A Natural System of Music

based on the Approximation of Nature

by

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Chapter I

Music

Music is a combination of art and science in which both complement each other in a marvelous fashion. Although it is true that a work of music performed or composed without the benefit of Art comes across as dry and expressionless, it is also true that a work of art cannot be considered as such without also satisfying the principles of science.

Leaving behind for a moment the matters of harmony and the art of combining sounds, let us concentrate for the moment only on the central matter of the basis of music: The unity tone and it's parts in vibrations, which can be divided into three groups: the physical, the mathematical and the physiological. The physical aspect describes for us how the sound is produced, the mathematical it's numerical and geometrical relationships, the physiological being the study of the impressions that such a tone produces in our emotions..

Musical sounds are infinite. However we can break them down and classify them into seven degrees and denominations:

Do	*Re*	*Mi*	*Fa*	*So*	*La*	*Ti*
1°	2°	3°	4°	5°	6°	7°

Climbing to 'Ti' we reach the duplicate of 'Do', that is to say Do^2, the unity note of the previous series. Ascending in this same manner, we reach Do^3, Do^4, etc.

We shall define the musical scale in seven steps, not because it invariably must be seven; we could also choose to use five or nine divisions, perhaps they would be more practical. There are as many different scales as there are a profusion of different musical sounds. However, as a homage to the past and as a indication of our devotion to the great musicians of the past, let us continue with the seven step division presented above. All of the various musical scales, the above as well as those classified as 'irregular' have their own lasting and immortal positions, appreciated in all of their grandeur by those who hear them presented within the Natural System, and choosing one particular scale within which to work and tune has no affect upon the validity of the others – in fact a great deal of the expressive nature of music radiates from the interaction between all of these scales.

In the same vein one could say that would be more precise to discard the usage of the syllables do, re, mi, fa et al. , as musical sounds are better described using only their letter representations; however discarding these syllables, in part, we destroy a beautiful part of the concept of music.

The history of music says that these syllables were taken from the first syllables of the lines of a prayer to Saint John:

Ut queant laxis
Resonare fibris
Mira gestorum
Famuli tuorum
Solve polluti
Labii restum

Being a prayer and taking it, as such, as directed to the Creator, it is as if in pronouncing said syllables we are repeating the text of the prayer itself: "Because our voices can sing of your admirable deeds, guide our lips in your service”. For what exactly is music, even if when guided by science, it does not as well maintain a bit of the divine?

Chapter II

The Harmonics

Sound is produced by the means of vibrations, and each sound is composed of a determined number of said vibrations. Depending upon their relationship to the unity note, they are considered more or less agreeable or disagreeable to the ear, and can be played simultaneously or in succession.

In producing any musical sound they follow a distinct series of sounds that depend, in the principal factor, on the form in which the sound itself was obtained. These 'sounds' which become more and more weak are given the name *Harmonics*, and constitute one of the most important parts of music.

The harmonic scales which are formed by said harmonics are simply the division of the vibrations of the unity note in equal parts, considering them in this order:

First	1 and 2 (the octave)
Second	1, 3/2, 2
Third	1, 4/3, 5/3, 2
Fourth	1, 5/4, 3/2, 7/4, 2
Fifth	1, 6/5, 7/5, 8/5, 9/5, 2
Sixth	1, 7/6, 4/3, 3/2, 5/3, 11/6, 2
Seventh	1, 8/7, 9/7, 10/7, 11/7, 12/7, 13/7, 2
Eighth	1, 9/8, 5/4, 11/8, 3/2, 13/8, 7/4, 15/8, 2

and so on in succession, conserving the relationship between the unity and it's parts in vibrations, we have demonstrated by study of these harmonics that an 'isolated' musical sound does not exist.

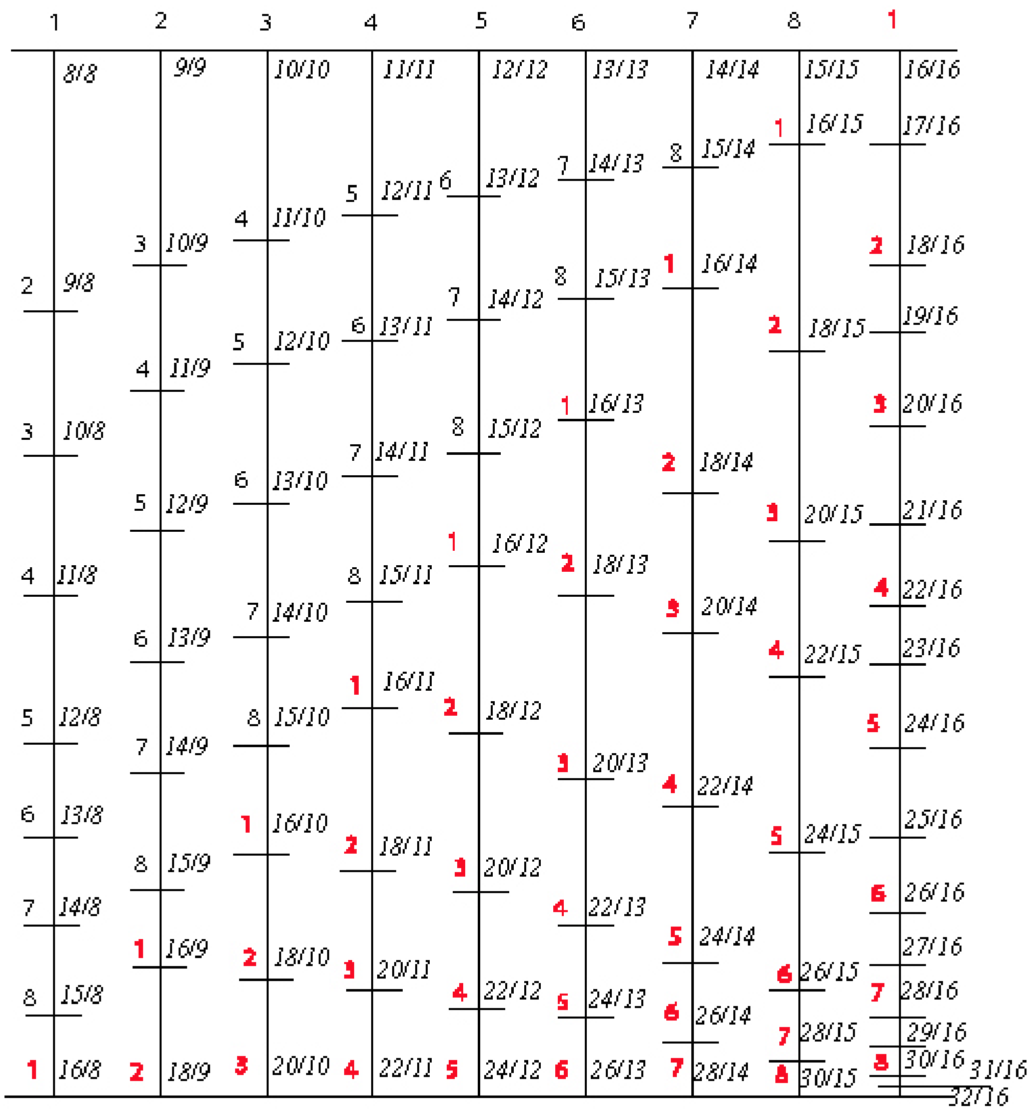
In all of the harmonic scales it will be perceived that the intervals stretch further each time, to the point that they are the immediate inferior of the octave, at a little more than half way from the first. This is due to the following: Suppose that we are dealing with the eighth harmonic scale – 1, 9/8, 5/4, 11/8, 3/2, 13/8, 7/4, 15/8, 2 or in whole numbers a relationship of 32, 36, 40, 44, 48, 52, 56, 60, 64. One will note that in-between the Unity and the Octave, we are left with unequal values. The ratio of 36 to 32 is 9/8, between 40 and 36 leaves use with 10/9, 44 and 40 is 11/10, 48 and 44 gives 12/11, between 52 and 48 is 13/12, and in such a manner we are left with smaller ratios in proportion to the octaves and each harmonic's immediate neighbor.

From the last example we can see that all sounds have two values: The absolute and the relative. The absolute is represented by the number of vibrations that it possesses and the relative by how these vibrations relate to the rest of the sounds in a given series.

TUNINGS:

We shall abstain from the use of the ear as the principle means of adjusting the sounds, and obtain them mathematically, in the same way nature has indicated in one of their many forms:

Being constant the tension of a string, the frequency of the transverse vibrations is inversely proportionate to it's length. This law of physics referring to the transverse vibrations of a string provides us with a simple method for figuring out the relationships of the sounds and the exact number of vibrations, if before hand we know the length of the string being played and assuming that it is being played freely.



Eight harmonic scales, showing the location of the intervals on a string. On the string notated as the octave *(octaves are shown in red)* is elaborated the sixteenth harmonic scale.

(This diagram is similar to others which have been omitted from the text in that it was originally intended to be used as a 'gauge' for construction of an instrument – needless to say a highly ineffective way of doing things. It was deemed necessary to include this diagram however, as it clearly lays out the patterns formed by the intervals in the different harmonic scales. Ratios and annotations by kind permission of Mr. Erv Wilson)

Let us proceed to figure the relationships. Divide the length of the string by the numerator and multiply the product by the denominator: 24 divided by 3 is 8 and 8 by 2 is 16, assuming that the string is 24 inches long and we are trying to find the 3/2 ratio, or it would be that the ratio of 6/5 would be 20 inches, as 24 divided by 6 leaves 4 by 5 equals 20. This process is exact, and we will well use for the isolation sounds; as they are numerous it is convenient to proceed in this fashion, being equally precise and of easiness of use.

Supposing that we are trying to figure the fourth harmonic scale: 1, 5/4, 3/2, 7/4, 2; and that the string in question is 24 inches long. The relationships of this scale in whole numbers is, as we have seen; 32, 40, 48, 56, and 64; now, multiplying the unity by the length of the string – 32 by 24 equals 768 and dividing this result in succession for the number of vibrations of each sound will indicate the place that each is found on said string. So we can see that notes of the fourth harmonic scale would be divided as such; 40, 19 ½ inches; 48, 16 inches; 56, 13 and 5/7 inches; and 64, 12 inches.

Furthermore, if we wish to tune another string to any of the other sounds mentioned and we wish to obtain the same results, it is necessary to perform the transformation in a relative fashion. For this multiply the length of the string by the number of vibrations possessed by the new tuning. For example, if we wished to use 40 it would be 40 by 24 equals 960 and divided 960 by 48, 56, 64 and 80, would come up with 20, 17 and 1/7, 15 and 12 inches respectively. These are the same intervals obtained in the string based on 32, but taking 40 as the unity.

It should be noted at this point that these examples are not the only ones possible for the relationships between sounds. It would be the same using 32, 40, 48, 56, 64 as if we where to use 1024, 1280, 1536, 1792 and 2048. They are the same notes respectively, just in different octaves, however in these examples we are using smaller numbers just for ease of computation.

Because these operations are being performed on an ideal string, it is necessary to carry it into practice with the minimum amount of alteration possible. We do this by assuming the tension of the string, it's elasticity, the resistance at the end of the string, the height of the bridge and all of the other factors that affect how sound is produced. With care and precision they can be used to form an acoustic box as in represented in the photograph and whose principle details are the following: The string is completely horizontal , it's frets are movable and have a small slot into which a small piece of tin has been fitted that lightly touches the string; one of the bridges has the ability to slide and has two screws that function as an aid to adjusting it's position.

Once the frets are fixed exactly in their corresponding positions, depending on the previously mentioned factors, one of the halves of the string will be more or less longer than the other. The experiments conducted demonstrate that in a string whose length is 24 inches and being a 4 gauge string, which corresponds to approximately 4 thousandths of an inch, the difference between the two halves will vary by approximately 1/50 to 1/64 of an inch. These proportions are the best results obtained for accurately achieving sounds in a string.

The Do that we shall use as the fundamental for tuning should be 1024 vibrations per second.

This text was originally accompanied by a photograph of the instrument described.

Chapter III

The Intervals

The formation of the intervals constitutes the harmonic scales. Due to it's importance in the world of music we shall use as the fundamental base of our calculations the fourth harmonic scale:

1	5/4	3/2	7/4	2

in which are contained the most beautiful and complete relationships, not only with each interval to the unity, but also in the relationships between the intervals themselves.

The intervals that compose the fourth harmonic scale, or we could even call them the four columns upon which the musical world is supported, are the following:

1	5/4	3/2	7/4	2
5/4	6/5	7/5	8/5	2
3/2	7/6	4/3	5/3	2
7/4	8/7	10/7	12/7	2

Changing the relatives 5/4, 3/2 and 7/4 into absolutes, as done in the above table, we form the primary intervals: 1, 8/7, 7/6, 6/5, 5/4, 4/3, 7/5, 10/7, 3/2, 8/5, 5/3, 12/7, 7/4, 2.

To continue with even greater clarity, before finishing with the formation of the intervals which we are going to use, let us concentrate on the inversions, chords and positions of said intervals.

Inversions:

Every interval has an inversion which can easily be figured in the following manner: Multiply the dominator of the ratio by 2 and invert the rational terms. By this rule the inversion of 3/2 comes out as 4/3, the inversion of 5/4 is 8/5, etc. etc.

Studying the inversions formed in such a fashion one notices that each interval has it's inversion:

interval	1	8/7	7/6	6/5	5/4	4/3	7/5
inversion	2	7/4	12/7	5/3	8/5	3/2	10/7
interval	10/7	3/2	8/5	5/3	12/7	7/4	2
inversion	7/5	4/3	5/4	6/5	7/6	8/7	1

Chords:

The joining of various tones is called a chord, which can be divided into two classes, consonants and dissonance. The consonants are derived from the third harmonic scale. In total, there are six:

1-4/3-5/3	1-5/4-5/3	1-4/3-8/5
1-6/5-8/5	1-5/4-3/2	1-6/5-3/2

These chords are considered as simple relationships. All chords falling outside these relationships are considered dissonant. The dissonant chords are of incredible importance, not only for there great expressiveness, but also because if we where to use only the consonant chords, regardless of how beautiful they are, we would be left with nothing more than monotonous music.

Positions

Every chord has as many positions as it has tones in it's formation. For example 1-5/4-3/2 which corresponds to do-mi-so in it's first position is so-do-mi in second and mi-so-do in it's third position. Reducing all to the unity we observe that each position gives a new chord:

First	1-5/4-3/2
Second	1-4/3-5/3
Third	1-6/5-8/5

Which is to say that every chord has it's inversion and consequentially, every chord composed of three sounds forms a circle of six, depending on the first in position if they are consonant or dissonant.

Let us continue the study of the fourth harmonic scale. The chords with which we shall work are ordered in their respective potions by the following form:

Fifth Major Chords

			inversion		
1	5/4	3/2	4/3	8/5	2
1	4/3	5/3	6/5	3/2	2
1	6/5	8/5	5/4	5/3	2

Fifth Minor Chords

			inversion		
1	8/7	10/7	7/5	7/4	2
1	7/5	8/5	5/4	10/7	2
1	5/4	7/4	8/7	8/5	2

Fourth Minor Chords

			inversion		
1	7/6	4/3	3/2	12/7	2
1	3/2	7/4	8/7	4/3	2
1	8/7	12/7	7/6	7/4	2

Fourth Major Chords

			inversion		
1	6/5	7/5	10/7	5/3	2
1	10/7	12/7	7/6	7/5	2
1	7/6	5/3	6/5	12/7	2

Seventh Minor Chords

				inversion			
1	5/4	3/2	7/4	8/7	4/3	8/5	2
1	8/7	10/7	12/7	7/6	7/5	7/4	2
1	7/6	4/3	5/3	6/5	3/2	12/7	2
1	6/5	7/5	8/5	5/4	10/7	5/3	2

As is shown above, from the fourth harmonic scale we derive 32 chords: 24 of which are triads, and 8 which are composed of four tones; which, because of their importance, are of the utmost usefulness in music. If we begin creating partial inversions, they can be considerably augmented. For example, the inversion of 1-5/4-3/2 is 4/3-8/5-2 and in inverting only one of the tones that form said chords we can obtain yet another new chord. For example: 1-5/4-3/2 gives us 1-5/4-4/3 if we invert the 3/2, or as so, 1-3/2-8/5 if the inverted tone is 5/4.

Also, we must take the following into account: A consonant chord will never produce a dissonance when inverted, nor will a dissonant chord produce a consonance. To produce this effect it is necessary that the inversion be only partial. This result can be seen by taking the consonance 1-4/3-5/3, if we invert only the 5/3 it produces the dissonance 1-6/5-4/3.

Let us continue the formation of intervals. In the Natural System we are limited to the use of the maximum of the fifth harmonic scale: 1, 6/5, 6/5, 8/5, 9/5, 2.

If we proceed to transform each relative to unity, it is noted that this scale provides us with six new intervals: 9/5, 9/7, 9/8, 10/9, 14/9, and 16/9.

Subtracting from those we can acquire the simple intervals. Of the diverse amount of intervals that we are provided by nature, of considerate note are 21/20 and 16/15; to acquire these with their respective inversions, 40/21 and 15/8 we obtain the complete set of the intervals, that is to say 23 tones:

1	21/20	16/15	10/9
9/8	8/7	7/6	6/5
5/4	9/7	4/3	7/5
10/7	3/2	14/9	8/5
5/3	12/7	7/4	16/9
9/5	15/8	40/21	2

Chapter IV

Notation

To each of the seven steps correspond the following values:

Do	*Re*	*Mi*	*Fa*	*So*	*La*	*Ti*
1	10/9	7/6	4/3	3/2	12/7	9/5

The formation of this irregular scale has the object of allowing us to use conventional notation for the representation of all tones.

Setting the value of the seven different steps to include all of the different intervals, we arrive at the following:

Do	1	21/20	16/15	
Re	10/9	9/8	8/7	
Mi	7/6	6/5	5/4	9/7
Fa	4/ 3	7/5	10/7	
So	3/2	14/9	8/5	5/3
La	12/7	7/4	16/9	
Ti	9/5	15/8	40/21	

The tones are written with the standard symbols known as notes, alterations and rests.

The notes and rests are the following:

Musical Notation Here!!!

We shall define alterations to notes where the alteration is ascending as the number that corresponds to a given interval. For example: 1 is Do, 21/20 is 2Do, 16/15 would be 3Do, etc. Utilizing a parenthesis to the left side indicates that the alteration is descendant i.e.: 10/9 is Re; 16/15 is 2)Re, 21/20 3)Re and so on. We will exclude notation with the number one, as this would be simply the step itself with no alteration.

All alterations shall remain valid for the period of one whole note. We shall rely on the 'natural' sign from standard musical notation to return any note to it's 'primitive', which is to say base 1, state.

In five parallel, horizontal lines, called a staff, we shall fix the spaces corresponding to each note. Note placement is fixed and invariable, and as a rule, will always be with Do represented by third space, and begin counting the series which will vary based on the height of the note and notating in this formula: I, II, III, IV, etc. We will take as the first in the series as the lowest note on the Piano, but if it necessary to descend even further, it will be notated with the negative sign: -I.

The notes correspond to the following time values: We shall take as a whole note the first note indicated, the second it's half, the third note it's quarter and the fourth, fifth, sixth and seventh notes will be respectively: the 8, 16, 32, and 64^{th} part of the whole. Each of these values can be augmented by half by placing a point next to the note (a dot) and by three-quarters by the placement of a double point.

In order to provide an indication of which tone is to be taken as unity for a given composition, it's place shall be marked by an 'X' at the beginning of the staff, or by marking the necessary alterations to the piece.

Having named several different scales, we arrive at the following:

A scale is a determined succession of sounds, classified into two different classes: harmonic and irregular. Those scales are harmonic which, as we have seen here, originate in the harmonics, for example: 1, 4/3, 5/3, and 2; and irregular those scales which in their formation occur different musical values e.g.:1, 8/7, 6/5, 4/3, 3/2, 8/5, 40/21, and 2.

Chapter V

Geometric Progressions

In the few words placed as an epigraph to this work: 'The Goal of this work will be to hear the True Music' one can find the reason for the previously laid out theory; however in order to gain a better understanding, it is necessary to proceed, although just briefly, into an explanation of the musical system in most common usage now, which is known as equal temperament.

The dilemma, which has often been argued in the study of music, can be condensed into the following statement: Musical sounds are a simple progression, but should said progression be arithmetical or geometrical?

The first with give us a perfect relationship between the unity and it's parts in vibrations. The second, will provide us the same between the sounds themselves. Let us compare both in the space understood as an octave:

Arithmetical Progressions:

Half	1	1.5000	2			
Third	1	1.333	1.6666	2		
Fourth	1	1.2500	1.5000	1.7500	2	
Fifth	1	1.2000	1.4000	1.6000	1.8000	2

Geometric Progressions:

Half	1	1.4142	2			
Third	1	1.2600	1.5874	2		
Fourth	1	1.1892	1.4142	1.6818	2	
Fifth	1	1.1487	1.3195	1.5157	1.7411	2

Even under superficial analysis, one notes that all of the geometric progressions form a determined succession of tones, leaving out the remainders, and originating in a partial which should not exist without having isolated them musically.

In respect to the arithmetical progressions, which we classify as harmonic scales, they progress 'tying' in to one another constantly. In this manner the second harmonic scale 1, 3/2, 2 –inverting the 3/2 to reach 4/3, these scales in turn prepare the fourths and the fifths; from the fourths we derive the sixths and the sevenths; and if we complete the intervals that we can derive from the fifth harmonic scale we reach the octaves and the ninths, and so on.

As an example, the third geometric progression: 1, 1.2400, 1.5874, 2. gives us only two intervals and only a singular, dissonant chord. The third harmonic scale: 1, 1.333, 1.6666, 2., provides us with six chords of perfect consonance. This shows us the poverty of the geometric progression and the disproportinality of the comparison.

There are many considerations that can be made between the two manners of proceeding, beyond the obvious conclusion that the geometric scales do not have the consistency necessary to provide the basic foundations of music. In another light, their harmonic and melodic confusion is manifest, which is easy enough to prove, leaving all logic behind, by simply tuning a string to each scale and then listening to them. However, if we ignore the sounds that form the fundamental basis of music, they can be of great usage if employed as imitations; if they do not stray to far from the place in which they belong, based upon how much is the maximum difference in an interval that the ear perceives as being identical.

This principle serves to reconcile the differences in the geometric progression, whose base is .01163 and which is proportionate to a division of four, five and six equal sounds within the octave, imitating as it where the fourth harmonic scale, even though in study it results to be unacceptable. This base forms the following series of sixty even tones within the octave:

1	1.01163	1.0234	1.0353
1.0473	1.0594	1.0718	1.0842
1.0968	1.1096	1.1225	1.1355
1.1487	1.1620	1.1775	1.1892
1.2030	1.2170	1.2312	1.2455
1.2600	1.2746	1.2894	1.3044
1.31915	1.3348	1.3503	1.3660
1.3819	1.3979	1.4142	1.4306
1.4473	1.1643	1.4811	1.4983
1.5157	1.5334	1.5512	1.5692
1.5874	1.6058	4.5245	1.6434
1.6625	1.6818	1.7013	1.7211
1.7411	1.7613	1.7818	1.8025
1.8235	1.8447	1.8661	1.8877
1.9097	1.9319	1.9543	1.9771
2			

Simply comparing the 5/3: 1.6666, one can appreciate the approximation that these relationships can have to the third harmonic scale, seeing that 1.6625 is unacceptable as an imitation of 1.6666 as it is far too low, and 1.6818 being excessively high. Nonetheless, if music depended on the geometric progressions, this scale would be the best, as it maintains the relationships to the unity en equal values amongst it's parts without being of an excessive number of notes.

Without the approximation of the above series into to the third harmonic scale, it is useless to compare it to the fourth harmonic scale, as they do not proceed in an interconnected fashion, as do the arithmetical scales, as each scale must be recalculated using a new base.

The Equally Tempered Scale is a geometric progression whose base is .05946, and which divides the octave into twelve notes:

1	1.05946	1.1224	1.1892
1.2600	1.3348	1.4142	1.4983
1.5874	1.6818	1.7818	1.8877
2			

In achieving a consonance, it is not acceptable that any imitation should fall in excess of positive or negative .003. From this we can see that the approximation of 1.5000 to 1.4983 is an acceptable one. Having 1.5000, which is to say 3/2 whose inversion is 4/3, between 4/3 and 3/2 we have 9/8, whose inversion is 16/9. These four notes, whose roots exist in said progression are the only ones acceptable out of the twelve, pretending to imitate with these the first order relationships with the unity.

Upon further examination, if we cannot acquire an imitation of the thirds: 1, 4/3, 5/3, 2, upon which depend the chords of perfect consonance, the music produced will be continuously confused, and vague. In the twelve notes that are in proportion to the 4/3, that which pretends to substitute for the 5/3, or in decimal 1.6666, is the note 1.6818, which is far too high to be considered for the consonances. This is the reason that all of the chords derived from the notes called 'Tempered' being dissonant and resolving indefinitely into more dissonance are missing the indispensable equilibrium that should exist in music.

We can also say that since we have no decent approximation for the ratio 7/4; we have eliminated all of the principle contrasts indispensable in the coloration of music. Adding on top of this the bothersome and continual succession of differences, due to the usage of equal distances between the intervals; is it really possible to accept the place of this system in music?

In theory, the six even tones are further divided into nine fractions that are called commas. If we put this theory into practice, we are left with an octave divided into 54 notes, with which we will achieve a complete and useless disorder.

In order to not extend this brief analysis, let it be said that we have sufficiently shown the inaccuracy of the Equally Tempered System; but just to show that the division least faulty that can be deduced with the fewest number of notes, it is necessary to go to fifty-three equal divisions of the octave, or a base of .013164, in order to come up with a fair approximation of the third harmonic scale which, not to down play it's importance, is non the less, insufficient. Whatever geometric progression with less than the above listed number of notes is musically useless, as it cannot represent the most basic intervals and it shows, as in the twelve tone system which lacks consonance and only amplifies it's shortcomings through the usage of additional divisions of the octave, only to fall into the same errors.

The mechanical difficulties that perhaps obligated the majority of musicians to accept an ambiguous system hundreds of years ago, do not exist any longer or at least are no longer relative, in which case we can say that the Equally Tempered System has served it's purpose and that it is necessary that the elements of music undergo a transformation, embellishing themselves to become more precise in their relation to the unity in the respect to vibration.

The 'perfect' tuning is a unique thing, more so due to the fact that the human ear cannot distinguish the difference of a single 'point', and as such, as we have shown, if the approximation is close enough, we will perceive the interval as being exact. Using this principle we shall continue to develop a new system that includes the core nature of the Natural System, but without it's inherent difficulties of tuning. These difficulties do not exist in Nature; the vocal chords can modulate as they see fit, and can easily produce the twelfth harmonic scale and reach as far as the sixteenth; however trying to produce the same results with the musical instruments in usage today would be a physical impossibility.

If we try to imitate only the third harmonic scale, one the best suited geometric progressions would be one of sixty-five even notes within the octave, or a base of .01072:

1	1.01072	1.0215	130325
1.0436	1.0547	1.0661	1.0775
1.0890	1.1007	1.1125	1.1244
1.1365	1.1487	1.610	1.1734
1.1860	1.1987	1.2116	1.2246
1.2377	1.2510	1.2644	1.2779
1.2916	1.3055	1.3195	1.3337
1.3480	1.3624	1.3770	1.3918
1.4067	1.4218	1.4370	1.4524
1.4680	1.4837	1.4996	1.5157
1.5320	1.5484	1.5650	1.5818
1.5988	1.6159	1.6332	1.6507
1.6684	1.6863	1.7044	1.7226
1.7411	1.7598	1.7786	1.7977
1.8170	1.8365	1.8561	1.8760
1.8962	1.9165	1.9370	1.9578
1.9788	2		

Or if we use the base .013164, which will provide us with fifty-three even notes within the octave, the result with be more or less an exact facsimile of the second harmonic scale:

1	1.013164	1.0265	1.0400
1.0537	1.0676	1.0816	1.0957
1.1103	1.1249	1.1397	1.1547
1.1699	1.1853	1.2009	1.2157
1.2327	1.2490	1.2654	1.2821
1.2989	1.3160	1.3334	1.3509
1.3687	1.3867	1.0450	1.4235
1.4422	1.4612	1.4804	1.4999
1.5197	1.5397	1.5599	1.5805
1.6013	1.6224	1.6437	1.6654
1.6873	1.7095	1.7320	1.7548
1.7779	1.8013	1.8250	1.8490
1.8734	1.8980	1.9230	1.9483
1.9740	2		

Even more elements are necessary to complete the fourth harmonic scale, however we can proceed to formulate a geometric progression that will imitate it;   proving that it is indispensable to study the Natural System in order to give the appropriate amplitude to the Natural-Approximated.

If the in the Natural System are written the immutable principles of music, logically in it’s imitation we will not find the last word on the subject. However this study has been done with the desire that those who are devotees of music might find in it the Ideal, that moving away from all prejudices they should be inclined to value and realize it’s considerations, and that the theory should be given with clarity, and in it’s entirety to the public.

Chapter VI

The Natural-Approximated System

Defining the System

The Natural-Approximated system is the imitation of the Natural System. It is composed of the following elements:

The Point – The unity and smallest division out of which the intervals are formed. To the human ear, it is 1/72 of an octave. In geometric progression it is a base of .00968

The Tones – The smallest interval that can be melodically used in music are called Tones. There are four: The largest is formed by six consecutive points; the major by five; the minor by four and the minimum by three.

The Intervals – The distance understood between two notes and classified, in addition to those previously listed, as the following:

Seconds – There are four seconds. The largest is 14 points; the major 12; the minor, 11 and the smallest 7

Thirds – There are four thirds. The largest is 26 points; the major is 23; the minor 19, and the smallest 16.

Fourths – There are three. The major is 35 points; the minor 30, and the smallest 28.

Fifths – Also three. The largest is 44 points; the major 42 and the smallest 37.

Sixths – There are four sixths. The largest is 56 points; the major of 53, the minor is 49 and the smallest is 46.

Sevenths – Four. The largest is 65 points. The major 61, the minor, 60 and the smallest is 58.

Eighth – Which is to say the octave; 72 points.

The intervals can be augmented or diminished in so many different ways, that it is not in our interest to classify them here.

Inversions – Every interval has an inversion. In order to figure them easily, keep in mind that the steps must add up without exception to nine, with the variation that the largest changes into the smallest, the major into the minor, and at the same time, the smallest into the largest and the minor into the major.

Comparisons – In order to maintain clear the reasons behind classifying the intervals in this fashion, we can compare them to those already introduced in the study of the Natural System. In the following table we list the approximations, both in vibrations and in their precise location on a string with a length of 24 inches.

It goes without saying that the approximations of the notes derived from the fourth harmonic scale are the most necessary.

	Natural		Natural-Approximated		
1	1	24	24	2	1
21/20	1.0500	22 6/7	22.87	1.0494	5
16/15	1.0666	22 ½	22.44	1.0697	7
10/9	1.1111	21 3/5	21.57	1.1117	11
9/8	1.1250	21 1/3	21.38	1.1224	12
8/7	1.1428	21	20.97	1.1443	14
7/6	1.1666	20 4/7	20.57	1.1665	16
6/5	1.2000	20	19.99	1.2007	19
5/4	1.2500	19 1/5	19.23	1.2479	23
9/7	1.2857	18 2/3	18.68	1.2845	26
4/3	1.3333	18	17.98	1.3348	30
7/5	1.4000	17 1/7	17.13	1.4007	35
10/7	1.4285	16 4/5	16.81	1.4279	37
3/2	1.5000	16	16.02	1.4983	42
14/9	1.5555	15 3/7	15.42	1.5572	46
8/5	1.6000	15	14.97	1.6027	49
5/3	1.666	14 2/5	14.41	1.6657	53
12/7	1.7142	14	14	1.7145	56
7/4	1.7500	13 5/7	13.73	1.7479	58
16/9	1.7777	13 ½	13.47	1.7818	60
9/5	1.8000	13 1/3	13.34	1.7990	61
15/8	1.8750	12 4/5	12.82	1.8697	65
40/21	1.9040	12 3/5	12.59	1.9060	67
2	2	12	12	2	72

In the Natural-Approximated system one can imitate as well, the sixth, seventh, eighth and ninth harmonic scales by using these intervals, which in relation to those previously classified form part of their augmentations and dimuniations.

	Natural		Natural-Approximated		
					Points
11/6	1.8326	13 1/11	13.08	1.8340	63
11/7	1.5714	15 3/11	15.26	1.5722	48
13/7	1.8571	12 12/13	12.96	1.8518	64
11/8	1.3750	17 5/11	17.47	1.3740	33
13/8	1.6250	14 10/13	14.83	1.6183	50
11/9	1.2222	19 7/11	19.60	1.2240	21
13/9	1.4444	16 8/13	16.64	1.4417	39
17/9	1.8888	12 12/17	12.71	1.8877	66

Steps: There are seven steps, forming in succession an irregular scale. Each step is named by it's syllable: Do, Re, Mi, Fa, So, La, Ti, which correspond, respectively, to 1º, 2º, 3º, 4º, 5º, 6º and 7º steps.

The Tonic: The tonic is the note which serves as the fundamental of any scale. In relation to the tonic, Do, the steps follow this relationship: Do-Re, minor second; Do-Mi, major third; Do-Fa, minor fourth; Do-Sol, major fifth; Do-La, major sixth; Do-Si, largest seventh and Do-Do², the octave.

The interval that exists in succession between these diverse steps is a second, differing in value: Do-Re, minor second; Re-Mi, major second; Mi-Fa, smallest second; Fa-Sol, major second; Sol-La, minor second; La-Ti, major second and Si-Do^2, smallest second.

Notation:

We shall continue to use the system of notation laid out in the Natural System, varying on the numerical alterations. It is not necessary in the Natural System that the steps are ordered in the distinct order that defines the Natural-Approximated system, and as these do not change except in value, the place that belongs to them in the staff shall remain the same.

The Chords:

There are thirty-two principle chords; which are derived from the fourth harmonic scale. Four are composed of four notes each:

And twenty-four of are triads:

Tuning:

We shall use as the fundamental for tuning the step *Mi*, having a constant of 1280 vibrations per second.

In Geometric Progression, each of the 72 points correspond to the following values:

Do-1	1.00968	1.0194	1.0293
1.0392	1.0493	1.0594	1.0697
1.0800	1.0905	1.1010	Re-1.1117
1.224	1.1333	1.1443	1.1553
1.1665	1.1778	1.1892	1.2007
1.2123	1.2240	1.2359	Mi-1.2479
1.2600	1.2722	1.2845	1.2969
1.3094	1.3221	Fa-1.3348	1.3477
1.3608	1.3740	1.3872	1.4007
1.4142	1.4279	1.4417	1.4556
1.4697	1.4840	Sol-1.4983	1.5128
1.5275	1.5423	1.5572	1.5722
1.5784	1.6027	1.6183	1.6339
1.6497	La-1.6657	1.6818	1.6981
1.7145	1.7311	1.7479	1.7648
1.7818	1.7990	1.8164	1.8340
1.8518	Ti-1.8697	1.8877	1.9060
1.9245	1.9431	1.9619	1.9808
Do^2-2			

The noted proportions relate respectively to a string, 24 inches long, as follows:

Do-24.00	23.77	23.54	23.32
23.12	22.87	22.65	22.44
22.22	22.00	21.79	Re-21.57
21.38	21.18	20.97	20.77
20.57	20.38	20.18	19.99
19.80	19.60	19.42	Mi-19.23
19.05	18.86	18.68	18.50
18.33	18.15	Fa-17.98	17.80
17.64	17.47	17.30	17.13
16.97	16.81	16.65	16.49
16.33	16.17	Sol-16.02	15.86
15.71	15.56	15.42	15.26
15.12	14.97	14.83	14.69
14.55	La-14.41	14.27	14.13
14.00	13.84	13.73	13.60
13.47	13.34	13.22	13.08
12.96	Ti-12.83	12.71	12.59
12.47	12.35	12.23	12.11
Do²-12.00			

These divisions must continue to stretch themselves a little bit further each step, due to the fractions resulting from the decimal process and the small length of the string used as an example. In the frets whose distances are noted above arises the case of having the space understood as a point just slightly greater than the space of it's immediate predecessor – as this should not be the case, we divide the difference in half, if we did not we would have to proceed to divisions of thousandths of an inch which, it goes without saying, is useless; to say that one could divide a string into such minute fractions would be lying to oneself.

Scales – The scales are divided into three classes. In the Natural-Approximated system we must add to the two previously mentioned scales: harmonics and irregulars, a third: the regular. They are regular scales which are produced by being equally divided within themselves. By their nature, they are without color, and should only be used when one is trying to produce this effect; avoiding them when the object is to give color to a musical figure. Various of the harmonic or irregular scales can be called chromatic for the richness of color which they produce. (*The word used by Novaro 'cromaticás', in Spanish implies a thing which has color, whereas the English term 'Chromatic' simply implies that a scale contains incidentals. Novaro is providing a 'double-proof' of the richness of these scales)* .

Chapter VII

Closing Notes on the Natural-Approximated System

The use of the point is indispensable in harmony, however in a melodic succession, there is no reason for it's usage. The intervals of two or three points, which are perfectly perceptible, can be used in determined cases; however in music for voice, one should not employ an interval of less than four points, which imitates the ratio of 25/24.

It is of great worth to study the intervals 7/6, 7/5, 10/7 and 12/7 which correspond to 16, 35, 37 and 56 points respectively for their incredible importance to consonance.

Of note also are the following intervals and ratios:

> 8/7, fourteen points, amongst other reasons, for being a factor of the triad 1-8/7-10/7.
>
> 21/20 and 25/20, whose imitations correspond to the major and minor tones.
>
> 12/11, nine points, with which can be formed an entirely new form of organizing music.
>
> 7/4, fifty-six points, which has a great importance within musical interactions.
>
> 10/9 and 9/8, eleven and twelve points, for their successive alterations and ability to produce musical consonance with the other steps.
>
> The smallest second, seven points, as it is a good imitation of 15/14 and 16/15, intervals of great importance in music, factor of the first division of 9/8 and 10/9. These first divisions are from 10/9; 16/15 and 25/24 and derived from 9/8; 15/14 and 21/20.
>
> The irregular scale 1, 8/7, 6/5, 4/3, 3/2, 8/5, 40/21, 2; which corresponds to: Do, 4 Re, 5) Mi, Fa, So, 5) La, 3 Si, Do and characterized by the triad 8/7-4/3-40/21. Taking as unity 8/7 we can obtain 1-7/6-5/3
>
> The irregular scales 1, 6/5, 4/3, 3/2, 5/3, 2 or Do, 5) Mi, Fa, So, La, Do; and 1, 5/4, 4/3, 3/2, 8/5, 2 which corresponds to Do, Mi, Fa, So, 5) La, Do both of which are formed by a consonant chord and the inversion of the same.

From the study of the previous example, it becomes obvious the transcendental importance *(T.N. – Novaro's wording)* of the continuous study of all of the chords in the system.

Final Examination – The finalization of the synthesis of this theory rests solely upon exciting it's continued study amongst those who aspire to know perfect harmony. Or in the words of Berlioz 'This mysterious sweetness that gives birth to brilliant fantasy and carries the imagination to the most gracious fictions in the worlds of poetry and the supernatural.'

Augusto Novaro
Mexico, D.F.
1927