267	249
F	315.8	315.6	315.6	315.6	315.6	315.6	310.3	309.7	322 F	311.4	316	315.6	316	317.0
F♯	379.0	386.3	386.3	386.3	386.3	386.3	386.3	387.1		384.8	401	386.3	386	385
G♭	442.1	427.4	427.4	427.4	427.4	427.4	427.4	425.8	456 X	430.3	471	427.4	435	453.9 bad
G	498.0	498.0	498.0	498.0	498.0	498.0	503.4	503.2	526 G	503.8	520	498.0	498	498
G♯	568.4	568.7	568.7	568.7	568.7	568.7	579.5	580.6		577.3	583	568.7	582	566
A♭	631.6	631.3	631.3	631.3	631.3	609.2	620.5	619.4	584 Y	622.7	632	631.3	631	634.0
A	694.7	702.0	702.0	702.0	702.0	702.0	696.6	696.8	708 A	696.2	702	702.0	702	702
A♯	757.9	772.6	772.6	772.6	772.6	772.6	772.6	774.2		769.7	787	751.2	765	747.1
B♭	821.0	813.7	813.7	813.7	813.7	813.7	813.7	812.9	814 Z	816.2	814	813.7	814	815
B	884.2	884.4	884.4	884.4	884.4	884.4	889.7	890.3	874 B	888.6	899	884.4	884	883
B♯ or C♭	947.3	946.9	955.0	925.4	925.4	925.4	930.8	929.0		934.1	969	955.0	969	951.0
C	1010.5	1017.7	1017.7	996.1	996.1	996.1	1006.8	1006.5	1018 C	1007.6	1018	1017.7	1018	1018.9
C♯	1073.7	1088.3	1088.3	1088.3	1066.8	1066.8	1082.9	1083.9		1081.1	1103	1066.8	1088	1067.1
D♭	1136.8	1129.3	1129.3	1129.3	1129.3	1129.3	1124.0	1122.6	1164 V	1126.5	1130	1129.3	1137	1132.
A.D.		8.8	8.2	9.8	8.9	10.3	8.2	9.2	16.0: 14.5	6.6	14.2	5.7	8.4	
M.D.		9.5	9.5	11.7	9.5	16.3	16.3	17.9	25.7: 15.5	13.0	17.4	7.9	10.7	Δ68 Δ45

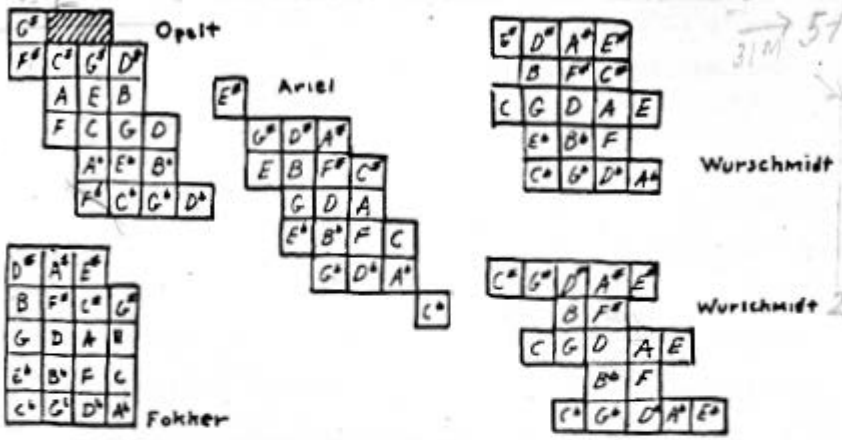
(1) Should be 457.0 to correspond with 2nd system layout on p 324

Example 45;

X See 2nd system p. 324 and p. 385

example 45 shows. Example 46 shows the five systems offered by those five authors in diagram form.

Example 46: Five 19-Tone Systems



Ariel's system uses the same constructing intervals as Opelt's (hence the same mean deviation), but its intervals are better when measured from a single central tone (hence Ariel's average deviation of intervals is somewhat smaller than Opelt's). The mean deviation of the two systems is 9.5 cents, a figure which compares favorably with the mean deviation for just intonation in the 12-tone system (over 20 cents). Würschmidt's second system is similarly constructed (of twelve instances of the interval 25:24, four of 128:125 and three of 648:625). The latter interval represents the difference between four 6:5's and

an octave, and is particularly close to 1/19. Würschmidt's first system involves four constructing intervals; the additional one is 135:128, representing three fifths and a third (less two octaves). The diesis, 128:125, the most out-of-tune of the intervals from the standpoint of equal temperament, is used six times instead of four, causing the mean deviation to increase to 11.7 cents.

As to the average deviation of the different intervals of the system, Ariel's involves the least due to maximal use of superimposed minor thirds from the central tone, D. The minor third being a diagonal, \ , it is clear from Example 46, that this interval is most evident in Ariel's system, less in Opelt's, still less in Würschmidt's second system, and least present in Würschmidt's first and Fokker's systems. The average deviation parallels the use of the minor third, as might be expected. Ariel's is smallest, 6.6 cents, followed by Opelt's, 8.2, Würschmidt's second and first, 8.9 and 9.8, and Fokker's 10.3. Of the just 19-tone systems proposed which are based on fifths and thirds, Ariel's is the closest to equal temperament.

As mentioned in several earlier chapters, 1/3-comma temperament is almost exactly equal to 19-tone temperament. The approximation is sufficiently close as to require no chart for 1/3-comma temperament in Example 45. As the alteration of the fifth is decreased from 1/3-comma, the deviation of the system from 19-tone equal temperament is

increased. In Aron's ¼-comma temperament, as shown in Example 45, the mean deviation is 16.3 cents, a high figure, owing to the disparity of the two constructing intervals, the 76-cent chromatic semitone of the meantone system and the 41-cents diesis. The average deviation of the intervals from a central tone in ¼-comma temperament is relatively small, however, owing to the regularity with which the large and small constructing intervals alternate in forming as equal as possible a scale.

The deviation of ¼-comma temperament within the framework of 19-tone temperament is interesting for two reasons. In the first place, it is often postulated that the earliest 19-tone instruments, those mentioned by Zarlino and Praetorius, were constructed for ¼-comma temperament.[1] The chart discloses that while the smallest intervals within the two systems are quite different, the larger ones, especially those having simple, diatonic relationships, are quite similar in the two systems. The second reason why the relationship of ¼-comma to 19-tone temperament is interesting

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[1]A comparison with Barbour's figures for the mean deviation of Aron's temperament in the 12-tone system is revealing. The mean deviation of ¼-comma temperament is 20.0 cents according to Barbour, a figure only slightly larger than that for the mean deviation in 19-tone temperament. These figures suggest that the advantages of a 19-tone instrument for the performance of ¼-comma temperament involve greater range within the cycle of fifths only, in the same way 18, or 20 pitches would. On the other hand Zarlino's 2/7-comma temperament would involve much smaller discrepancies on a 19-tone instrument. The system in Example 45 which most closely resembles Zarlino's is Kornerup's and the deviations are about the same.

is that many advocates of 19-tone temperament have suggested that 19-tone temperament is but a natural step in an evolution toward 31-tone temperament. As was shown in chapter 2, ¼-comma and 31-tone temperaments are nearly identical. There is sufficient difference however, to have justified the inclusion of both systems in Example 45. 31-tone temperament is a bit farther removed from 19- than is its relative, ¼-comma temperament.

The discrepancies between the intervals of 31- and 19-tone temperaments are large enough to suggest that should 19-tone equal temperament ever become as standard a tuning practice as 12-tone equal temperament is now, it may again be difficult to break through the acoustical barrier to the new temperament. Yet the discrepancies are not large enough to preclude the possibility of organic relationship between the two temperaments. The average deviation between the various intervals of the two systems (as calculated from a central tonic) compares favorably with most of those of the just systems offered as the basis for 19-tone temperament.

Since the twelve tones of Yasser's supra-diatonic scale use F as their tonic, the average deviation is calculated from that tone. The figure would be even larger if D were used as the starting point, but 16.0 is high enough to indicate that Yasser is not concerned with acoustical precision in the same way as Opelt and Ariel. The mean

deviation, calculated in this instance by comparing each of the 12 stepwise intervals with their unequal equivalents in 19-tone temperament, is likewise extremely high. Considering that Yasser emphasizes the modal contours of melody and the use, without octave displacement, of the 12-tone scale as a scale, the difference between that scale justly rendered and that scale as part of the temperament Yasser advocates is large indeed.

Of the two sets of figures involved in Example 45, the first and higher set is probably the more accurate as it assumes Yasser's "Y" to be the equivalent of Ab, an assumption clearly justified by Yasser's other letters and the equivalents they form. It is also justified by Yasser's other charts and by his assertion that the auxiliary tones will form a diatonic scale, an event which does not occur if "Y" is held to represent G♯. But so out-of-tune is "Y" (with respect to the Ab of 19-tone equal temperament) that it is much closer to G than to A both in Yasser's system and in the tempered tuning. The second set of figures assumes G♯ to be the equivalent to "Y" and shows a resulting improvement in the averages.

It is quite clear from the figures that Yasser's just supra-diatonic scale and the 19-tone temperament he advocates are two very different systems. Why, then, does he advocate them concurrently as one? It is possible, of course, that the temptation to build a musical system out of the hitherto

unexploited higher partials proved too much to resist and interfered with his reasoning. He is quite critical, however, of Schönberg's use of the upper partials to explain the 12-tone system because of the great discrepancies between the two.[2]

The explanation for Yasser's tolerance of such large discrepancies in his own system can probably be more accurately explained in light of his doctrine of melodic determination of a musical system. It is Yasser's basic conviction that consonance is psychological and not acoustical, and his conviction that it is based on the simultaneous use of alternate members of a musical scale, that lead him into the dilemma of the disparate systems. For, with the perfect fifth and minor third falling outside the realm of consonances, Yasser actually feels compelled to seek out-of-tune intervals in the just representations of 5/19 and 11/19. While other writers might regret that 19-tone temperament does not offer a sufficiently pure fifth or major third, Yasser's greatest concern is that 19-tone temperament might offer too pure a fifth and too pure a minor third. This forces him to turn to less suitable higher partials which involve him in large discrepancies between the just and tempered systems.

Yasser's just system should be judged with the

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[2] A.M.S. Journal, 1953: p. 56. Schönberg's views are set forth in Modern Music XI, p. 170.

realization that it is the most ambitious of the systems considered, and that it involves a revolutionary change in our aspect toward the consonances and dissonances of music. It is the first theory of 19-tone temperament which takes cognizance of Schönberg's dodecaphonic theories and attempts to construct a parallel alternative. Its many premises are rigorously applied throughout. If the deviations do indeed represent a defect, the defect is the result not of careless application but of a possibly faulty premise: that consonance within a musical system is melodically determined.

Kornerup's version of just tuning is based on his Golden System. It shares with 4-tone and 31-tone temperaments the characteristic of having only two constructing intervals for the 19-tone scale, and of having a great distinction between the mean deviation of the single steps (high) and the average deviation of intervals measured from a single central tone (low). In the latter category it compares well with Ariel's as the most suitable system for representation by 19-tone temperament. If the mean deviation is considered, however, it is inferior to the systems of thirds and fifths which possess three and four constructing intervals.

The golden system by its very nature is designed for that series of equal temperaments 12-19-31- etc. and is so constructed that its accuracy will improve greatly with each increase in the number of tones belonging to this

series. By Kornerup's chosen standards 19-tone temperament is a distinct improvement over 12-tone temperament. The only questionable aspect of his theory is that Kornerup selected his temperament eight years before he evolved the standards on which its defense is based (1922 vs. 1930).

Perrett's system alone among the 19-tone systems cited is intended as a tuning in itself, and not as a representation for 19-tone temperament. Perrett eschews the principle of equal temperament, and his tuning system is included only as a basis for comparison. Despite its having been conceived from a viewpoint opposed to equal temperament, its deviation from equal temperament on both counts is less than that of Yasser's system, which is conceived with equal temperament in mind.

THREE ADDITIONAL SYSTEMS PROPOSED

In addition to the numerous systems, derived from many sources and viewpoints, which have been presented and compared on the preceding pages, I should like to present three new interpretations of 19-tone temperament. I do not consider them to be better than the systems already shown, but each one shows a slightly different aspect of the particular properties of 19-tone temperament, and together they reinforce the conclusion that for any and every specific purpose a different just intonation can be found.

Example 47 shows diagrams of the three systems. The third is intended to be three dimensional and so uses points instead of squares. The first system is a structural absurdity. It consists simply of a system of 19 minor thirds. Taken together, the 19 minor thirds leave a comma of

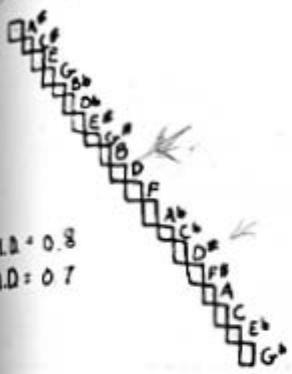
$$\frac{19,073,486,328,125}{19,042,491,875,328}, \quad = \frac{5^{14}}{3^{19} \times 2^{14}}$$

which is small enough (about 3 cents) but rather complex as a "just" ratio. The system is therefore a kind of Pythagorean system without the logical argument of the Pythagoreans that their one interval has an acoustical right to primacy. The system is presented, however, because it is the most accurate approximation of 19-tone temperament among the "just" systems. Just as Barbour concluded that the "best" just intonation among 12-tone systems was that which approached Pythagorean tuning, so the most accurate "just" representation of 19-tone temperament is that which approaches a single string of minor thirds. In each case the limiting system is based on the exclusive use of that interval which is least out of tune in the temperament.

The average and mean deviations of the first system are well under 1 cent. I offer this as an artificial system. Dodecaphony is, acoustically, an artificial system, and it is possible that in 19-tone temperament a meaningful if artificial system of music could be constructed based on cycles of minor thirds.

Example 47: 3 New 19-Tone Systems

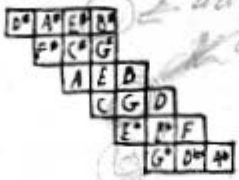
1st System



M.D. = 0.8
A.D. = 0.7

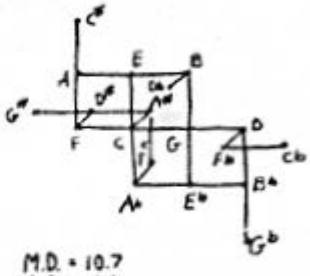
2nd System

added center



M.D. = 7.9
A.D. = 5.7

3rd System
Optimal



M.D. = 10.7
A.D. = 8.4

See opposite for rendering in 53-Tone

The second system is constructed, as was the first, with the primary purpose of limiting the average and mean deviations from equal temperament to the lowest possible figures. It is a compromise between the all-but-perfect-but-absurd first system, and the seriously proposed system with the lowest deviation (Ariel's). Its basic tone lattice consists of five minor thirds, with each group of six tones so created separated by one of two perfect fifths. This accounts for 18 tones, and one tone is added. The system requires four constructing intervals and is not symmetrical. Nevertheless, because of its heavy reliance on minor thirds, it possesses the great advantage of relative equality of intervals. The mean deviation of 7.9 cents is smaller than that for any of the seriously proposed systems, as is the

See also p 385

average deviation of its intervals from a more-or-less central tone, 5.7 cents. The system presents the closest approximation to 19-tone equal temperament in which just intervals can be perceived as the basis of at least the consonances.

For a third system, I should like to propose a tuning involving ratios using the seventh partial. The deviations will have to be larger in this instance, for this partial is approximately 21 cents out-of-tune in 19-tone temperament. Nevertheless, this does represent an improvement of 10 cents over 12-tone temperament, and it is possible that some musicians might prefer to make corrections of this magnitude when singing or playing stringed instruments in order to make use of the 7th partial as a consonance. Although Yasser and Perrett have built 19-tone systems with the 7th partial, this third system is offered to demonstrate that it is possible to reduce the deviations of their systems considerably without relinquishing this acoustical phenomenon.

The compactness which, at least to some extent, characterizes Fokker's 31-tone and 53-tone septimal systems is missing here.[3] The seventh partial, like a soft but

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[3]It is all but impossible to construct a plan for 19-tone temperament involving the seventh partial which is compact in the manner of Euler's genera, because the 7th partial is made from the bisection of the perfect fourth, and the two intervals are therefore too interdependent. In 19-tone temperament, Fifth = 2 sevenths minus octave, and seventh = two fifths plus two thirds. No parallelepiped is possible.

Example 48: Septimal Just Tuning: 19-Tone System

	Ratio	Cents	Deviation
C	1:1	00.0	0.0
C#	25:24	70.67	7.5
Db	15:14	119.5	6.8
D	9:8	203.9	14.4
D#	7:6	266.8	14.1
Eb	6:5	315.6	0.2
E	5:4	386.3	7.3
Fb	9:7	435.1	7.0
F	4:3	498.0	7.4
F#	7:5	582.5	14.1
Gb	36:25	631.3	0.3
G	3:2	702.0	7.4
G#	14:9	764.9	7.0
Ab	8:5	813.7	7.3
A	5:3	884.4	0.2
A#	7:4	968.8	21.5
Bb	9:5	1017.7	7.2
B	15:8	1088.3	14.6
Cb	27:14	1137.1	0.3
C	2:1	1200.	
		average	8.4

Constructing intervals:

21:20 , 84.4 cents (used 3 times)
25:24 70.7 cents (used 5 times)
28:27 62.9 cents (used 4 times)
36:35 48.8 cents (used 7 times)

Mean Deviation: 10.7 cents.

valuable metal, is only once used in its pure state, while six times it is used as an alloy. The other intervals used with it are such as will invariably improve the intonation. The system is offered merely as a sample of septimal tuning in a 19-tone just system which is so manipulated as to have a relatively small deviation from equal temperament.

THE IMPORTANCE OF JUST TUNINGS TO TEMPERAMENT

Since, with the exception of Perrett, all of the theorists proposed equal temperament for 19-tone systems, why, it may be asked, worry about the just basis for these systems? Even were the tempered tuning always fixed and unalterable, it would be of considerable use in evaluating the tempered system to know what kind of intervals it was designed to represent, and how accurately it did so. But, as we know from our 12-tone system, a temperament is only the starting point for intonation. One's interpretation of what the just system may be in any given key of a temperament will determine whether and how far he raises or lowers the tempered pitch when singing, playing a violin or trombone, or even, in certain instances, tuning a piano. That this question may have practical importance can be seen from the ambiguity which presently affects the intonation of the major third. The present tendency seems to be to extend the major thirds toward the ideal of Pythagorean tuning, although a minority of musicians continue to prefer just thirds. One of the obstacles which advocates of 19-tone temperament must overcome, if theirs is to become a system in general use, is the direct result of this tendency. Were the compensations in 12-tone temperament in the direction of just rather than Pythagorean thirds, the margin between the present system and 19-tone or any of the negative temperaments

would be smaller. In such an instance a transition from one system to the other would be much easier to contemplate.

Similarly, if 19-tone temperament is not to be regarded as the terminal tuning system of music (and none of its advocates have been so rash as to suggest it as such), then the just system toward which the tempered structure is allowed to lean might well influence the ease with which 19-tone temperament could yield to further evolution. If, as many suggest, 31-tone temperament is the logical successor to 19-tone temperament, then tunings resembling 31-tone temperament in Example 45 would appear to have an advantage over those which do not. Septimal tunings in 19-tone temperament would also have special relevance when considered with respect to possible evolutions toward 31-, owing to the excellent representation of the 7th partial in 31-tone temperament.

The determination of the most useful or the most ideal just system for 19-tone music may have to wait for a period of experimentation in practice. It is also possible that if 19-tone temperament develops it will do so with such serial practices as are used in present dodecaphony, and this will render just intonation a superfluous concept. Further acoustico-psychological research, especially on the 5th and 7th partials, would appear warranted, at least in eliminating some of the systems proposed.

CONSONANCE AND DISSONANCE IN 19-TONE TEMPERAMENT

Only two of the authors whose systems of 19-tone temperament have been presented have made studies having as an end a determination of the consonances and dissonances of the system. These authors are Ariel and Yasser, and their views clash on several of the intervals. Example 49 shows how Ariel and Yasser regard each of the nine basic intervals of 19-tone temperament (each considers the inversions of every interval to possess equivalent consonant values), together with the traditional concept of the nearest equivalent interval in 12-tone temperament (where a sufficiently equivalent interval exists).

As Example 49 shows, the three sources agree on only three intervals of the nine, two of them being the two smallest intervals, on which all are agreed that they form dissonances. The other agreement is on the major third, 6/19, which is considered a consonance. Yasser and Ariel further agree on the consonance of the major second and tritone, which represents a departure from traditional practice. This departure seems warranted, however, in view of contemporary developments in 12-tone music (it is somewhat more debatable in the case of the tritone than in the case of the major second).[4] The two authors disagree in

[4]Hindemith in particular among modern theorists continues to regard the tritone as the dissonance par excellence. I am inclined to agree with him and to disagree with both Ariel and Yasser as regards this interval when used in 19-tone temperament. See supplement, below p. 385.

Example 49: Consonance and Dissonance in 19-Tone Temperament

Interval	Ariel	Yasser	12-Tone equivalent
1/19	D	D	D
2/19	D	D	D
3/19	C	C	D (mild)
4/19	D	C	none
5/19	C	D	C
6/19	C	C	C
7/19	D	C	none
8/19	C	D	C
9/19	C	C	D

regard to the perfect fourth and the minor third, which Ariel regards as consonant, and the two intervals unparalleled in 12-tone temperament, which Yasser considers to be consonant. Clearly it is Yasser on whom the burden of proof must lie. For further discussion of this matter the reader is referred to the chapters on Ariel and Yasser and to the first supplement.

MISCELLANEOUS DATA AND CONJECTURES ABOUT 19-TONE TEMPERAMENT

Augusto Novarro has published a complete list of interval ratios for 19-tone temperament which may prove useful to anyone seeking to build a 19-tone scale on a monochord or on a multi-stringed instrument built on monochordal principles. The ratios are as follows:[5]

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[5]Novarro, op. cit., p. 161.

1.0000	1.4403
1.0372	1.4938
1.0757	1.5493
1.1157	1.6068
1.1571	1.6665
1.2001	1.7285
1.2447	1.7927
1.2909	1.8593
1.3389	1.9284
1.3887	2.0000

Novarro has completed similar tables for nearly every other possible equal temperament, and the reader is referred to his excellent book for the ratios of other temperaments.

A rather unusual 19-tone scale has been proposed by R. M. Frye and Esther Tipple,[6] consisting of the following degrees of the 53-tone tempered system: 1, 4, 6, 9, 13, 15, 18, 20, 23, 26, 28, 31, 35, 37, 40, 43, 45, 59, 52. It is not the closest approximation to 19-tone equal temperament which is possible in the 53-tone system, since it contains intervals of 2, 3, and 4 units of the latter system. It would be possible to build a more equal 19-tone scale using only 2 and 3-unit intervals. Assuming the tone numbered 1 to be the proposed tonic of the scale, the best fifth would be 32, which is missing. Complete diatonic scales can be built within the 19-tone framework beginning only on tones 6, 18, 23 and 40. Unfortunately there is no detailed explanation of the aesthetic behind Frye and Tipple's choice of a 19-tone

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[6] Frye, R. M., and E. W. Tipple, Data Supplement to A Graphic Introduction to the Harmon.

Typo - not consulted. LAH 7/20/78

scale drawn from 53-tone temperament.

Among the scales occasionally mentioned as a desirable aspect of multiple division systems is the quasi-equal pentatonic. Among the seriously proposed multiple divisions, only 50-tone temperament has a real equal-tempered pentatonic scale within it. But Wyschnegradsky mentions with some enthusiasm the quasi-equal 5-tone scale as one of the unique and attractive scales possible in 24-tone temperament. A scale quite similar to that which Wyschnegradsky cites is available within 19-tone temperament as is shown by Example 50.

Example 50: Three Pentatonic Scales

5-tone equal.	Approx. in 24-tt	Approx. in 19-tt
0	0	0
240	250	253
480	500	505
720	700	695
960	950	947
1200	1200	1200

Both 19- and 24-tone temperaments offer the composer a bridge between the Western diatonic scale and practices of the <u>salendro</u> <u>gamelan</u> of Indonesia. It is possible that an eventual criterion for a suitable multiple division system might be that it represent a common denominator between the scales of different musical cultures.

CRITICISMS OF NINETEEN-TONE TEMPERAMENT

Among the writers interested in multiple division, several, including Partch, Barbour, and Fokker, are quite critical of 19-tone temperament. Partch stresses the weakness of the perfect fifths (nearly 7.3 cents in error). Of the interval 694.7 cents, Partch says,[7] "It somehow borders on the absurd to be asked to consider it as the strongest consonance in music next to 2:1." Novarro is likewise hesitant about endorsing the fifths of 19-tone temperament. Opelt and Handschin admit to uncertainties about the fifths but conclude that the error is not great enough to disturb the sense of consonance.

It is the thirds, rather than the fifths, which draw the strongest criticism from Barbour.[8] "To modern ears, accustomed to the sharp major thirds of equal temperament, the thirds of 379 cents, 1/3-comma flat, would sound insipid in the extreme."

While Opelt, Würschmidt and Ariel advocate 19-tone temperament based entirely on thirds and fifths, Yasser, eloquent advocate of 19-tone temperament that he is, ques-

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[7]Partch, op. cit., pp. 304-5. In spite of his fundamental disagreement with 19-tone temperament, however, Partch remains sympathetic to its being tried. "Ambitious composers have, in Yasser's book, a plan that has been widely approved, and it can be had for the asking. From the standpoint of slow evolution of existing materials, ideas, and thought habits, Yasser's proposal is . . . credible and adaptable."

[8]Tuning and Temperament, p. 116.

tions the value of the temperament in dealing with traditional diatonic music which is, after all, based on thirds and fifths. "With reference to the diatonic system, an impartial opinion would probably deny much, if any, harmonic advantage in the 19-tone over the 12-tone temperament, while the mechanical disadvantage of the increased number of frequencies to the octave is unquestionable, in this instance."[9]

Fokker, who treats 19-tone temperament at some length in his book, Rekenkundige Bespiegeling der Muziek, takes issue with the representations of the harmonic seventh, the tritone, and the two kinds of semitone. The seventh partial is a whole comma too small, he asserts, the tritone (which he evidently thinks should be based on 7:5) is 2/3 of a comma too small, and the two kinds of semitone are too different in size, the one being twice the size of the other.[10] Fokker also shows his awareness of the inaccuracy of the representation of many of the higher partials (as does Partch) and includes the following chart of the sizes and errors of various intervals involving small-number ratios. The intervals are measured in hundredths of a unit of the temperament, that is to say 1/1900^{ths} of an octave.

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[9] Letter from Yasser to John Redfield, May 1, 1933.

[10] "De harmonische zevende is een heele komma mis, wanneer hij hierin gespeeld wordt, te klein. De tritonus valt 2/3 komma te klein uit. Het verschil tussen grote en kleine halve toon wordt in deze stemming overdreven, de ene wordt twee maal zo groot als de andere." Rekenkundige Bespiegeling der Muziek, p. 185.

Example 51: Fokker's Chart of Small-Number Ratios

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	20
2	0	1112														
3	-12	0	788	1400												
4		12	0	611		1535										
5		00	-11	0	500	924	1290	1610								
6				00	0	424				1660						
7			-35	-24	-24	0	364	687	975	1230	1470	1700				
8				10		36	0	323		873		1330		1720		
9				-10		13	-23	0	288	551		1010	1210		1575	
10						25		12	0	263		622				
11					+40	-30	-27	49	37	0	237	459	661	848	1030	1640
12						30				-37	0	221				
13						00	-30	-10	-22	41	21	0	203	389	566	1180
14								-10		39		-03	0	187		
15							-20			-48		11	13	0	177	
16								15		-30		34		23	0	

In the face of these valid criticisms it is difficult to claim for 19-tone temperament anything better than the compromise status which is traditionally claimed for 12-tone temperament, not that this is not enough. Leigh Gerdine, in writing to Yasser, lists the following as assets of 19-tone temperament: it includes within its framework the "good" music of the past; it allows for indefinite extension; it affirms the validity of the natural harmonic system; it makes possible a future in vocal as well as instrumental music. It seems to me that with a slight reservation on the first of Gerdine's four points, they make the most effective and valid case for 19-tone temperament that can be made with words. The writing of and familiarization with 19-tone music would have to be the next step in the dialogue.

CHAPTER 13

GENERAL DISCUSSION OF METHODS FOR CHOOSING A SYSTEM OF MULTIPLE DIVISION AND SOME CONCLUSIONS

Elliott Carter, in speaking of the problem of diversity within contemporary music,[1] makes a point which is completely valid when applied to multiple division. Carter states that the composer "is free . . . to do what he likes . . . but . . . whatever he chooses to do, radical or conservative, his music will further divide into small sub-groups the handful of people who will listen to contemporary music at all. . . . While diversity of opinion is much to be welcomed, where so little support exists such decimation of interest, one hesitates regretfully to conclude, can lead to cancelling of efforts and ultimately to their negation."

In what field could this be more true than multiple division, where, in spite of the very limited public attention and support which is accorded, the few active composers and theorists work in seemingly contradictory directions? With such limited support is there room for just tunings and equal temperaments, for systems using and not using the upper partials, for systems using quarter-tones and systems not using quarter-tones, for positive systems and negative systems, for systems using a few more

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[1] Musical Quarterly, April 1960, pp. 200-1.

than twelve tones and systems using many more than twelve tones, for systems employing modifications of current instruments and systems demanding new instruments? In such a broad field, is there a system or group of related systems whose superior merits could gain general approval if they were generally known? Would such a system or group of related systems have any chance of becoming generally known? What kinds of research projects or attempts at composition or instrument construction or modification might help to bring about that state of affairs wherein musicians of all kinds might know what they need to know in determining the next general system of intonation, if indeed there is to be a next system of intonation? It is the purpose of this concluding chapter to deal (mostly indirectly) with questions of this kind. The method to be employed will be to try to associate specific acoustical and aesthetic premises with the systems of multiple division which best meet these premises and to attempt to identify accurate processes through which such associations can be made. I shall not try to persuade anyone to a specific point of view since it is my conviction that there is not yet enough experimental evidence to warrant an attempt to close ranks behind any one system of multiple division.

There are four fundamental questions of aesthetic values which, it seems to me, must be answered by each in-

dividual interested in multiple division and concerned with finding that system which will most appeal to him. With his answer to each question he can alter the substance of each of the succeeding questions.

Question One: Is there any acoustical validity to small-number ratios as the basis for a musical system? If so, which small-number ratios are to be included unconditionally among those with acoustical validity for a musical system, which ones conditionally, and under what conditions? This is probably the most fundamental question facing music today, and it is probably farther from having a generally accepted answer today than at any previous time in the history of the art. This question was taken up interval by interval in the first chapter; a summary of the present views of major composers and theorists is as follows: The Cologne school and other groups representing similar views apparently hold that no small-number ratio possesses any special intrinsic musical value. The rest of the musical world seems to remain convinced of the value of at least one small-number ratio, the octave, although several proposals have been made to modify its size slightly.

The serial dodecaphonists reject all small-number ratios except the octave. As serial dodecaphony possesses a substantial following among composers and musicians, their views on small-number ratios can not be rejected outright. However, since serial dodecaphonists, of all groups of

knowledgeable musicians, tend to be the least inclined toward the altering of the present tuning system,[2] their views can be dismissed for the purpose of the study of multiple division as not germane to the subject. Except for the serial dodecaphonists, all musicians appear to consider 3:2 and 4:3 to have fundamental importance. The fifths of many non-Western musical cultures are considerably distorted, sufficiently so to cause debate as to whether the fifth is or is not an element of all musical systems. Let it suffice that it is an essential consonant element in all Western musical systems, past and present, except those cited above.

At this point anything approaching unanimity ceases completely. The remaining three questions will be framed in such a way as to assume an affirmative answer to question one as it involves ratios of the numbers 1 through 4, since the advocates of fixed systems of multiple division appear to be completely in accord on this point (Yasser is the only possible exception).

Question two: Is it preferable to have equal temperaments, or is it preferable to use exact small-number ratios (besides the octave)? If question one, above, is the most fundamental question dividing musicians today, question two is the most fundamental question dividing advocates of

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[2]See, for example, the quotation from Babbitt, above, at the beginning of Chapter 3.

multiple division of the octave (who tend to find themselves in agreement on a fundamental level regarding question one). If this study has tended to stress the work of the advocates of equal temperament it is because it is easier to analyze and explain their systems and to relate them to one another. Until now, with the exception of several of the Pythagorean systems, systems of multiple division not involving a basis in equal temperament have been strictly individual affairs, leading to no formation of schools and to no accumulation of works by more than one composer. The emphasis in this study should not be construed as indicating any negative judgment upon the works of the advocates of untempered systems. Perrett, Partch, Groven, and their predecessors have made important contributions to musical theory and practice.

Question two can also be put as follows: are approximations valid as representatives of small-number ratios? If this can be answered affirmatively a second question can also be asked: are the advantages of equal temperament sufficient enough to warrant the substitution of approximations of pure intervals for the real thing? The formation of all melodic and harmonic combinations possible within a system of any and all tones of the system is, of course, possible only in an equal temperament. Music, tonal and atonal alike, has relied on this resource for so long that the retraction of such a possibility might seem

to many an intolerable retrogression. In any case, as Partch himself recognizes, the discontinuation of the principle of equal temperament would represent a far more revolutionary change in the fabric of music than the change from one temperament to another.[3] The untempered systems based on Pythagorean tunings must be excepted from this generalization, because as mono-intervalic systems they resemble equal tempered systems, and because their basic interval, the fifth, is so closely approximated in 12-tone temperament.

Among the untempered systems of multiple division which have been used, Perrett's 19-, Partch's 43-, and the Pythagoreanists' 24- and 36-tone systems have been noted in particular. To this might be added a 41-tone untempered system which Louis Greensway apparently employed in at least one work performed during World War II.[4] It might be noted that every one of the numbers cited above has been proposed by others as the basis for an equal temperament. It is rather curious that there is a numerical association between systems based on such completely differing concepts of proper intonation. A glance at the chart in which Partch's and Sauveur's 43-tone systems are compared (Example 39) shows how great the differences between them are.

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[3]Partch, op. cit., pp. 305-6.

[4]The only data I have on this is an announcement of a coming concert of 41-tone untempered music by Greensway in Philadelphia, March 13, with year not given.

Perrett's component intervals vary from 27 to 85 cents, and his system can hardly be said to resemble 19-tone temperament.

Nevertheless, it may not be entirely a coincidence that their systems employ the same number of tones per octave as important equal-tempered systems. Both Partch and Perrett "fill out" their systems to correct the largest variations in size among the component intervals. It is by this process that each reaches his final number of tones. Partch adds 14 to what was originally a 29-tone system; Perrett adds 5 to what was initially a 14-tone system. In a sense these men are, by this process, attempting "from the other side" to find the same equilibrium between perfection and approximation which the advocates of equal temperament are seeking. While the advocates of equal temperament insist on perfect equalization within the system and seek to approximate pure intervals as closely as possible, their counterparts insist on the perfection of the pure intervals and seek to approximate equality between the intervals. But these two processes tend naturally to lead to the same number of tones, because departure and error are complementary aspects of the same phenomenon. If with a given number of tones an equal temperament can be constructed with relatively small error in the approximation of the pure intervals deemed important, these same pure intervals can be used in an untempered

system with equally small departures from the intervals of the equivalent tempered system. It should finally be noted that the number of tones present in Partch's and Perrett's systems before the additions of tones needed to equalize the intervals, represent far less desirable numbers for equal-tempered systems than the numbers of tones employed by the final systems.

To some extent, then, the problems involved in choosing the number of tones for an equal temperament apply also to choosing the best number of tones for an untempered system. Beyond this, very little can be said about the latter, as the possibilities are almost infinite. This is even evident in the vast number of "just" systems shown in the previous chapter as bases for 19-tone temperament. It would seem far more difficult to expect large numbers of theorists and composers ever to close ranks behind a single specific just system than behind a single equal tempered system which might continue to mean different things to different users. It is necessary, owing to the foregoing, to concentrate on the equal-tempered systems in attempting to find answers to the remaining questions, even though these answers may apply indirectly to untempered systems as well.

Question Three: What degree of approximation will be accepted as tolerable for each of the intervals present in a tuning system, and what methods can be used to

evaluate degrees of "satisfactoriness" in the approximation of various intervals within a tempered system? With untempered systems a similar question must be posed concerning the degree of difference in the size of the constructing intervals that will be tolerated.

It is with this question that the crux of the problem of choice within the field of multiple division, narrowed down to equal temperament, is first met. Answers have varied from a refusal to accept even an error of two cents in the perfect fifth to Barbour and Kuttner's statement that commatic errors are no longer disturbing.[5] If a musician is able to name which small-number ratios he deems essential to his system and the maximal error which he will tolerate in his system, he can be told the number of tones in the simplest equal-tempered system that will meet his demands. If, however, he wishes to use a graduated series of tolerances, whereby some intervals must be more closely in tune than others, the problem becomes more difficult.

Another complicating factor is the role of divergence in the sizes of the errors. A single system may contain particularly good approximations of some intervals to compensate for barely tolerable approximations of others. Theorists have debated over the proper method for evaluating

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[5]Introductory notes for <u>The Theory and Practice of Just Intonation</u>, col. 31.

① Squares

the relative error in different systems involving more than one approximated interval. A method favored by many is calculation by the sum of the squares of the discrepancies. The discrepancies are calculated in logarithmic units such as cents and the squares of each taken. The system having the lowest sum of the squares of the intervals of discrepancy is then called the best system. Fokker cites this method to support his advocacy of 31- and 53-tone temperaments. According to Fokker, M. F. K. Ligtenberg, whose figures he uses, used the squares of the intervals of discrepancy (calculated in units of 1/100 octave or 12 cents), rather than the intervals themselves, simply to nullify the mutually cancelling effect of positive and negative signs.[6] This could, however, be done quite arbitrarily simply by disregarding the signs. The use of squares of the logarithmic values of the intervals can produce results drastically different from the results obtained by the use of the logarithms of the interval ratios themselves. The use of squares tends to favor the systems involving relatively equal discrepancies over those whose average discrepancy may be smaller but which contain one somewhat larger discrepancy.

The use of squares leads to a judgment in which the largest error among the basic intervals of a system is

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[6] . . . since both positive and negative numbers possess positive squares. Fokker, La Gamme..., p. 156.

taken into greater account than the other errors, but not into exclusive account. This seems a logical and reasonable solution to the dilemma caused by intervals involving errors of different sizes. But it leaves unsolved problems arising from differences in the importance of the intervals considered. What if the largest discrepancy involves the least important of the intervals considered? The interval having the largest error may indeed be justifiably considered of special importance because of that error, but there may be other factors tending to emphasize the importance of other intervals.

It is possible to emphasize the perfect fifth, as many theorists have done. Possibly because of the conditioning effects of the long use of 12-tone temperament, many surveys have shown musicians to be far more sensitive to slight deviations in the intonation of the perfect fifth than to similar deviations in thirds or other intervals not as well represented in 12-tone temperament. With the more distant partials, one encounters conflicting theories as to whether it is more or less important that they be represented as accurately as the lower partials. Partch insists, for example, that the 7th and 11th partials create consonant effects sufficiently subtle to be spoiled by even small errors in intonation, while other theorists claim to see the seventh partial (and even the 11th partial) as present in the 12-tone tempered system.

The sum of squares method, as it has been applied to date, has shown no allowances for inequalities in the importance of the accuracy of the various intervals involved. Woolhouse, in 1835, used the method with three intervals, the perfect fifth, the major third, and the minor third. Fokker applied the same method to the perfect fifth, the major third, and the natural seventh. Wedell and Bertelson applied the same method, although Kornerup does not tell us what intervals they used.[7] Elsewhere they applied the sums of simple deficiencies without squares to 35 intervals within each system, involving one for each letter name with natural, flat, sharp, double-flat, and double-sharp.[8]

Below are the results of Ligtenberg's and Wedell-Bertelson's computations, as reported by Fokker and Kornerup, respectively.

Ligtenberg-Fokker
(in 1/100 octaves squared)

10-tone temperament	7.4663
12-tone temperament	8.1533
15-tone temperament	4.1802
17-tone temperament	3.8901
22-tone temperament	1.6447
31-tone temperament	0.2044
41-tone temperament	0.2682
46-tone temperament	0.3318
53-tone temperament	0.1646
56-tone temperament	0.3304

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[7]Kornerup, Acoustic Valuation of Intervals, p. 7. His citation is of articles by Wedell and Bertelson in Musik, Copenhagen, 1917 and 1919.

[8]Kornerup, Acoustic Methods of Work, p. 27.

Wedell & Bertelsen-Kornerup
(in cents, not squared)

12-tone temperament	1174 cents
19-tone temperament	458 cents
31-tone temperament	174 cents
50-tone temperament	67 cents
Pythagorean (12-tone)	1780 cents.

It is quite evident that Ligtenberg and Wedell are looking for two quite different things. Except for 12- and 31-, the temperaments deemed worthy of examination and listing are completely different. Agreement is reached only on (1) the excellence of 31-tone temperament which is held in high esteem by Wedell and Bertelson, Ligtenberg, and Kornerup and Fokker alike; and on (2) the particular weakness of 12-tone temperament, which Ligtenberg shows to have an even greater deficiency than 10-tone temperament, owing to the relatively good 7th partial in the latter system. Were Ligtenberg to evaluate 19-tone temperament by his method, the deficiency would be approximately 3.7 square centioctaves.

Question Four: Assuming that a means can be found to evaluate the various deficiencies of a temperament, what relative importance is to be given to the difficulty entailed by the number of tones employed? Musicians have offered answers radically at variance with one another. This element, which might be called the difficulty factor, is hardest of all to evaluate. Is the difficulty of 12-tone temperament close to the limit of the ability of human

hands to play and human ears to absorb? Does doubling the number of tones in a system double its difficulty, or does it come nearer to quadrupling the difficulty? Or, on the contrary, would it require a quadrupling of the number of tones to effect a doubling of the difficulty? These questions are very difficult even to attempt to answer without accumulated experimental data, and their documented answers would represent a major contribution to the study of the feasibility of many systems of multiple division. It is only when reliable information on this subject is available that it will be possible to make a reasonably valid decision on "how high to go" in the number of tones per octave to insure the maximum possible accuracy of intervals without overtaxing human artistic capacities.

In order to simplify this problem for the present, it will be useful to divide the field of multiple division into distinct areas of complexity. It is of little value to compare the acoustical merits of 19- and 53-tone temperaments, for example, because the difficulty factor is so different in the two cases. The boundaries between the areas of complexity must be arbitrary, of course, but I have tried to select them as logically as possible. Since we are used to thinking in a 12-tone pattern, I have selected twice 12, or 24, as the upper limit of the first area of complexity. Besides being a multiple of 12, 24 commends itself as an upper limit because above it there is

a void of advocated systems, none being mentioned until 29- and none being widely advocated until 31-tone temperament. The first and second areas of complexity are therefore clearly separated.

For an upper limit to the second area of complexity I have selected 36 for precisely the same reasons. 31-, 34-, and 36-tone temperaments are the principal systems in this group. Because of the special credentials of 53-tone temperament, I have extended the limits of the third area of complexity to include this system. While admitting the possible injustice to such systems as 41- and 43-tone temperaments by the extension of this category to 53, I feel justified as it represents less than a 50 percent increase over 36 tones whereas every other category represents an increase of 50 percent or more.

All systems involving more than 53 tones to the octave will, for practical purposes, be discounted by being considered as lying within a fourth area of complexity.

Within each of the areas the demands upon a system of temperament are properly quite different. Within the first area one might reasonably expect some acoustical improvement over 12-tone temperament in addition to the obvious increase in harmonic diversity rendered possible. Within the second area a considerable increase in the accuracy of represented intervals and in the number of partials involved might reasonably be expected. Within

the third area excellent accuracy and/or multiple partials would seem an altogether reasonable expectation.

METHODS OF CHOICE: TRIAL AND ERROR

Owing to the finite limits on the number of equal temperaments within the first groups, and to the extent of the explorations of these temperaments which have already been completed, it is quite possible to select one's desired temperament through elimination by trial and error. Following this procedure, it is probably best to begin by eliminating all equal temperaments in which the perfect fifth is not obtained with a sufficient degree of accuracy, then to repeat the process for each of the other essential intervals in the order of their importance. Owing to the different demands on the temperaments in each area of complexity, they are considered separately.

Area 1: The errors for the fifths in each of the equal temperaments in this area are shown in Example 52 in cents and in percentage of possible error.

To find the best means of evaluating the relative differences of the two kinds of error shown below again requires a value judgment on the role of the difficulty factor. Accepting a compromise view in order to have some basis for evaluation, I shall evaluate the intervals according to the simple product of the two factors which

Example 52: The Fifths in Area One

Number of tones	Size of fifth	Error-cents	Percent of possible error
12	700	2.0	04
13	738.5	36.5	79
14	685.7	16.3	38
15	720	18.0	45
16	675	27.0	72
17	705.9	3.9	11
18	733.3	31.3	94
19	694.7	7.3	22
20	720	18.0	60
21	685.7	16.3	57
22	709.1	7.1	26
23	678.3	23.7	91
24	700	2.0	08

I shall call the Combined Error Factor. The seven best temperaments in area one from the standpoint of their fifths only are:

System	Combined Error Factor
12	.08
24	.16
17	.43
19	1.61
22	1.85
14	6.19
15	8.10

One can draw the line where one chooses on the tolerability of the fifth. Probably the two most logical places are either somewhere between 17- and 19-tone temperaments for a strict limit, or between 22- and 14-tone temperaments for a loose limit. 14- and 15-tone temperaments seem out of the question if the approximation of the perfect

fifth is to be of any importance whatsoever. So are all other temperaments in area one.

The thirds for the five temperaments shown above to have discussable fifths are:

Number of tones	Size of major thirds	Error-cents	Error-percent
12	400	13.7	27
24	400	13.7	55
17	352.9	33.4	93
19	378.9	7.4	22
22	381.8	4.5	16

The consideration of the major thirds tends to invert the relative values shown for the fifths. 17-tone temperament would have to be eliminated from any further consideration if thirds are to be held of any importance whatsoever. The Combined Error Factor for the major thirds of the remaining four temperaments in area one is shown below.

Number of Tones	Combined Error Factor for Major Thirds
22	0.62
19	1.63
12	3.70
24	7.53

If we now consider the combined error factor of the perfect fifths and the major thirds, the result varies according to the process we employ. We can use the product of the errors to favor the single very small error. This

is a worthless method, however, since it is the larger rather than the smaller error that poses the greater danger to a tempered system. We can use the sum of the errors, or the sum of the squares of the errors. The latter process emphasizes the importance of the larger error.

We can weigh the two intervals unequally if we so desire. In the following chart, the combined error factors of the two intervals are placed in various combinations.

Example 53: Combined Errors of Fifths and Thirds

Method Used	12-	19-	22-	24
Product	0.3	2.6	1.2	1.2
Sum	3.8	3.2	2.5	7.7
Sum of squares (approximate)	14.0	5.3	3.8	56.0
Sum with fifth rated 2 to 1 over third	3.9	4.8	4.4	7.9
Sum of squares with fifth rated 2 to 1 over third	14.0	7.9	7.2	56.0
Sum with fifth rated 3 to 1 over third	4.0	6.5	6.2	8.0
Sum of squares with fifth rated 3 to 1 over third	14.0	10.5	10.6	56.0

According to the method used, 22-tone temperament is shown to be the best in some of the measurements, 12- in others, and 19- in yet another. 24-tone temperament runs last in all but one measurement.

The extremely favorable showing of 22-tone temperament in the statistics compiled above is dissipated con-

siderably if the minor third, 6:5 is also taken into account. The errors of the minor third in each of the remaining temperaments are as follows:

Number of tones	Size of minor third	Error-cents	Percent
12	200	15.6	31
19	315.8	0.2	1
22	327.3	11.7	41
24	300	15.6	62

The combined error factor for this interval is as follows:

12	4.84
19	0.00
22	4.80
24	9.68

The addition of the minor third, even with very slight weight, turns the results strongly in favor of 19-tone temperament. For example, considering a weighting of 4-2-1 involving the sums of the squares of the errors of fifth, major third and minor third respectively, the result proves to be:

12	19	22	24
51.3	15.8	37.5	202.0

The same weighting, using the simple sums of the combined error factor multiples produces the following values:

12	19	22	24
12.6	9.7	13.4	25.3

Consideration of the harmonic seventh in the first area of complexity is probably out of the question, as is demonstrated by the errors of the interval 7:4 in each of the four surviving temperaments in this group.

Number of tones	Size of seventh	Error-cents	Percent of possible error
12	1000	31.2	62
19	947.4	21.4	68
22	981.8	13.0	48
24	950.0	18.8	75

The prospects of 22-tone temperament are thereby improved somewhat with respect to those of the other temperaments, but an error as great as 13 cents in a temperament involving intervals as minute as those of 22-tone temperament is a highly questionable one.

Conclusions on area one: There is no equal-tempered system with 24 or fewer tones with satisfactory coexistant intervals for the 3rd, 5th, and 7th partials. If the 3rd partial alone is to be admitted, 12-tone temperament retains its hegemony. If the 5th partial is to be admitted only in the form of the major third, 22-tone temperament appears the best, with 19-tone temperament as close second. If, however, the minor third is also to be considered, 19-tone temperament is substantially the best of the tempera-

ments in this area.

Area 2: Within area two, the only temperaments whose fifths are sufficiently accurate to merit further consideration are 29-, 31-, 34-, and 36-tone temperaments. 29- and 31- are prime numbers and their systems are therefore entirely new. 34-tone temperament contains the same good fifths as 17-tone temperament, along with a great improvement in its thirds. 36-tone temperament contains no improvement on the thirds of 12-tone temperament despite the great increase in complication involved. Below is a comparison of the fifths in the four systems in this group which contain possibly adequate fifths.

Number of tones	Size of fifth	Error-cents	Percent	Combined Error Factor
29	703.5	1.5	7	.11
36	700	2.0	12	.24
34	705.9	3.9	22	.85
31	696.8	5.2	26	1.35

With the perfect fifth taken alone as the basis of a temperament, 29-tone temperament replaces the multiples of 12 as best. But it is highly doubtful that an increase in complication into the second area can be justified on the basis of the perfect fifth alone. Following are the figures for the major third.

Number of tones	Size of major third	Error-cents	Percent	Combined Error Factor
29	372.4	13.9	66	9.5
36	400	13.7	82	11.2
34	388.2	1.9	11	0.2
31	387.1	0.8	4	0.0

The error for the major thirds of 29- and 36-tone temperaments is so great that once that interval is considered, even if weighted lightly as compared with the fifth, 31- and 34-tone temperaments emerge as the only satisfactory prospects in this area. Of the two, 34-tone temperament appears to be the better on the basis of its fifths. When the minor third is brought into consideration, the superiority of 34- to 31-tone temperament becomes more pronounced.

Number of tones	Size of minor third	Error-cents	Percent	Combined Error Factor
31	309.7	5.9	30	1.8
34	317.6	2.0	11	.2

Within area of complexity two, among systems built on fifths and thirds only, 34-tone temperament appears a clear winner, involving no error larger than 4 cents. It would appear under the circumstances to be a very much underrated system for, although it is mentioned favorably among other temperaments by many writers, it appears to have obtained the specific advocacy of none. This may be

due in part to the non-cyclic character of the fifths. Bosanquet's rule for forming thirds from fifths does not apply; the third of 34-tone temperament is never obtainable by a series of fifths from the tonic. 34-tone temperament possesses the further liability of an unequal partition of the major third into whole tones whose sizes relate to one another as 6 to 5. 6 + 5 = 11

With the consideration of the harmonic seventh, the balance swings heavily to 31-tone temperament. Below are the figures for the interval 7:4.

Number of tones	Size of seventh	Error-cents	Percent	Combined Error Factor
31	967.7	1.1	6	0.1
34	952.9	15.9	90	14.3

It is especially because of this last set of figures that 31-tone temperament thrives today in Holland and in many theoretical works, while 34-tone temperament is ignored. The former system possesses the 7th partial in very close approximation, the latter does not possess it at all. Once the 7th partial is brought under consideration, all competition for 31-tone temperament in this area of complexity vanishes.

With the addition of the 9th partial, and especially of the 11th, 31-tone temperament proves distinctly inadequate, leaving no adequate system in the second difficulty area.

Area 3: The relative merits of the temperaments in area three can also be shown by trial and error. The process can be simplified for the reader, however, because of the towering dominance of 53-tone temperament at every stage of consideration involving fifth (error less than 0.1 cent), fifth and third (the major third is about 1.4 cents too small), and fifth, third, and seventh (although the seventh is nearly 4.8 cents too large and taken alone does not make an especially good showing in this temperament). It is only when the 11th partial is added to the less-than-perfect 7th that the system appears to have passed its reasonable limits (error of the 11th partial, 8 cents or about 70 percent).

41-tone temperament is of some note in this group because it is the first temperament to approximate all of the first 11 partials within six cents. The thirds are the weakest intervals in this temperament; the minor third is a trifle over six cents too large, while the major third is nearly six cents too small. The fifths are excellent, and the 7th partial is less than three cents deficient. With interval distinctions as minute as those of 41-tone temperament, a 6-cent error must be considered quite substantial, but as this is the only temperament in the third group admitting the possibility of all partials through 11, it is worth some consideration. In both 41- and 53-tone temperaments the diatonic scale can be produced

either with equal whole-tones and Pythagorean major thirds or with unequal whole-tones and thirds which approximate just thirds.

Conclusion on Tempered Systems as Obtained by Trial and Error: The 7th partial being unattainable in the first area of complexity with the lower partials also represented, the most complete system would represent all intervals in the senario as closely as possible. This is 19-tone temperament. In the second area, 31-tone temperament dominates by its inclusion of the 7th partial together with a reasonable approximation of the intervals in the senario. In the third area, 53-tone temperament is excellent in its rendition of the senario but weak on the upper partials. 41-tone temperament is the first instance where the 11th partial might play a role in an equal temperament together with the lower partials. A rather poor 5th partial mars this otherwise excellent temperament.

MATHEMATICAL METHODS

Even while using trial and error in their selection of temperaments theorists have attempted to find a more scientific basis for choosing one temperament over another. Since trial and error has provided all of the information needed by most musicians in the selection of a system of equal temperament from approximations of the pure small

number ratios, one might be justified in asking why mathematical methods need be employed at all. Besides the obvious advantage a workable mathematical formula would possess in saving calculating time for a theorist who does not have available all of the figures he might need for comparison and proof, there are, it seems to me, other advantages to using a mathematical formula. An accurate and significant mathematical formula speaks with an authority which no system supported only by trial and error can achieve. It may well be the lack of this very authority which has prevented multiple division from occupying a prominent place in that part of the modern musical world which is preoccupied in certain respects with mathematics. The search for mathematical authority behind systems of multiple division has met, in most cases, however, with very little success. The formulas have tended to derive their existence from favored systems rather than the reverse. And the formulas when applied in series often have led to patently inferior temperaments. Nevertheless, at least one hitherto unpublicized method has led to a large measure of success, and a workable union between mathematical formulae and musical systems may well at long last exist.

Mathematical methods for the determination of the most desirable temperament have been varied; many different mathematical formulae have been explored, even by single authors. Barbour surveys several of these in Tuning and

Temperament, beginning on page 128. I shall discuss a few which have been proposed by the current disciples of multiple division, concentrating on those not mentioned by Barbour.

Yasser and Kornerup arrive, by different methods, at what amounts to a mathematical formula for the determination of an equal temperament. The desirable temperaments must all, according to their theory, belong to the Fibonacci series 2-5-7-12-19-31-50.... To Yasser the reasons are more sociological than mathematical, while Kornerup relates his choice of a Fibonacci series closely to his Golden Tone theory. It is true that the golden ratio, ω , equals the limit, as $n \to \infty$ of $\frac{n+1}{n}$ where n and n + 1 are consecutive members of a Fibonacci series.

The existence of 5- and 7-tone temperaments among Eastern musical systems, the current use of 12-tone temperament, and the favorable prognosis of 19- and 31-tone temperaments place the theory of a Fibonacci series as the mathematical basis for desirable temperament in a most favorable light. From the standpoint of evolution, the Fibonacci theory is also attractive, because it enables a temperament to develop, as it were, out of the two immediately preceding systems in combination. It is this aspect of the Fibonacci theory which is so attractive to Yasser.

Nevertheless, it is very hard to see how the Fibonacci series theory can be held to have any mathematical

validity. The golden mean is an admirable act of artifice, but it does not derive its sanction from the harmonic series which appears more irregular in its favors.[9] However much Kornerup may insist that the ideal fifth is 696 cents, the ideal fifth in the real world of acoustics is 702 cents. That 31-tone temperament contains exactly the combined number of tones of 12- and 19-tone temperaments does indicate that it will possess intervals representing a mean of those intervals of the two simpler systems. But this does not guarantee that it will be a better system than the two simpler ones. This latter event occurs only if the errors of the two simpler systems tend to cancel one another out. Since the major thirds of 19-tone temperament are too small, while the thirds of 12-tone temperament are too large, one might expect that the thirds of 31-tone temperament will represent an improvement over both, and they do. The success of the members of the Fibonacci series up to 31 is the result of the opposition of positive and negative intervals in the 5- and 7- systems. This opposition creates a pendular balancing relation from system to system, up to 31-tone temperament, which tends to make each successive system more accurate in its representation of the principal acoustic values than the one preceding. At this point, however, the characteristics of the limit

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[9]For a detailed discussion of this subject, see Chapter 9, above.

values for both fifth and third prove too small, especially in the case of the fifth. Neither in its fifth, its third, nor its seventh, is the next member of the series, 50-tone temperament, an improvement over 31-tone temperament. A comparative chart of the errors of these three values is shown below:

	31-tone		50-tone	
	size	error	size	error
3:2	696.8	5.2	696.0	6.0
5:4	387.1	0.8	384.0	2.3
7:4	967.7	1.1	960.0	8.8

Further demonstration of the failure of the Fibonacci series on the 50-level lies in the comparison of the errors of 50- and 53-tone temperaments. Through the seventh partial, the latter interval is better for every possible interval except for 7:6, where the excesses of 50-tone temperament's 3rd and 7th partials tend to cancel one another out.

The supporters of adducing temperaments by the Fibonacci series are right in their basic premise that the best temperaments with large numbers of tones are based on the adding together of the numbers of tones of already good temperaments. But their theory neglects to insure that the systems so combined will possess the opposite characteristics which cancel one another out in the additive process. 19-

tone temperament, a negative temperament, complements far better 34-tone temperament which is positive, than 31-tone temperament which is also negative. Hence 19 plus 34 produces 53, an excellent temperament, while 19 plus 31 produces 50-, a decidedly inferior temperament from the standpoint of the small number ratios.

Würschmidt's use of mathematics in formulating his theory of rational tone systems is described above in Chapter 1.

Fokker makes many uses of mathematical formulae as they relate to musical systems. His article, Les Mathematiques et la Musique, explores many procedures for measuring a temperament, including a mode of evaluation in which unity is the sum of the squares of octave, fifth, third and natural seventh. His chief contribution to the art of selecting a temperament by mathematics is in his use of analytical geometry to ascertain the number of tones in a temperament from the properties of the defining intervals.

The various defining intervals are arranged according to the number of fifths, thirds, and sevenths, so that they will equal either zero or an exact octave or multiple of an octave. The syntonic comma, for example, which Fokker uses as a defining interval in a number of instances, is expressed as follows: four fifths minus one third (equals two octaves), or 4, -1, 0 as a function of its fifths,

thirds, and sevenths, respectively.

Fokker proceeds to derive that temperament which uses the following three defining intervals: the syntonic comma; 225:224 (two fifths plus two thirds, minus a seventh); and 1029:1024 (three sevenths plus a fifth). A system based on these three defining intervals is expressed in analytical geometry as follows:

Interval	Fifths	Thirds	Sevenths
81:80	4	-1	0
225:224	2	2	-1
1029:1024	1	0	3

In analytical geometry the solution is found by adding all products of groups of three numbers that would be formed by the descending diagonals if the formula were continued, and deducting from it all similar products of ascending diagonals. Thus, the equation above becomes $(24 + 1 + 0) - (-6 + 0 + 0)$ equals $25 + 6$ or 31.

Of course, with other data the same mathematical process suggests other temperaments. Substituting the schisma for the syntonic comma, the pattern becomes

8	1	0
2	2	-1
1	0	3

which is equal to 41. Using the first two intervals above,

as in 41-tone temperament, but substituting 6144:6125 (the difference between three thirds plus two sevenths, and the perfect fifth) for 1029:1024, 53-tone temperament is obtained by the formula

8	1	0
2	2	-1
-1	3	2

The system Fokker has demonstrated cannot be used until the defining intervals have been established. The establishment of defining intervals is an intermediate process between the establishment of acoustical standards and their realization in a temperament. Attempts to eliminate this intermediate procedure have, however, been made.

Barbour has made such an attempt with ternary continued fractions. By virtue of what Barbour calls a "mixed expansion" he is able to obtain the best of the systems using fifths and thirds as a series.[10] Barbour's performance is as ingenious as it is complex. He first applies the standard or Jacobi ternary continued fractions, obtaining a series of supposedly ideal temperaments using thirds and fifths. Finding the series badly wanting, he reverses the terms, obtaining another expansion on the same prin-

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[10] Barbour, Music and Ternary Continued Fractions, American Mathematical Monthly, 1948, p. 545.

principle: better but still not entirely satisfactory.[11] Finally he works out a formula whereby the two processes are alternated, the choice between them to be determined by which of the two would involve the larger devisor. The result of this complicated procedure is almost identical with Brun's somewhat similar calculation based on ternary continued fractions but using subtraction instead of Barbour's division. Barbour's method is available to all who are interested; as Brun's is generally unknown in this country, it will be examined in considerable detail.

PROFESSOR BRUN'S METHOD

The most impressive mathematical method yet proposed for the derivation of systems of equal temperament from a series of just intervals (or from any other intervals if they should be desired) was first presented in 1919 by Viggo Brun, a Norwegian mathematics professor. He has since expanded on his views in a series of articles, the most recent and complete of which will appear this year (c.1961) in the Nordisk Matematisk Tidskrift. I am convinced that it provides the music theorist with the most useful apparatus

[11] The Jacobi expansion tends to favor temperaments with excellent thirds but questionable fifths: 3, 25, 28, 31, 87. The last two terms listed have good fifths as well. The expansion through the reversal of terms favors the fifths: 1, 2, 5, 12, 41, 53... The mixed expansion alternates on a principle similar to Brun's where the largest of the possible terms is used for each new stage of the expansion.

yet devised for probing the reaches of multiple systems of equal temperament.

Brun's method employs Euclidean algorithms. To begin with, the logarithms of all just intervals on which the system is to be based are computed. Any number of intervals may be employed (although Brun concedes that the method begins to lose its accuracy if too many factors are involved) and any members of the harmonic series may, for experimental purposes be omitted. Let us follow Brun's process, using the octave, fifth, and major third as values for our example.

He calls the logarithms of the three values a, b, and c respectively, and places them, in order from the highest to the lowest, in the left hand column. The right hand column shows values for x_1, y_1, z_1, x_2, y_2, z_2, and x_3, y_3, and z_3, as shown in Figure 1. These represent the number of times the ratio whose logarithm is on the left will go exactly into the ratios of the intervals x, y, and z, which in this case represent the octave, fifth, and major third. Figure 2 shows the same formula with numerical values.

FIG. 1

	x	y	z
a	x_1	y_1	z_1
b	x_2	y_2	z_2
c	x_3	y_3	z_3

FIG. 2

(2:1)0.3010	1	0	0
(3:2)0.1761	0	1	0
(5:4)0.0969	0	0	1

Brun's basic process is to substitute "a - b" for "a" in the left hand column, producing with

each stage logarithms representing smaller intervals, while in the right hand columns he substitutes "x_1 plus x_2" for "x_2," which is to say that he increases the number of times the intervals (now smaller) are used. Throughout the process, a, b, and c represent the sizes of intervals while x_1, x_2, and x_3, etc., represent the number of occurances of each of the respective intervals such that ax_1 plus bx_2 plus cx_3 equals exactly an octave, ay_1 plus by_2 plus cy_3 a perfect fifth, etc.

The substitution shown in Figure 3 represents no basic change in the values, since $(a - b)x_1$ plus $b(x_1 + x_2) = ax_1 - bx_1 + bx_1 + bx_2 = ax_1 + bx_2$, exactly the same as in Figure 1.

FIG. 3

a - b	x_1	y_1	z_1
b	x_1+x_2	y_1+y_2	z_1+z_2
c	x_3	y_3	z_3

FIG. 4

0.1249	1	0	0
0.1761	1	1	0
0.0969	0	0	1

The smallest interval, c, is a kind of carried-over remainder which remains outside of the main calculations until one of the other figures becomes smaller, whereupon it is renamed b, and the new smallest term becomes the remainder. In Figure 4, the numbers in Figure 2 are modified according to the process shown in Figure 3. In place of the octave, log (a - b) representing the perfect fourth, appears. The x column to the right of the line

shows that one fourth plus one fifth comprises an octave. In Figure 5, the terms of Figure 4 are simply rearranged to place the three intervals in positions a, b, and c according to size. In Figure 6, the basic process is repeated. The fourth, b, is deducted from the fifth, a, leaving "a - b," a 9:8 major second. The octave is shown to consist of two fourths and a major second, the fifth of one fourth and a major second. The third is yet to play a role in the process.

In Figure 7, the terms in Figure 6 are rearranged to place the interval sizes in descending order. In this process, the major third is advanced to position b. Figure 8 shows a repetition of the basic procedure, with the diatonic semitone 16:15 resulting from the subtraction of 5:4 from 4:3. As this is the smallest interval so far produced, it assumes position c in the rearrangement which is depicted by Figure 9.

FIG. 5

(3:2)0.1761	1	1	0
(4:3)0.1249	1	0	0
(5:4)0.0969	0	0	1

FIG. 6

0.0512	1	1	0
0.1249	2	1	0
0.0969	0	0	1

FIG. 7

(4:3)0.1249	2	1	0
(5:4)0.0969	0	0	1
(9:8)0.0512	1	1	0

FIG. 8

0.0280	2	1	0
0.0969	2	1	1
0.0512	1	1	0

In Figure 9, the octave is shown as comprised of two just major thirds, a major tone, and two diatonic semitones. This can be stated as follows: $\frac{5}{4}\times\frac{5}{4}\times\frac{9}{8}\times\frac{16}{15}\times\frac{16}{15} = \frac{2}{1}$. In Figure 10, it is further broken down into traditional units of the just diatonic scale, three 9:8 tones, two 10:9 tones, and two 16:15 semitones. The other right hand columns show the traditional breakdowns of third and fifth among these intervals.

Figure 11 shows the necessary rearrangement to restore the values on the left hand side to numerical order, and Figure 12 shows the next step in the subdivision with a new small value, the syntonic comma, emerging to assume the role of "c" for quite some time.

FIG. 9

(5:4) 0.0969	2	1	1
(9:8) 0.0512	1	1	0
(16:15)0.0280	2	1	0

FIG. 10

(10:9) 0.0457	2	1	1
(9:8) 0.0512	3	2	1
(16:15)0.0280	2	1	0

FIG. 11

0.0512	3	2	1
0.0457	2	1	1
0.0280	2	1	0

FIG. 12

(81:80)0.0055	3	2	1
(10:9) 0.0457	5	3	2
(16:15)0.0280	2	1	0

We are now at the point where recognizable tempered systems are about to emerge, and it is time to ask ourselves how one is to interpret the figures obtained. As has been stressed, $ax_1 + bx_2 + cx_3$ will always equal an

octave. Do we, then, add the number of terms in the first column to the right of the line to obtain the number of tones in our temperament? No; this is not done, because the third value is external to the main process and is likely to be so small as to be negligible to the process as a whole.

In Figure 13 (which is a simple rearrangement of Figure 12) the octave is composed of ten units, three of which are mere syntonic commas. To propose a 10-tone temperament with units representing everything from minor tones (10:9) to syntonic commas (81:80) would be absurd. What Figure 13 does offer, in its top row, (derived from row 2 of Figure 12) is an instance of 5-tone temperament in which the fifth and major third have their proper values. A comparison of this row with the sum of the terms in Figure 9, shows the superiority of this process. The sum of the terms in Figure 9 are 5, 3, and 1 respectively. If this were indeed supposed to represent 5-tone temperament, the third would be much too small. 386 cents is considerably closer to 480 cents than it is to 240 cents; the row 5, 3, 2 represents this temperament better than the rougher, earlier estimate, 5, 3, 1.

FIG. 13

(10:9) 0.0457	5	3	2
(16:15)0.0280	3	1	0
(81:80)0.0055	3	2	1

Figure 14 carries Brun's process a step farther, with a new temperament of 7 tones to the octave emerging.

Figure 15 represents the characteristic rearrangement of terms, and Figure 16 carries the process another step, showing the emergence of 12-tone temperament. It might be mentioned at this point that the progression from 5- through 12- has begun to follow Yasser's Fibonacci series. This it will continue to do as long as every new interval value (a - b) remains larger than the syntonic comma, which is presently value c. As soon, however, as the new value has been reached which is smaller than the syntonic comma, the syntonic comma will pass from the role of "defining" interval to that of "constructing" interval and the values to the right of it in the chart will play a role in determining the number of tones in the temperament. It is Figure 15 which best shows the 12-tone system under Brun's approximation. In the octave are seven diatonic semitones (16:15) and five chromatic semitones (25:24). Three syntonic commas are "left over," which are generally added to three of the diatonic semitones in the theory of just intonation.

FIG. 14

0.0177	5	3	2
0.0280	7	4	2
0.0055	3	2	1

15

FIG. 15

(16:15)0.0280	7	4	2
(25:24)0.0177	5	3	2
(81:80)0.0055	3	2	1

FIG. 16

128:125	0.0103	7	4	2
25:24	0.0177	12	7	4
81:80	0.0055	3	2	1

22

Figure 17 represents the rearrangement of Figure 16

in the descending order of the intervals, and Figure 18 represents the next step: 19-tone temperament. The syntonic comma remains the smallest figure, but the new interval, representing the difference between the chromatic semitone and the diesis of the major thirds, 128:125, is nearly as small.

FIG. 17

25/24	.0177	12	7	4
128/125	.0103	7	4	2
81/80	.0055	3	2	1

R

FIG. 18

3125/3072	.0074	12	7	4
128/125	.0103	19	11	6
	.0055	3	2	1
		34		

Figure 19 shows the rearrangement, and Figure 20 the next step, whereby 31-tone temperament is shown, and where a new interval, smaller than the syntonic comma is formed. It is this stage which alters the Fibonacci series, as 81:80 assumes position b in the rearrangement which is shown in Figure 21.

FIG. 19

128/125	.0103	19	11	6
3125/3072	.0074	12	7	4
	.0055	3	2	1

R

?

FIG. 20

abyzma

3187/3186	.0029	19	11	6
	.0074	31	18	10
	.0055	3	2	1
		53		

Thus it is not 19, but rather 3 tones, representing the number of syntonic commas carried by the simpler systems as a remainder, which must be added as the syntonic comma is advanced to the rank of constructing interval. Figure 22 shows the emergence

FIG. 21

3125/3072	.0074	31	18	10
81/80	.0055	3	2	1
	.0029*	19	11	6

R

* Abyzma: 0.00287146

Kleisma = .00203378

of 34 as the next system through the combining of 31- and 3-tone temperaments, the latter shown, by Brun's method, to have the best features to balance the weaknesses of 31-tone temperament.

FIG. 22

214/213	.0019	31	18	10
	.0055	34	20	11
	.0029	19	11	6
		84		

Figure 23 shows the rearrangement which follows; a new smallest interval assumes position c. In Figure 24, it is 53-tone temperament which is shown to arise out of the combination of ultrapositive 34- and negative 19-tone temperaments. With the evolving of 53-tone temperament, the syntonic comma has completed its role as constructing interval and will itself be subdivided.

FIG. 23

91/86	.0055	34	20	11
3187/3166	.0029	19	11	6
214/213	.0019	31	18	10

R

FIG. 24

1208/1201	.0026	34	20	11
	.0029	53	31	17
	.0019	31	18	10
		118		

Figure 25 represents a rearrangement and Figure 26 introduces the next temperament of the series, possessing 87 tones to the octave. The excellent merits of this system are considered briefly in Chapter 6, above.

FIG. 25

3187/3166	.0029	53	31	17
1208/1201	.0026	34	20	11
	.0019	31	18	10
		171		

FIG. 26

1251/1250	.0003	53	31	17
	.0026	87	51	28
	.0019	31	18	10
		171		

The next temperament to emerge, after the rearrangement shown in Figure 27, is 118-tone temperament in Figure 28. This is the "perfect system" of so many advocates of temperaments based on fifths and thirds, and it is also shown in Chapter 6.

FIG. 27

.0026	87	51	28
.0019	31	18	10
.0003	53	31	17

FIG. 28

.0007	87	51	28
.0019	118	69	38
.0003	53	31	17

The algorithm can be continued, in which case it yields 205, 323, 441, and 494-tone temperaments. The temperaments agree almost completely with those derived by Ariel and by Barbour by different methods, with the latter members of the series showing the greatest divergence between them. The error of third and fifth alike in 441-tone temperament is less than .05 cents and compares well with the errors in systems of many tones which are advocated by other theorists.

APPLYING BRUN'S PROCESS TO OTHER INTERVALS

A process similar to the one outlined above can be followed for any combination of intervals. The theorist may choose to use additional values in the harmonic series. Brun himself has carried the process through using the above values plus the seventh partial, obtaining as temperaments for the octave, 13-, 15-, 18-, 31-, 35-, 53-,

and 68-tones.[12] The agreement of this series with the one studied above at the 31- and 53- levels strengthens the case for these temperaments. Brun has also used the same process with all partials through 11, and obtained the following results: 15, 17, 22, 37, 41, 63, 72.... Of these, his own preference lies with 41- and 72-tone temperaments, both of which have been found quite attractive by other theorists, usually for very different reasons.[13] Carrying this algorithm farther, I find the next temperaments to be 89-, 161-, 233-, and 270-tone temperaments. The intervals of 270-tone temperament show the potential value of Brun's method for locating exceptional temperaments.

Interval	3:2	5:4	7:4	11:4
Size of interval	702.22	386.67	968.89	551.11
Size of error	0.27	0.36	0.[illegible]	0.2

The smallest unit of the system being 4.44 cents, the error never exceeds 18 percent of the possible; this is a very good achievement for a system with so many disparate factors.

It is also possible to use Brun's method with

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[12] In my own calculations using Brun's methods, I arrive at slightly different figures: 49 instead of 35, and 71 instead of 68.

[13] 41-tone temperament is discussed above in the first section of Chapter Five, and 72-tone temperament is discussed near the end of Chapter Four.

intervals lying outside of the harmonic series, should it be the conviction of anyone seeking a more complex equal temperament that substitute intervals (such as a larger major third) are preferable. The method can also be used with members of the harmonic series but omitting one or more, as Yasser suggests with regard to the 3rd partial. Brun's procedure, in short, provides a means for converting any set of preferred intervals into a series of equal temperaments tending to increase in accuracy as more tones are added. There is a shortcoming, in that there is no room for differentiating the importance of the various intervals involved. This same weakness has been noted in other methods for calculating advantageous temperaments. A further shortcoming, if one wishes to call it that, is that it still does not perform a musician's thinking for him. He must still decide which intervals he is to use, and he must select from the series which the method he employs yields that single system which is the best compromise, for his purposes, between accuracy and expediency.

CONCLUSION

If one accepts the acoustical validity of small-number ratios on the one hand and the principle of equal temperament on the other, one appears all but bound to conclude that the most desirable temperaments belong to the

following series: 12, 19, 22, 31, 34, 41, 53, 72.... The underlined tempered systems seem, in particular, to offer distinct acoustical refinements over all those which precede them in the series. The systems not underlined display important disadvantages as well as some advantages over those which immediately precede them. In general, accuracy and scope increase with number of tones, but so does difficulty. It is possible to measure the degree of accuracy of a temperament minutely, and we have attempted to do so in the foregoing pages. It is only possible to guess at the difficulties involved in learning to perform and to hear music based on 19-, 31-, 53-, or 72 sounds to the octave. Certainly the generalized keyboard deserves a generalized test.[14] Ear-training experiments with selected subjects may afford a partial answer to another important aspect of the question, "how difficult?"

The generalized keyboard might afford a beginning for a new tempered system; the ear might be expected gradually to digest the new pitch relationships manually produced. With the mastery by the ear would come the availability of the vocal and string media as well as the slide trombones. Valved brass instruments could be easily adapted by the addition of one or two microtonal crooks. Keyed wind instruments would present the greatest problem,

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[14] See Appendix 1, below, for diagrams and descriptions of proposed generalized keyboards.

but multiple tiers of keys on a wind instrument are hardly out of the question, and their exploitation has only just begun with the present woodwind instruments.

There seems little point in adapting many instruments until the direction of further evolution, if there is one, becomes clearer. The absence of sufficient information for an imminent decision must be conceded. There appears, therefore, to be little chance that multiple division will replace 12-tone temperament in the near future.

But there would appear to be no reason why multiple division on an experimental scale shouldn't co-exist with 12-tone temperament. There is ample justification for further experimental composition, experimental hearing tests, and experimental attempts at obtaining performance virtuosity in various systems, not only in the isolated studies of a Partch, a Fokker, and a Wyschnegradsky, but wherever creative musicians are dissatisfied with the materials currently at their disposal.

PART III

HYPOTHETICAL AND EXPERIMENTAL MATERIALS

CHAPTER 14

A THEORY OF HARMONY BASED ON SYMMETRICAL SCALES WITH QUASI-EQUAL INTERVALS IN NINETEEN-TONE TEMPERAMENT

In this section, I present what I consider, from careful examination of the proposals already made and from my own laboratory experiments, to be a particularly fruitful line of inquiry into the possibilities of 19-tone temperament. It is, needless to say, subject to disagreement at any point, and it makes no pretense at exhausting any but one branch of the possibilities inherent within 19-tone temperament. It is hoped that the inclusion of such a theory will further stimulate thought and experimentation in 19-tone temperament.

THE GAMUT

The gamut is drawn from natural intervals, which are tempered to facilitate transposition and modulation. In its origin the system is bi-intervalic (tri-intervalic counting the octave). The two basic intervals are the perfect fifth (3:2) chosen because of its dominant position in the overtone series, and the minor third (6:5) chosen because among the intervals of the senario it is most superbly suited to the peculiar features of 19-tone temperament, falling short

of the interval created by five units of the temperament by only 0.15 cent. Starting from a central tone, the gamut is created by adding single fifths above and below, and by adding two superimposed minor thirds above and three below each tone thereby obtained. Eighteen tones, including the central tone, are produced in this manner. The nineteenth tone may be derived in any of several ways. The 19-tone system derived in this manner is shown, above, in Example 47, p 324 Chapter 12. Its intervals are compared with those of 19-tone equal temperament in Example 45. p 314

See also p 324

CONSONANCE AND DISSONANCE

The two basic intervals are consonances, as are their octave inversions. Also consonant are all combinations of the basic intervals used singly, and their inversions. All other intervals are either neutral or dissonant.[1] The eight consonances within the 19-tone system are:

3:2 Perfect fifth (11/19)
4:3 Perfect fourth (8/19)

6:5 Minor third (5/19)
5:3 Major sixth (14/19)

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[1]There are several reasons for the selection of this particular group of intervals as consonant. They are, (1) structural: since these are the intervals made from the starting point of two basic intervals each used singly; (2) acoustic: these intervals represent unquestionably the simplest ratios which can be approximated with any degree of accuracy in 19-tone temperament; and (3) evolutional: these are the intervals which most closely resemble their counterparts in 12-tone temperament, counterparts which themselves are regarded as the least dissonant intervals in 12-tone temperament.

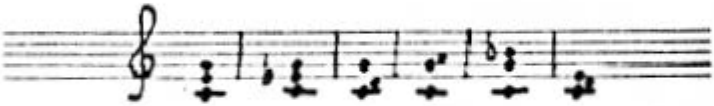
9:5 Minor seventh (16/19) (Perfect fifth plus minor third)
10:9 Major second (3/19)

5:4 Major third (6/19) (Perfect fifth minus minor third)
8:5 Minor sixth (13/19)

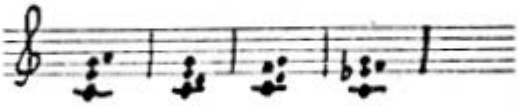
No differentiation is made between perfect and imperfect consonances as concerns the single intervals.

HARMONIC SONORITIES

All chords containing only consonant intervals are considered perfect. This includes the following six triads, with their inversions,



the following four tetrads with their inversions,



and a single pentad, representing what is generally considered to be the pentatonic scale. The perfect sonorities all have close equivalents in music based on 12-tone temperament.

Dissonant are the two smallest intervals of the system and their inversions. On these Yasser and Ariel agree, and through an agreement to treat them as dissonances the basic melodic and harmonic units are made distinct.[2] The remaining six intervals, representing 4/19, 7/19, and 9/19 and their inversions, are regarded as neutral. Sonorities containing neutral intervals but no dissonant intervals shall be regarded as imperfect consonances, while those containing dissonant intervals shall be regarded as dissonances. The imperfect consonances, which include sonorities with those intervals unique to 19-tone temperament, are the characteristic sounds of this temperament which distinguish it from other musical systems. They will be analyzed later in some detail.

THE FORMATION OF SCALES

Scales can be formed, of course, by any number of tones within the octave, at any sequence of distances from one another. The systems considered here will all have in common that they are based on scales containing intervals as equal as is possible within a 19-tone system. Since 19 is a prime number, 19-tone temperament contains no equal-interval scale within it such as might be likened to the

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[2]Just as the major and minor seconds in 12-tone temperament are distinguished by the intensity of their dissonances (the latter considered the more severe), 1/19 and its inversion are considered to be more dissonant than 2/19 and its inversion.

whole-tone scale in 12-tone temperament. But, as the diatonic scale in 12-tone temperament is composed only of the intervals 1/12 and 2/12 (as opposed to the harmonic minor scale which also contains an interval 3/12), so all of the scales to be considered here contain a range of two intervals of adjacent size; the size of the intervals will depend on the number of tones in the scale. This is what is meant by the term "quasi-equal" in the title of this supplement. "Symmetrical" means that where two different sizes of scale intervals are used they will be spaced as evenly as possible throughout the scale. This is how the standard diatonic scale differs from the melodic minor. In the former, the semitones are separated by 2 and 3 whole steps, while in the latter they are separated by 1 and 4 whole steps. The scales considered in this section will be symmetrical in this special sense whereby the major scale is symmetrical but the melodic minor is not.

In 19-tone equal temperament, quasi-equal-interval-symmetrical scales are formed by the superposition of a single interval. It is one of the basic theorems of the system that when a given scale is quasi-equal-interval-symmetrical, the tones which are not used, taken in their totality, also create a scale which is quasi-equal-interval-symmetrical. Such a pair of scales will be considered complementary. The sum of the number of tones in any two complementary scales is always 19. The two complementary scales of which Yasser writes, involving 12 and 7 tones (QE19)

respectively, are quasi-equal-interval-symmetrical. They are created by the superposition of perfect fifths until a quasi-equal-interval-symmetrical pattern is attained. Example 54 shows the 7- and 12-tone scales of 19-tone temperament resulting from the superposition of perfect fifths and the number of units of the system in each of their intervals.

Example 54: The 7- and 12-Tone Scales

Fb Cb Gb Db Ab Eb Bb F C G D A E B F# C# G# D# A#

C D E F G A B C
3 3 2 3 3 3 2

C C# D D# E F F# G G# A A# B C
1 2 1 2 2 1 2 1 2 1 2 2

The harmonic units of which Yasser writes belong not to the system "12 + 7", but rather to the system "13 + 6", which is the second scale group of the system and is based on the superposition of major seconds. Example 55 shows the scale-group "13 + 6" with enharmonic pitch names freely used. It should be remarked that since the scale formations rest purely on a tempered basis, the enharmonic names are perfectly suitable.

Example 55: Scale Group "13 + 6"

C# D# Fb Gb Ab Bb C D E F# G# A# Cb Db Eb F G A B 3/19

C Db D Eb E F F# G G# A A# B Cb C C D E F# G# A# C
2 1 2 1 2 1 2 1 2 1 2 1 1 3 3 3 3 3 4

MOS 3/19 MOS 16/19

It will be noted that the six-tone scale above forms a Yasser-hexad; this hexad would appear to be a reasonable basis for harmony involving the 13-tone scale. The consonances are imperfect, owing to the presence of all of the "neutral" but none of the "dissonant" intervals in the hexads.

The third system is comprised of the 11- and 8-tone scales, and is derived from the augmented third (diminished fourth), a neutral interval, totally unfamiliar to present-day ears. The characteristic harmony of this scale-system is dissonant to neutral. The 11-tone scale shows a preponderance of "whole-steps" over "half-steps" not unlike that of our present diatonic scale. 7/19

Example 56: Scale Group "11 + 8"

D Gb B Eb G# C Fb A Db F# Bb D# G Cb E Ab C# F A# 7/19

C Db D Eb Fb F# Gb G# A Bb B C Cb C# D# E F G Ab A# Cb
2 1 2 2 2 1 2 2 2 1 2 2 3 2 2 3 2 2 3

12/19 MOS 12/19 MOS 7/19

The fourth scale system is comprised of the 14- and 5-tone scales. The traditional Western pentatonic scale is, of course, a satisfactory scale, and is the basis for consonant harmony in the 19-tone system. However, it is not a quasi-equal-interval scale, since it possesses intervals of 5- and 3- units. While it is possible to build such a pentatonic scale, and the 14-tone scale which is its complement, by juxtaposing perfect fifths, such a group of scales, although excellent, falls outside of the scope of this particular theoretical study. The quasi-equal-interval 5-tone scale is composed of intervals of 3 and 4 units, one of the former and four of the latter. It sounds nearly equal-tempered and suggests the Indonesian salendro scale. The subtended interval is the augmented second (diminished third) which does not exist as an independent interval in 12-tone temperament, and possesses a strikingly novel sound. 4/19

Example 57: Scale Group "14 + 5"

B Db E Gb A Cb D Fb G A# C D# F G# Bb C# Eb F# A

15/19 4/19

C C# D D# Eb Fb F F# G G# Ab A# Bb Cb C B Db E Gb A B
1 2 1 1 2 1 1 2 1 1 2 1 2 1 4 4 4 4 3

MOS 17 MOS

The fifth scale system is comprised of the 10- and 9-tone scales. The two component scales are almost identical. The smaller scale can be produced from the larger by the

omission of a single tone. The intervals of the two scales are almost entirely equal as well, suggesting a rather limited possibility of use, except for a single effect or color. This scale system might play a role in 19-tone music somewhat akin to the role of the whole-tone scale in 12-tone temperament. It might also be quite effective for serial exploitation since the intervals are larger than those of 12-tone temperament and the serial pattern is likely to be at once less dissonant and easier to follow.

Example 58: Scale Group "10 + 9"

Fb F# G Ab A# B C Db D# E F Gb G# A Bb Cb C# D Eb

C Db D# Fb F# G Ab A# B C
2 2 3 2 2 2 2 2 2 2

C# D Eb E F Gb G# A Bb Cb C#
2 2 1 2 2 2 2 2 2 2

The sixth scale system is comprised of the 15- and 4-tone scales. It is probably less useful than the preceding systems because of the greater disparity between its component scales. It is produced by a series of minor thirds, and takes on additional significance from the importance of this interval in 19-tone temperament. It is the only one of the scale systems which may be considered to be just, as well as tempered, in 19-tone temperament. When minor thirds are subtended, the first four tones produce a diminished seventh chord (albeit quite different in sound from the

diminished seventh of 12-tone temperament); the fifth tone exceeds the octave, however, by one unit of the system. The resulting sonority is quite effective, provided that the first and fifth tones are not sounded within the same octave. Since the minor third represents the basic unit in the scale, pentachordal configurations whose outer tones form that interval might well be the principal melodic materials of this scale system.

Example 59: Scale Group "15 + 4"

G# B D F Ab Cb D# F# A C Eb Gb A# C# E G Bb Db Fb

C C# D D# Eb E F F# Gb G# Ab A A# B Cb C Db Fb G Bb Db
1 2 1 1 1 2 1 1 2 1 1 1 2 1 1 5 4 5 5

The remaining scales are probably of interest only as curiosities, involving as they do so many or so few tones. The 16- and 3-tone scales result from the juxtaposition of major thirds, the 17- and 2- from combining augmented fourths, and, if one wishes to call them such, the 18- and 1-tone scales result from single units of the temperament, the diminished seconds.

It will be noted from the above that 19-tone temperament contains nine different scale classes, just as there are nine different intervals (plus their octave inversions). For each interval there is a scale class. For each scale

class there is, correspondingly, an interval. Because 19 is a prime number, any interval in the system creates a complete cycle of 19 tones, when repeated, before returning to a replica of the original tone.

The correspondence of interval to scale class is significant, because the interval which, juxtaposed, produces a given scale class, is the basic interval of modulation for music composed utilizing the scale class in question.[3] Example 60 shows the distance in number of steps, ascending, from C to each member of the system, given each of the various possible intervals as the basis for scale and modulation.

Example 60 shows the multiplicity of function possible for each tone with respect to a central tone C. Each of the 18 tones other than C can be any of the first 18 members of a cycle, depending on the interval chosen as the basis for the cycle. There is no duplication of function whatsoever.

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[3]On this basis the fifth is, of course, the determining interval for diatonic music in both 12- and 19-tone systems, as well as for the 12-tone scale in 19-tone temperament and for certain 14-tone scales in the latter temperament. It follows that these are likely to be the most consonant scale systems in 19-tone temperament; more dissonant intervals produce scale systems requiring more dissonant idiomatic harmonic treatment. The one-to one correspondence of scale to interval to harmonic type is one of the most challenging possibilities of this theoretical system.

Example 60: Distance from C of Each of the Tones by Cycles of the Various Intervals

Basic Interval	C#	Db	D	D#	Eb	E	Fb	F	F#	Gb	G	G#	Ab	A	A#	Bb	B	Cb	C
dim. 2nd	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2/19 minor 2nd	10	1	11	2	12	3	13	4	14	5	15	6	16	7	17	8	18	9	19
3/19 major 2nd	13	7	1	14	8	2	15	9	3	16	10	4	17	11	5	18	12	6	19
4/19 aug. 2nd	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19
5/19 minor 3rd	4	8	12	16	1	5	9	13	17	2	6	10	14	18	3	7	11	15	19
major 3rd	16	13	10	7	4	1	17	14	11	8	5	2	18	15	12	9	6	3	19
7/19 aug. 3rd	11	3	14	6	17	9	1	12	4	15	7	18	10	2	13	5	16	8	19
perf. 4th	12	5	17	10	3	15	8	1	13	6	18	11	4	16	9	2	14	7	19
9/19 aug. 4th	17	15	13	11	9	7	5	3	1	18	16	14	12	10	8	6	4	2	19
10/19 dim. 5th	2	4	6	8	10	12	14	16	18	1	3	5	7	9	11	13	15	17	19
perf. 5th	7	14	2	9	16	4	11	18	6	13	1	8	15	3	10	17	5	12	19
12 aug. 5th	8	16	5	13	2	10	18	7	15	4	12	1	9	17	6	14	3	11	19
min. 6th	3	6	9	12	15	18	2	5	8	11	14	17	1	4	7	10	13	16	19
maj. 6th	15	11	7	3	18	14	10	6	2	17	13	9	5	1	16	12	8	4	19
aug. 6th	14	9	4	18	13	8	3	17	12	7	2	16	11	6	1	15	10	5	19
16/19 minor 7th	6	12	18	5	11	17	4	10	16	3	9	15	2	8	14	1	7	13	19
17/19 major 7th	9	18	8	17	7	16	6	15	5	14	4	13	3	12	2	11	1	10	19
aug. 7th	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	19

7/8/91

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A MORE DETAILED TREATMENT OF HARMONIC SONORITIES

The number of possible tone combinations in 12-tone equal temperament is sufficiently large to fill an entire book,[4] and, needless to say, the number of possible combinations of tones in 19-tone temperament is far larger. There are, for example, 43 distinct tetrads possible in 12-tone temperament, with inversions and transpositions excluded. In 19-tone temperament there are 201. In order that any harmonic order be made of so myriad a field of possibilities, a drastic reduction must be made in the same way that harmonic usage cut the field of tetrads in traditional 12-tone music to the seven seventh chords and the French sixth.

Because of the increase in the number of available tones and because of the general tendency toward increased complexity, it seems reasonable to assume that the function of the simple triad will be taken over by a more complex chord (Yasser has suggested the hexad). The four triads generally accepted in traditional 12-tone music possess one essential characteristic: no interval is smaller than 3 units of the system. The four established triads represent every possible combination of three intervals within the octave such that no interval is smaller than 3 units. The pattern 6-3-3 produces the diminished triad, the patterns

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[4] I am thinking of Howard Hanson's <u>Harmonic Materials of Modern Music</u>, and there may well be others.

5-4-3 and 4-5-3 produce major and minor triads, while the pattern 4-4-4 yields the augmented triad. There is no other triad in the 12-tone system without an interval smaller than 3 units.[5]

Were 3 units of 19-tone temperament to be used as the smallest interval in the construction of triads in that system, however, no fewer than 22 distinct triads would become part of the harmonic vocabulary, including one (13-3-3) whose largest interval would exceed four times the size of the smallest. With tetrads the number is increased; there are 30. Classification into chord types, whereby all chords consisting of a given group of intervals (such as 6, 5, 4, and 4 units) would be treated together, would reduce to a manageable 11 the number of distinct classifications.

Hexads, on the other hand, offer too few possibilities; there is only one hexad (4-3-3-3-3-3) which, with its inversions, is possible without the use of intervals smaller than 3 units. Furthermore, there is no consonant hexad according to our theory of consonance, although the hexad cited here is the generating basis for consonance according to Yasser.

Offering the best compromise between too few and too many possibilities is the pentad devoid of intervals smaller

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[5]It should be noted that in forming triads, I use three rather than two intervals, thereby completing the octave. Triads such as C-E-B are eliminated from consideration by the smallness of that third, octave-completing interval. A similar procedure is used for the harmonic sonorities of 19-tone temperament, causing the sum of the intervals therein represented always to equal 19.

than three units. There are five types of pentads, containing 14 possible chords. Of these, as will be seen, 2 types, containing 5 chords, can to all intents and purposes be eliminated. Here are the five types of pentads:

7-3-3-3-3	yielding one chord
6-4-3-3-3	yielding four chords (6-4-3-3-3, 6-3-3-3-4, 6-3-4-3-3, 6-3-3-4-3)
5-5-3-3-3	yielding two chords (including the consonant 5-3-5-3-3)
5-4-4-3-3	yielding six chords (5-4-4-3-3, 5-4-3-4-3, 5-4-3-3-4, 5-3-3-4-4, 5-3-4-3-4, 5-3-4-4-3)
4-4-4-4-3	yielding one chord

The first two types can be eliminated because they represent essentially incomplete hexad structures wherein the range of sizes of the intervals equals or exceeds two to one. Such chords may be used as neutral pentads in some instances, but essentially they belong not to the realm of pentads but rather of hexads. The remaining three pentad types yield only nine different chords plus their inversions. These can be considered the nucleus of non-dissonant harmony in 19-tone temperament.

The determination of the possible roots of these chords offers one of the greatest obstacles to the development of a theory of harmony. Three possibilities exist: that these chords have no roots; that they have variable roots which are determined by the position of the chord, as is often the case in Hindemith's theory; and that they have distinct roots as do the triads in Rameau's theory. For

experimental purposes I have tentatively adopted the third alternative, seeking to assign to a single tone in each sonority the function of the root, regardless of the inversion. Since no such simple matter as tertian construction is available as a basis, the selection of the root depends on Hindemith's procedure up to a point.

The procedure is as follows: (1) Determine which members of the sonority possess porfect fifths above or perfect fourths below. (2) Where one member of the sonority is so endowed it is the root. (3) Where two or more members of the sonority are accompanied by the perfect fifth above or the perfect fourth below, eliminate all other tones from consideration; the root is the remaining sonority which also possesses a major third above. (4) Where no perfect fifths or fourths exist, the root shall be that member of the sonority which possesses a major third above. (5) Where it is still impossible to determine the root after consideration of the major thirds, continue the process of considering the lesser intervals eliminating from consideration, however, those tones which lack the aforementioned consonances that are possessed by other tones. (6) The other intervals shall be considered in the following order: minor third above, perfect fourth above, major second above.

The nine chords, stated numerically by intervals taken upwards from the root are:

a	3-3-5-5-3	d	4-3-4-3-5	g	5-3-3-4-4
b	3-3-5-3-5	e	4-3-4-5-3	h	3-3-4-5-4
c	3-3-5-4-4	f	3-5-3-4-4	i	4-4-3-4-4

The spelling of the same nine pentads, considering C to be the root, is as follows:

a	C-D-E-G-Bb	d	C-D#-Fb-G-A	g	C-Eb-F-G-A#
b	C-D-E-G-A	e	C-D#-Fb-G-Bb	h	C-D-E-Gb-A#
c	C-D-E-G-A#	f	C-D-F-G-A#	i	C-D#-F-G-A#

Of the above, pentad b is the most consonant, while pentads h and i are probably the least stable, h alone possessing no perfect fifth whatsoever.

The pentads are most effective when used in open position. Because the roots are less pronounced in their character than in traditional triadic harmony, it is advisable that any composer wishing to induce a sense of root progression in these harmonies be careful to use a preponderance of pentads in root position, at least for the present.

THE TENDENCY OF INTERVALS

In traditional harmony dissonances are said to create tendencies toward intervals of resolution. The tritone and its inversion, in particular, are said to have strong outward and inward tendencies respectively, while seconds and sevenths create (in one of the voices only, as a rule) outward and inward tendencies respectively. While it is generally assumed,

especially in the case of the tritone, that harmonic context creates the specific tendency of an otherwise neutral interval (by determining whether it will be, for example, an augmented fourth or a diminished fifth), it is necessary, in 19-tone music, to have a concept of tendency which exists independent of total chordal context. This is because the traditional enharmonic relationship between such intervals as the augmented fourth and the diminished fifth no longer exists in 19-tone temperament. The tritone and the diminished fifth are two different intervals. In order to provide some basis for the consideration of tendency in temperaments other than 12-tone, I have evolved the following theory of tendency, which I believe to be applicable to traditional practice in 12-tone temperament as well as to a possible practice in 19-tone temperament.

This theory might be called the theory of repulsion. When a consonance is perfect (or very, very close to perfect) a sense of rest is achieved. When a consonance is substituted for by a near-lying dissonance, the implied consonance repells the closely approximating dissonance, causing the dissonance to possess a tendency <u>away</u> from the said consonance.[6] Two principles affect the strength of this magnetic repulsion. The first is that the more perfect the consonance

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[6] It is this tendency which may well be the reason why players tend to sharpen their thirds and upward-tending leading tones. The nearby consonant third imparts to the Pythagorean third an outward tendency through "repulsion."

the greater its repelling force on nearby dissonances. The second is that except for intervals so close to the consonance that the ear substitutes the consonance for the actual dissonant interval played, the closer the interval to the consonance, the greater the force of the repulsion.

In 12-tone music the tritone is more or less neutral until it is harmonized, for the inward repulsion created by the fifth is all but cancelled out by the outward repulsion created by the fourth. The dissonant interval is suspended between these two consonances and the tendencies neutralize, requiring the creation of harmonic context for the development of a tendency. In 19-tone temperament, the augmented fourth, being closer to the perfect fourth than to the perfect fifth, has a natural outward tendency, while the diminished fifth has a natural inward tendency. These tendencies are independent of the harmonization which may either reinforce them or weaken them at the discretion of the composer.

The second, especially the minor second, obtains its outward tendency from its nearness to the prime, while the seventh is repelled inward by the octave. With these intervals, 19-tone temperament possesses the same tendencies as 12- except that since the major second and minor seventh are treated as consonances, no tendencies are considered to affect them.

The augmented second in 19-tone temperament might better be called the diminished third, as it derives its

tendency from its nearness to the minor third. Since the major second is too weak to offer a counter-repulsion and too close to serve as a target for the resolution, the diminished third (4/19) tends to want to resolve inward to the prime. The dissonant interval 7/19 is, like the tritone in 12-tone temperament, suspended between two relatively equal consonances, the major third and perfect fourth. It can resolve with equal effectiveness in either direction. Somewhat different, however, is the tendency of this interval when it is inverted. 12/19 is repelled with the greatest strength by the perfect fifth and so tends to resolve outward.

It is postulated that where any of the above dissonances are used, their resolutions should, where possible, follow the patterns as set forth above.

CONCLUSION

It is hoped that the above suggested principles may offer the experimental composer a meaningful starting point for his exploration of 19-tone temperament. In the following supplement, some but not all of the principles are illustrated in the compositions. It will require, of course, a far larger body of works to evolve a workable language, let alone an accurate measure of the aesthetic wealth of 19-tone temperament. It is hoped that those who feel impelled to write additional music in 19-tone temperament will inform

me, and if possible send scores and tapes, in order that a central source be available to which those seeking information about progress in 19-tone temperament might come.

CHAPTER 15

NINE PRELUDES FOR TWO PIANOS IN 19-TONE EQUAL TEMPERAMENT

The nine short pieces on the pages which follow are offered as tentative explorations of some of the possibilities of this temperament. I have chosen to probe many possible styles rather than focus on one. The nine preludes are recorded on the tape which accompanies this dissertation. Franklin Ulrich Fanning is my collaborator in the performance.

The notation herein employed is as shown in the Preface to the entire dissertation. The traditional staves are used, and the 19 pitch names, in ascending order from C, are: C, C#, Db, D, D#, Eb, E, Fb, F, F#, Gb, G, G#, Ab, A, A#, Bb, B, Cb, C. The two pianos are tuned as follows:

First Piano:	C#	D#		F#	G#	A#		
	C	D	E	F	G	A	B	C
Second Piano:	C#	D#		F#	G#	A#		
	Cb	Db	Eb	Fb	Gb	Ab	Bb	Cb

Brief notes on the use of the 19-tone system in each of the preludes appear on the pages following the music.

I

Allegro maestoso

f

mf

ff

molto dim

espressivo

mf

f

ff>mf

mf

f

ff

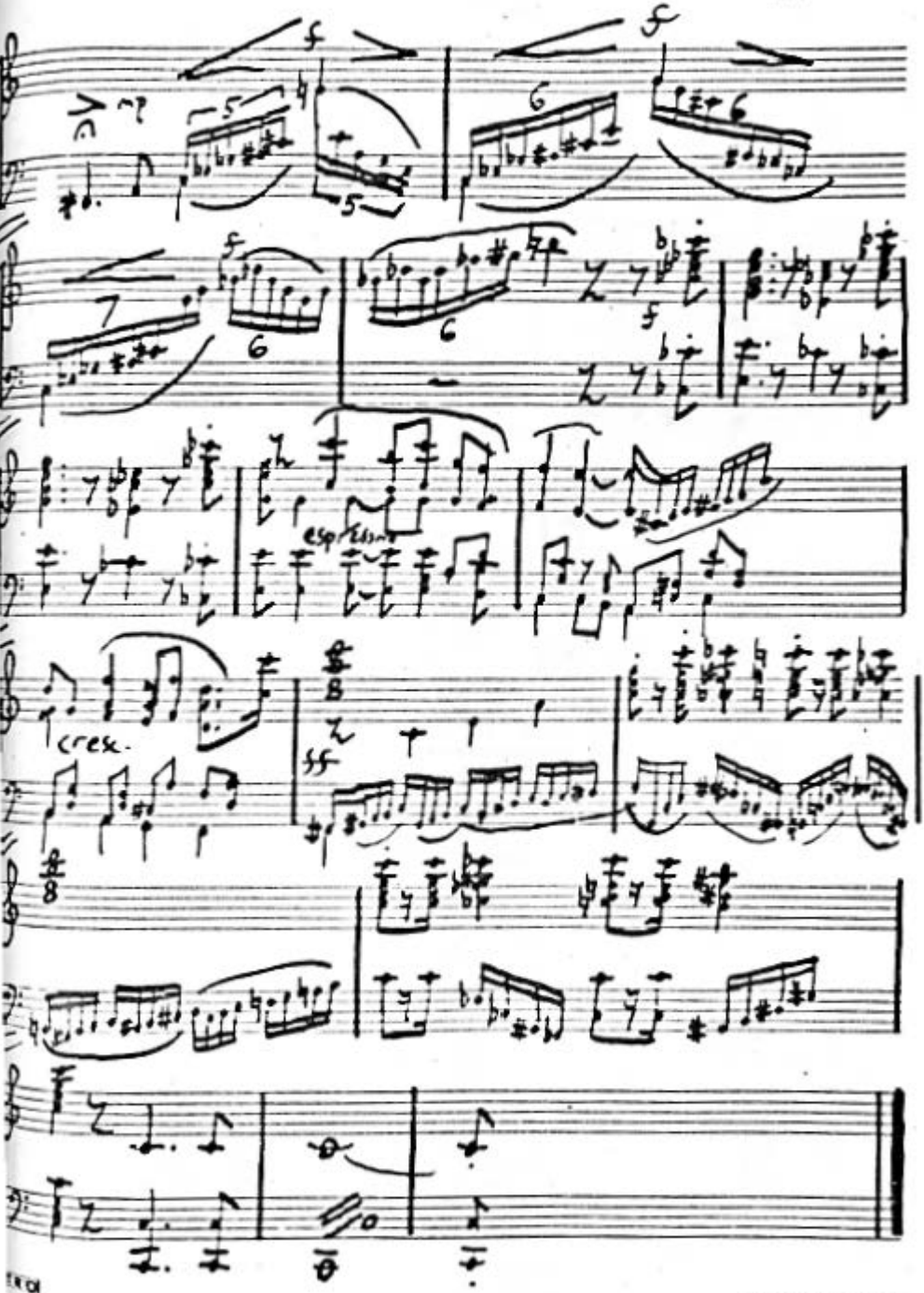
5

5

ad libitum

5

5


f
f
esp.
cresc.
ff

II.

Un poco lento

sempre legato

dim

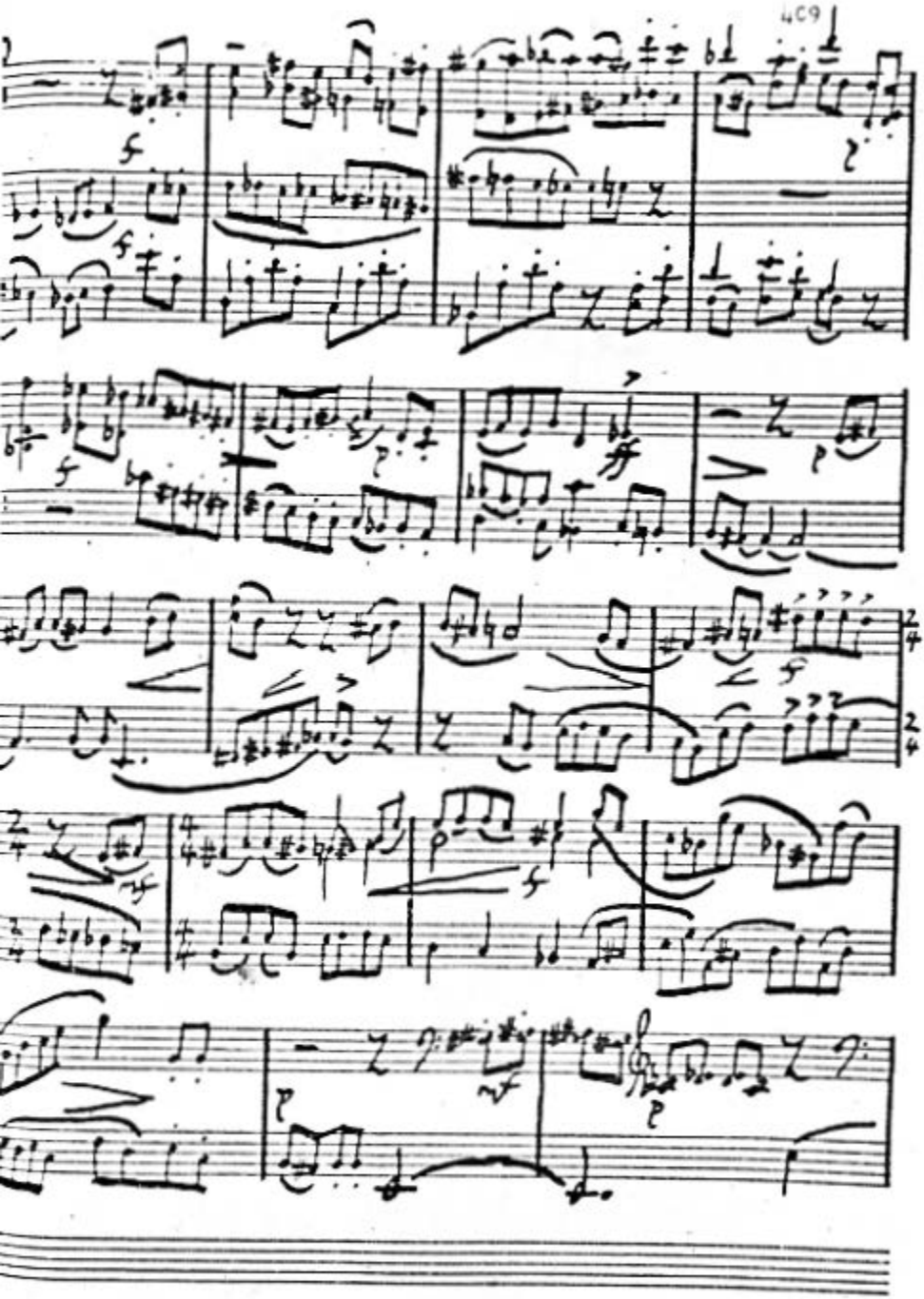
cresc.

poco dim

poco rit.

III.

Allegretto


f
f
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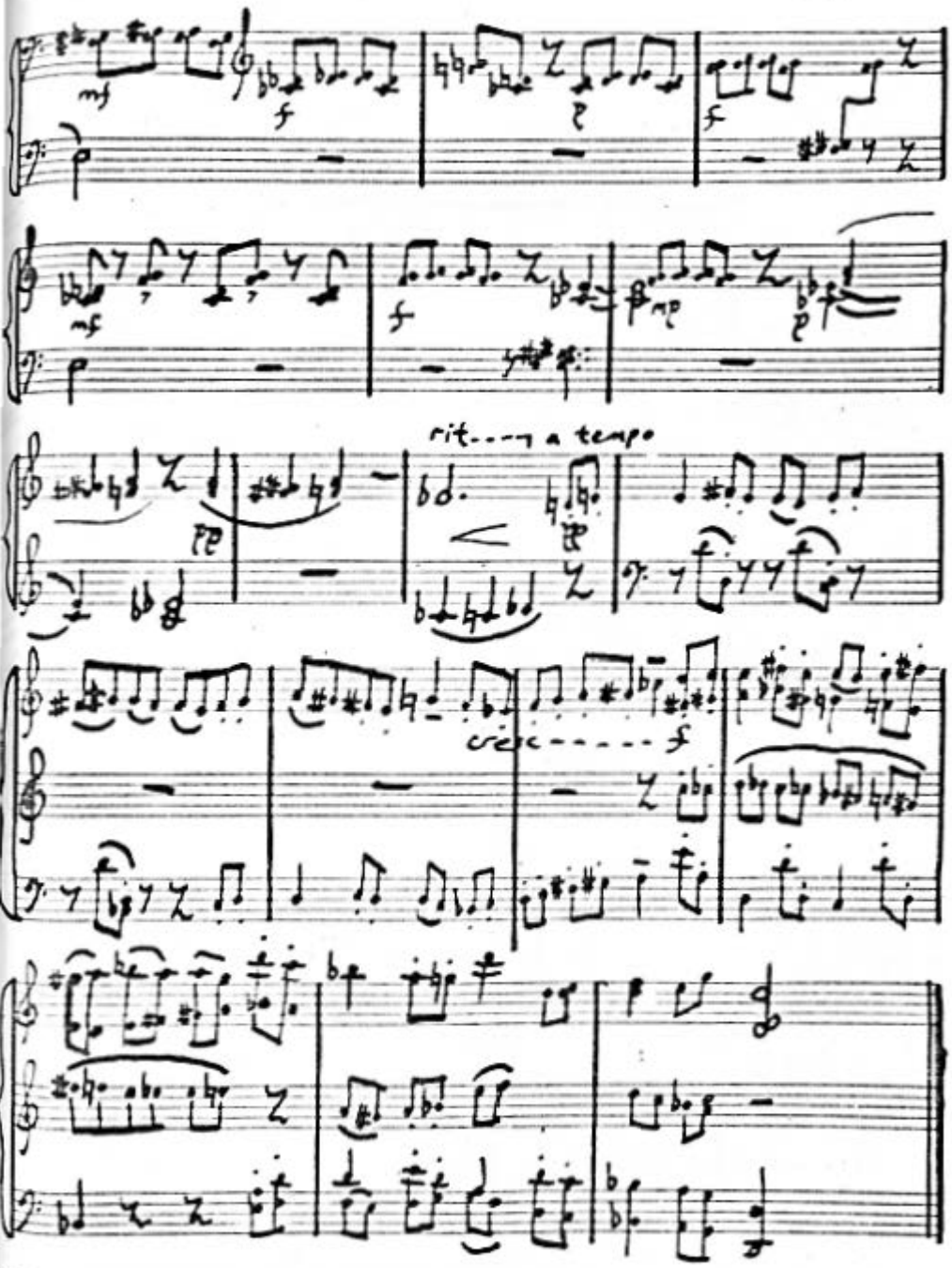
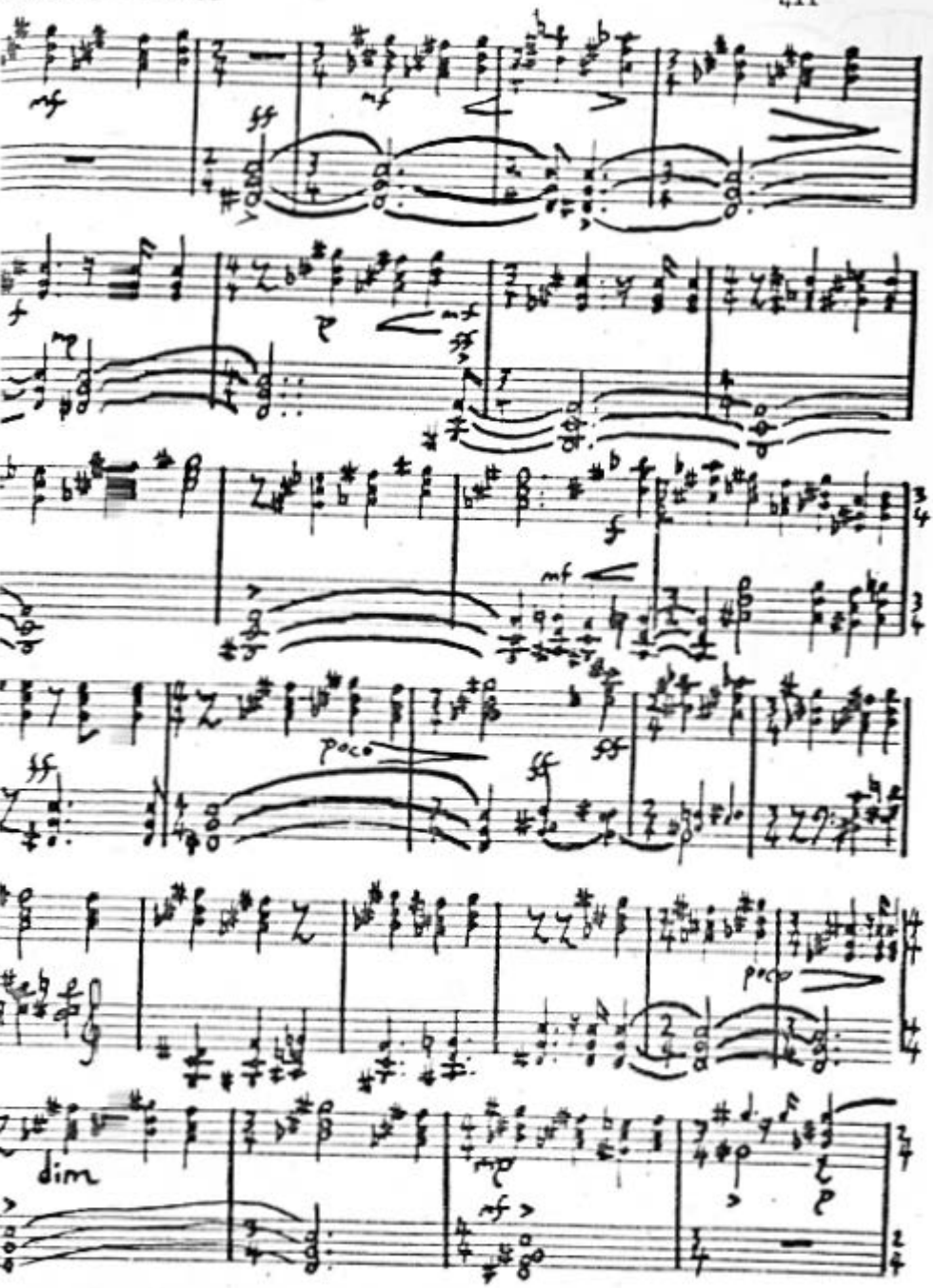
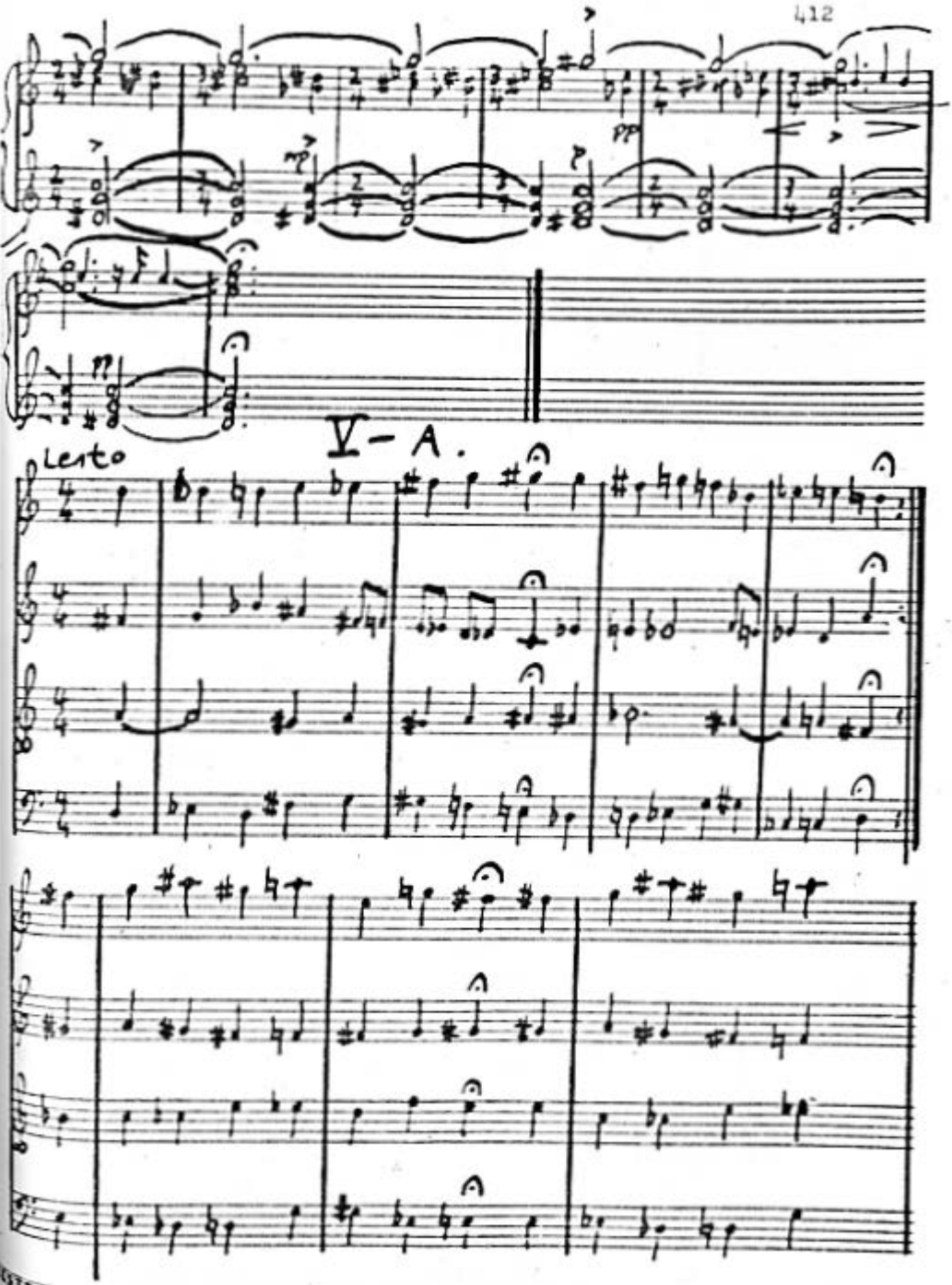
mf
f
mf

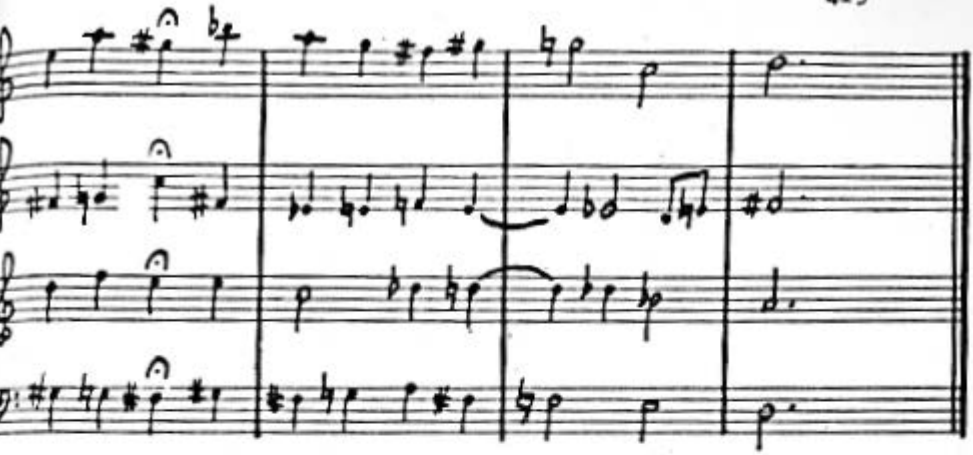


ESTRO
PLAIN


mf
ff
mf
f
mf
ff
f
mf
ff
poco
ff
ff
poco
dim
mf
mf

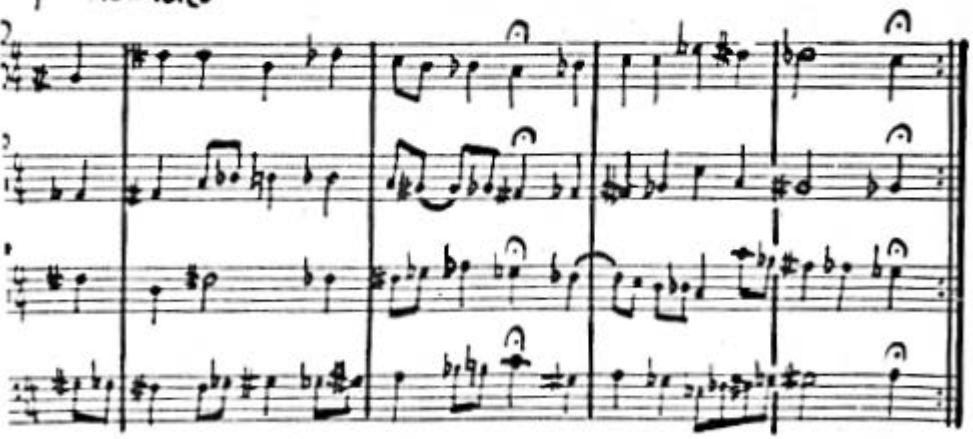


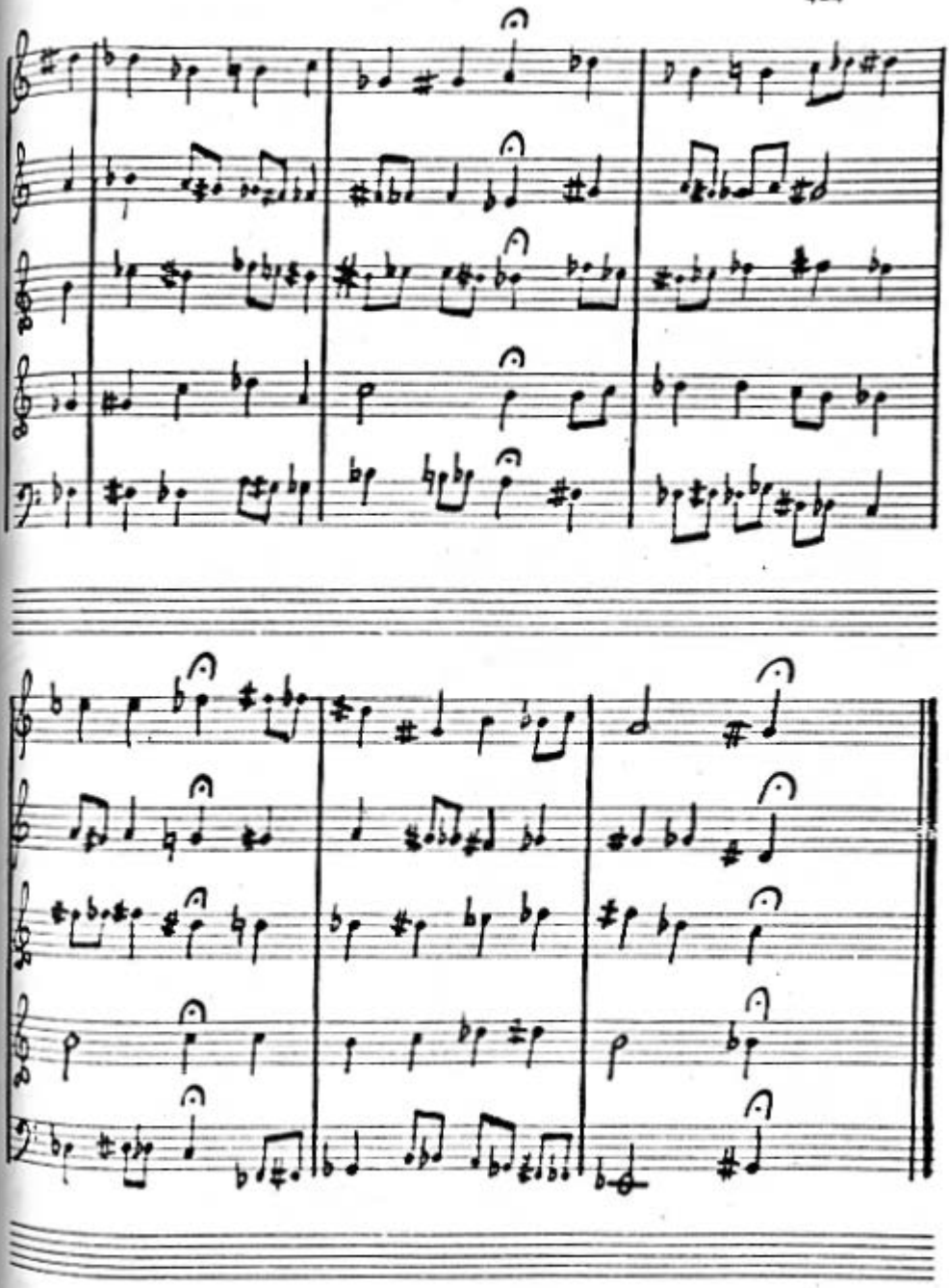
ESTEROL
TI-PLAIN



V - B.

Un poco meno lento





VI.

Moderato espressivo

f p mf

mf espressivo

3 mp f mf mf

mp

cresc. ff

accel

cresc

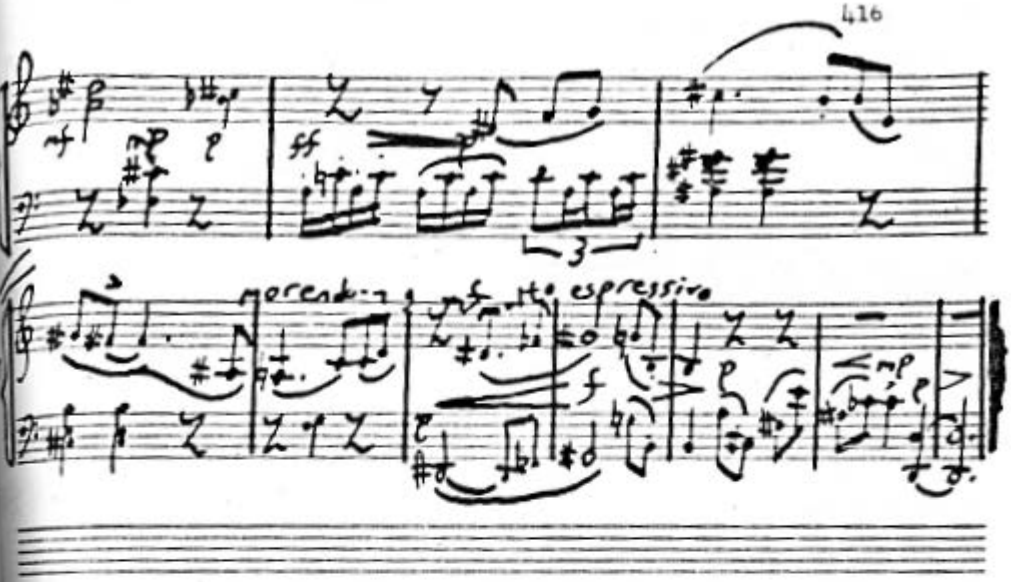
ff

accel

a tempo

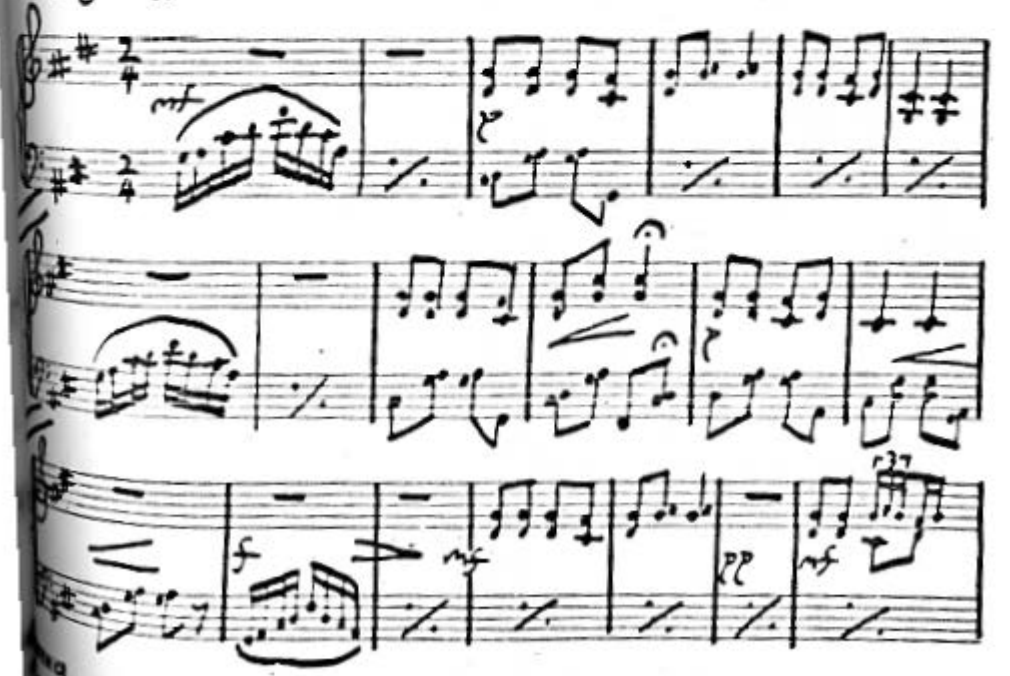
3 3

mf ff



VII.

Allegretto



Poco rit. a tempo

ritardando

f

mf

f

rit.

Molto moderato

mf

ff

VIII.

Allegro moderato

cresc.

mf

mp

cresc.

dim.

cresc.

mf

mp

simile

cresc

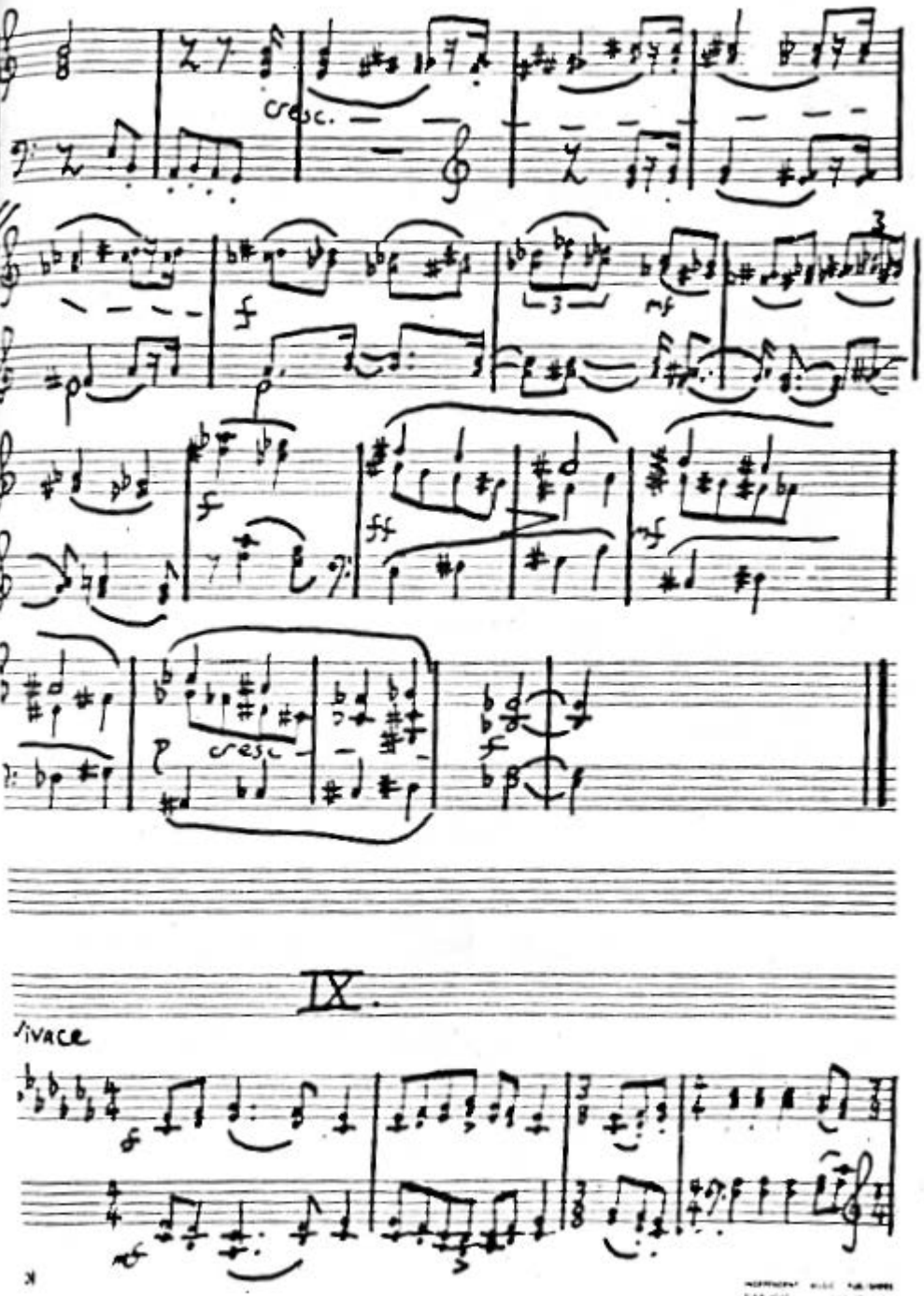
f

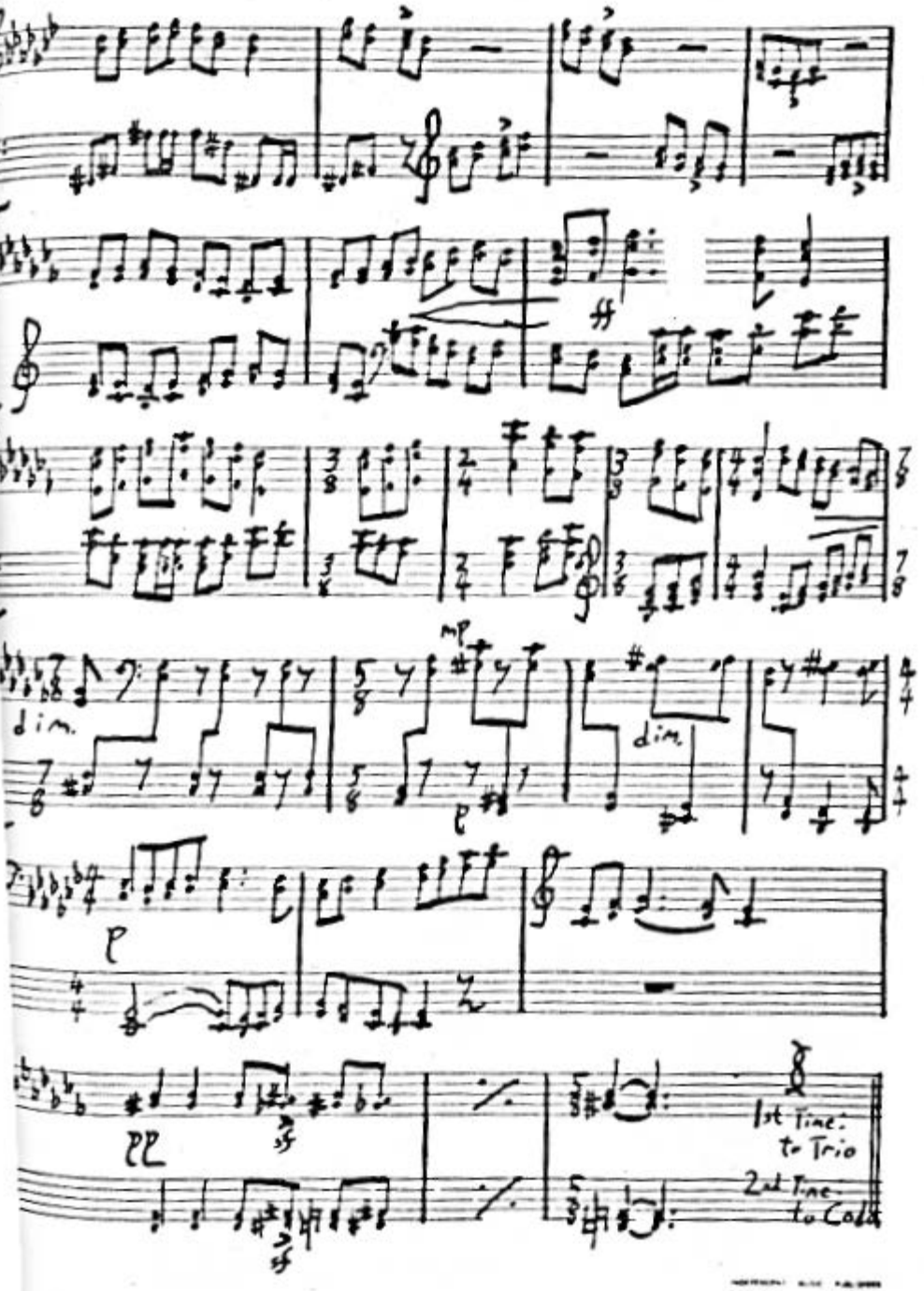
dim.

f

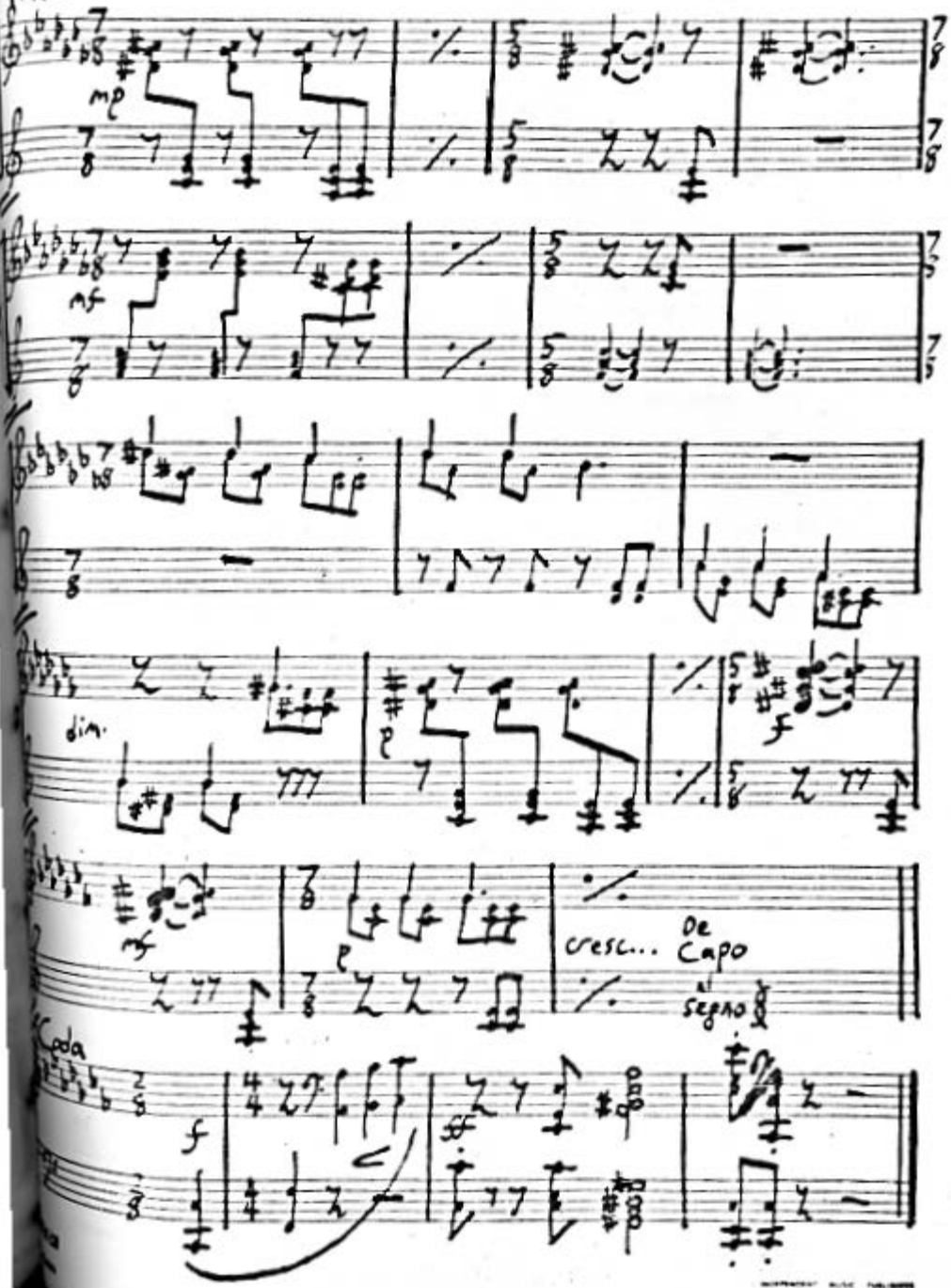
ff

TRO
PLAIN


cresc.
f
mf
ff
mf
cresc
f
IX.
vivace
f
mf


ff
dim.
mp
dim.
pp
sf
sf
1st Time: to Trio
2nd Time: to Coda

TRIO



BRIEF ANALYTICAL NOTES

Number I: This Prelude makes full use of the interval embodying the "golden ratio."[1] The motivic interval C-Fb is based on Kornerup's assertion that 7/19 is almost exactly .382 of an octave. The interval Fb-C is larger than its inversion by almost precisely the same ratio by which the octave is larger than Fb-C. In the ensuing measures C is made the root of a major triad, and Fb of a minor triad, in which also is present G# which forms, with C, another "golden" interval. In measure 6, the two "golden" tones are permitted to associate freely, and a simple tonal theme in C# major evolves. Of interest are the notational difficulties in measures 7 and 8. If for "Fb" and "Cb" are read "E#" and "B#" respectively, the musical sense is much clearer. The performing pianos, however, are so arranged that all "flats" are sounded by white keys of the same letter name, and all "sharps" by the black keys. There is therefore no "E#" key, and the "Fb" which is its enharmonic equivalent is sounded by the "F" key on one of the two pianos.

In measure 9, a modulation is made to Db minor, using the common third Fb as a pivot. It is Ariel who suggests the validity of the "Terzendifferenz" as the basis for a single unit of the temperament. Measure 10 demonstrates one of the curiosities of 19-tone temperament. By alternating

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[1] For a discussion of the "golden ratio" see Chapter 9. It has been suggested that this prelude be titled "The Golden Mean Justifies the Ends."

Bb with B while Db, Fb, and G# are repeated, two different diminished sevenths are sounded. When they are given the following spellings they become identical pyramids of minor thirds: Bb-Db-Fb-G#; and Db-Fb-G#-B. There is absolutely no comparable phenomenon in 12-tone temperament.

In measure 11, Fb becomes a temporary tonic; A and C are the two new "golden" tones.

In measure 14 a free middle section is begun, with quasi-cadenza passages based on cyclic pyramids of some of the intervals. There is a circle of 16 fifths to Eb in measure 19. The "golden" progression is then sounded between Eb and G#. Measure 23 is a run of 20 consecutive descending major seconds. Instead of a repeating 6-tone scale, 19-tone temperament produces a 19-tone cycle of major seconds. A re-transition is provided by the next four measures, built entirely on superimposed minor thirds; a total of 13 from the low F# to the high B. In the process, no tone is used twice.

The recapitulation is begun 1/19 lower, so that the emerging "theme" will be in the "tonic," C. In the final four lines, traditional romantic harmony is interspersed with instances of the "golden" progression.

<u>Number II</u>: In this miniature, simple traditional triads, most of them minor, appear in unexpected relationships to one another. The "golden" progression is present in the often repeated juxtaposition of Ab minor and E minor.

Note that the "Neapolitan sixth" in measure 14 is on the raised tonic rather than the lowered supertonic. Note also the illusory effect produced by two consecutive downward progressions by 1/19 as at the very beginning. The interval between E and the D# of the third chord seems much greater than it really is.

Number III: This is a rather free composition based on the "13 plus 6" scale group outlined in Chapter 14, above. The major second is the generating interval of much of both the melody and harmony. Where further subdivisions of this interval play a large melodic role, it is the intervals produced by two or more major seconds which assist in the harmonic structure. Much of the prelude is in two-part style, but a contrasting full section beginning in measure 5 presents, among other things, a 13-tone "chromatic" scale in an inner voice.

A middle section begins in measure 12 in E. As in the first prelude, the generating interval is also employed as the chief interval of modulation. In measure 21, a touch of C, the "sub-supertonic" or "sub-generator" is employed. From measure 22 to measure 29 a single chain of superimposed seconds is exploited. The harmony, especially where it thickens, is suggestive of Yasser's hexads. The piece ends with an abbreviated statement of the main subject.

Number IV: This is a neo-Impressionistic tone-painting, suggestive of morning bells and perhaps sunshine.

A single melodic line, with an occasional second contrapuntal part, is reinforced in parallel harmony by a series of sonorities unique to 19-tone temperament as they make use of the interval 4/19. A very "dark" minor and a very "bright" major result. The clashes between harmonies oriented to C and C# seem to be highly suggestive of bells. Otherwise the greatest influence on the structure of this piece is probably the Bartok of the Mikrokosmos.

Number V: The two "Chorales," A and B, are purely intellectual exercises, rigidly following the "laws" of chord construction of the 13- and 11-tone scales in pieces A and B respectively. Unlike the four preceding preludes, the two "Chorales" offer a complete break from the harmonic practices of the present and past. They present, in simplest form, some of the progressional possibilities inherent in 19-tone temperament.

In piece A, the vertical sonorities are generally produced by superimposed major seconds. A fifth and/or sixth part might well improve the sound, as Yasser suggests. In piece B, the generating interval is 7/19, "golden," but dissonant. Where convenient the perfect fifth has been added, especially at the end of phrases. The final sonority is typical: over the root, G#, are a perfect fifth, a "high" third (7/19 above the root), and a "low" third (4/19 above the root). This "low" third is 7/19 below the fifth, creating a symmetry of sorts.

Number VI: This represents a first probing of what appears to be a particularly promising line of inquiry. It is entirely serial, and built on the 12-tone row represented by the first twelve melodic pitches. The seven "flat" tones are withheld and serve as a field for modulation. The 19-tone resources thereby permit serial dodecaphony with modulation.

With the last tone in measure 4 begins an inversion of the row. C# is the only tone with which the inversion could begin without calling into use pitches outside of the original row. Thus a feature (and it seems to me to be an advantageous one) of serial dodecaphony within a 19-tone field is that the "home" or "tonic" position of the inversion is inherent in the row rather than a matter open to arbitrary choice.

In measure 9 a variation begins; the original row is in the bass, the inversion in the treble. Modulation begins in measure twelve, with the original row transposed downward by 3/19. The new tones are Bb and Eb, the latter appearing in the lower voice as it completes the row, starting with the 7th tone (D). A modulating sequence presents the original row beginning on A (measure 13), and involving one "foreign" tone, Fb.

A second variation offers the untransposed original (beginning with the last tone in measure 14) in the lower voice, against which the inversion, transposed down a major

second, is presented. The D# in measure 16 represents a possible alteration of the row. The actual pitch called for by the row is Eb. The upper voice here replaces the called for pitch with a "tonal answer," the nearest available (unused elsewhere in the row) pitch in the "key" of the moment. This alteration leads to a rather strong tonal feeling (B major) at the moment in which it occurs.

The D# in measure 19 is both the final tone of a row and the first tone of a new row, a direct transposition of the original. The C in the upper voice at the end of the same measure begins a retrograde inversion of the row. The reader interested in completing a row analysis can, I am sure, do so without further aid. Note that in order to finish on the starting tone, D, a transposed version of the inverted row is used at the end with two "foreign" tones. The melodic and rhythmic contours are the same as in measures 5-8 wherein the inversion was first sounded. Prelude number VI suggests a possible path toward the achievement of what Yasser calls the synthesis of tonality and atonality through "supra-tonality."

<u>Number VII</u>: Several writers on multiple division have spoken of the quasi-equal-tempered pentatonic scale as a possibly desirable goal. Prelude number VII is an <u>Orientale</u> employing such a scale.

The greatest structural problem in this piece is sustaining interest with such a narrow range of materials. I

have attempted to do this through the modulations. As the interval 4/19 is the generator of this scale, the modulation at measure 29 is from C to D#. At measure 43 a new and distant key, containing only the common tone G# with the preceding key, is introduced for contrast.

Number VIII: This prelude is written with the instrumental media particularly strongly in mind. Its basic structure is provided by a dialogue between the two pianos, one of which plays only "flats" and "sharps," and the other only "naturals" and "sharps." The prelude might be said to be in a kind of sonatina form with coda. In the recapitulation, the second theme first appears in a new key, G (measure 46), then reverts back to its own original key, C. In the quasi-developmental coda, the two modes as represented by the two pianos are brought into closer apposition, with the progression that twice had been used to reach distant keys (C and G) finally returning to the original tonic, Eb. The second piano is, however, symbolically present in the final sonority with the G natural.

Number IX: Yasser is of the view that there is essentially no such phenomenon as bi-tonality; that what is supposed to be bi-tonality is more accurately a kind of presage of a more complicated kind of single tonality. In this piece the white keys of the two pianos supply the materials for a rather dissonant counterpoint which is not altogether

unpleasant. Neither C nor Cb is ever able to establish itself as sole tonic; the contest must end as a draw.

Note the rather odd spelling for the D♯ major triad in the Trio. Perhaps it would be simpler to call Gb "F double sharp."

In the coda, the "golden" formula of Kornerup is again alluded to. The octave is broken down into its "golden" components, a perfect fourth, a minor third, a tone, a diatonic semi-tone, and a chromatic semitone. This represents the descending form of a Fibonacci series (8:5:3:2:1). The last sonority of measure 71 is comprised of the two "golden" chords of the first prelude. Here the upper component has a dual function, for Fb is also the plain subdominant of Cb.

SUPPLEMENT: THE GENERALIZED KEYBOARD

The literature on multiple division abounds in illustrations and descriptions of instruments producing more than twelve sounds per octave. There are the famous historical examples: Vicentino's 31-tone Archicembalo; the 19-tone "Clavicembalo Universale" described by Praetorius; the special instruments built under the instructions of Zarlino and later of Mersenne. There are the numerous efforts of 19th century theorists to build the perfect instrument for just intonation. The methods of tuning these instruments varied, as did the means of tone production and the design for the keyboards. The 20th century has added its own vast number of instruments with more than 12 tones to the octave. There is the collection of quarter-tone instruments dating from the first three decades of this century. There is Harry Partch's arsenal of stringed and percussion instruments.

Many of these instruments can be found in various museums around the world. Detailed studies of their operating principles and the state of their preservation would be extremely useful. It is beyond the scope of this paper to make such a detailed study of the instruments of multiple division. However, because of its direct bearing on the central problem of reducing the difficulty of performing music involving many tones to the octave, a brief consideration of a small group of instruments based on a single principle of keyboard design is here appended.

The principle of the generalized keyboard is essentially simple. The keys must be arranged in such an order that for any given sonority, regardless of key, a single basic position for the hands is employed. Attempts have been made, most notably by Paul von Jankó around 1900, to incorporate this principle into pianos for everyday use. The widespread technical mastery of the regular piano keyboard has rendered such an instrument unnecessary. However, the complexities of multiple division would probably make the luxury of a keyboard as inefficient as that of the piano impossible.

In order to achieve a generalized keyboard three fundamental principles must be observed, none of which is applied to our present-day instruments. (1) The keys must be arranged in an absolutely regular pattern from bank to bank (they would probably best be of uniform shape as well). (2) The succession of keys along any bank or in any given direction must involve always the same interval. (3) There must be a substantial number of extra keys in order that any pitch in the system can be sounded by any part of a hand configuration.

Three keyboards of actual instruments will be examined. In each, the interval between adjacent members of a bank of keys is a whole-tone of the system in use. Example 61 is a diagram of the keyboard of a 19-tone electric organ built by Tillman Schafer and James Piehl during the late 1940's. The arrangement of keys is reminiscent of the typewriter keyboard,

a keyboard which has also fascinated Partch.[1] There are five parallel rows of keys, separated from one another by a single unit of the temperament, 1/19. Consecutive keys in a given row are separated by a whole-tone or 3/19. The arrangement of black and white keys is based on the hexads of Yasser's supra-diatonic scale, 12 of the 19 tones in each octave being white. The 7 black keys form a scale equivalent to our present diatonic scale. The basic hand configuration for the diatonic scale is the same, no matter on what tone it is begun. It will be further observed that this basic pattern remains essentially unchanged on the 31- and 53-tone instruments to be examined later.

Assuming the black keys in Example 61 to represent the scale of C major (for purposes of illustration--in actuality they do not), the key at the upper left-hand corner is D. The scale is formed by proceeding horizontally for whole-tones and diagonally for semitones. It should be pointed out that the two different diagonal directions involve different intervals. Consecutive tones along the diagonal from the top left are diatonic semitones (2/19) from one another, while consecutive tones along the diagonal from the bottom left are a full 4/19 from one another.

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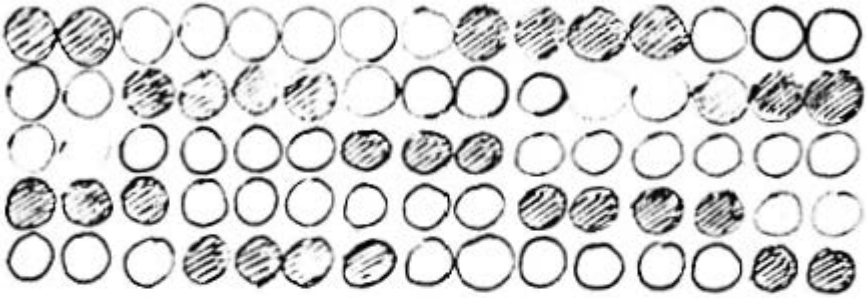
[1]Partch has temporarily abandoned experiments involving the use of an instrument with a keyboard resembling that of a typewriter. In Genesis of a Music, however, he compares favorably the efficiency of the keyboard of the typewriter (the instrument which any stenographer can learn to play in six weeks) with that of the piano.

Example 61 shows the full depth of the keyboard (five rows) but only a cross-section of its full length, which is about four octaves. All of the tones could be represented by three banks (rows) of keys, but the extra two rows are essential to the generalization of the keyboard, as they permit any standard combination of sounds to be produced in any key by a single configuration of the hand.

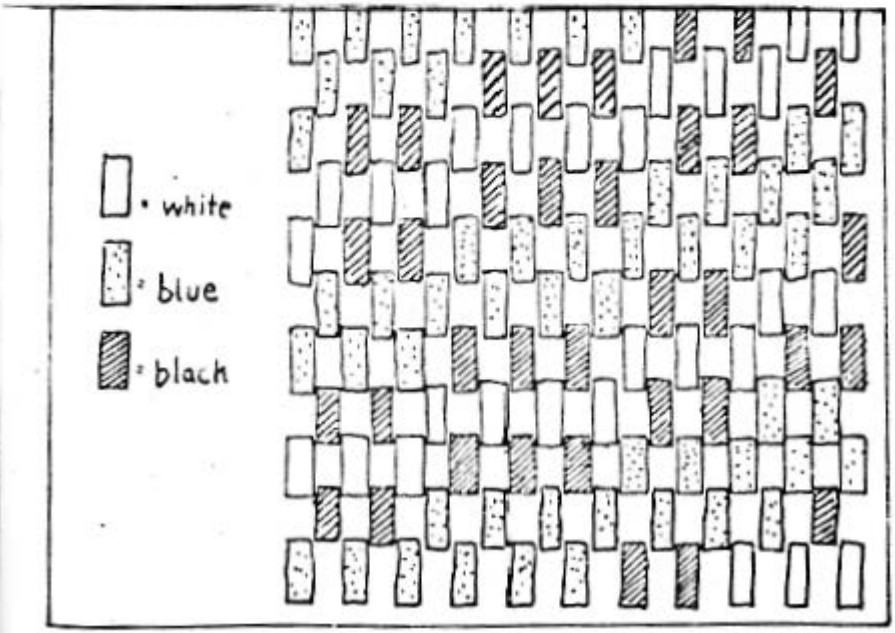
Example 62 shows a similar cross-section of one of the two identical manuals of Fokker's 31-tone organ in Haarlem, built during the middle 1940's. This instrument has received the most widespread use among the modern multiple division instruments. A special console with traditional 12-tone keyboard is also attached to the pipes of the instrument for the rendering in meantone temperament of ancient music. The main console, shown in Example 62, contains 11 banks of keys on each manual. The white keys form the diatonic scale of C major, just as they do on the piano. It will be noted that Fokker's keyboard differs from Schafer's in that the ascending diatonic semitone uses the upward instead of the downward diagonal. The result is a kind of mirror inversion of the pattern Schafer uses.

Unlike Schafer's keyboard, in which the keys are placed on parallel planes perpendicular to the rows of keys, Fokker's keyboard places the keys in alternate rows diagonally in such a way that a vertical line of keys (that is to say a line of keys proceeding perpendicular to the rows or banks)

Example 61: Schafer and Piehl's 19-Tone Keyboard--Cross-Section



Example 62: Fokker's 31-Tone Keyboard--Cross-Section

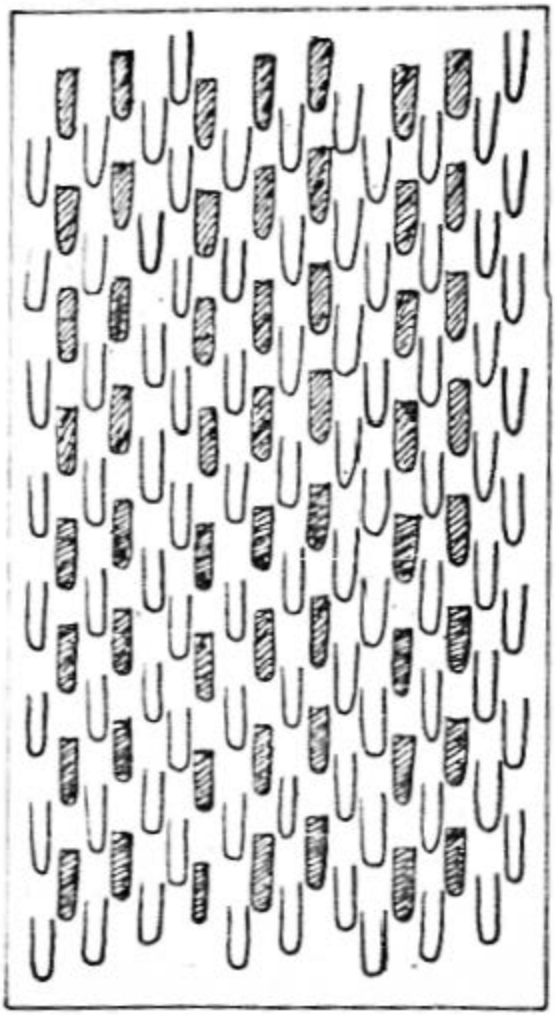


uses alternate rows only. Such a vertical line gives the smallest interval, 1/31. The rising diagonal yields the diatonic semitone, 3/31, while the falling diagonal yields the chromatic semitone, 2/31. The interval between consecutive tones on a given bank is a whole-tone, 5/31.

The black keys represent each sharp and flat currently obtainable on the black keys of the piano keyboard. There are ten black keys in each octave, since there are separate keys for F# and Gb, etc. The fourteen keys in each octave which are neither black nor white are blue; they represent the tones which lie between the tones of the present piano. The 11 banks of keys offer ample alternative fingering possibilities for complete generalization. Five banks would be adequate for the single rendering of each tone.

Bosanquet's keyboard, shown in Example 63, some 70 years earlier than the other two, is the most complex of the three, but the principle is much the same. Where the other two contain level banks of keys in which the tones are a whole-step apart, Bosanquet's whole-tones form a slanting row. This is the result of recessing each fifth slightly (the whole-tone has twice the recess of the fifth). After twelve fifths the recess is sufficient that a new bank of keys is begun. As the fifths are almost exactly pure in 53-tone temperament, the next higher bank sounds one comma higher than the first bank or, more precisely, 1/53 higher. The result is rather like the Fokker system slightly tilted

Example 63: Bosanquet's 53-Tone Keyboard--Cross-Section



and inverted (as in Schafer's instrument, the diatonic semitone is achieved by a downward diagonal). The downward diagonal yields the diatonic semitone (4/53 in Bosanquet's Pythagorean system), while the upward diagonal yields the larger chromatic semitone (5/53). The row of whole-tones (9/53) is itself a gently sloping diagonal.

Counting every sloping row of semitones as a separate bank, there are about 16 banks of keys at most parts of the instrument (9 would suffice for every pitch), providing for a high level of generalization. The sloping character of the whole-tone rows is particularly efficient for diatonic scale-production, for in completing such a scale the hand remains at a constant distance from the edge of the keyboard nearest the player.[2]

Example 63 shows a cross-section of Bosanquet's keyboard. The seven-to-five ratio of white keys to black is retained; comma alterations do not change the color of the keys. Bosanquet appears to have seven sets of twelve keys to the octave, 84 in all. By an effective arrangement, however, Bosanquet has an even greater number of keys per octave in his middle registers.

It should be added that on Bosanquet's instrument (and on Fokker's as well) as the keys are recessed from the player

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[2]In contrast, the ascending diatonic scale on Fokker's instrument brings the hands away from the player (and eventually off the keyboard unless the player leaps to a new position), while a similar scale on Schafer's instrument brings the hands toward the player.

they are elevated. In the case of Bosanquet's instrument this means that no two keys within an octave are at precisely the same elevation.

The three examples shown in this short supplement offer, it seems to me, the best basis for a starting point in the performance of tempered music using more than 12 tones per octave. It would be quite possible to use the more complex instruments, such as Bosanquet's or Fokker's, for the simpler musical systems such as Schafer's, although the reverse, of course, would be impossible. A keyboard of about the complexity of Fokker's could be tuned to yield a more complex temperament, such as 53-tones to the octave, without any essential changes in the hand positions for the approximations of given just sounds. However, the use of a keyboard for a more complex system than that for which it was designed will result in the sacrifice of some or all of the generalization. Certain chords can then no longer be transposed to certain keys without a radical change in the position of the hand.

The School of Music of Indiana University has just purchased the equipment to build an instrument with up to 60 different (variable) pitches to the octave. I strongly recommend that a generalized keyboard be built for this instrument along the lines of Bosanquet's or possibly Fokker's. The successful completion of such an instrument would represent a valuable contribution to the mastery of the complex field of multiple division by offering the opportunity to many

students to experiment with all of the recommended tuning systems. Such an instrument, for maximum effectiveness, should possess at least 90 keys to the octave.

GLOSSARY

Area of complexity.... used in Chapter 13 to group together tempered systems possessing approximately the same number of tones. There are four such areas considered, with 24, 36, and 53 tones representing the maximum number of tones in each of the first three areas.

Autochthonous.... a term used by Fokker and others in referring to already existing Folk music.

Average Deviation.... a statistical measurement of the differences between two musical tuning systems computed by measuring the intervals between each of the tones of each system and a central tone, finding the differences between each interval of one system and the comparable interval of the other system, and taking the average of these intervals. The method is applicable only where the two systems have the same number of tones to the octave, and is meaningful only where one system is intended as a representative of the other, as when a tempered system is proposed as the representative of a just system.

Cents.... in music this term is used in the measurement of intervals. Cents are computed by multiplying the logarithm of the interval-ratio by a constant approximately equal to 3986.3, in order that an octave contain exactly 1200 cents; each semitone in 12-tone equal temperament thus contains 100 cents. Cents are

used throughout this paper as a means of measuring intervals, since they enable the reader to relate most easily the relationship between the interval discussed and the intervals of 12-tone temperament which are the most familiar to him.

Combined error factor.... a concept used in Chapter 13 in which the errors of intervals are measured both in terms of absolute size and in terms of how large they are in relation to the size of each unit of the system. The combined error factor is obtained by multiplying the size of an error (measured in cents) by the fraction whose numerator is this same error and whose denominator is half the size of one unit of the system. Where E represents the error of an interval measured in cents, and n is the number of tones in a tempered system, the combined error factor can be represented by the formula

$$\frac{E^2 n}{600}$$

Constructing interval.... a just interval which is made theoretically equal to one unit of a tempered system. Such an interval, in 12-tone temperament, is 25:24. (from Wörschmidt)

Defining interval.... a just interval of small size which is made to equal zero in the establishment of a tempered system. Wörschmidt uses this term because he defines

each tempered system by the largest just intervals which must be considered equal to zero. In 12-tone temperament it is the various dieses which are the largest defining intervals.

Departure.... Bosanquet's term for the difference between a given just interval and a specified tempered interval used as a point of reference. The departure of the just fifth is about two cents (in the 12-tone tempered system).

Difficulty factor.... in the selection and evaluation of tempered systems, the role played by the difficulty in recognizing and performing all of the tones, hence determined by the number of tones in the system.

Ditonic comma.... The Pythagorean comma, the difference between 12 fifths and 7 octaves; about 23.5 cents, or 2 cents larger than the syntonic comma. The term "comma" when used alone refers not to the ditonic but to the syntonic comma.

Dodecaphony.... the term applied to serial 12-tone composition by the Schönberg school and its successors.

Error.... Bosanquet's term for the difference between a given tempered interval and a specified just interval used as a point of reference. The error of the fifth in 12-tone temperament is about two cents.

Fibonacci series.... a series of numbers so arranged that any term represents the sum of the two preceding terms. An example is 1,1,2,3,5,8,13,21,34....

Golden ratio.... the numerical ratio between two terms such that the same ratio exists between the larger term and the sum of the two terms. $\frac{a}{b} = \frac{b}{a + b}$ By cross-multiplication, $b^2 = a^2 + ab$. Dividing both sides ' b^2, $\frac{b^2}{b^2} = \frac{a^2}{b^2} + \frac{ab}{b^2}$. Simplifying, $1 = \frac{a^2}{b^2} + \frac{a}{b}$. Deducting 1 from both sides, $\frac{a^2}{b^2} + \frac{a}{b} - 1 = 0$. Considering (the ratio) to be equal to $\frac{a}{b}$, $w^2 + w - 1 = 0$. $w = .618$.

Golden tone system.... Kornerup's "ideal" musical system in which the intervals of different sizes, diesis, chromatic semitone, diatonic semitone, tone, minor third, and perfect fourth, relate to the next larger interval by the golden ratio .618...

Infra-diatonic.... Yasser's name for what he alleges to be the musical system from which ours is essentially evolved. It consists of a five-tone scale with two additional auxiliary tones.

Mean deviation.... a concept used by Barbour in evaluating various tuning systems according to how close they are to equal temperament. The difference between each constructing interval of a system and a unit of the equivalent tempered system is taken, and the average found.

Meantone temperament.... in its narrowest sense that particular temperament in which the fifth is made smaller by exactly 1/4 of a syntonic comma so that the tone

represents the mean between 9:8 and 10:9. In the general sense that the term is used in this paper, meantone temperament is any regular but non-cyclical system wherein the fifths are made smaller by from 1/3 to 1/6 of a syntonic comma.

Negative systems.... Bosanquet's term for regular systems whose fifths are smaller than 700 cents (the size of the fifth in 12-tone temperament).

Ornamental scales.... scales based on the arithmetical division of a string or pipe. Such a scale might have as consecutive tones, $\frac{1}{12}\ \frac{1}{11}\ \frac{1}{10}\ \frac{1}{9}\ \frac{1}{8}\ \frac{1}{7}\ \frac{1}{6}$.

Positive systems.... Bosanquet's term for regular systems whose fifths are larger than 700 cents (the size of the fifth in 12-tone temperament).

Pythagorean tuning.... any system composed entirely of just perfect fifths.

r.... Bosanquet's symbol for the order of a regular cyclical system, representing the number of units of the system by which 12 perfect fifths (as represented in the system) exceed or fall short of 7 octaves.

Regular cyclical system.... an equal-tempered system. (from Bosanquet)

Regular system.... a system whose perfect fifths are always of the same size. If, as in meantone or Pythagorean tuning, the tones at the two ends of such a chain of equal fifths do not themselves form a fifth of the

same size, the system, although regular, is not cyclical. (from Bosanquet)

senario.... the numbers from 1 through 6. According to theorists representing much of the best writing on music for several centuries, all true consonance in music resides in intervals formed by ratios between numbers in the senario.

senarist.... a person who believes all musical consonance to derive from the relationships of numbers in the senario.

Small-number ratio.... A ratio between two rational numbers. There is no exact limit to what can be considered a "small" number in this sense, but the implication in the use of this term is that the ratio will involve numbers sufficiently small that a perceptible sense of consonance will result from the simultaneous use of two sounds whose vibration frequencies are related by such a ratio.

Superparticular ratio.... a ratio involving consecutive integral numbers such as 16:15, 81:80, 3:2.

Supra-diatonic.... a term used by Yasser to describe what he considers to be the next musical system in an evolving series. The supra-diatonic scale is a 12-tone scale and 7 additional auxiliary tones complete a 19-tone system.

Syntonic comma.... the interval 81:80, or 21.5 cents; the difference between the just third and the third formed by four fifths. Where the term "comma" is used without further explanation, it may be assumed that the reference is to the syntonic comma.

Tricesimoprimal.... relating to the number 31. Fokker uses this term in reference to music using 31 tones to the octave.

12-tone system.... the twelve-tone equal-tempered tuning. In this paper, the expression "12-tone system" does <u>not</u> refer specifically to the procedures of the serial technique of Schönberg's disciples.

Unit.... the smallest interval in any given tempered system.

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