CHAPTER 2

HISTORICAL PRECEDENTS FOR MULTIPLE DIVISION

"Greek music of the best period . . . was a thing of beauty unsurpassed and unattained in any subsequent age."[1] Opinions of this kind lead their holders to seek out ways of reestablishing the beauties of bygone eras. The alleged beauty may be of the music of ancient Greece, of the Renaissance, of the Baroque, or of a non-Western culture. In general the pattern of the arguments is the same. The older system gave way to newer systems not owing to any inferiority of its intervals or sonorities. Quite the contrary, the older system gave way to newer, coarser systems because the older system was not flexible enough to satisfy the expanding demand for harmonic and modulational resources. Thus, the argument continues, a better intonational system was lost because musicians of an earlier epoch were incapable of adding the number of extra tones to the octave which might have been necessary to preserve the advantages of their intonational systems while expanding harmonic resources. Today, however, with our vastly increased technical resources, it would be possible to recreate the beauties of earlier intonational systems without sacrificing the present modulational possibilities demanded by musicians. By this line of reasoning, multiple

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[1] Perrett, _op. cit._, p. 7.

division becomes a potential time-machine, bringing back the lost intonational subtleties of the past to be enjoyed with or without the full sophistication of the present. Chief among the systems of the past that recent advocates of multiple division have singled out to imitate have been the harmoniae of the ancient Greeks, the just intonation of the Renaissance, and the meantone systems of the late Renaissance and the Baroque.

THE GREEKS

Musical theory today, in studying the theory of the Greeks, seems to be interested primarily in the Greater-Perfect System and the network of modes which theorists of the Christian epoch tried unsuccessfully to imitate. As far as intonation is concerned, the Greek mathematician and theorist Pythagoras is recognized for his system of tuning by perfect fifths, a practice still widely adhered to today. Pythagorean intonation is, as Partch points out, a very direct ancestor of 12-tone equal temperament, a temperament in which the fifths are nearly pure at the expense of the other small-number intervals.[2] But while large numbers of theorists have concerned themselves with the diatonic modes and with Pythagorean intonation in Greek music, a smaller group has concerned itself with a much thornier area in

[2] Partch, *op. cit.*, p. 280.

Greek musical theory: the genera, enharmonic and chromatic as well as diatonic; and the tuning systems enumerated by Ptolemy as part of the Greek practice.

The neatness of Pythagorean systematization is weakened when applied to the chromatic genus, and Pythagorean tuning breaks down completely when an attempt is made to apply it to the enharmonic genus. In standard explanations the chromatic tetrachord is diagnosed as "semitone, semitone, sesquitone."[3] But in Pythagorean tuning the size of the "semitone" is not precisely half the size of the tone; it is smaller. It is with the second semitone that the difficulties of the Pythagorean system commence. If both semitones are 90 cents (the recognized size of the Pythagorean semitone) then the remaining interval in the tetrachord must be 316 cents, the interval subtended by no less than 9 fifths. This is far less economical than the diatonic tetrachord which requires but 5 fifths. The seven-tone diatonic scale is produced from a series of six contiguous fifths. The seven-tone chromatic scale requires twelve fifths and is not a self-contained completed system when produced by fifths, as is the diatonic scale.

[3]In order to conform to the pattern established elsewhere in this paper, I am counting from the bottom up and hope that this will not cause too much confusion in this chapter since, with the music of the Greeks, it is more customary to count from the top down.

The Pythagorean's problems are compounded when he tries to explain the enharmonic genus as based on fifths alone.[4] The standard explanation of the enharmonic genus is "quarter-tone, quarter-tone, di-tone." How large is the quarter-tone to be? Assuming, for a beginning, that the large interval is to be the usual size of the Pythagorean third, each of the quarter-tones would be 45 cents. This interval, however, does not exist within the Pythagorean series, although a fair approximation is offered by the double-comma (about 47 cents) subtended by 24 fifths. An approximation is, however, against the precise spirit of the Pythagoreans. If the large interval is to be the size of the third produced when 8 fifths are deducted from 5 octaves, 384 cents, then each quarter-tone must be 57 cents in size. This interval is also not produced within the Pythagorean system. Therefore the enharmonic genus cannot consist of equal quarter-tones produced by the Pythagorean process.

Evidence seems to suggest that two radically conflicting views on musical intonation existed side by side for quite some time; the one was Pythagorean and excluded all genera other than the diatonic; the other was based on

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[4]Perrett states, "The problem which the Pythagoreans failed to solve was how to divide a major semitone melodically and harmonically." He adds, "The correct solution gives us an 'open sesame' back to the world of genuine and beautiful Greek music." op. cit., p. 11.

various combinations of small-number ratios and included all three genera as well as hybrids of them.[5] Besides Pythagoras, Aristoxenos of Tarent is considered by many as a disciple of simplification, although some of his writings and those of his disciples leave his position somewhat ambiguous. Aristoxenos is known for suggesting the possibility of 12-tone temperament, which makes him possibly its first advocate. He is also supposedly known for his diatribes against the harmonists, who advocated music based on small-number ratios; this is one of our ways of learning that the harmonists did exist, and that they did have some influence.[6]

The harmonists seem to have had a martyr in Timaeus, whose devotion to the enharmonic genus and to small-number ratios was sufficient to have induced him to defy the populace of his community so that he was banished.[7] Katheleen Schlesinger has attempted to reconstruct his scale. The martyred Timaeus stands as a kind of culture-hero to those who feel cut off by the simplifications of 12-tone temperament much as Timaeus was cut off by the Pythagoreans.

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[5] Kathleen Schlesinger, a major musicologists who devoted much of her work to a study of Greek music, holds this view, as do many exponents of multiple division.

[6] Schlesinger and Perrett emphasize the apparent antipathy between Aristoxenos and the harmonists.

[7] Kinkeldey, Otto, The Harmonic Sense, M.T.N.A. Proceedings, 1927.

Positive evidence of a considerable impact by the proponents of small-number ratios exists in the writings of a number of theorists and historians, among them Archytus of Tarent, Didymus, Plutarch, and Ptolemy, whose catalogue of tunings has received wide circulation through its publication in Barbour's _Tuning and Temperament_. Ptolemy lived several centuries after the crest of Greek civilization and after the supposed decline of the non-diatonic genera. For this reason his catalogues are sometimes discounted as mere speculation based on the reading of what even in his day was ancient theory. Nevertheless, as reliable an authority as the late Otto Gombosi takes the view that Ptolemy's figures were based on actual practice.[8]

Probably the strongest evidence of the use of small-number intervals in Greek music comes from surviving instruments. In her major work, _The Greek Aulos_, Schlesinger represents measurements of many such instruments as proving conclusively that the wind instruments of ancient Greece were attuned to small number ratios. Gombosi also indicates quite a number of scales based on septimal intervals.

Along with questions of the extensiveness of the use and influence of small-number ratios and the non-diatonic genera, there is the question of their artistic quality as

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[8] Gombosi, Otto, _Tonarten und Stimmungen der Antiken Musik_, 1939, p. 103. "Man darf nicht vergessen dass sie nicht von Ptolemaios geprägt worden sind, sondern aus der Praxis stammen."

opposed to that of the music of the Pythagoreans. Perrett suggests that "Perhaps . . . Pythagoras and Aristoxenos between them were the death of music as an art."[9] He admits that the contrary view--that they helped clarify and ennoble the art--is more prevalent. Nevertheless, no less an authority than Gevaert, perhaps the first of the great modern musicologists to have concentrated his attentions on Greek music, reports favorably on the enharmonic. He states quite emphatically that the best sources considered the enharmonic to be the genus par excellence, the most distinguished, the best ordered, and the most exact.[10] It is opinions such as this that interest many proponents of multiple division.

The 16th and 17th centuries produced a number of theorists who sought to reproduce the genera of the Greeks through multiple division on an experimental basis. Some of these theorists have, in turn, had a profound influence on contemporary advocates of multiple division. Vicentino, one of the first builders of a multiple-division instrument, is said to have attempted in theory a reconciliation of Greek ideas with 16th century practice.[11] Giovanni Battista

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[9] Perrett, op. cit., p. 9.

[10] Gevaert, Francois Aug., Histoire et Théorie de La Musique de L'Antiquité, 1875, p. 296. "Le Jugement que les Grecs portent sur le genre enharmonique a de quoi nous surprendre davantage. Selon les écrivains les plus compétents 'c'est le genre par excellence, le plus distingué le mieux ordonné, le plus exact.'"

[11] Cited in Barbour, Tuning and Temperament, p. 117.

Doni in the 17th century built an elaborate organ with three keyboards, each tuned to a different mode. By a combination of tones from the different keyboards the genera were also obtainable.[12] Perrett ascribes the origin of the enharmonic genus to just such a combination of diatonic modes, although he proceeds quite differently from Doni. Where Doni separates the "tonics" of his modes by major thirds, Perrett separates them by the interval 21:20, derived from the interval 7:5 (a fifth lower) which Perrett considers an exceptionally fine consonance. Perrett, like Doni, constructs his modes in just intonation, and a curious feature in the interval construction is that he chooses, like Doni, to use different modes. In Perrett's scheme the modes are symmetrically complementary, the one being the inversion of the other.

Leonhard Euler was another 17th century theorist deeply interested in the Greek genera. Euler's ideas have been influential among an important group of 20th century theorists interested in multiple division. He reduced the various Greek scales to mathematical formulae involving combinations of thirds and fifths and sometimes sevenths. It is this systematic attempt to codify the practice of the Greeks which Fokker seizes upon as the basis for his 31-tone system. "Euler has given us our scale," asserts Fokker on several occasions. According to Barbour, Euler

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[12] Ibid., p. 111.

was also one of the first to suggest an instrument based on tiers separated in pitch by a comma.

Euler's greatest influence on advocates of multiple division derives from his method of constructing (or postulating the reconstruction of) genera from groups of intervals. The diatonic genus is, according to Euler, produced by the formula $3^3 x 5$. The 3^3 means that from a starting pitch a cycle of three fifths is marked off (as for example from F -- to C, G, and D). The "5" indicates that for each of the notes produced a major third above is added (in the case of the pitches named above the added tones would be A, E, B, and F#). Euler's diatonic genus contains 8 pitches and yields not one but two diatonic scales (they are not identical).

Kirnberger, the great 18th century theorist, spoke out for the inclusion of the enharmonic intervals. His statement on the subject was made by Bosanquet the frontispiece for his Treatise. In Bosanquet's translation, it reads as follows: "Greater certainly would be the gain of Song if we really had the enharmonic intervals in our system. For then singers would accustom themselves, from their youth up, to sing correctly the smallest harmonic intervals, and the ear of the listener to appreciate them; and thereby would it be possible, in many cases, to make the expression of the passions very much stronger."[13]

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[13]Bosanquet, *op. cit.*, Frontispiece.

The Greeks have supplied the imaginations of musicians with interest in the septimal intervals, the very small intervals of the enharmonic genus[14] and scales of unequal intervals produced by harmonic and arithmetic series. But they have provided even more as a *mystique*: an unknown music associated with one of the greatest cultures history has known. Those who interest themselves in this *mystique* are drawn quickly beyond the limiting sphere of the systems of tone relations of the present.

JUST INTONATION

"Chords formed by the notes ordinarily in use are much inferior in excellence to chords which are in perfect tune,"[15] states Bosanquet. The tuning known as just intonation permits the principal chords to be in "perfect tune." When the dominant is exactly a perfect fifth above the tonic and the subdominant a perfect fifth below, and the major triads of the tonic, dominant, and subdominant, contain both pure thirds and pure fifths, the scale thereby

[14] Typical of the interest in the Greeks by advocates of multiple division is an article by Edoardo Cavallini, *Il Pluricromatismo nell' evoluzione musicale*, in the Rivista Musicale Italiana 1946, p. 130. Cavallini emphasizes the importance of the Greek enharmonic to contemporary multiple division, and reconstructs the extant fragment from the *Orestes* according to enharmonic theory with rather pleasing results.

[15] Bosanquet, *op. cit.*, p. 6.

formed is said to be in just intonation. Technically, however, any scale based entirely on unaltered rational-number intervals is in just intonation.

Designed to make more perfect the consonances of the Renaissance, this tuning, involving two different sizes of whole-tone, had a short life-span owing to its inflexibility. Since the early death of just intonation, however, attempts at its resuscitation have filled volumes. Still more volumes have been filled in refutation.

A number of controversies regarding just intonation still smolder. They concern its history, its merits, and its present use, if any. How extensive was the use of just intonation? How early in Western history can it be found, and how long did it last? Was it generally discarded because it failed to do what it was supposed to do or because it failed to do what it was not supposed to do? And, finally, do choruses and brass instruments use just intonation when they are able to do so, as some writers allege? Although these questions are quite distinct from one another, they tend to be treated as one, because just intonation has managed to lend itself extraordinarily well to opinion-forming. Those who favor just intonation argue that just intonation had an exalted and extended history, that its replacement by meantone and then equal temperament was due to changes in the musical requirements rather than to any inherent weakness in just intonation. They further argue that it is ideal, at least for harmony, and that it is still used for some music by the best choruses. Their opponents argue that the history of just intonation was, at best, short and undistinguished, that just harmonies sound insipid, that just intonation was condemned by its own inherent weaknesses, and that choruses foolish enough to try to use just intonation will go radically flat.

It is possible to make all kinds of hypotheses about the use of just intonation at the end of the Middle Ages and the beginning of the Renaissance. It is generally acknowledged that the Englishman Odington[16] claimed that the proper tuning for the thirds should be 5:4 and 6:5 in about the year 1300. Whether this represented theoretical wishful thinking or actual musical practice can only be guessed at. It is also quite possible that the 5:4 third represented a local practice. England may have used a just third before the continent; musicologists generally acknowledge that England, through the practice known as gymel, used the third as a consonance earlier than the continent.[17]

With Bartolomeo Ramos de Pareja in the 15th century

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[16]Kornerup, who likes just intonation, tends to emphasize Odington's role, while Barbour, who does not, tends to minimize it.

[17]Besides being possibly the first, England was probably the last country in the West to use the just third. Organs in meantone temperament were still extant and in use in England in the late 19th century. (Bosanquet)

begins a long series of theorists who tuned their monochords to just intonation. Barbour notes that Ramos cited expedient simplicity rather than musical necessity as the reason for including the intervals of the 5th partial in his tuning. Barbour also points out that Ramos' tuning is more weighted in favor of the perfect fifth over the third than are the later, more standard, just tunings. From this Barbour deduces that at the time of Ramos just intonation had behind it neither the support of widespread practice nor the emphatic endorsement of theorists on the basis of its superior sound.

As to the early end to the practice of just intonation, if such practice there was at all, Barbour points to the difficulty of singing the more chromatic works of Marenzio, Cipriano de Rore, and Gesualdo without instrumental accompaniment. Barbour points out in this connection that a capella did not mean "unaccompanied" in its earlier usage and that choruses must have adopted the intonation systems of whatever instruments were used as accompaniment.

Whether musicians unhampered by fixed intonations could distinguish just intonation at all is another subject for contention. Recent experimentation has indicated a wide span of preferences among musicians concerning the thirds to be employed. According to the preliminary findings in a study by Lewis Richardson at Indiana University,[18]

[18] Conversation with Mr. Richardson. His findings will become part of an unpublished dissertation at Indiana University.

some excellent musicians demonstrated great inconsistency in the thirds they "preferred." Many theorists have postulated different thirds for melody and harmony and it is conceivable that some of Richardson's subjects may have been thinking "melodically" and "harmonically" on different occasions when they were tested. Fokker has recognized the difficulty contemporary musicians have at forming the intervals of just intonation, and has written a graded series of exercises aimed at developing these skills.[19]

Proponents of just intonation assert that the current general inability to measure thirds accurately is the direct result of atrophy caused by the long disuse of just intonation. At the same time other proponents of just intonation (in some cases the same ones)[20] insist that choruses do, indeed, use just intonation. There appears to be an inconsistency here.

A similar contradiction can be noted on the other side, however. Barbour and Kuttner deny that unaccompanied choruses use just intonation[21] and then proceed to attribute the tendency of unaccompanied choruses to fall in pitch to

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[19]Fokker, Just Intonation is the work in question. The exercises also attempt to train the musician to hear and use the seventh partial.

[20]Norden argues that choruses use just intonation and that those who don't prefer just intervals to tempered ones through the 7th partial demonstrate insufficient ear-training. Organ Institute Quarterly, 5:1, p. 31 ff.

[21]Introductory notes to Musurgia Records A-2, op. cit., columns 29-30.

the use of just intonation.

The argument that just intonation causes flatting is one of the classical and oft-repeated points in the case against the use of pure fifths and thirds. It is based on the double employment of the supertonic. In chords in which the supertonic is employed with subdominant function, the supertonic is derived by thirds from the subdominant, and has the value of 10:9 above the tonic (4:3 - 6:5 = 10:9). In chords of dominant function, however, the supertonic is derived by fifths above the dominant and has the value of 9:8 above the tonic (3:2 x 3:2 ÷ 2:1 = 9:8). Since the normal course of harmonic progression is from subdominant to dominant rather than the reverse, harmonic music is full of instances in which the supertonic is first sung (or played) as a member of subdominant harmony and held over in an upper voice to become a member of dominant harmony. Assuming that the pitch of the supertonic is held at a constant level, the pitch of the tonic must move downward as the interval between the two increases from 10:9 to 9:8. Thus, for every progression from ii to V in just intonation, where the supertonic is sustained at a constant pitch, the tonic falls by the interval 81:80 or 21.5 cents. In one of the great tours de force of this branch of musical theory, Gustav Engel has analyzed the entire score of Don Giovanni for progressions in which the double employment of held-over tones would force the music upwards or downwards. Engel concludes that were Don Giovanni to be performed in its entirety in just intonation, the pitch would slip by about an augmented fourth in the course of the evening.[22] The many audiences which have heard choruses slip a whole tone in less than five minutes can hardly be alarmed at Engel's estimate.

In refutation Partch suggests that the supertonic need not be held constant. According to Partch, if the pitch of the supertonic is raised a comma during the progression ii-V, the result is not disturbing and the pitch of the original tonic is thereby retained. In any case, the tendency of choruses to drop in pitch may demonstrate that left to their own devices singers will tend to adopt some kind of just intonation.

Despite the vehemence with which Barbour and Kuttner attack just intonation, and especially its thirds, they leave the door open to those who would re-establish just intonation through multiple division when they conclude, "Of course it is the severe limitation of just intonation to 12 notes in the octave, with enharmonically equivalent notes forbidden, that has caused its outright rejection by musicians."[23] The record which accompanies these remarks

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[22]Engel, Gustav, Eine mathematisch-harmonische Analyse des Don Giovanni von Mozart, VMw, III, 1887, p. 491 ff.

[23]Op. cit., column 31.

amply demonstrates that more than twelve tones would indeed have to be available if just intonation were to be applied to music having any substantial harmonic range.

It is from just such a premise that Bosanquet and Fokker began their speculations on multiple division. It is also from this premise that the vast number of unequal scales such as Helmholtz' (24-tone), Eitz' (50-tone) and Groven's (36-tone) have arisen. Partch also makes just intonation his starting point, although insisting on the extension of the principle to include the 7th and 11th partials. Würschmidt and Ariel used just intonation as the theoretical basis for 19-tone temperament much as Zarlino, Mersenne and Rameau used just intonation as the basis for musical systems for which they too advocated temperament.[24] The coincidence of the just minor third with its equivalent interval in 19-tone temperament (the discrepancy is a small fraction of one cent) is one of the strongest arguments used for that particular system, especially by Ariel.

Technically, just intonation supplies the proponent of multiple division with a model whose consonant thirds differ substantially from those of 12-tone temperament. It also interests him through its greater variety of basic intervals, which require smaller subdivisions of the octave

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[24] The temperament was meantone in the former case, a sub-commatic temperament not unlike equal temperament in the second, and equal temperament in the third.

than 1/12. It provides him with such relatively minute intervals as 25:24 the difference between major and minor thirds (which Ariel considers to be the basic unit for 19-tone temperament) and 81:80, the difference between the major and minor whole-tones (which is widely held to offer the basic unit for 53-tone temperament).

But, as with Greek tunings, just intonation is equally important for the intangibles it brings to the modern proponent of multiple division. There is the <u>mystique</u> of small rational numbers, and the ideal of perfect consonance. And there is, as with Greek tunings, the association with a golden age in the history of the world--in this case with the Renaissance. The attempt to achieve just intonation in more complex music than was obtainable in the Renaissance has stimulated many projects in multiple division and will no doubt continue to do so.

MEANTONE AND OTHER SUB-COMMA TEMPERAMENTS

Used literally, the term "meantone temperament" applies only to Pietro Aron's ¼-comma temperament of the fifth, wherein the major thirds are exact and the major seconds (tones) are the mean (geometric) between 9:8 and 10:9.[d] In the more generic sense in which "meantone temperament" is sometimes used, it has come to mean any temperament of the fifth, for the purpose of improving the thirds, wherein

[d] 19:17 = 192.56¢ or $\sqrt[2]{\frac{9}{8}\cdot\frac{10}{9}}$ = 193.16

the fifth is smaller than in 12-tone equal temperament. In this sense there are nearly as many varieties of meantone temperament as there are 16th, 17th and 18th century theorists combined. The generic meaning will be used here.

Meantone temperament figured heavily in the theoretical writings of the 16th through 18th centuries. Its importance arose out of the realization that pure fifths and pure thirds could not exist side by side in a musical system containing a small number of tones. Theorists, therefore, returned for convenience to the Pythagorean practice of producing the third by combining four fifths and deducting two octaves. However, whereas the Pythagoreans used perfect fifths, allowing the thirds to be a comma larger than the just thirds, the proponents of meantone temperament reduced the size of the fifths in order to improve the thirds. In Aron's ¼-comma temperament, the most significant and the only literally correct "meantone temperament," the fifths are altered about 5½ cents in order that the thirds can be pure.

The shift in emphasis from the perfect fifth to the major third as the basic unalterable interval within the octave is partly explained by the development of the third as a consonance in musical usage. But there is a logical reason for preferring meantone temperament to Pythagorean intonation. Since the third is composed of four fifths (less two octaves), a minute alteration of a single fifth will have a fourfold effect on the size of the third. By making each fifth smaller by ¼ comma, the third is improved one whole comma, which is enough to make it perfect. Therefore, whereas a system containing pure fifths will have thirds in error by a whole comma, a system with perfect thirds will have fifths in error by only ¼ comma. If thirds and fifths are considered equally important in a musical system, then clearly it is better to choose the system involving ¼-comma of error than the system involving a discrepancy of an entire comma.

Other so-called meantone temperaments have arisen from attempts to compromise the differences between fifth and third, making neither perfect while holding to a minimum the disagreeable discrepancies in both. Still others have developed from the shortening of both the fifth and the major third in order to improve other intervals in the temperament, most notably the minor third. Meantone temperament is looked upon with disfavor by those who consider the pure fifth to be more important than the pure third or who look upon the pure thirds as inferior to altered thirds. Nevertheless, it represents an important phase in the history of intonation and one which saw the birth and development of harmonic practice.

There are, as will be shown, two important reasons why there is a close kinship between meantone temperament and multiple division. In the first place there is a theoretical relationship between any given meantone temperament and an equal temperament involving multiple division. This is because meantone temperament is a perfect but incomplete system. The sharps on the one hand and the flats on the other are never joined together, each meantone system having its own particular size of "wolf" fifth where the two ends "meet." But it is possible to continue the series of fifths in theory until, with the aid of only negligible additional tempering, they <u>do</u> meet. This produces an equal temperament which may be thought of as the fulfillment or basis or background for the particular kind of meantone temperament. Theorists of the meantone period, such as Sauveur, actually did use multiple division as a theoretical basis for meantone temperament. Conversely, today, this relationship is used to justify multiple division as a means of restoring the advantages of meantone temperament.

The second close relationship between meantone temperament and multiple division is in the practical realm. Many of the proponents of meantone temperament actually conceived it for instruments possessing more than twelve keys. Renaissance instruments were built and amply described which made multiple division, particularly those systems involving 19- and 31-tones, a reality. As Charles Kent asserts,[25]

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[25]Kent, Charles, <u>An Introduction to Tuning and Temperament</u>, p. 46. This hitherto unpublished document is particularly valuable for the beginner due to the thoroughness of its information and the lucidity of its presentation.

referring to Aron's ¼-comma temperament, "If one were to design an instrument capable of playing the complete meantone system . . . (involving both the sharps and the flats in common use) . . . it would require 19 keys to the octave." It might be added that if one wished to make of ¼-comma temperament a closed system, he would require 31 tones to the octave.

Meantone temperament gradually gave way to 12-tone equal temperament (which is, itself, a form of meantone temperament when the term is used in its broadest sense). 12-tone equal temperament is the result of the alteration of the fifth by approximately 1/11 comma, which temperament leaves the third 7/11 comma too large, and the minor third 8/11 comma too small. There appears to have been a substantial period between the maximum influence of Aron's ¼-comma temperament and the triumph of 12-tone equal temperament when a compromise temperament of 1/6 comma, sometimes called Silbermann's temperament, enjoyed widespread use.[26]

Of importance to the present study are evidences of close historical ties between meantone temperament and theories of multiple division. Equally important is the question whether the deficiencies in meantone temperament which caused its downfall were of the kind that multiple

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[26]Sauveur undoubtedly meant this temperament when he says that the 55-tone temperament is in general use by musicians as the intervals are essentially the same. See Scherchen, The Nature of Music, p. 42.

division could have prevented. If it can be established that such close ties as have been suggested did exist, and if it can be further established that meantone temperament fell because of its "wolves"[27] and not because of its small fifths and thirds, then a strong historical case can be made for multiple division as a justifiable extension of meantone temperament.

Of particular relevance is the work of the deaf-mute theorist, astronomer and mathematician, Joseph Sauveur. Although himself unable to hear, he worked with the constant corroboration of musicians, and obtained such insight into music that Hermann Scherchen was led to place him with Bach and Beethoven among the great innovators of music.[28] Sauveur's writings on music appear in several articles in the Mémoires of the Royal Academy of Sciences of Paris and, except for the brief summary by Scherchen, have not been translated into English. Scherchen alludes to Sauveur as a proponent of 43-tone temperament and as an opponent of 12-tone temperament. The latter is quite true, as the following statement, translated by Scherchen, bears out.[29] "This

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[27]That is, the bad fifths between the tones at the ends of the system. These could be corrected by multiple division, while the small thirds and fifths are basic to the meantone system and are not corrected by the addition of more tones.

[28]The Nature of Music. In this book, Scherchen honors Sauveur, Bach, and Beethoven with an essay apiece, using their transcending greatness and their physical disabilities as a central theme.

[29]The English is not by Scherchen but by William Mann; Scherchen's translation was to German. The Nature of Music, p. 43.

system (12-tone temperament) is preferred by our less competent instrumentalists; it allows one to transpose easily without changing of interval. But more sensitive musicians have rejected it, because its intervals deviate from the diatonic norm."[30]

It is also true that Sauveur proposes 43-tone temperament, but there is no evidence that he ever envisaged music with 43 actual tones to the octave. Sauveur evidently meant 43-tone temperament to be the theoretical basis for a meantone temperament involving the flatting of the fifth by 1/5 comma. The crucial statement by Sauveur which Scherchen passes over without comment is that "55-tone temperament is in universal use by musicians." It is obvious that taken in the literal sense this statement is absurd. 55 tones to the octave is a radical projection today and a 55-tone octave has never enjoyed "universal use." What Sauveur evidently had in mind is 1/6 comma temperament which, in the early 18th century, is known to have enjoyed widespread use. The fifth in 55-tone temperament is altered by almost precisely 1/6 comma, rendering 55-tone temperament the theoretical completion of the

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[30]"Ce systême à son usage chés les Joüeurs d'Instrumens les moins habiles, a cause de sa simplicité et de la facilité pouvant transposer les notes ut re mi fa sol la si sur telle touche qu'ils veulent, sans aucun changement dans les intervalles: mais les différences des intervalles de ce systême avec ceux du systême Diatonique juste étant trop grandes, les habiles Joüeurs d'Instrumens l'ont rejetté." Mémoires, 1701, p. 214.

meantone system involving fifths altered by 1/6 comma. By the same logic, the 43-tone temperament advocated by Sauveur can be construed as the theoretical completion of a meantone system involving fifths altered by 1/5 comma. Consideration of Sauveur's reasons for advocating this temperament bear out the assumption that this is what he intended. He lists 9 reasons:

1. It preserves the just minor second of the diatonic system.[31]

2. It preserves a major second and chromatic semitone that represent arithmetical means between the just alternatives.[32]

3. The deviations of the other intervals are equal, except that of the minor third which is double; equal discrepancies bother the ear less than do unequal ones.[33]

4. If the unit of 43-tone temperame t is itself divided into 7 parts, all of the discrepancies fall very close to units of this smaller subdivision.

5. The most perfect interval, the 4th (5th), is better represented than the thirds.

6. 43-tone temperament is a mean between the other legitimate systems. Its whole-tone is an

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[31] This and the succeeding 8 points are taken from Table Generale des Sistèmes Temperez de Musique, Mémoires, 1711, pp. 316-17.

[32] This seeming error is clarified by Sauveur's sixth point.

[33] This point, somewhat alien to our age, is made repeatedly in the 18th century. Our 12-tone system has most unequal discrepancies as between 5th and 3rd, while the fifth and major third of 19-tone temperament are in error by almost precisely the same amount.

arithmetical mean between two minor and three major tones, precisely the proportion which appears in the octave.

7. The sum of all deviations of the intervals, taken positively, is least of all the systems.

8. Eptamérides (the smaller subdivision mentioned above in no. 4) are equal to the 301st part of the octave and represent ordinary logarithms.

9. I have tried all of the temperaments on my clavecin, and my colleagues have preferred the sound of 43-tone temperament.

Sauveur's third point, that he prefers equal deviations to unequal ones, is probably at the root of his preference for 1/5 comma temperament. The perfect fifth and the major third, representing the intervals from the fundamental to the upper consonant partials, are the important ones to most of the theorists. It is self-evident that ¼-comma temperament favors the third completely while Pythagorean tuning favors the fifth. The arithmetic mean between them would be 1/8 of a comma, but the arithmetic mean is hardly valid here, since discrepancies of the third would then be four times as great as discrepancies of the fifth. Sauveur's 1/5 comma temperament is the equivalent of a geometric mean between the Aron meantone temperament on the one hand and Pythagorean tuning on the other. The third as well as the fifth is in error by 1/5 comma.[34]

[34]According to Kent's figures in op. cit., the fifth in 43-tone temperament is 4.28 cents flat while the third is 4.39 cents sharp. The discrepancy between the two deviations is a reminder that 43-tone temperament is only an approximation (although a good one) for 1/5 comma temperament, for in the latter system the deviations would be exactly equal.

An aspect of Sauveur's case which emphasizes that his interest in 43-tone temperament was as a basis for a musical system rather than as a complete system is in his emphasis in the advantages of 43-tone temperament for measurement. 1/301 has been found by a number of theorists to be a useful unit of measurement because the logarithm of 2 (the octave) is 30 103. 1/301, called an <u>éptaméride</u> by Sauveur, is called a <u>savart</u> by others who use it. Use of <u>savarts</u> makes possible the measurement of intervals by ordinary logarithms of their ratios, without transposing by a complicated constant as in the case of cents or milioctaves. Theorists as far removed from the advocacy of 43-tone temperament as Würschmidt and Danielou have made use of the savart for purposes of measurement. But this has little relevance to the musical merits of 43 tones to the octave. Neither do Sauveur's other points, which stress the diatonic intervals but in no way pertain to a system containing many extra tones.

As Sauveur's 1/5-comma and Silbermann's 1/6-comma temperaments have their close equivalents in 43- and 55-tone temperaments, so Aron's ¼-comma temperament has its close equivalent in 31-tone equal temperament. The Dutch mathematician Huygens pointed out this equivalence in the 17th century. A practical connection between the two was established, at least for experimental purposes, by Vicentino, late Renaissance composer, theorist and musician extra-ordinary. His archicembalo was tuned, he says, to the prevailing intonation of his day, which was ¼-comma temperament.[35] His instrument possessed 31 keys to the octave and, if it was attuned to the prevailing intonation of his day, was nearly equal-tempered. This is shown by example 23, below, in which the 17 pitches shown by Kent for the meantone system are contrasted with the equivalent pitches for 31-tone equal temperament. As will be observed, there is no discrepancy as large as a schisma. The two systems are nearly identical, so that one might say with full justification that ¼-comma temperament, which enjoyed widespread use, is the direct ancestor of 31-tone temperament, which has its very strong advocates today.

During the earliest period of meantone tuning, some theorists proposed to alter the fifth by more than ¼ comma in the interest of rendering the minor third and major sixth closer to the ratio 6:5. The minor third is created in these systems by three superimposed fifths. Where the fifth is pure the minor third, like its complement within the fifth the major third, is out of tune by a comma. For the minor third to be in perfect tune, the fifth must be altered by 1/3 comma, which will also be the error of the major third. Two important Renaissance theorists, Zarlino and Salinas, proposed such a temperament. It should be stressed that both theorists also proposed other temperaments and that they did not single out 1/3-comma temperament for special favor.

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[35] Barbour, op. cit., p. 117.

Example 23: Meantone and 31-tone Temperaments Compared[36]

31-tone temperament	Meantone tuning		Discrepancy
0.00	A	0.00	
38.71			
77.42	A#	76.05	1.37
116.13	Bb	117.11	0.98
154.84			
193.55	B	193.16	0.39
232.26			
270.97			
309.68	C	310.27	0.59
348.39			
387.10	C#	386.31	0.79
425.81	Db	427.37	1.56
464.52			
503.23	D	503.42	0.19
541.94			
580.65	D#	579.47	1.18
619.35	Eb	620.53	1.18
658.06			
696.77	E	696.58	0.19
735.48			
774.19			
812.90	F	813.69	0.79
851.61			
890.32	F#	889.74	0.42
929.03	Gb	930.79	1.76
967.74			
1006.45	G	1006.84	0.39
1045.16			
1083.87	G#	1082.89	0.98
1122.58	Ab	1123.95	1.37
1161.29			
1200.00	A	1200.00	

1/3-comma temperament can well be considered a direct forbear of 19-tone equal temperament. According to Barbour, Salinas never realized the possibility of closing his system through the use of 19 tones to the octave. Zarlino, who

[36] All figures are from Kent, op. cit.

proposed 1/3-comma temperament, but only as one of many and without the enthusiasm he reserved for ¼- and 2/7-comma temperaments, had a 19-tone instrument constructed, very probably the first of its species, but there is no evidence that he tuned it to 1/3-comma temperament at any time. Nevertheless, the correspondence between 1/3-comma temperament and 19-tone equal temperament is extraordinarily close, as Example 24 shows, and 1/3-comma temperament can be considered a direct ancestor of the later system.

Example 24: 1/3-Comma and 19-Tone Temperament Compared

19-tone temperament		1/3 comma temperament	Discrepancy
00.00	A	00.00	0.0
63.16	A#	63.52	0.36 cent
126.32	Bb	126.06	0.26 cent
189.47	B	189.57	0.10 cent
252.63			
315.79	C	315.64	0.15 cent
378.95	C#	379.15	0.20 cent
442.11	Db	441.69	0.42 cent
505.26	D	505.21	0.05 cent
568.42	D#	568.73	0.31 cent
631.58	Eb	631.27	0.31 cent
694.74	E	694.79	0.05 cent
757.89			
821.05	F	820.85	0.20 cent
884.21	F#	884.36	0.15 cent
947.37	Gb	946.90	0.47 cent
1010.53	G	1010.43	0.10 cent
1073.68	G#	1073.94	0.26 cent
1136.84	Ab	1136.48	0.36 cent

A number of compromises between 1/3- and ¼-comma temperaments were proposed. Zarlino's name is generally associated with 2/7-comma temperament, while the 18th century

English theorist Robert Smith endorsed 5/18-comma temperament. 50-tone equal division might be used to complete either of the above systems, as the fifth is very close to 696 cents in both. Smith's is the nearer one to 50-tone temperament, having been calculated with the 50-division in mind.

Smith is also on record as opposing 12-tone equal temperament which, during his lifetime, had begun to dislodge meantone temperaments from their position of supremacy. "The harmony in this system of 12 hemitones is extremely coarse and disagreeable."[37] Smith's and Sauveur's opposition at least suggest that the approaching triumph of 12-tone equal temperament was viewed with alarm by some musicians who fully understood its implications.

A number of modern advocates of multiple division have taken particular interest in the theory and practice of meantone temperament. Fokker regards meantone temperament as having been a highly satisfactory system of intonation whose undoing was caused by the inconvenience of requiring separate keys for G# and Ab.[38] He is of the opinion that the music of Sweelinck sounds best when performed in meantone temperament.[39] Of the shift from

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[37]Smith, Harmonics, p. 166. (1759)

[38]Fokker, La Gamme... p. 152.

[39]Fokker, Ibid., p. 161.

meantone to equal temperament, Fokker writes, "There was a growing tendency to sacrifice the refinement of the variety of semitones in order to make them all equal and thereby to gain the complete freedom of modulation."[40]

Fokker acknowledges his debt to Huyghens who offered 31-tone temperament as a means of completing Aron's meantone temperament.[41] In Fokker's final summation, historical and acoustical considerations (the latter relating primarily to the 7th partial) share equally in his choice of 31-tone temperament. "Tricesimoprimal equal temperament has two faces. One is turned to the past. It recalls into being the all-but forgotten beauties of the music of the great old masters by restoring the temperament they breathed in. The other face is turned to the future, with its possibility of coping with new harmonies and with autochthonous music outside the compass of traditional secular Western music."[42]

Kornerup is another advocate of multiple division who is well aware of its close relationship with meantone temperaments. He considers Aron's ¼-comma temperament to be the forerunner of 31-tone temperament and Salinas' 1/3-comma temperament the forerunner of 19-tone temperament.[43]

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[40] Fokker, Equal Temperament with 31 Notes, Organ Institute Quarterly, Vol. 5:4, p. 41.

[41] Fokker, Un Orgue a Demitons Majeurs et Mineurs, Recherches Musicales, p. 162.

[42] Fokker, Equal Temperament with 31 Notes, p. 44.

[43] Kornerup, Die Vorläufer der Gleichschwebenden Temperaturen mit 19 oder 31 Tönen in der Oktave, p. 6. Kornerup attributes both meantone temperaments to Arnold Schlick, rather than to Aron or Salinas.

These two systems of multiple division are the ones in which Kornerup has the greatest interest.

Meantone temperament left behind specimens of instruments with "split-keys" and other devices for realising more than 12 tones to the octave. Lacking, perhaps, the intangible prestige of the supposed Greek tunings or the illusion of perfection in just intonation, meantone temperament offers a background of practical multiple division as well as a theory suggesting greater multiple division. It is hardly surprising that advocates of multiple division such as Fokker, Kornerup and Bosanquet, having determined to their satisfaction that meantone temperament was not dropped because of its basic intervals,[44] cite meantone temperament as precedent for their proposals. While the split keys on the instruments possessing 13 to 16 tones per octave could push the "wolves" away from the door, they can be eliminated entirely by a completed system of multiple division.

Periods other than the Greek, Renaissance, and Baroque, have seen isolated phenomena suggestive of multiple division. The famous Montpelier codex of the 13th century

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[44]Bosanquet devotes several pages to an extended discussion of Bach's apparent reasons for preferring equal temperament, and concludes that Bach's only objection to meantone temperament was to the "wolf" which can be removed through multiple division. Except for the "wolf", says Bosanquet, "no claim to equal temperament (over meantone) can be made in Bach's name." <u>op</u>. <u>cit</u>., p. 36.

possessed marks generally thought to indicate mictrotonal alterations in pitch. Joseph Gmelch carefully shows why the signs ⊢ , ⊣ , ⌈ , ⌉ and ⌋ cannot indicate rhythmic or dynamic alterations but must apply to pitches.[45] At least one quarter-tone composer, Alois Haba, refers to the Montpelier codex as an historical precedent for multiple division.[46] Sachs has written of a most unique medieval instrument with septimal tuning[47] but as far as is known this instrument has not yet been cited as a precedent for multiple division.

A BRIEF NOTE ON ORIENTAL AND FOLK MUSIC

Closely linked with the intonational systems of the past as models for multiple division are the present systems of other cultures. Such leading advocates of multiple division as Fokker, Bosanquet, Kornerup, Yasser, Partch, and Wyschnegradsky have shown considerable interest in the musical systems of the Orient. The presence within the world of intonational systems other than our own offers a chance to test other possibilities and a challenge to

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[45] Gmelch, Joseph, Die Vierteltonstufen im Messtonale von Montpellier, 1911, pp. 9-19.

[46] Haba, Kwart-tonen-Problemen, De Muziek, 1927, p. 109.

[47] Sachs, Curt, A Strange Medieval Scale, A.M.S. Journal, Fall, 1949, p. 169. According to Sachs, two instances show that "in the time around 1100, the wide-meshed 8:7 scale was used in addition to the customary . . . scales."

enlarge our own system so as to incorporate the best of other systems. No less a musician than Bartok is cited[48] as urging finer divisions than those provided by 12-tone temperament in order to realize the autochthonous folk-music of Eastern Europe. Fokker attributes Haba's preoccupation with multiple division to a similar interest in Eastern European music.[49] Fokker himself is greatly interested in the salendro scale of Indonesia, which he considers to be based on the trisecting of the perfect fifth.[50] Such a trisection is possible in 31-tone temperament where the fifth is 18 units in size. Wyschnegradsky also cites the near-equal 5-tone scale as a feature of his system (24-tone temperament), and he mentions Arabic music as a basis for his system.[51] Bosanquet concerns himself at length with Indian music. While many musicians argue that large areas of the Orient possess music with less subtle regard for nuances of pitch then Western music, some Eastern cultures, particularly those in the near-East and India, have paid considerable attention to precision in the intonation of

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[48] By Fokker in La Gamme... p. 152. Bartok is here cited as advocating in particular the seventh partial. Bartok uses tones lying outside the 12-tone system in his Violin Concerto.

[49] Ibid., p. 152.

[50] Just Intonation, p. 34.

[51] Wyschnegradsky, La Musique à Quarts de Ton, Revue Musicale, Jan. 1937, p. 26.

their intervals.[52] The fact that Arabian and Indian musical systems employ more than twelve tones to the octave serves as an example to musicians of the West that sensitivity to 12 distinct pitches in the octave can hardly be the limit of human capacity.

CONCLUSION

Multiple division has firm roots in the actual music of the distant and near past, and in other musical cultures of the present. Some of the proposed systems of multiple division, such as 31-tone equal temperament, involve intonations which (with negligible deviations) were actually used in music. Others, such as 19-tone temperament, involve intonations which were seriously proposed by musical scholars to meet problems of an earlier age but were never extensively used. The case for multiple division is unquestionably enhanced by the extent to which it can draw on historical precedent.

But the presence of various tunings in the music of the past is not in itself proof that all or any of them should be revived today. In the final analysis one's own reading of the implications of history depends upon his views of the acoustical properties of 12-tone temperament.

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[52] Prof. Paul Matthen of Indiana University tells of a visiting Arabian singer who expressed astonishment at how many different musical intervals were notated and regarded in Western music simply as a major third.

Is it a better tuning than those which preceded it, or rather only a more convenient one? For those who have accepted the latter alternative, history has supplied a stimulus for further study, and for the proposal of systems of multiple division.