

CHAPTER 8

THE ANTECEDENTS AND EARLY ADVOCACY OF 19-TONE
TEMPERAMENT

The advocacy of 19-tone equal temperament has been for the most part a 20th century phenomenon. Three writers in this century have presented extended treatises favoring its adoption--two of them in book form and one in a series of related pamphlets. Each of these writers will be studied in some detail in a separate chapter.

19-tone temperament is referred to without advocacy by at least one theorist in the 18th century. It is modestly advocated by several writers in the 19th century of which the first is evidently the Englishman, Wesley Woolhouse, and is the subject of an enthusiastic report by the German, Melchiorre Sachs, to the 4th Congress of the Internationalen Musikgesellschaft in London, in 1911. Finally, it is written of with favor by a number of authors in scattered periodicals leading to its espousal in the 1920's and 1930's by Kornerup, Ariel, and Yasser.

In two important respects 19-tone temperament has its roots in the Renaissance, although it appears never to have been directly advocated before the 19th century. Theoretically, it resembles very closely one of the tempered tunings which gained the qualified endorsement of several of the leading Renaissance theorists. Practically, 19-tone temperament can claim an antecedent in the several instru-

ments using 19 tones to the octave which were built during the Renaissance. We do not know precisely how these instruments were tuned, but the evidence suggests that they were tuned neither to 19-tone temperament nor to the meantone temperament which most closely resembles it ($1/3$ -comma temperament). Nevertheless, the existence of these instruments has been widely known, and the possibility that they were tuned in such a way as to anticipate 19-tone temperament has been recognized by several of the advocates of 19-tone temperament.¹ As these instruments were easily retunable cembali, it is possible that they were experimentally tuned and retuned to many possible temperaments including $1/3$ -comma.

SALINAS AND $1/3$ -COMMA TEMPERAMENT

In his famous treatise De Musica, the blind Spaniard Salinas, 1513-1590, takes up at some length the subject of temperament which he considers a necessity. He lists three temperaments which he deems to be worthy of consideration; the first alters the fifths by $1/3$ of a comma, the second by $2/7$ of a comma and the third by $1/2$ of a comma.¹ In each case he carefully lists the various tunings of the tetrachords--diatonic, chromatic, and enharmonic--which he considers renderable in the temperament. In each case he

¹De Musica, Liber III

also diagrams the temperament, showing the position of each tone with respect to its position in the diatonic genus of Ptolemy and with respect to the others. Salinas discusses the three alternatives as though they were equally known (or unknown) to musicians of his time, but where scholars have found earlier instances of the advocacy of κ - and $2/7$ -comma temperament, they have apparently found no earlier reference to $1/3$ -comma temperament.² If Salinas was the inventor of $1/3$ -comma temperament, he bears his achievement modestly. He treats it first among the three possible temperaments, at somewhat shorter length than the other two, and he acknowledges the superiority of the major thirds and the fifths in the other two temperaments. Nevertheless, Salinas did not take a stand against $1/3$ -comma temperament, and as long as he appears to have been the first major writer to acknowledge it as a reputable temperament, his role as an unwitting grandfather of 19-tone temperament appears secure. The two temperaments are almost identical, and it appears that Salinas was interested in a 19-tone application of the temperament to the three genera.³

Barbour points out that Salinas never stressed the

²Barbour refers to $1/3$ -comma temperament as the "invention of Francisco Salinas." Tuning and Temperament, p. 33. It appears likely that it was of earlier invention. Kernerup attributes it to Arnold Schlick, substantially earlier, claiming $1/3$ -comma temperament to be included in the supplement of his Speigel der Orgelmacher und Organisten. Attribution of a number of meantone temperaments to Schlick is traditional and is strongly disputed by Barbour, who claims Schlick's tunings to have been irregular.

³Barbour, op. cit., p. 34.

advantage that $1/3$ -comma temperament possessed in being a closed system (through 19-tone equal temperament). This is quite understandable, since Salinas predated the general concurrence that there is any advantage a priori to a closed system as such. Salinas could not have failed to notice the tendency of $1/3$ -comma temperament to produce approximately equal intervals, for he meticulously diagrammed each of his temperaments, and $1/3$ -comma temperament produces intervals which appear exactly equal on his chart.⁴ It is unfortunate that Salinas did not record his reaction to what he must have noticed on his diagrams.

The impression left by Salinas and other writers who included mention of $1/3$ -comma temperament is that it was an accepted, though junior, partner of $1/4$ - and $2/7$ -comma temperaments which, because of their better thirds and fifths, had more prestige and more widespread use.

It was Zarlino who is generally credited with ordering the construction of the first known instrument with 19 tones to the octave. Another more famous 19-tone instrument was the "clavicembalo universale," mentioned by Praetorius in his highly renowned work Syntagma Musicum. Praetorius reports that the instrument was built by Elsass and was in the service of the duke of Prague. Elsass has accordingly been given occasional credit for building the first 19-tone instrument and even for originating 19-tone

⁴ De Musica, op. cit.

temperament,⁵ although it is highly unlikely that his instrument was tuned to a very close approximation of 19-tone temperament.

What music could be performed on the clavicembalum universale? According to Praetorius, works in the enharmonic genus and works in the chromatic genus such as Marenzio's madrigals could be performed on such an instrument. Hauptmann has effectively disputed this assertion, pointing out that in such chromatic madrigals as Quel Suo Antico Stylo Marenzio uses C# and Db in different voices at the same time as apparent identities.⁶ A 19-tone instrument would be just what is not needed to perform such works of Marenzio, which clamor instead for 12-tone temperament.

Praetorius' apparent misunderstanding is itself revealing. That he could have made such a mistake in failing to see the semitonal implications of Marenzio's notation suggests how far 12-tone equal temperament remained from the consciousness of the average musician in 1600. To such a musician C# and Db were different tones and could only be realized together if instruments other than the conventional keyboard instruments played them.

The increase in the use of chromatic resources, and the rise of interest in modulation forced musicians gradually to a choice between additional keys to the octave on

⁵Kornerup, in Acoustic Valuation of Intervals, refers to 19-tone temperament as Elsass' 19-tone temperament, p. 11.

⁶Hauptmann, M., Temperatur, Jahrbücher für Musikalische Wissenschaft, I, 1863. The reference cited is from p. 40.

the one hand, and the gradual erosion of the differences in the semitone on the other. It was the latter course which was chosen, at the expense of the purity of the thirds. Throughout the 17th and 18th centuries, the fifths and thirds grew generally larger as meantone temperament of $\frac{1}{4}$ -comma was superseded by $\frac{1}{6}$ -comma temperament and finally by equal temperament. This brought music farther and farther away from the intonations of 19-tone temperament, and from that acoustic condition in which tones numbering more than twelve could make a substantial contribution to the mobility and consonance of the system. As the thirds and fifths grew larger and the semitones became more and more equal, instances of "split keys" became rarer and rarer. It is conceivable that there was a point around Salinas' and Zarlino's time when music might have evolved as easily toward 19-tone temperament as toward 12-. With the enlargement of the fifth and third in the 17th century, an unchangeable course toward 12-tone temperament was established. It is hardly surprising that in this atmosphere little or nothing about $\frac{1}{3}$ -comma or 19-tone temperament was said.

It is quite appropriate that England, the country which relinquished meantone temperament most belatedly and most reluctantly,⁷ produced whatever interest in 19-tone

⁷According to Bosanquet, at least two organs in major English cathedrals were still tuned to meantone temperaments in 1875. While Bach in Germany was advocating and using an almost equal temperament of 12 tones, Handel, in England, had been using an organ with 16 keys to realize a broad range in what may well have been a temperament of more than $\frac{1}{6}$ of a comma.

temperament existed before the middle of the 19th century. After the continent had long since established $\frac{1}{4}$ -comma temperament as an outer limit for the reduction of the fifth, and had passed through $\frac{1}{6}$ -comma temperament toward equal temperament, Robert Smith in England, in 1759, published his treatise Harmonics or The Philosophy of Musical Sounds, advocating the temperament of the fifth by $\frac{5}{18}$ -comma. On page 158, Smith includes 19-tone temperament in a table also listing 12-, 31-, 50-, and meantone temperaments. 19-tone temperament is dismissed by Smith because of the discrepancy between the degree of accuracy obtained by the major sixth (almost exact) and that of the fifth and major third (about $\frac{1}{3}$ -comma error). It is Smith's thesis that insofar as possible the errors involved in a temperament should be equally shared by the three principal consonances. While excluding Smith from any list of advocates of 19-tone temperament, it is worthwhile noting that his inclusion of the major sixth among the basic consonances shows a point of view in common with almost all of the later supporters of this octave division, one of whose peculiar features is the almost perfect representation of that interval.

Another Englishman, Wesley S. B. Woolhouse, may well have been the first advocate of 19-tone temperament as such. Presumably a mathematician, he was Head Assistant on the Nautical Almanac. His Essay on Musical Intervals, Harmonics,

and the Temperament of the Musical Scale, 1835, contains a direct espousal of 19-tone temperament as a practical musical system.

In his over-all view of temperament, Woolhouse is essentially a disciple of Smith. Regarding the perfect fifth, the major third, and the minor third as the three basic consonant intervals of music (not counting the octave), Woolhouse seeks to find that temperament in which the total dissonance involved in these three intervals will be a minimum. Although this represents a somewhat different emphasis from that of Smith, whose primary concern was in equality of error, it produced a similar result, because of Woolhouse's method, which was to calculate dissonance by the square of the error of each interval. The use of squares renders preferable that system in which the discrepancies tend to be equal, because a large discrepancy in any single interval would produce an exceptionally large square.⁸

By a complicated algebraic process Woolhouse relates the errors of each of the three intervals to the alteration

⁸Consider, for example, two systems involving two intervals, the errors for one system being each 3 units in size, the errors for the other system being unequal, 1 and 5 units respectively. Calculating without squares the two systems are of apparently equal error, the sum of each being 6 units. But calculated by the squares of the errors the system with unequal errors is much the worse, containing a total of 26 square units of error while the system of equal errors contains only a total of 18 square units.

of the 9:8 whole-tone in each of the systems. He then finds the ratio of tone to semitone which will produce the minimum sum of the squares of the errors, and finds it to be close to 8:5, whereupon he suggests that in the ideal musical system the tone would have 8 units and the semitone 5. This would give 50 units to the octave, the very temperament advocated by Smith.

Woolhouse, however, is conciliatory in nature, finding much to praise in both 12- and 31-tone temperaments, and especially in 53-tone temperament. After giving serious concern to nearly all of the temperaments which have since been advocated, Woolhouse selects 19-tone temperament as the most useful. "The scale of 19 sounds in the octave, which gives great accuracy to the harmony, though not to such theoretical minuteness as the scale of 53 sounds, would, on account of its simplicity and easy adaptation to the construction of the instrument, be a useful improvement in the correctness of the harmony, without infringing on its practicability as regards the performer. It is to be hoped that some persevering experimental musician will construct an instrument on this scale and give it a fair trial."⁹

Such is Woolhouse's view, and the points made by later advocates have tended to be elaborations and substantiations of Woolhouse's position, with only occasional and scattered arguments of a really different nature.

⁹Woolhouse, op. cit., p. 54.

From an English Nautical Almanac assistant, 19-tone temperament turned next to a German finance minister for support. In 1852 Friedrich Opelt published his Allgemeine Theorie der Musik. Beginning with a theory of consonance based on the simplicity of the polyrhythm set up by two or more fundamental waves, Opelt proceeds to measure various musical systems by the standards of proximity of pitches to consonances.

An interest in the 7th partial causes Opelt to consider seriously the merits of 31-tone temperament, but he rejects 31-tone temperament as too difficult for use.¹⁰ He therefore chooses not to use the 7th partial in constructing his musical systems, and asserts that only 12- and 19-tone temperaments can provide representation for the remaining natural consonances.

As between 12- and 19-tone temperaments, Opelt considers 19- in many respects to be the better of the two. The chief advantage of 19- which he stresses is the absence of any error greater than $1/3$ -comma in the representation of the basic consonances. His favoring of 19- over 12-tone temperament is supplemented by a consideration of the likelihood of a change from the one to the other. He concedes that such a change would be difficult, but he does not consider such an alteration in keyboard instruments to be

¹⁰Opelt, op. cit., p. 66.

impossible (unsusführbar) or even improbable (unwahrscheinlich).¹¹

Between the dates of Woolhouse's and Opelt's publications, a harmonium with 19 tones to the octave was constructed by P. S. Munck, a professor at Lund, Sweden.¹² This instrument may well have been the first effort at a consciously equal-tempered instrument with 19 sounds to the octave. The work of Prof. Munck, whose instrument is presently at the Stockholm Museum, represents the beginning of very fruitful labor on multiple division by inhabitants of the Scandinavian countries. Scandinavians have remained in the forefront of the advocacy of multiple division to this day.

Unfortunately we do not have any of the music Prof. Munck played on his harmonium. The first known descriptive information about the performance of music written for a 19-tone instrument comes from the report of Prof. Melchior Sachs to the 4th Congress of the International Music Society in London in 1911. The actual music is again missing (although a manuscript may well exist somewhere), but we have enough descriptive material to obtain a fairly good idea as to what it was like. It probably involved traditional harmony with an abundance of 7th chords employing the

¹¹Ibid., p. 53.

¹²Kornerup, Musical Acoustics based on a Pure Third System, Munck is said by Kornerup to have built his 19-tone harmonium about the year 1845.

unique enharmonic relationships of 19-tone temperament, wherein the "common tone" of 12-tone temperament is replaced by "resolution" in 19-. Sachs' emphasis is on the playability of 12-tone music on 19-tone instruments. This playability he supposedly demonstrated at the Congress in London. It is easy to disagree with his conclusions, however, especially in chromatic music, where the different systems may involve discrepancies approaching and even exceeding half a semitone.

Sachs' presentation to the congress included a theory of the development of musical systems which closely anticipates Yasser's. Sachs sees the development of music as having progressed from 3 to 5 to 7 to 12 tones per octave, with 19 as the next system. That Sachs believes in the same additive concept of tonal evolution as Yasser is evident from the following statement: "As the 12-tone system arose out of the combining of the 5- and 7-tone systems, so, in the further course of development, the 19-tone system, which must also be tempered, will come out of the union of the 7- and 12-tone systems."¹³

To Sachs the advent of 19-tone temperament will result in a greater degree of purity in the triads, but of much

¹³ Report of the 4th Congress, International Music Society, 1911, p. 279. "Wie aus der Verbindung des Fünf- und Siebentonsystems das Zwölftonsystem entstand, so entsteht im weiteren Verlauf der Entwicklung aus der Verbindung des Sieben- Zwölf- und Neunzehntonsystem, welches auch ein temperiertes sein muss."

greater importance to him is the increase in the richness of musical expressive means (Bereicherung der musikalischen Ausdrucksmittel) that can be expected through 19-tone temperament. As an example, Sachs cites the addition, through 19-tone temperament, of a third form of the deceptive cadence. In the key of C, he points out, one can, in the 12-tone system, resolve the dominant chords to either A minor or Ab major and produce what is called a deceptive cadence. In 19-tone music a third deceptive resolution to G# is possible. It seems unlikely that anyone today would consider this of great importance, but Sachs' arguments make sense for their time, and point out a possible use for multiple division which has not gone altogether unnoticed by its later advocates. It might be possible with 19-tone temperament, or with another multiple division, to expand functional harmony, beginning at the point where it transcended its own limits within a 12-tone framework and disintegrated. Such a theory of the disintegration of functional harmony is implied by Yasser, although he does not state it directly.

Finally, Sachs should be noted for several fascinating suggestions for the reform of musical notation. Briefly, they include the use of multicolored staff lines to simplify pitch differentiation and obviate the need for accidentals even in a 19-tone system, and a graphic notation of pitches by horizontal strokes whose length gives exact duration and

whose thickness gives an exact dynamic indication which would, among other things, simplify the designation of principal voices.

During the years immediately following Sachs' report to the International Music Society a flurry of activity in support of 19-tone temperament was begun. In Denmark, P. S. Wedell built an instrument capable of both 19- and 31-tone temperament, and later published, with Bertelson, a series of articles showing the purported advantages of 19- and 31-tone temperaments over 12-tone temperament by the sum of the squares of the errors in the consonances.¹⁴ The approximate errors of the fifth, major third and minor third in 12-, 19-, and 31-tone temperaments are, respectively, 2, 14, 16; 7, 7, 0; and 5, 1, 6 cents. The sums of the squares of these intervals are 456, 98, and 62 square cents. Using this method, the jump from 12- to 19-tone temperament would be emphatically justified by the improvements of the consonances.

In 1919 and 1921 Prof. Valentin Kowalenkow published articles (in German in the Russian publication Iswestija Akademitschenkowo) which went almost unnoticed outside of Russia, but which formed the basis for an article which Jacques Handschin, the noted musicologist, published several

¹⁴ According to Kornerup, Wedell and Bertelson calculated by smallest squares in 1915 and published their findings in Gottfried Skjerne's monthly Danish-language periodical, Musik, in 1917 and 1919.

years later in a book in honor of Dr. D. P. Scheuerleer.¹⁵ Handschin shows Kowalenkow's chart of the intervals of what he considers to be the just enharmonic scale (21 tones, representing the seven lettered degrees each three times, once each with natural, sharp, and flat). The intervals are given in decimal ratios, and are shown together with the equivalent intervals in 12- and 19-tone temperament. The error in each of these temperaments is approximated in commas and half-commas. The table and approximations are shown, below, in Example 40.

The emerging pattern shows that 19-tone temperament is nearly perfect for those interval relations built of minor thirds, better than 12-tone temperament for those built of major thirds, and not as satisfactory for those relations built on perfect fifths.

Wedell and Bertelson prefer 19- to 12-tone temperament for its consonances. Kowalenkow prefers it on the basis of a table of all of the intervals. The case would be all but closed were it not for one basic weakness in both arguments. Neither provides for the possibility that the intervals he considers are of unequal importance. It is the retrogression in the quality of the perfect fifth, deemed the most important of the intervals within the octave, that imposes the gravest doubts on the acoustical

¹⁵Handschin, Jacques, Akustisches aus Russland, published in the Gedenkboek aangeboden aan Dr. D. P. Scheuerleer, p. 143.

Example 40: Kowalenkow's Chart of 12- and 19-Tone Temperaments

<u>Distance</u>	<u>12-tone</u>	<u>Tone</u>	<u>Pure</u>	<u>19-tone</u>	<u>Distance</u>
-----	1.000	C	1.000	1.000	-----
over 1 comma	1.059	C#	1.042	1.037	under 1/2 comma
over 1 comma	"	Db	1.08	1.075	under 1/2 comma
under 1/2 comma	1.122	D	1.125	1.116	over 1/2 comma
over 1 comma	1.189	D#	1.172	1.157	1 comma
over 1/2 comma	"	Eb	1.200	1.200	under 1/2 comma
over 1/2 comma	1.260	E	1.250	1.244	under 1/2 comma
2 commas	1.335	F#	1.302	1.291	over 1/2 comma
over 1 comma	1.260	Fb	1.28	"	over 1/2 comma
under 1/2 comma	1.335	F	1.333	1.339	under 1/2 comma
over 1 comma	1.414	F#	1.389	1.388	under 1/2 comma
over 1 comma	"	Gb	1.44	1.440	under 1/2 comma
under 1/2 comma	1.498	G	1.500	1.494	under 1/2 comma
over 1 comma	1.567	G#	1.5625	1.549	over 1/2 comma
over 1/2 comma	"	Ab	1.600	1.607	under 1/2 comma
over 1/2 comma	1.682	A	1.667	1.666	under 1/2 comma
over 2 commas	1.782	A#	1.736	1.728	under 1/2 comma
over 1/2 comma	"	Eb	1.8	1.792	under 1/2 comma
over 1/2 comma	1.888	B	1.875	1.859	over 1/2 comma
over 1 comma	"	Cb	1.92	1.928	under 1/2 comma
2 commas	2.000	B#	1.953	"	1 comma

quality of the temperament. Handschin turns his attention directly to this problem, accepting in principle the priority of the fifth but not to the point where it precludes the consideration of other factors in the selection or rejection of 19-tone temperament.

In comparing 19- with 24-tone temperament, which was already quite well known at the time of the publication of his article, Handschin takes a very well-defined position: 24-tone temperament would add only to the chromatic element in music, while 19- would also result in an increased proximity to just intonation.

Handschin partially disassociates himself from the views expressed in his article in a preface which establishes his role as a transcriber rather than a convinced partisan.

The articles of Jose Würschmidt show a gradual development from an interest in multiple division in general to the advocacy of 19-tone temperament in particular. Citing Busoni, Würschmidt agrees that 12-tone temperament is near the end of its usefulness. But he disagrees with Busoni about 36-tone temperament and with the advocates of 24-tone temperament on the grounds that their proposals involve no organic growth for music, but rather, pure mechanical growth.¹⁶ He follows his criticism of 24- and 36-tone temperaments

¹⁶ Würschmidt, "Viertel- und Sechsteltonmusik," Neue Musik-Zeitung, 42: p. 183.

with an article extolling 19-tone temperament.¹⁷ In comparing the acoustical values of 12- and 19-tone temperaments, he declares for the latter despite a chart which places the former in a far better light than did those of Wedell or Kowalenkow. Where Wedell restricted himself to three consonances, and where Kowalenkow considered all of the intervals in the enharmonic scale of 21 tones, Würschmidt presents the major scale as his unit, comparing the intervals from the tonic in 12- and 19-tone temperament. He uses the just scale of Ptolemy as the basis for comparison. As three of the intervals, 3:2, 4:3, and 9:8, are derived from fifths alone, 19-tone temperament does not fare well in this comparison. Würschmidt's chart, shown below as Example 41, uses the ordinary logarithms of the interval ratios.

Example 41: Würschmidt's Chart of 12- and 19- Systems

<u>Pitch</u>	<u>Just</u>	<u>12-tone</u>	<u>deviation</u>	<u>19-tone</u>	<u>deviation</u>
C	0 000	0 000	000	0 000	0
D	5 115	5 017	-98	4 753	-362
E	9 691	10 034	+343	9 506	-185
F	12 494	12 543	+49	12 675	+181
G	17 609	17 560	-49	17 428	-181
A	22 185	22 577	+392	22 181	-4
B	27 300	27 594	+294	26 934	-366

¹⁷ Würschmidt, Die 19-stufige Skala, Neue Musik-Zeitung, vol. 42: p. 215.

It would appear that with the simpler scale possessing the advantage, 4 intervals to 2, and possessing the smaller total discrepancy when compared with the just diatonic scale, Würschmidt would favor that system. But Würschmidt sees no advantage in simplicity in this case. Having already stated his conviction that the resources of 12-tone temperament are near exhaustion, he also asserts that he can see no impossible obstacles to surmount in the shift to the more complex temperament. 19-tone temperament will be no problem at all for strings, he asserts, while for the piano it need cause no major changes. The white keys can be kept as they are; the existing black keys can be subdivided and two new ones added.¹⁸ Where others might read the statistics in his chart as an endorsement of 12-over 19-tone temperament, Würschmidt sees in it support for 19-tone temperament, on the grounds of its near equality to 12-tone temperament on an acoustical basis. This near-equality, combined with the assumption of the exhaustion of the resources of 12-tone temperament (which is fundamental to Würschmidt's viewpoint) produce as strong a leaning toward 19-tone temperament as that of those authors whose acoustical surveys found 19-tone temperament well ahead.

Würschmidt's case for 19-tone temperament rests, then, on its being the most useful temperament containing more than 12 tones and not on its being intrinsically better than

¹⁸Ibid., p. 216.

12-. Wärschmidt's theory of rational tone systems (discussed above in Chapter 1) shows why he considers this to be so and also suggests a possible subsequent course of evolution to 22-tone temperament, which Opelt had also declared possible, but which later advocates of 19-tone temperament were strongly to oppose.

The relative weakness, in 19-tone temperament, of the representation of the diatonic scale, which Wärschmidt inadvertently exposes without being particularly concerned, appears to have caused considerable concern to the later advocates of 19-tone temperament. None of the three major advocates of 19-tone temperament who follow uses the diatonic scale of Ptolemy as a musically basic phenomenon in the 19-tone system. Kornerup and Ariel both replace it with the diatonic of Didymus, thereby improving the rendition of the supertonic in 19-tone temperament considerably.¹⁹ Yasser goes much farther; he eliminates the diatonic scale as a point of reference for 19-tone temperament in favor of a "supra-diatonic" scale with 12 members. As Kowalenkow's chart (Example 40) demonstrates, the more non-diatonic tones used which are related to the diatonic members by thirds, the better 19-tone temperaments fares in comparison to 12-.

¹⁹The supertonic is 9:8 in the tetrachord of Ptolemy, and 10:9 in the tetrachord of Didymus. The respective cent values are 204 and 182. The supertonic in 12-tone temperament is 200 cents; in 19-tone temperament, 189 cents. By this comparison it is evident that the shift of this one tone from the tetrachord of Ptolemy to that of Didymus is sufficient to shift the balance of accuracy for the whole scale from 12-tone to 19-tone temperament.

With the above, our consideration of the theory of 19-tone temperament and its advocacy up to the time of Korschurp, Ariel, and Yasser is completed. The next three chapters will be concerned with the writings of this triumvirate, with whose work the cause of 19-tone temperament is closely associated.