1219 Poinsettia Drive
L.A. Calif 90046
26 April, 1975

Dear John

The basic structure of the "moments-of-symetry" is almost embarressingly simple. Unfortunately, what is simple is not always obvious, or visa-versa. And we use certain devices, repeatedly, without identifying them. Or we neglect to use them, when we might well have chosen to do so, had we recognized them.

Let me give a number of examples, starting with the classic moments-of-symetry of 12, where the "generating interval" is the Fourth; (5 units) (and 12 is equal)

0	1	2	3	4	5	6	7	8	9	10	11	12		
0												0	1	(?)
0					1							0	2	
0					1					2		0	3	
0			3		1			4		2		0	5	
0	5		3		1	6		4		2		0	7	
0	5	10	3	8	1	6	11	4	9	2	7	0	12	

These are all highly coherent patterns, and that is probably why we like them. But why do we stop exactly where we do, at 5 instead of 4 or 6, at 7 instead of 8 or 9 or — ? These structures are so rich in properties that one is hard put to isolate one, and say "this is the raison-d'être".

I think that as part of the pattern making process we latch onto cycles. That anywhere a cycle has the potential of forming, it will tend to do so. Having asserted itself its own inertia re-affirms it, and gives it remarkable durability. Now these are not acoustic cycles in the strict scientific sense. These are functional cycles, or effective cycles, based on melody and rythm, rather than on harmony. That is to say that if an interval is "functioning" as a Fourth it effectively "is" a Fourth. Our perception of Fourth-ness is not just acoustic, i.e. 4/3 determined, it is melodic and/or rythmic influenced to a high degree. We may learn melodically/rythmicaly to 'expect the Fourth to subtend 2 scale-steps as it does in the pentatonic. We then experience the pentatonic above as a conjunct series of 2-step Fourths, with a "disjunct" and also 2-step Fourth completing the cycle of melody/rythmic Fourths.

0	1	2	3	4	5	6	7	8	9	10	11	0/12	1	2	3	4	5	6	7	8	9	10	11	12
0			3		1			4		2		0			3		1			4		2		0

$\underbrace{0\ \ 3\ \ 1}_{\text{2 steps}}\underbrace{\ 1\ \ 4\ \ 2}_{2}\underbrace{\ 2\ \ 0\ \ 3}_{2}\underbrace{\ 3\ \ 1\ \ 4}_{2}\underbrace{\ 4\ \ 2\ \ 0}_{\text{2 (disjunct Fourth)}}$

When the Fourth subtends 3 scale steps we get this cycle of "tetrachords".

0 1 2 3 4 5 6 7 8 9 10 11 0/12 1 2 3 4 5 6 7 8 9 10 11 0/12 1 2 3 4 5 6 7 8 9 10 11 12
0 5 3 1 6 4 2 0 5 3 1 6 4 2 0 5 3 1 6 4 2 0

3-steps 3 3 3 3 3 3-steps

generating Fourths "disjunct" Fourth

And the final, unifying factor is that the disjunct Fourth is functioning melody/rythmically in the same capacity as the generating Fourths, hence the cycle, the cyclic feedback which lifts this organization up into an autonomous state. The deletion or addition of a single linear member would disrupt the cyclic continuity.

A moment-of-symetry may be used as a matrix, as a rythmelodic cycle analogous to an acoustic cycle, for which sub-moments may be derived. A spectrum of 5-member sub-moments may be taken from the 7-member moment of symetry, thus:

0 1 2 3 4 5 6 7 8 9 10 11 12/0 1 2 3 4 5 6 7 8 9 10 11 12/0 1 2 3 4 5 6 7 8 9 10 11 12/0
7/0 5 3 1 6 4 2 7/0 5 3 1 6 4 2 7/0 5 3 1 6 4 2 7/0

0 3 1 4 2 5/0 3 1 4 2 5/0

0 3 1 4 2 5/0 3 1 4 2 5/0

0 3 1 4 2 5/0 3 1 4 2 5/0

0 3 1 4 2 5/0 3 1 4 2 5/0

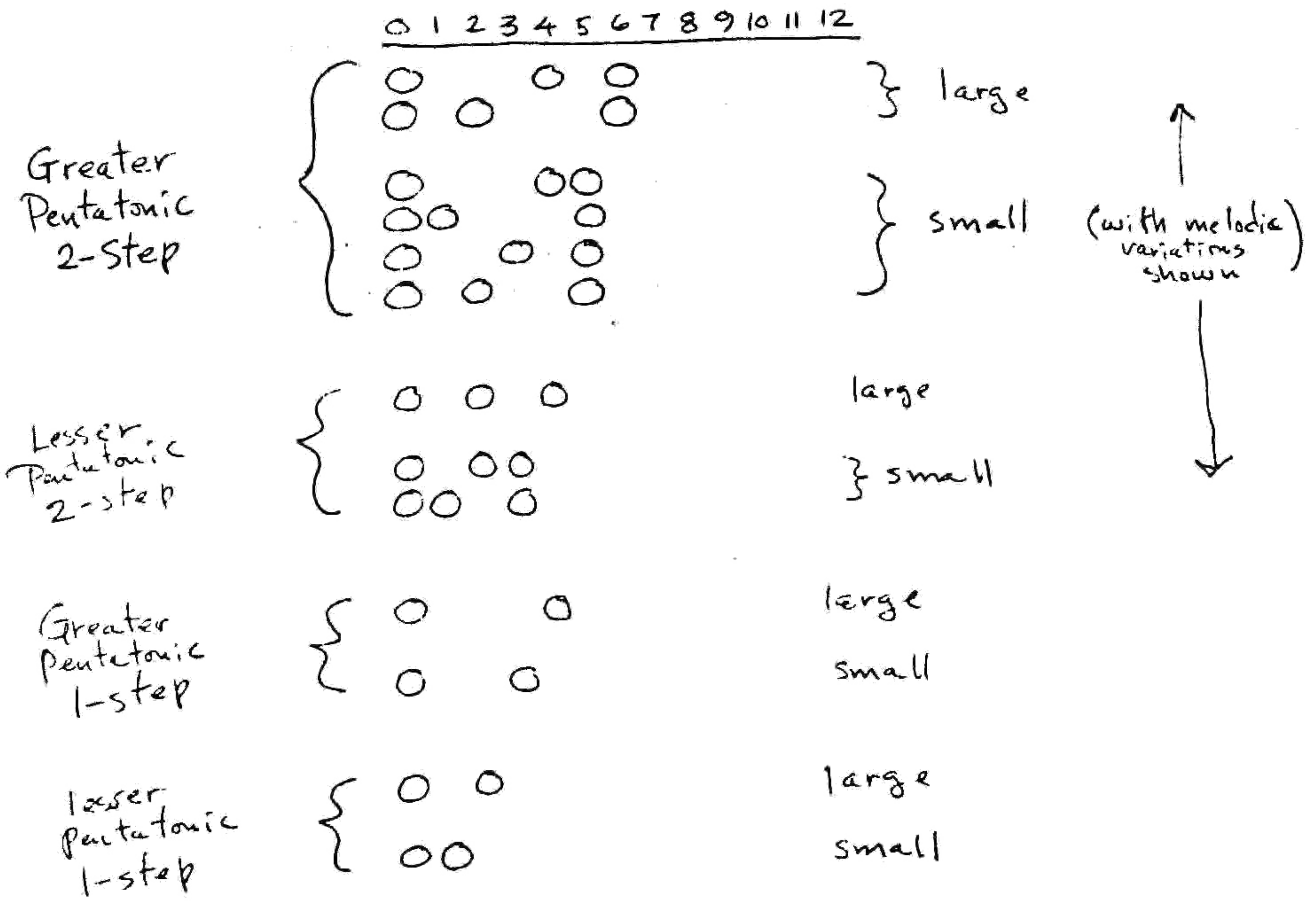
0 3 1 4 2 5/0 3 1 4 2 5/0

0 3 1 4 2 5/0 3 1 4 2 5/0

0 3 1 4 2 5/0 3 1 4 2 5/0

← chain of "Trichords" → ↑ disjunct "Trichord"

And we may observe, with musical interest, the variations occurring to the chain of "Trichords" and disjunct "Trichord". The Fourths of the chain have a "Tolerance" which charecterizes them. The disjunct fourth has a different and distinctive tolerance. We have sort of a double-layered depth. Each Interval has 2 size categories, each of which is again divided into 2 sizes.



We have "Binary Depth". And perhaps a kind of perceptual analog to binocular vision; I dont know. But it does seem to me that these "Japanese Pentatonics" carry more information, not less, than the simple, geometric pentatonic. We may observe that the Greater 1-Step does not "interfere" with the Greater 2-Step. Nor does the Lesser 1-Step interfere with the Lesser 2-step. There is a correspondence between the sizes of the Greater 1-steps and the Lesser 2-steep, but the psyche experiences no ~~c~~ confusion. Indeed, the large Greater 1-step overlaps the small lesser 2-step. But I cannot, for the life of me, experience a so-called "contradiction". It seems to me that the organizing principle must override absolute acoustic value of the intervals, in this case, insofar as the perceptual apparatus is concerned.

Let me tabulate some moments of symetry in 17, for example;

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0								1									0	2
0								1								2	0	3
0							3	1							4	2	0	5
0						5	3	1						6	4	2	0	7
0					7	5	3	1					8	6	4	2	0	9
0				9	7	5	3	1				10	8	6	4	2	0	11
0			11	9	7	5	3	1			12	10	8	6	4	2	0	13
0		13	11	9	7	5	3	1		14	12	10	8	6	4	2	0	15
0	15	13	11	9	7	5	3	1	16	14	12	10	8	6	4	2	0	17

by 8 units

Complements

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0							1										0	2
0							1							2			0	3
0				3			1				4			2			0	5
0	5			3			1	6			4			2			0	7
0	5	10		3	8		1	6	11		4	9		2	7		0	12
0	5	10	15	3	8	13	1	6	11	16	4	9	14	2	7	12	0	17

Generated by intervals of 7 units

complements

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0						1											0	2
0						1						2					0	3
0	3					1	4					2					0	5
0	3	6				1	4	7				2	5				0	8
0	3	6	9			1	4	7	10			2	5	8			0	11
0	3	6	9	12		1	4	7	10	13		2	5	8	11		0	14
0	3	6	9	12	15	1	4	7	10	13	16	2	5	8	11	14	0	17

by 6 units

complements

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0					1												0	2
0					1					2							0	3
0					1					2					3		0	4
0			4		1			5		2			6		3		0	7
0	7		4		1	8		5		2	9		6		3		0	10
0	7	14	4	11	1	8	15	5	12	2	9	16	6	13	3	10	0	17

by 5 units

complements

By 4 units

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0				1													0	2
0				1				2									0	3
0				1				2				3					0	4
0				1				2				3				4	0	5
0			5	1			6	2			7	3			8	4	0	9
0		9	5	1		10	6	2		11	7	3		12	8	4	0	13
0	13	9	5	1	14	10	6	2	15	11	7	3	16	12	8	4	0	17

complements

By 3 units

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0			1														0	2
0			1			2											0	3
0			1			2			3								0	4
0			1			2			3			4					0	5
0			1			2			3			4			5		0	6
0	6		1	7		2	8		3	9		4	10		5		0	11
0	6	12	1	7	13	2	8	14	3	9	15	4	10	16	5	11	0	17

complements

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0		1															0	2
0		1		2													0	3
0		1		2		3											0	4
0		1		2		3		4									0	5
0		1		2		3		4		5							0	6
0		1		2		3		4		5		6					0	7
0		1		2		3		4		5		6		7			0	8
0		1		2		3		4		5		6		7		8	0	9
0	9	1	10	2	11	3	12	4	13	5	14	6	15	7	16	8	0	17

complements

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
0																	0	1
0	1																	2
		2																3
			3															4
				4														5
					5													6
						6												7
							7											8
								8										9
									9									10
										10								11
											11							12
												12						13
													13					14
														14				15
															15			16
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	0	17

complements

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41

17-tone moment of Sum by 17 units of 41 (equal)

7 tone moments of Symetry by 5 units of above 17 Tone moment

0	5	10-2	3	8-4	1	6	11-1	4	9-3	2	7	12 0
0		4		1		5	2		6		3	0/7
	6	3		0/7		4	1		5		2	
	5	2		6		3	0/7		4		1	
	4	1		5		2		6	3		0/7	
	3	0/7		4		1		5	2			6
	2		6	3		0/7		4	1			5
	1		5	2			6	3	0/7			4
	0/7		4	1			5	2		6		3
		6	3	0/7			4	1		5		2
		5	2		6		3	0/7		4		1
		4	1		5		2		6	3		0/7
6		3	0/7		4		1		5	2		6
5		2		6	3		0/7		4	1		5
4		1		5	2		6		3	0/7		4
3		0/7		4	1		5		2		6	3
2		6		3	0/7		4		1		5	2
1		5		2		6	3		0/7		4	1

return to beginning of Cycle

Here 41 is shown as master set for convenient frame of reference. However the 17 tone moment could well be Helmholtz' 1/8 skhisma temperament to 17 places. When that is done, these scales correspond to attached sheet.

This "attached sheet" was reconstructed by memory and inference by Erv Wilson in 1994. Bracketed header* was not on original. Erv Wilson Feb 6, 1995

	0	5	10-2	3	8-4	1	6	11-1	4	9-3	2	7	12 0	7/17 moment of sym.
*	|	|	| |	|	| |	|	|	| |	|	| |	|	|	| |	
	0	7	14 4	11	1 8	15	5	12 2	9	16 6	13	3	10 0	5/17 Cycle, ref

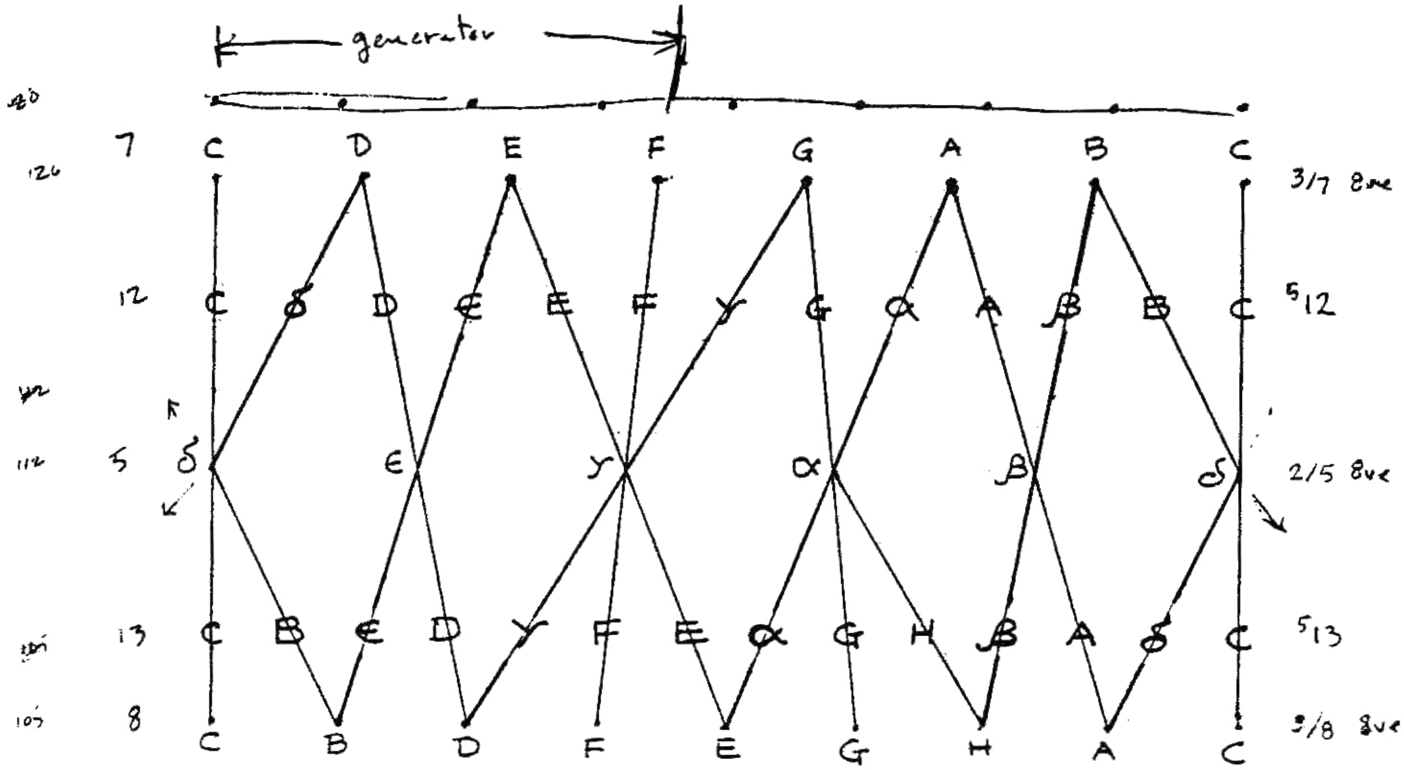
9/8	10/9	9/8	16/15	9/8	10/9	16/15	7
16/15	10/9	9/8	16/15	9/8	10/9	9/8	
16/15	9/8	10/9	16/15	9/8	10/9	9/8	
16/15	9/8	10/9	9/8	16/15	10/9	9/8	
16/15	9/8	10/9	9/8	16/15	75/64	16/15	
9/8	16/15	10/9	9/8	16/15	75/64	16/15	
9/8	16/15	75/64	16/15	16/15	75/64	16/15	
9/8	16/15	75/64	16/15	9/8	10/9	16/15	14
16/15	16/15	75/64	16/15	9/8	10/9	9/8	
16/15	9/8	10/9	16/15	9/8	10/9	9/8	
16/15	9/8	10/9	9/8	16/15	10/9	9/8	
10/9	16/15	9/8	10/9	9/8	16/15	9/8	
10/9	9/8	16/15	10/9	9/8	16/15	9/8	
10/9	9/8	16/15	9/8	10/9	16/15	9/8	
10/9	9/8	16/15	9/8	10/9	9/8	16/15	
9/8	10/9	16/15	9/8	10/9	9/8	16/15	
9/8	10/9	9/8	16/15	10/9	9/8	16/15	

A Beautiful Cycle of 7-tone scales derived from Helmholtz' 1/8 skhisma temperament to 17 places

9/8	10/9	9/8	16/15	9/8	10/9	16/15
16/15	10/9	9/8	16/15	9/8	10/9	9/8
16/15	9/8	10/9	16/15	9/8	10/9	9/8
16/15	9/8	10/9	9/8	16/15	10/9	9/8
16/15	9/8	10/9	9/8	16/15	75/64	16/15
9/8	16/15	10/9	9/8	16/15	75/64	16/15
9/8	16/15	75/64	16/15	16/15	75/64	16/15
9/8	16/15	75/64	16/15	9/8	10/9	16/15
9/8	16/15	16/15	75/64	16/15	9/8	10/9
9/8	16/15	9/8	10/9	16/15	9/8	10/9
9/8	16/15	9/8	10/9	9/8	16/15	10/9
10/9	16/15	9/8	10/9	9/8	16/15	9/8
10/9	9/8	16/15	10/9	9/8	16/15	9/8
10/9	9/8	16/15	9/8	10/9	16/15	9/8
10/9	9/8	16/15	9/8	10/9	9/8	16/15
9/8	10/9	16/15	9/8	10/9	9/8	16/15
9/8	10/9	9/8	16/15	10/9	9/8	16/15

Return to Beginning

C, Db ($C_1\#$), Ebb D (D_1), Eb ($D_1\#$), Fb E (E_1), F, Gb ($F_1\#$), G, Ab ($G_1\#$), Bbb A (A_1), Bb, Cb (B_1), C



The boundaries of 12 tone terrain are determined by a generator which may vary between 3/7 8ve & 2/5 8ve. If the generator becomes smaller than 2/5 8ve the scale passes thru the looking-glass into 13-toneland which terrain continues until the generator reaches 3/8 8ve.

The conventional differences between the two scales are so minor that much of our notation and nomenclature may be transferred from 12 to 13. But we must allow some flexibility of meaning to these signatures. The pitch sequence gets juggled

I deem it appropriate to identify a scale with a numeral indicating the number of steps subtended by the 8ve, & with a left superscript indicating the number of steps subtended by the generating interval. In the ${}^{5}12$-scale there are 12 steps to the 8ve and 5 steps between each member of this series B E A D G C F β ε α δ γ; <u>THERE ARE, ALSO, 5 STEPS BETWEEN γ & B</u>, creating a functional 5-step cycle of 12 members. *(The end with the beginning of of the series). In the ${}^{5}13$ scale there are 13 steps to the 8ve and 5 steps between each member of the generating series H B E A D G C F β ε α δ γ. AND 5 STEPS BETWEEN γ & H. I have previously called scales of this type "moments of symetry". The left superscript e may identify an equal division of the 8ctave thus ${}^{e}12$ or ${}^{e}13$.

The complementary scales of ${}^{5}12$ are ${}^{3}7$ and ${}^{2}5$.

0 1 2 3 4 5 6 7 8 9 10 11 12 13
C B ꞓ D γ F E α G H β A δ C $\quad {}^{e}13$

C B D F E G H A C $\quad {}^{3}8)^{e}13$ $\qquad$) means "out of".

C B D F E G H A C $\quad {}^{3}8)^{e}13$

B D E G A (B)
C D E G A C
C D F G A C
C D F G H C
C B F G H C
C B F E H C
B F E H A (B)
B D E H A (B)
~~B~~

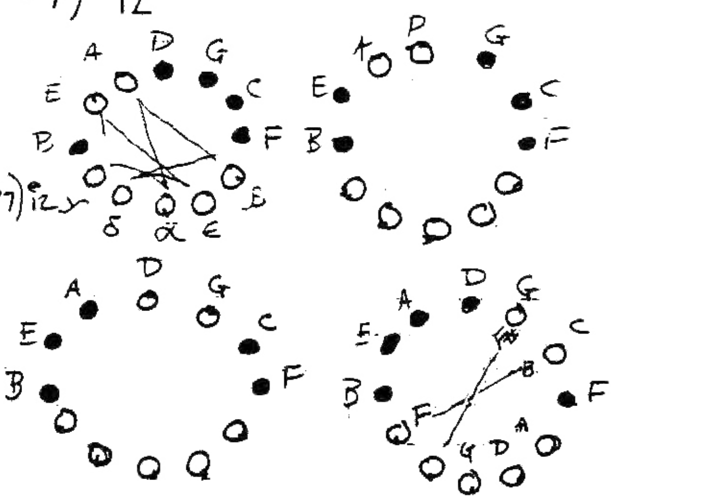
$\left.\right\}\ {}^{2}5)\,{}^{3}8)\,{}^{e}13$

0 1 2 3 4 5 6 7 8 9 10 11 12
C δ D ε E F γ G α A β B C

C D E F G A · B C $\quad {}^{3}7)^{e}12$

· · D · E · · · G · A · B ·
C · D · E · · G · A · · C
C · D · · F · G · A · · C
C · D · · F G B C
C E F G B C
C E F A B C
D E F A B

${}^{2}5)\,{}^{3}7)^{e}12$



Blanks

C D E♭ F G A♭ A C
C D♭ D F G A♭ A C
C D♭ D F G♭ G A C
C D F G♭ G A C♭ C

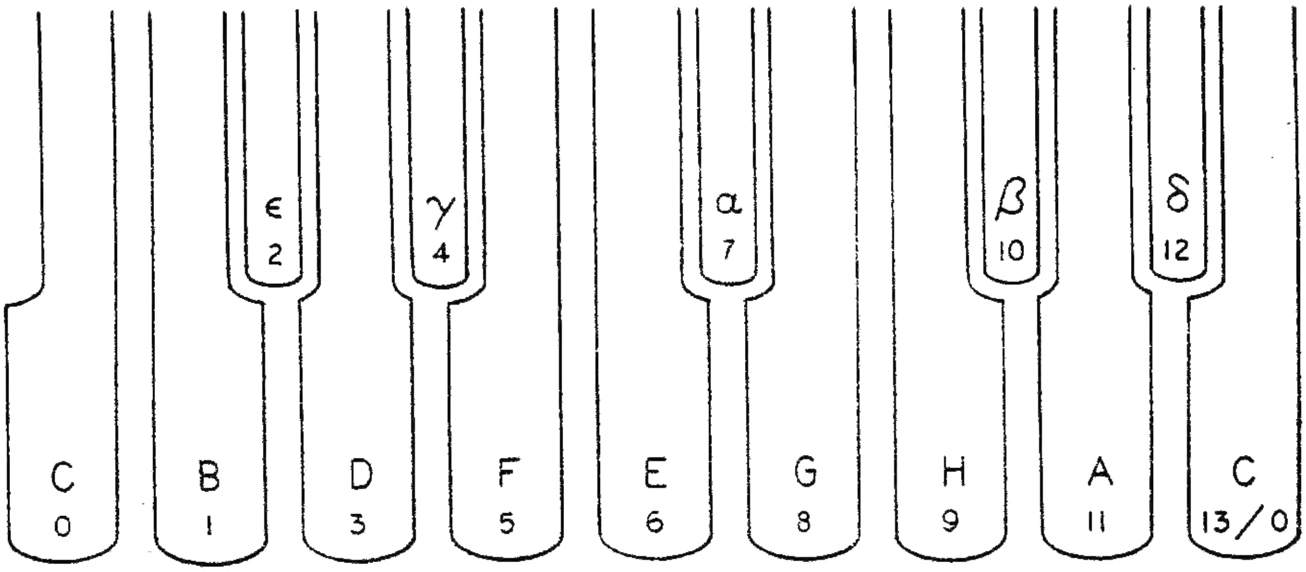
F G A# B C D E F

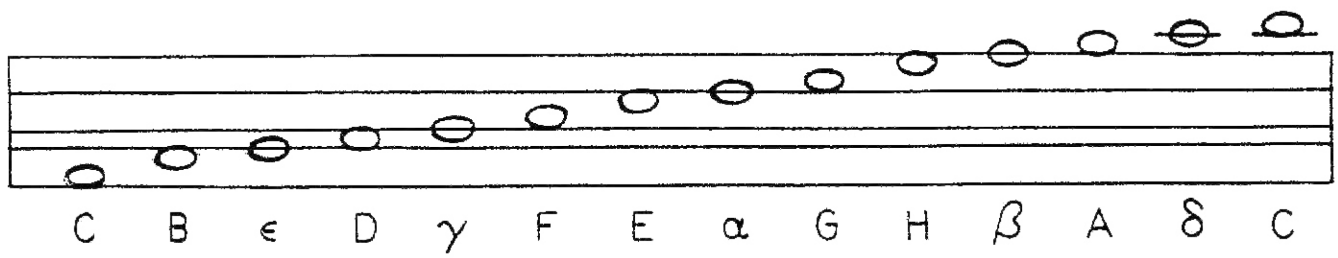
The ${}^{2}5)^{3}7$ scales seem to be logical, however the remaining $7)^{e}12$ defy explanation. ${}^{e}5)^{3}7$ is complemented by ${}^{1}2)^{3}7$. 7 becomes a true cycle independent of 12.

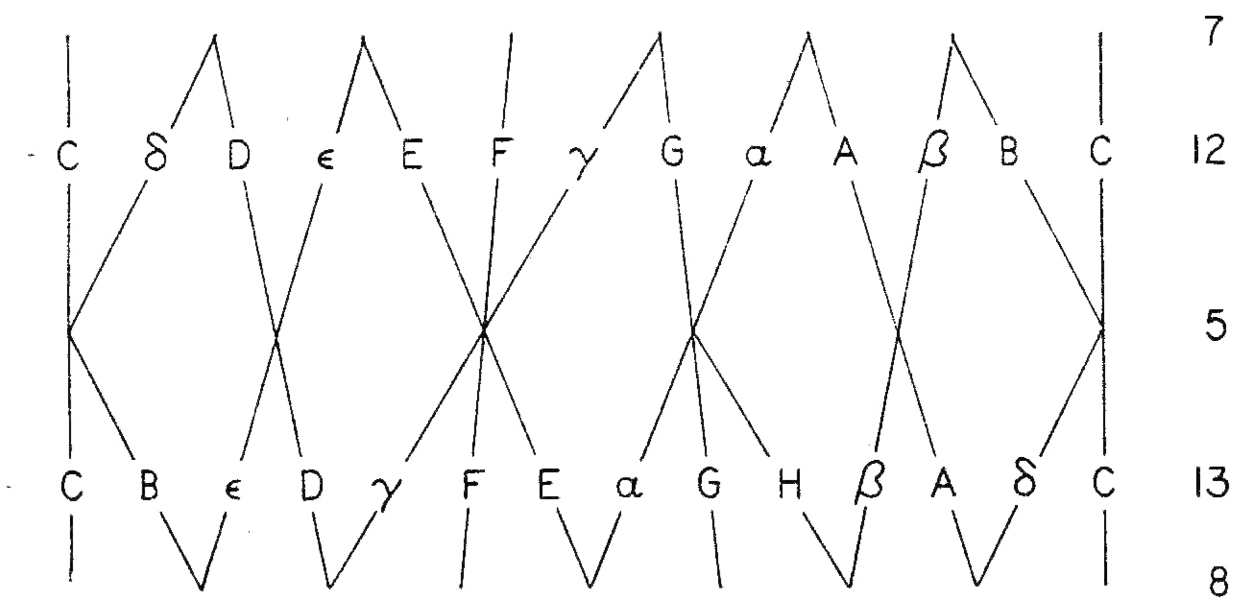
0	1	2	3	4	5	6	7	8	9	10	11	12	
C		D		E	F		G		A		B	C	
C		D	Eb		F		G		A		B	C	
C		D	Eb		F		G	Ab			B	C	+
C	Db		Eb		F		G	Ab			B	C	
C	Db		Eb		F	Gb		Ab			B	C	
C		D		E	F		G	Ab			B	C	
C	Db			E	F		G	Ab			B	C	3
C	Db			E	F	Gb		Ab			B	C	
C	Db			E	F		G		A		B	C	2
C	Db			E	F	Gb			A		B	C	
C		D		E	F	Gb			A		B	C	1

Sketch for 13-Tone Keyboard (8+5=13)

Circa 1975 by Erv Wilson

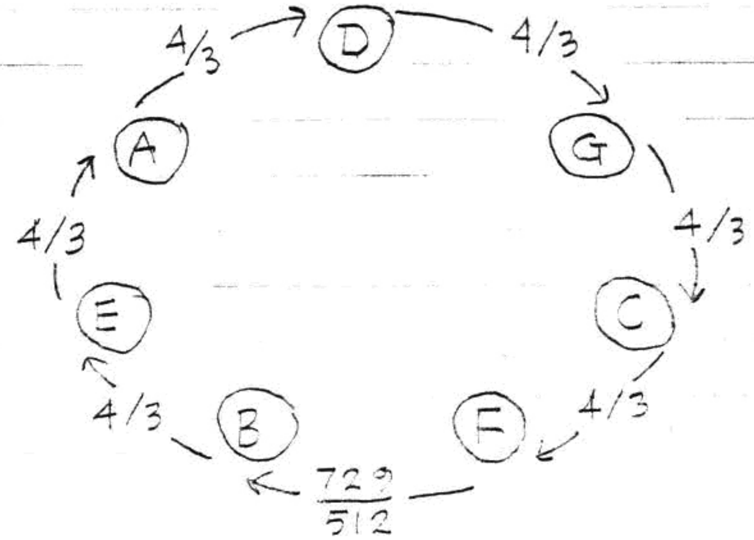


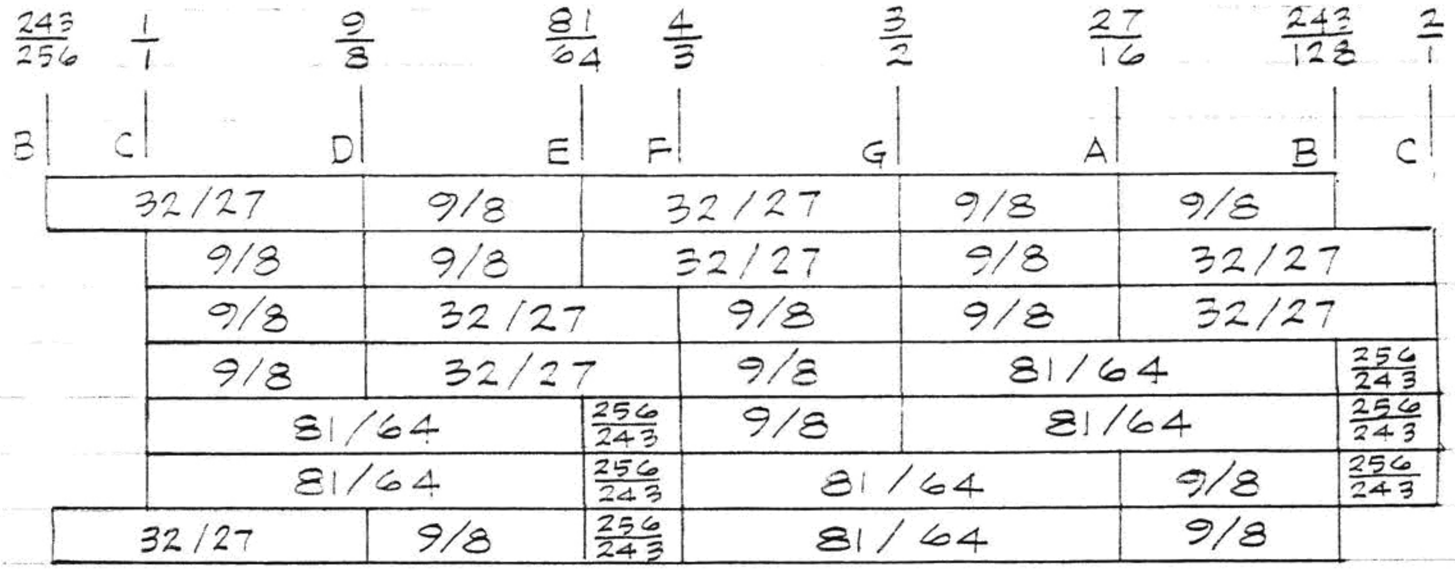




THE TANABE CYCLE

diagram by Erv Wilson, 1998





as explained to me by Dr. Hisao Tanabe about 1947. He stated also, that he was about to publish a book on the subject, which I have not seen. He emphasized that the tuning was by <u>pure</u> (Pythagorean) intervals.

PARALLELOGRAM FROM THE TANABE CYCLE

1	256/243	9/8	32/27	81/64	4/3	1024/729	729/512	3/2	128/81	27/16	16/9	243/128	2
C	D♭	D	E♭	E	F	G♭	F♯	G	A♭	A	B♭	B	C

G C F	9/8	9/8	32/27	9/8	32/27
D	9/8	256/243	81/64	9/8	32/27
A	9/8	256/243	81/64	256/243	81/64
E	256/243	9/8	81/64	256/243	81/64
B	256/243	9/8	32/27	9/8	81/64

F	9/8	81/64	256/243	9/8	32/27
D G C	9/8	32/27	9/8	9/8	32/27
A	9/8	32/27	9/8	256/243	81/64
E	256/243	81/64	9/8	256/243	81/64
B	256/243	81/64	256/243	9/8	81/64

F	9/8	81/64	256/243	81/64	256/243
C	9/8	32/27	9/8	81/64	256/243
A D G	9/8	32/27	9/8	32/27	9/8
E	256/243	81/64	9/8	32/27	9/8
B	256/243	81/64	256/243	81/64	9/8

F	81/64	9/8	256/243	81/64	256/243
C	81/64	256/243	9/8	81/64	256/243
G	81/64	256/243	9/8	32/27	9/8
E A D	32/27	9/8	9/8	32/27	9/8
B	32/27	9/8	256/243	81/64	9/8

F	81/64	9/8	32/27	9/8	256/243
C	81/64	256/243	81/64	9/8	256/243
G	81/64	256/243	81/64	256/243	9/8
D	32/27	9/8	81/64	256/243	9/8
B E A	32/27	9/8	32/27	9/8	9/8

white Key Position

verbal communication with Hisao Tanabe about 1947

Secondary Moments of Symetry
(sub-moments)

© by Erv Wilson Feb 9, 1995

Primary	0	5	3	1	6	4	2	0
	a	b	b	a	b	b	b	
Secondary	0		3	1		4	2	0
		4	2	0		3	1	
		3	1		4	2	0	
		2	0		3	1		4
		1		4	2	0		3
		0		3	1		4	2
			4	2	0		3	1

⇨

0 3	3 1	1 4	4 2	2 0
ab,	b,	ab,	b,	b
ab,	b,	ba,	b,	b
ba,	b,	ba,	b,	b
ba,	b,	bb,	a,	b
bb,	a,	bb,	a,	b
bb,	a,	bb,	b,	a
bb,	b,	ab,	b,	a

also

Examples;

Let a = 1 and b = ∅ (1.618..)

2.618	1.618	2.618	1.618	1.618
2.618	1.618	2.618	1.618	1.618
2.618	1.618	2.618	1.618	1.618
2.618	1.618	3.236	1	1.618
3.236	1	3.236	1	1.618
3.236	1	3.236	1.618	1
3.236	1.618	2.618	1.618	1

compare ⇦

* Let a = 1 and b = 2

3	2	3	2	2
3	2	3	2	2
3	2	3	2	2
3	2	4	1	2
4	1	4	1	2
4	1	4	2	1
4	2	3	2	1

Let a = ∅ and b = 1

2.618	1	2.618	1	1
2.618	1	2.618	1	1
2.618	1	2.618	1	1
2.618	1	2	1.618	1
2	1.618	2	1.618	1
2	1.618	2	1	1.618
2	1	2.618	1	1.618

compare ⇦

Let a = 2 and b = 1

3	1	3	1	1
3	1	3	1	1
3	1	3	1	1
3	1	2	2	1
2	2	2	2	1
2	2	2	1	2
2	1	3	1	2

These diagrams are intended to clarify my discussion of "sub-moments" in letter to John Chalmers dated 26 April 1975.

* The Japanese pentatonic permutations were shown to me by Dr Hisao Tanabe, U. of Tokyo, about 1947. He used the Pure "Pythagorean" 7-tone scale as the primary Moment-of-Symetry.

844 N. Ave 65
Los Angeles, CA 90042
December 14, 1992

Dear John,

The year 1949 found me at BYU. In their library I found Joseph Yasser's A Theory of Evolving Tonality. I was very much impressed with the potential of: Equal Division, Iterating Sequences, Moments of Symetry. Yasser's stamp is on everything I have done in that regard subsequently. I was caught up by broad, sweeping implications of what he was saying more than by the detail. This gave rise to my studies in equal division, organic scale-trees, nested moments-of-symetry, golden scales, and related pathways, key-board design, notation, etc.

Enclosed is my letter to you of 26 April 1975 where I set down a brief attempt to explain moments-of-symetry to you. These were considerations that I took for granted, and I certainly did not see any novelty in them at the time. Some of my usages of MOS are none-the-less quite creative.

A point that I utterly failed to clarify (to myself) sufficiently at the time was the pervading 2-interval MOS character of all "interval-category" cycles (cycles-of-Seconds, -Thirds, and -Fourths). Examples from "C Major"

Cycle-of-Fourths: B E A D G C F B (2-interal pattern at 3 8ve MOS)
S S S S S S L

Cycle-of-Seconds: B C D E F G A B (2-interval pattern at 1 8ve mos)
S L L S L L L

Cycle-of-Thirds: B D F A C E G B (2-interval pattern at 2 8ve MOS)
S S L S L S L

This leads into some relatively unexplored territory, oddly enough.

Joy and Good Will, yours
Erv Wilson

Cycle									MOS (2-Interval cycle)	S	L	range	generator
Ref.	0.	1.	3.	5.	6.	8.	10.	12/0.					
	O	O	O	O	O	O	O	O					
	B	C	D	E	F	G	A	B					
1-step	0.	1.	2.	3.	4.	5.	6	7/0	S L L S L L L	1	2	1 8ve (12)	5/12
	O	O	O	O	O	O	O	O					
2-step	0.	4	1.	5.	2.	6	3.	7/0.	S S L . S L S L	3	4	2 8ves. (24)	7/24
	O	O	O	O	O	O	O	O					
3-step	0.	5.	3.	1.	6	4	2.	7/0.	S S S S S S L	5	6	3 8ves. (36)	5/36
	O	O	O	O	O	O	O	O					
4-step	0.	2.	4.	6.	1.	3.	5	7/0.	S L L L L L L	6	7	4 8ves. (48)	7/48
	O	O	O	O	O	O	O	O					
5-step	0.	3.	6.	2.	5	1.	4.	7/0.	S L S L S L L	8	9	5 8ves. (60)	17/60
	O	O	O	O	O	O	O	O					
6-step	0.	6.	5.	4.	3.	2.	1.	7/0.	S S S L S S L	10	11	6 8ves. (72)	31/72
	O	O	O	O	O	O	O	O					

TRANSFORMS OF 7-TONE MOS (5/12 GENERATOR)

21 AUG 65

Dear John,

My appreciation, again, for the tables of 2/1. I dreamt last night you had done the 3/1's but a computer error had left off the decimal places?! what?

I have been working on the idea of 5-limit moments-of-symmetry which will account for some of the systems that 3-limit does not. It's too soon for generalizations; apparently the 3-limit M.O.S. have a minimum of 2 melodic intervals, 5-limit a minimum of 3, 7-limit a minimum of 4 — then we have to allow for a theoretical 9-limit moment-of-symmetry with a minimum of 5 melodic intervals before we go to 11-limit with a minimum of 6. It is quite a jump, then, from the introduction of 7 to the exhaustion of 7 and the introduction of 11 as the most compact and coherent procedure.

On p4 enclosed, one possible development of the 5-limit M.O.S. is shown to 15. At that level we have the option of interposing the 16/15's (there are 7) with 3-limit, 5-limit, or 7-limit intervals. 3-limit interposition is wasteful, 5-limit interposition indicates modulation of a lesser system, 7-limit interposition, as shown, reduces the system to rock-bottom simplicity and coherence and would seem the most musically productive.

I have plotted out some of the recognized tetrachordal species, and must say that in this respect the 7-limit configuration is amazingly versatile and coherent, more so than I had anticipated.

Later,

Erv

On the Construction of the General Moment-of-Symetry
©1994 by Erv Wilson
(Notes on Scale-Tree, work in progress)

Consider an 11-tone scale where 3-steps of the 11-step scale is the generator of a chain thru the scale.

0.	1,	2,	3,	4,	5.	6.	7.	8.	9,	10,	11/0.	Scale
0	+4	+8	+1	+5	+9	+2	+6	+10	+3	+7	0	3-step Chain
	+4	+4	-7	+4	+4	-7	+4	+4	-7	+4	-7	Relative Chain moves
	A	A	B	A	A	B	A	A	B	A	B	2-Interval Sequence

$A = (+4)$, $B = (-7)$. $(7\times(+4)) + (4\times(-7)) = 0$

Example 2: We choose a 5-note (5-step) scale with 2 steps as the generator

0	1	2	3	4	5/0	Scale:
0	+3	+1	+4	+2	0	2-step Chain
	+3	-2	+3	-2	-2	Relative chain moves
	A	B	A	B	B	2-interval Sequence

$A = (+3)$, $B = (-2)$. $(2\times(+3)) + (3\times(-2)) = 0$

Example 3: A 12-step scale where 5-steps form the generator

0.	1.	2,	3,	4.	5,	6.	7.	8,	9.	10,	11,	12/0.	Scale
0	+5	+10	+3	+8	+1	+6	+11	+4	+9	+2	+7	0	5-step Chain
	+5	+5	-7	+5	-7	+5	+5	-7	+5	-7	+5	-7	Relative Chain Moves
	A	A	B	A	B	A	A	B	A	B	A	B	2-interval Pattern

$A = (+5)$, $B = (-7)$. $(7\times(+5)) + (5\times(-7)) = 0$

Example 4: A 17-tone scale where $\left(\frac{3}{17}\right)$ is the Generator

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17/0. Scale

0 +6 +12 +1 +7 +13 +2 +8 +14 +3 +9 +15 +4 +10 +16 +5 +11 0 3-Step Chain

+6 +6 −11 +6 +6 −11 +6 +6 −11 +6 +6 −11 +6 +6 −11 +6 −11 Relative Moves

A A B A A B A A B A A B A A B A B General 2 interval Pattern

$A=(+6)$, $B=(-11)$. $(11\times(+6))+(6\times(-11))=0$

Example 5: A 19-tone MOS where $\left(\frac{5}{19}\right)$ is the Generator

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19/0. Scale

0 +4 +8 +12 +16 +1 +5 +9 +13 +17 +2 +6 +10 +14 +18 +3 +7 +11 +15 0 5-Step Chain

+4 +4 +4 +4 −15 +4 +4 +4 +4 −15 +4 +4 +4 +4 −15 +4 +4 +4 −15 Relative Moves

A A A A B A A A A B A A A A B A A A B General 2-Interval Pattern

$A=(+4)$, $B=(-15)$. $(15\times(+4))+(4\times(-15))=0$ Check

The melodic ratio of A:B can be as the scalemaker chooses,

If it is A:B = 2:1 then $(15\times2)+(4\times1)=34$, gen $(4\times2)+(1\times1)=9$

3:2 then $(15\times3)+(4\times2)=53$, $(4\times3)+(1\times2)=14$

5:3 $(15\times5)+(4\times3)=87$ $(4\times5)+(1\times3)=23$

4:3 $(15\times4)+(4\times3)=72$ $(4\times4)+(1\times3)=19$

1.618:1, (Φ) Then it is a 19-tone golden scale where (see next line)

.057233987:.035372549 and generator is .264308497

see scale-tree

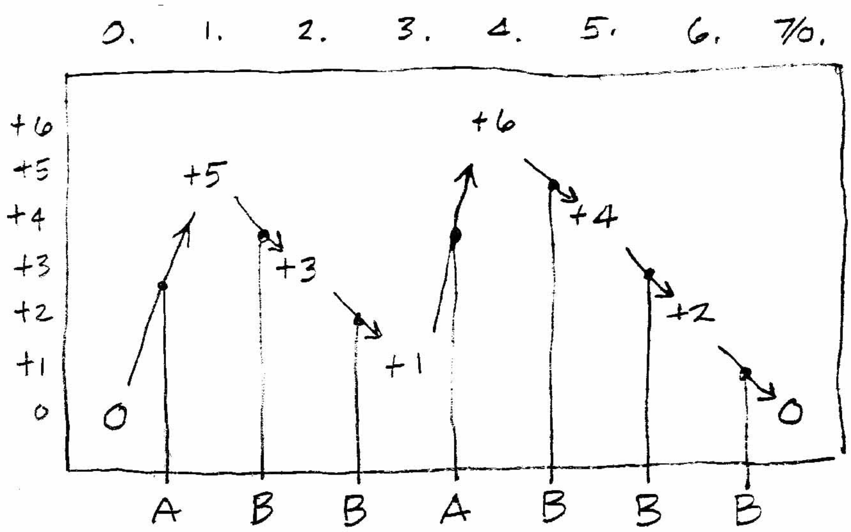
Example 6; A 7-tone scale where $\left(\frac{3}{7}\right)$ is the generator

0	1	2	3	4	5	6	7/0.	SCALE
0	+5	+3	+1	+6	+4	+2	0	3-step Chain
+5	-2	-2	+5	-2	-2	-2		relative moves
A	B	B	A	B	B	B		General 2-interval Pattern

$A = (+5)$, $B = (-2)$. $(2 \times (+5)) + (5 \times (-2)) = 0$ check

Table of scales

	Intervals A	Intervals B	Generator/System (A+2B)/(2A+5B)
equal	1	1	3 / 7
	1	2	5 / 12
	1	3	7 / 17
	2	3	8 / 19
	1	4	9 / 22
	2	5	12 / 29
	3	5	13 / 31
Limits	0	1	2 / 5
Limits	1	0	1 / 2
	5	1	7 / 15
enantio-morphs	6	1	8 / 17
enantio-morphs	1	6	13 / 32
	1	5	11 / 27



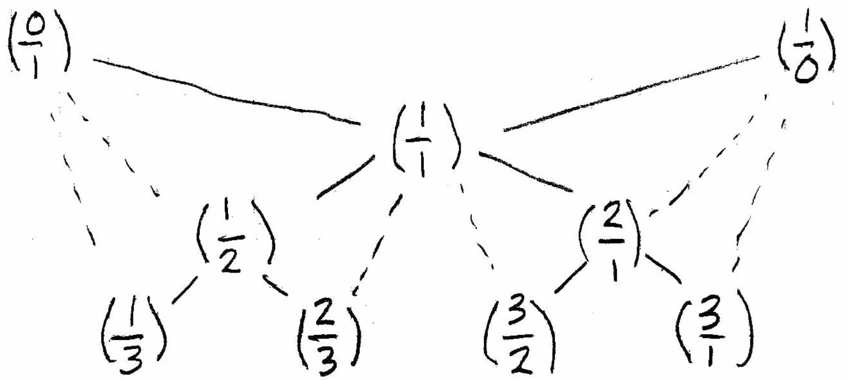
Modes of Above scale

Mode							
0.	A	B	B	A	B	B	B
1.	A	B	B	B	A	B	B
2.	B	A	B	B	A	B	B
3.	B	A	B	B	B	A	B
4.	B	B	A	B	B	A	B
5.	B	B	A	B	B	B	A
6.	B	B	B	A	B	B	A

Enantiomorphic pairs of scales

1	2	5 / 12
2	1	4 / 9
1	3	7 / 17
3	1	5 / 11
2	3	8 / 19
3	2	7 / 16

The ratio of the intervals A to B may be sorted in order of their magnitude by the use of the following Scale-Tree; Figure 1



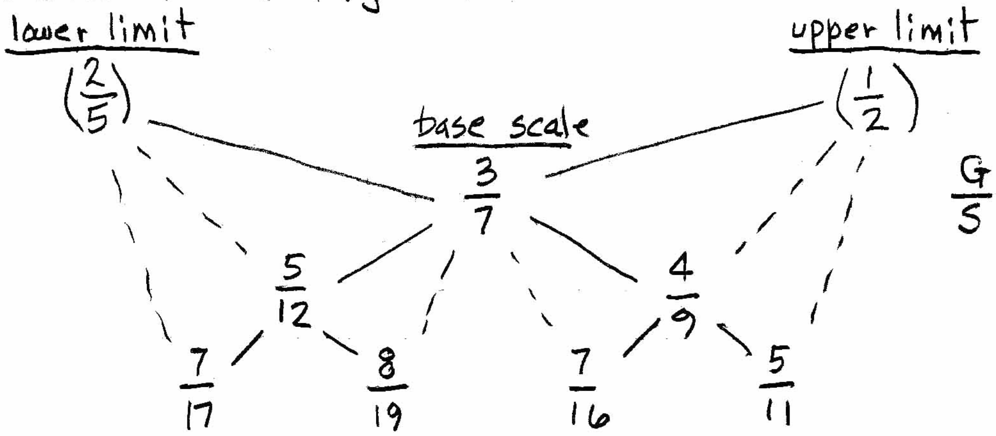
$\frac{A}{B}$ SUBSTITUTION TREE

(add the numerators & add the denominators To get the intermediate Pair of Terms)

etc

The corresponding generator to system $\left(\frac{G}{S}\right)$ of the $\left(\frac{3}{7}\right)$ scale (example 6) will then be as shown below. Fig 2.

$$\frac{(1A+2B)}{(2A+5B)} = \frac{\text{generator}}{\text{system}}\left(\frac{G}{S}\right)$$



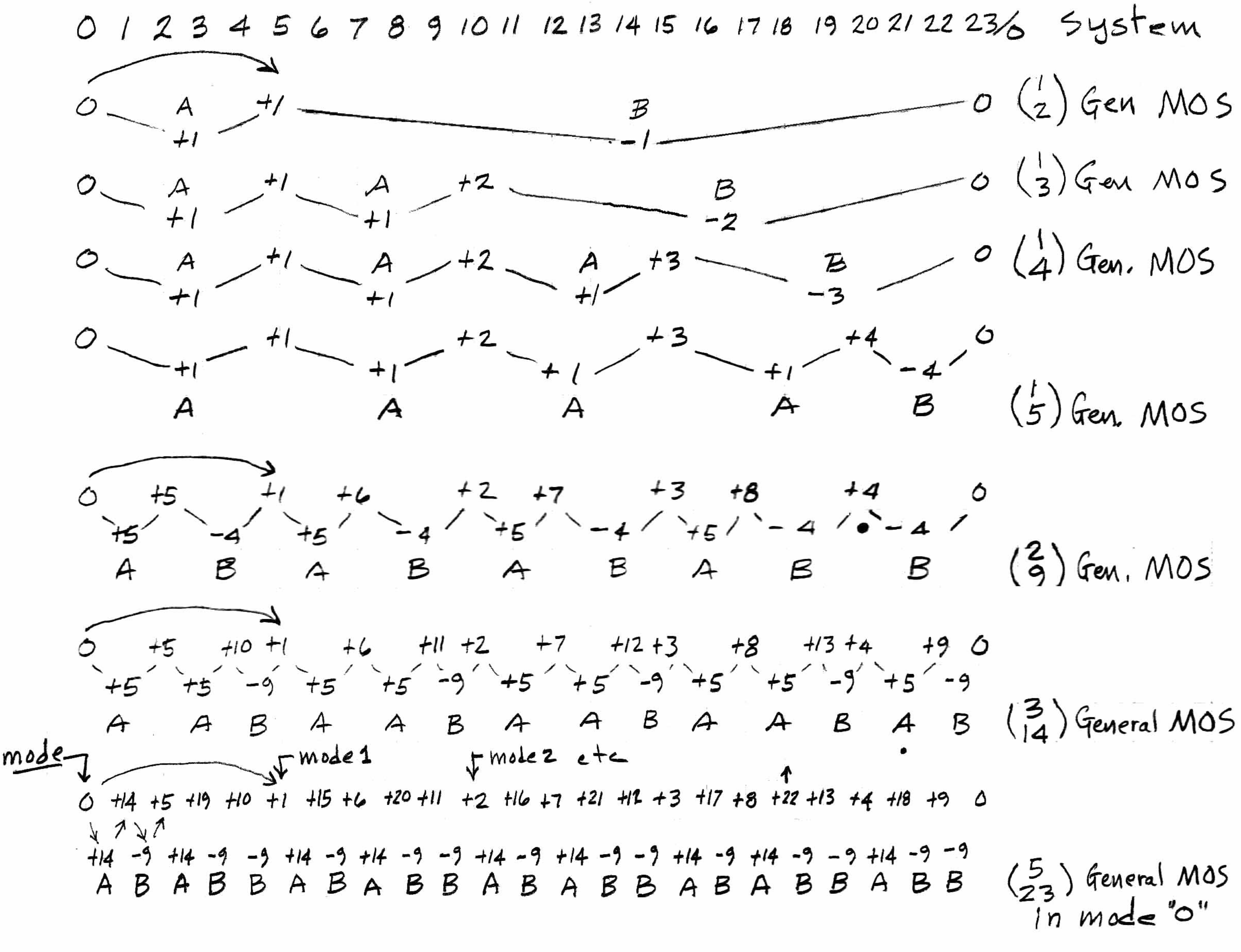
$\frac{G}{S}$ TREE

Thus is shown the beginning terms of a recurrent sequence of systems that will embody the base scale $\left(\frac{3}{7}\right)$.

Fig 3. a

	A	B	B	A	B	B	B	General MOS $\left(\frac{3}{7}\right)$
A=0, B=1	0	1	1	0	1	1	1	5-Tone, Lower Limit (2,5)
A=1, B=0	1	0	0	1	0	0	0	2-tone, Upper Limit (1,2)
A=1, B=1	1	1	1	1	1	1	1	7-Tone, Base Scale (3,7)
A=1, B=2	1	2	2	1	2	2	2	12-Tone system (5,12)
A=2, B=1	2	1	1	2	1	1	1	9-Tone System (4,9)
A=1, B=3	1	3	3	1	3	3	3	17-Tone system (7,17)
A=3, B=1	3	1	1	3	1	1	1	11-Tone System (5,11)
A=2, B=3	2	3	3	2	3	3	3	19-Tone System (8,19)
A=3, B=2	3	2	2	3	2	2	2	16-Tone System (7,16)

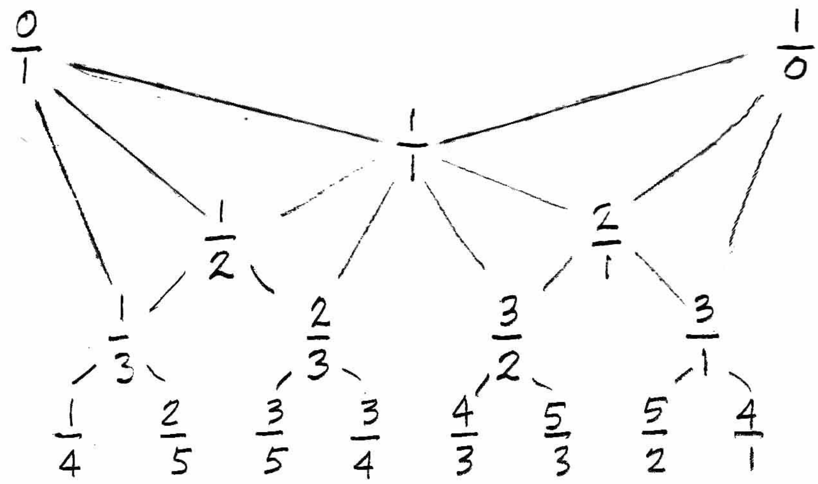
The MOS with Generator (5/23)



Example 7; A 7-tone scale where $(\frac{2}{7})$ is the generator

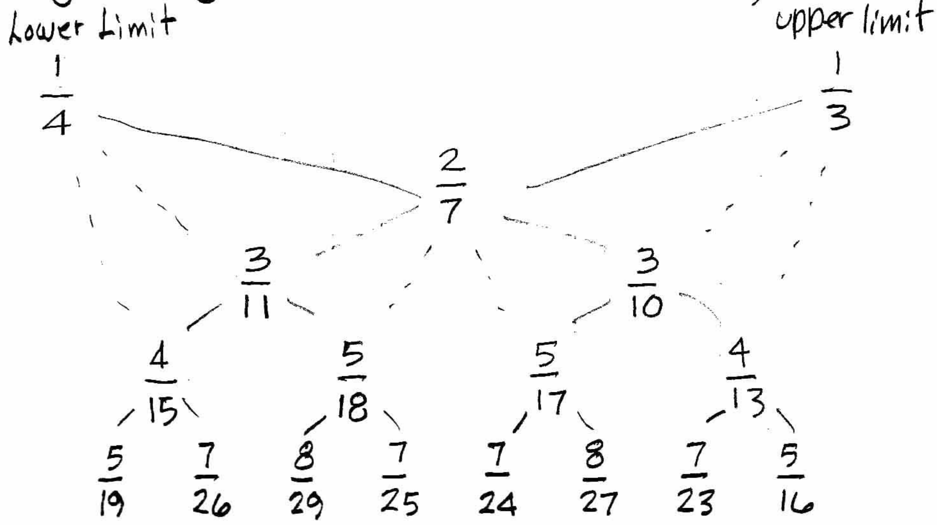
0.	1.	2.	3.	4.	5.	6.	7/0.	scale
0	+4	+1	+5	+2	+6	+3	0.	$(\frac{2}{7})$ chain
+4	-3	+4	-3	-4	-3	-3		relative moves
A	B	A	B	A	B	B		2-interval Pattern

Fig 1 (Scaletree) $\frac{A}{B}$ TREE



$\frac{A}{B}$ TREE

Fig 2 $\frac{G}{S}$ TREE FOR $(\frac{2}{7})$ MOS; $G=(1A+1B)$, $S=(3A+4B$



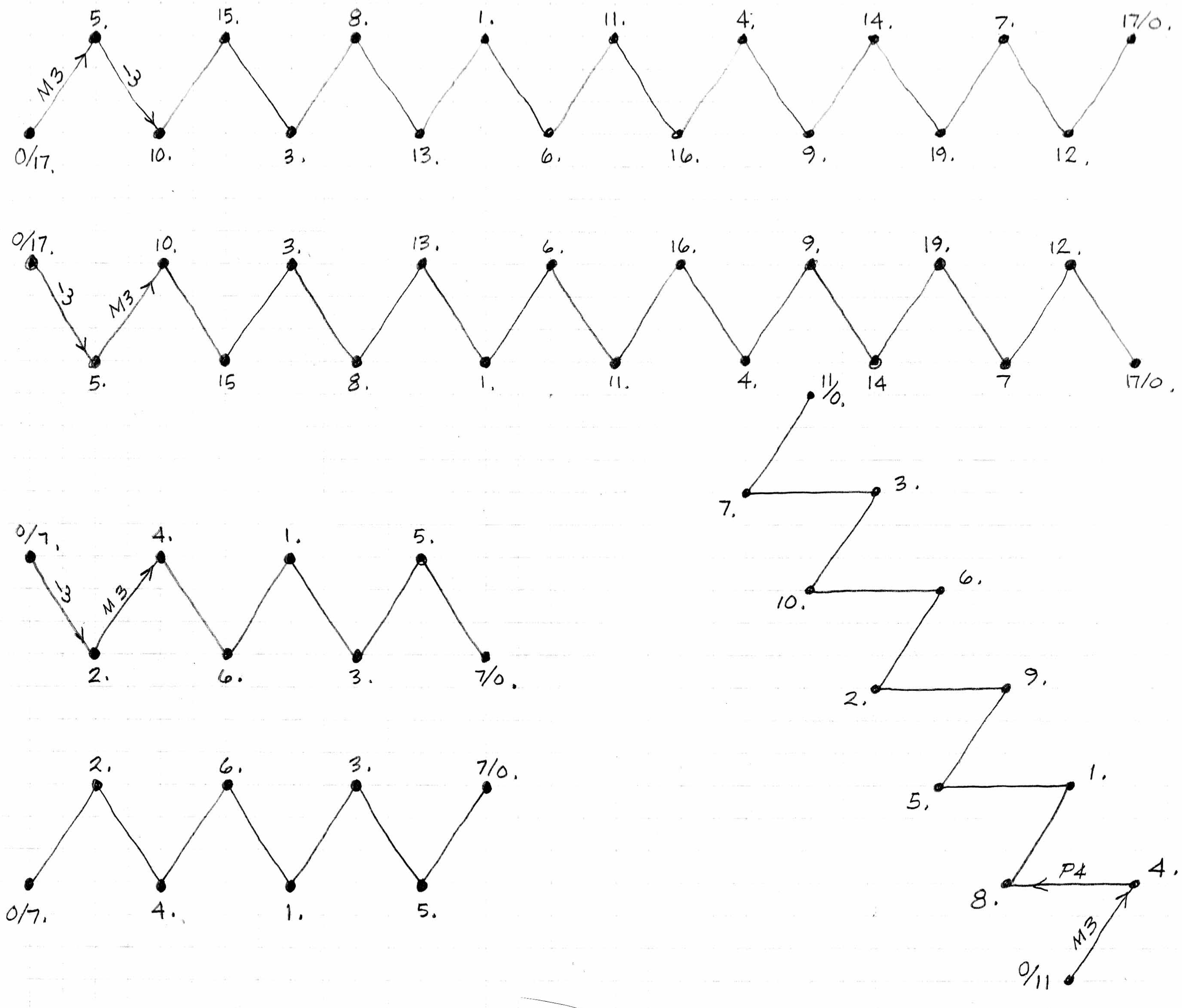
$\frac{G}{S}$ TREES $\left(\frac{\text{Generator}}{\text{Scale}}\right)$

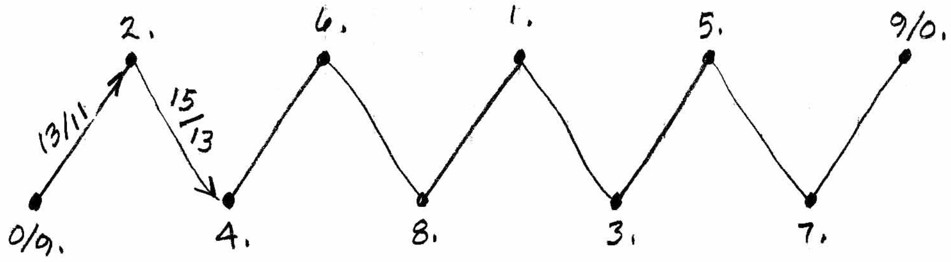
$\binom{12}{7}$ MOS

		0.	1.	2.	3.	4.	5.	6.	7/0.	Scale
upside down		0	4	1	5	2	6	3	0	2-step chain
		+4	-3	+4	-3	+4	-3	-3		Relative chain moves
		A	B	A	B	A	B	B		General 2-interval Pattern

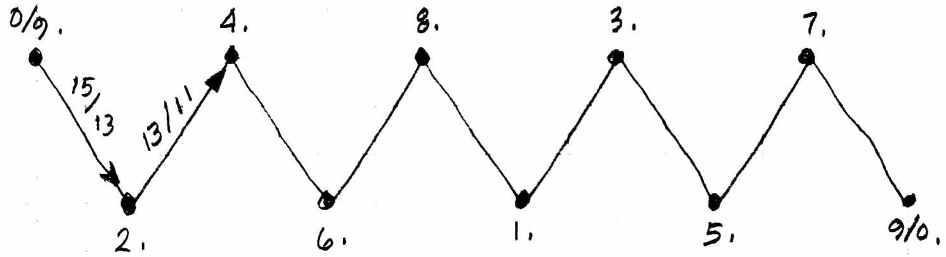
G

A,	B								G (1A+1B),	S (3A+4B)	Tree Level
0	1	0	1	0	1	0	1	1	1 /	4	0
1	4	1	4	1	4	1	4	4	5	19	4
1	3	1	3	1	3	1	3	3	4	15	3
2	5	2	5	2	5	2	5	5	7	27	4
1	2	1	2	1	2	1	2	2	3	11	2
3	5	3	5	3	5	3	5	5	8	29	4
2	3	2	3	2	3	2	3	3	5	18	3
3	4	3	4	3	4	3	4	4	7	25	4
1	1	1	1	1	1	1	1	1	2	7	1
4	3	4	3	4	3	4	3	3	7	24	4
3	2	3	2	3	2	3	2	2	5	17	3
5	3	5	3	5	3	5	3	3	8	27	4
2	1	2	1	2	1	2	1	1	3	10	2
5	2	5	2	5	2	5	2	2	7	23	4
3	1	3	1	3	1	3	1	1	4	13	3
4	1	4	1	4	1	4	1	1	5	16	4
1	0	1	0	1	0	1	0	0	1	3	0

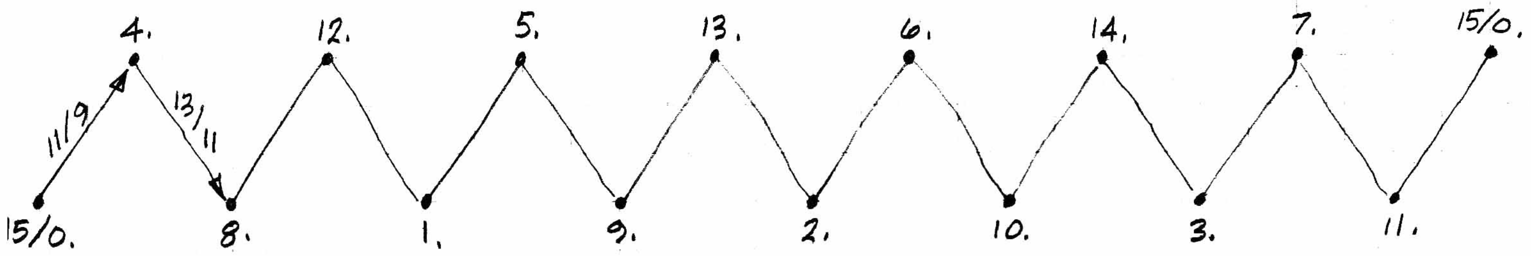

0/17.
M3
5.
m3
10.
15.
3.
8.
13.
1.
6.
11.
16.
4.
9.
14.
19.
7.
12.
17/0.
0/17.
m3
5.
M3
10.
15
3.
8.
13.
1.
6.
11.
16.
4.
9.
14
19.
7
12.
17/0.
0/7.
m3
2.
M3
4.
6.
1.
3.
5.
7/0.
0/7.
2.
4.
6.
1.
3.
5.
7/0.
0/11
M3
4.
P4
8.
1.
5.
9.
2.
6.
10.
3.
7.
11/0.

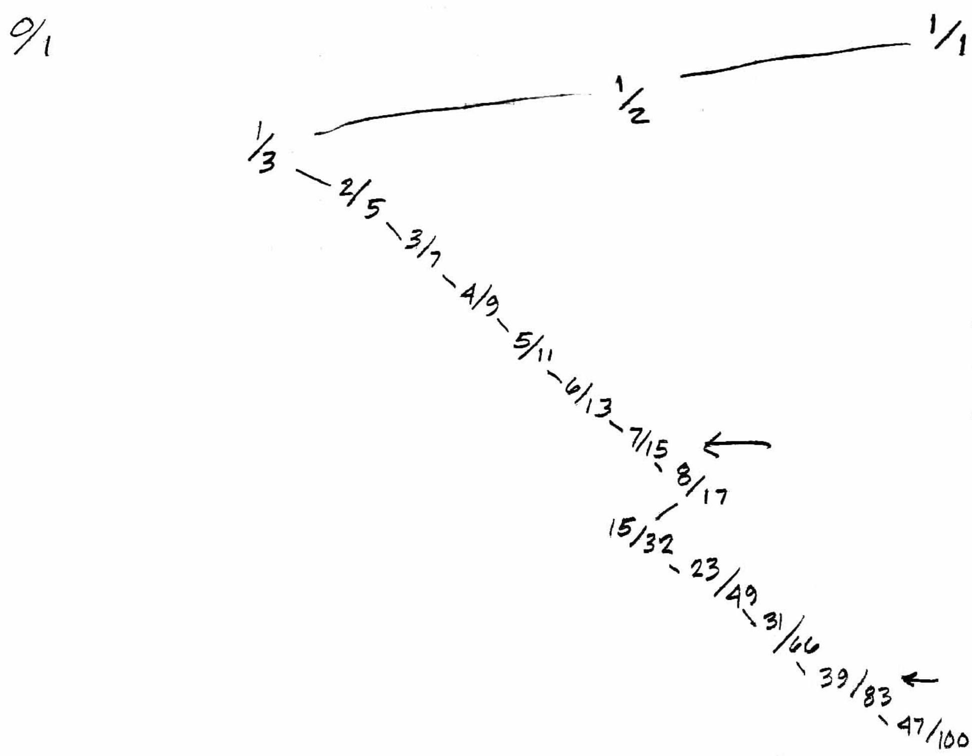


5(13/11)'s plus 4(15/13)'s
equal 2 octaves
(.030844007 shrink)



5 (15/13)'s plus 4 (13/11)'s
equal 2 Octaves
(.003713215 stretch)





8(11/9)'s plus 7(13/11)'s
equal 4 Octaves
(.003109634 shrink)
Shr.Gen. .530102610408
(.469897390)

<u>1/x Pattern</u>

←	2	.128
→	7	.804
←	1	.242
→	4	.126
	7	.917
	1	.089
	11	.127
	7	.842
	1	