# The Marwa Permutation



I've been held, spellbound, for an unusually long time by a group of 10 unique scales.

Forms of each were included in the scales listed by Fox-Strangways' in his Music of Hindustan. I sensed they should be in a special group, and so placed them this way;

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10. | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | S | R | | | G | | M′ | P | | D | | N | S |
| 2. | S | R | | | G | | M′ | P | D | | | N | S |
| 3. | S | R | | G | | | M′ | P | D | | | N | S |
| 4. | S | R | | G | | | M′ | P | D | | N | | S |
| 5. | S | | R | | G | | M′ | P | D | | | N | S |
| 6. | S | | R | G | | | M′ | P | D | | | N | S |
| 7. | S | | R | G | | | M′ | P | D | | N | | S |
| 8. | S | | R | G | | | M′ | P | | D | | N | S |
| 9. | S | | R | G | | | M′ | P | | D | N | | S |
| 10. | S | | R | | G | | M′ | P | | D | N | | S |

I saw them as a set of melodic variations of the Diatonic sequence;

S R G M′ P D N S

This point of view though helpful was also misleading, and prevented me from seeing the subtle thread that binds these 10 together.

Lately I've begun to think of these scales as those having the diminished Fourth, a feature which they share in common. That was a step in the right direction, and ultimately precipitated a more productive point of view. Is it possible that they can have more than the diminished Fourth in common? yes, indeed it is. They are all linear permutations of the series of fourths; 4, 4, 4′, 4, 4, 4 with the 4′ (diminished Fourth) taking the role of the "closing Fourth" that completes the cycle. The 2 4′ (Augmented Fourth's) must not be adjacent.

# The Marwa Permutations



Consider the 32 Raga-scales (Thats) of N. India (fig. 1a). These may expressed as chains of Fourths, distinguishing between Augmented Fourth (4′), Normal Fourth (4), and Diminished Fourth (4̲), (fig. 1b). 3 classes of That are thus revealed, (fig 1c) based on permutations of 3 basic chains of Fourths. In class A there are 6 Thats based on 4′,4,4,4,4,4,(4). (fig.1d). In class B there are 20 Thats based on 4′,4̲,4′,4,4,4,(4), (fig. 1e). In class C there are 6 Thats based on 4′,4̲,4′,4̲,4′,4,(4), (fig. 1f). All 6 of the Thats in class A are associated with popular ragas. 15 of the 20 Thats in class B are associated with popular ragas. None of the 6 Thats in class C is associated with a popular raga. Class A and Class B are the original source of the variations and permutations that follow;

The principal features of the basic chain (linear genus) may be ordered in several interesting ways by interpolation and by the selection of the Fourth which is to be "fixed", shown in parenthesis, ( ).

Example; $4'_1\ \underline{4}\ 4'_2\ 4\ 4\ 4\ (4)$
$4'_2\ \underline{4}\ 4'_1\ 4\ 4\ 4\ (4)$
$4'_1\ 4\ 4'_2\ 4\ 4\ 4\ (\underline{4})$
$4'_2\ 4\ 4'_1\ 4\ 4\ 4\ (\underline{4})$

also; $4'\ 4\ 4\ 4\ 4\ 4\ (4)$
$4\ 4\ 4\ 4\ 4\ 4\ (4')$

and; $4'\ 4_2\ 4_1\ 4_1\ 4_1\ 4_1\ (4_1)$
$4_2\ 4'\ 4_1\ 4_1\ 4_1\ 4_1\ (4_1)$
$4_2\ 4_1\ 4_1\ 4_1\ 4_1\ 4_1\ (4')$
$4'\ 4_1\ 4_1\ 4_1\ 4_1\ 4_1\ (4_2)$

A diversity of linear species (shown in ratios) may be associated with a linear genus. These are analogous intonational determinants; each would impart its own, unique color to the tuning of a given set of Thats.

Example;

| $4'_1$ | 4 | $4'_2$ | 4 | 4 | 4 | ($\underline{4}$) |
|---|---|---|---|---|---|---|
| $\frac{64}{45}$ | $\frac{4}{3}$ | $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{81}{64}$) |
| $\frac{35}{24}$ | $\frac{4}{3}$ | $\frac{81}{56}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{6}{5}$) |
| $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{18}{13}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{13}{10}$) |
| $\frac{63}{44}$ | $\frac{4}{3}$ | $\frac{11}{8}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{9}{7}$) |
| $\frac{36}{25}$ | $\frac{4}{3}$ | $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{5}{4}$) |

and;

| $4'_2$ | 4 | $4'_1$ | 4 | 4 | 4 | ($\underline{4}$) |
|---|---|---|---|---|---|---|
| $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{64}{45}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{81}{64}$) |
| $\frac{81}{56}$ | $\frac{4}{3}$ | $\frac{35}{24}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{6}{5}$) |
| $\frac{18}{13}$ | $\frac{4}{3}$ | $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{13}{10}$) |
| $\frac{11}{8}$ | $\frac{4}{3}$ | $\frac{63}{44}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{9}{7}$) |
| $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{36}{25}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | ($\frac{5}{4}$) |

Example;

| $4'$ | 4 | $4_2$ | 4 | 4 | 4 | (4′) |
|---|---|---|---|---|---|---|
| $\frac{11}{8}$ | $\frac{4}{3}$ | $\frac{27}{20}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{15}{11})$ |

and;

| $4_2$ | 4 | $4'$ | 4 | 4 | 4 | (4′) |
|---|---|---|---|---|---|---|
| $\frac{27}{20}$ | $\frac{4}{3}$ | $\frac{11}{8}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{15}{11})$ |

| $4_2$ | 4 | $4_2$ | 4 | 4 | 4 | (4′) |
|---|---|---|---|---|---|---|
| $\frac{27}{20}$ | $\frac{4}{3}$ | $\frac{27}{20}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{25}{18})$ |

| $4'$ | 4 | $4'$ | 4 | 4 | 4 | (4) |
|---|---|---|---|---|---|---|
| $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{32}{25})$ |

| $4_2$ | $4'$ | 4 | 4 | 4 | 4 | (4) |
|---|---|---|---|---|---|---|
| $\frac{27}{20}$ | $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |
| $\frac{21}{16}$ | $\frac{81}{56}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |

and;

| $4'$ | $4_2$ | 4 | 4 | 4 | 4 | (4) |
|---|---|---|---|---|---|---|
| $\frac{45}{32}$ | $\frac{27}{20}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |
| $\frac{81}{56}$ | $\frac{21}{16}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |

| $4_1'$ | 4 | $4_2'$ | 4 | 4 | 4 | (4) |
|---|---|---|---|---|---|---|
| $\frac{45}{32}$ | $\frac{81}{64}$ | $\frac{64}{45}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |
| $\frac{11}{8}$ | $\frac{9}{7}$ | $\frac{63}{44}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |

| $4_2'$ | 4 | $4_1'$ | 4 | 4 | 4 | (4) |
|---|---|---|---|---|---|---|
| $\frac{64}{45}$ | $\frac{81}{64}$ | $\frac{45}{32}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |
| $\frac{63}{44}$ | $\frac{9}{7}$ | $\frac{11}{8}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |

| $4_1$ | 4 | 4 | 4 | 4 | 4 | (4′) |
|---|---|---|---|---|---|---|
| $\frac{27}{20}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{45}{32})$ |
| $\frac{21}{16}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{81}{56})$ |

| $4'$ | 4 | 4 | 4 | 4 | 4 | (4) |
|---|---|---|---|---|---|---|
| $\frac{729}{512}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{4}{3})$ |

| 4 | 4 | 4 | 4 | 4 | 4 | (4′) |
|---|---|---|---|---|---|---|
| $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $(\frac{729}{512})$ |

These linear species are derived from 7-tone tetrachordal scales. The figures appearing on the following pages are permutation sets of these linear species, and excerpted from work in progress.

I am indebted to Amiya Dasgupta for giving me a copy of his book Applied Theory of Indian Music (North) 1977 (California Institute of the Arts, Valencia). Errors in interpretion are mine.

I am also indebted to John Chalmers for sending me a copy of his unpublished book The Divisions of the Tetrachord.

# Fig. 1

## fig. 1a

The 32 Thats of North India

| | S 0 | R -5 | G -3 | M -1 | P -6 | D -4 | N -2 | S 0 |
|---|---|---|---|---|---|---|---|---|
| Kalyan | S | R | G | M′ | P | D | N | S |
| Bilawal | S | R | G | M | P | D | N | S |
| Champakali | S | R | G | M′ | P | D | N | S |
| Khamaj | S | R | G | M | P | D | N | S |
| Madhubanti | S | R | G | M′ | P | D | N | S |
| Patdeep | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Kafi | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Nat Bhairav | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Mohan Kauns | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Chandra Kauns | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Asawari | S | R | G | M | P | D | N | S |
| Marwa | S | R | G | M′ | P | D | N | S |
| Anand Bhairav | S | R | G | M | P | D | N | S |
| Ahir Bhairav | S | R | G | M′ | P | D | N | S |
| | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Parameswari | S | R | G | M | P | D | N | S |
| Purvi | S | R | G | M′ | P | D | N | S |
| Bhairav | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Basant Mukhari | S | R | G | M | P | D | N | S |
| Todi | S | R | G | M′ | P | D | N | S |
| Jogiya Todi | S | R | G | M | P | D | N | S |
| | S | R | G | M′ | P | D | N | S |
| Bhairavi | S | R | G | M | P | D | N | S |

## fig. 1b

Chain of Fourths

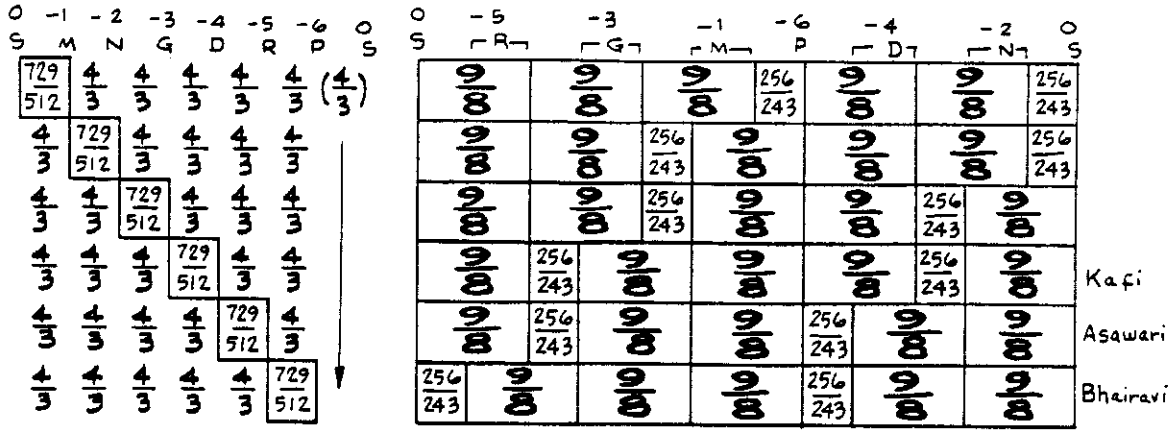
| | S 0 | M -1 | N -2 | G -3 | D -4 | R -5 | P -6 | S 0 | class |
|---|---|---|---|---|---|---|---|---|---|
| Kalyan | 4′ | 4 | 4 | 4 | 4 | 4 | (4) | | A |
| Bilawal | 4 | 4′ | 4 | 4 | 4 | 4 | | | A |
| Champakali | 4′ | 4 | 4′ | 4 | 4 | 4 | | | B |
| Khamaj | 4 | 4 | 4′ | 4 | 4 | 4 | | | A |
| Madhubanti | 4′ | 4 | 4 | 4′ | 4 | 4 | | | B |
| Patdeep | 4 | 4′ | 4 | 4′ | 4 | 4 | | | B |
| | 4′ | 4 | 4 | 4′ | 4 | 4 | | | B |
| Kafi | 4 | 4 | 4 | 4′ | 4 | 4 | | | A |
| | 4′ | 4 | 4 | 4 | 4′ | 4 | | | B |
| Nat Bhairav | 4 | 4′ | 4 | 4 | 4′ | 4 | | | B |
| | 4′ | 4 | 4′ | 4 | 4′ | 4 | | | C |
| Mohan Kauns | 4 | 4 | 4′ | 4 | 4′ | 4 | | | B |
| | 4′ | 4 | 4 | 4 | 4′ | 4 | | | B |
| Chandra Kauns | 4 | 4′ | 4 | 4 | 4′ | 4 | | | B |
| | 4′ | 4 | 4 | 4 | 4′ | 4 | | | B |
| Asawari | 4 | 4 | 4 | 4 | 4′ | 4 | | | A |
| Marwa | 4′ | 4 | 4 | 4 | 4 | 4′ | | | B |
| Anand Bhairav | 4 | 4′ | 4 | 4 | 4 | 4′ | | | B |
| Ahir Bhairav | 4′ | 4 | 4′ | 4 | 4 | 4′ | | | C |
| | 4 | 4 | 4′ | 4 | 4 | 4′ | | | B |
| | 4′ | 4 | 4 | 4′ | 4 | 4′ | | | C |
| | 4 | 4′ | 4 | 4′ | 4 | 4′ | | | C |
| | 4′ | 4 | 4 | 4′ | 4 | 4′ | | | C |
| Parameswari | 4 | 4 | 4 | 4′ | 4 | 4′ | | | B |
| Purvi | 4′ | 4 | 4 | 4 | 4 | 4′ | | | B |
| Bhairav | 4 | 4′ | 4 | 4 | 4 | 4′ | | | B |
| | 4′ | 4 | 4′ | 4 | 4 | 4′ | | | C |
| Basant Mukhari | 4 | 4 | 4′ | 4 | 4 | 4′ | | | B |
| Todi | 4′ | 4 | 4 | 4 | 4 | 4′ | | | B |
| Jogiya Todi | 4 | 4′ | 4 | 4 | 4 | 4′ | | | B |
| | 4′ | 4 | 4 | 4 | 4 | 4′ | | | B |
| Bhairavi | 4 | 4 | 4 | 4 | 4 | 4′ | | | A |

4′ = Augmented Fourth
4 = diminished Fourth

## Fig. 1c

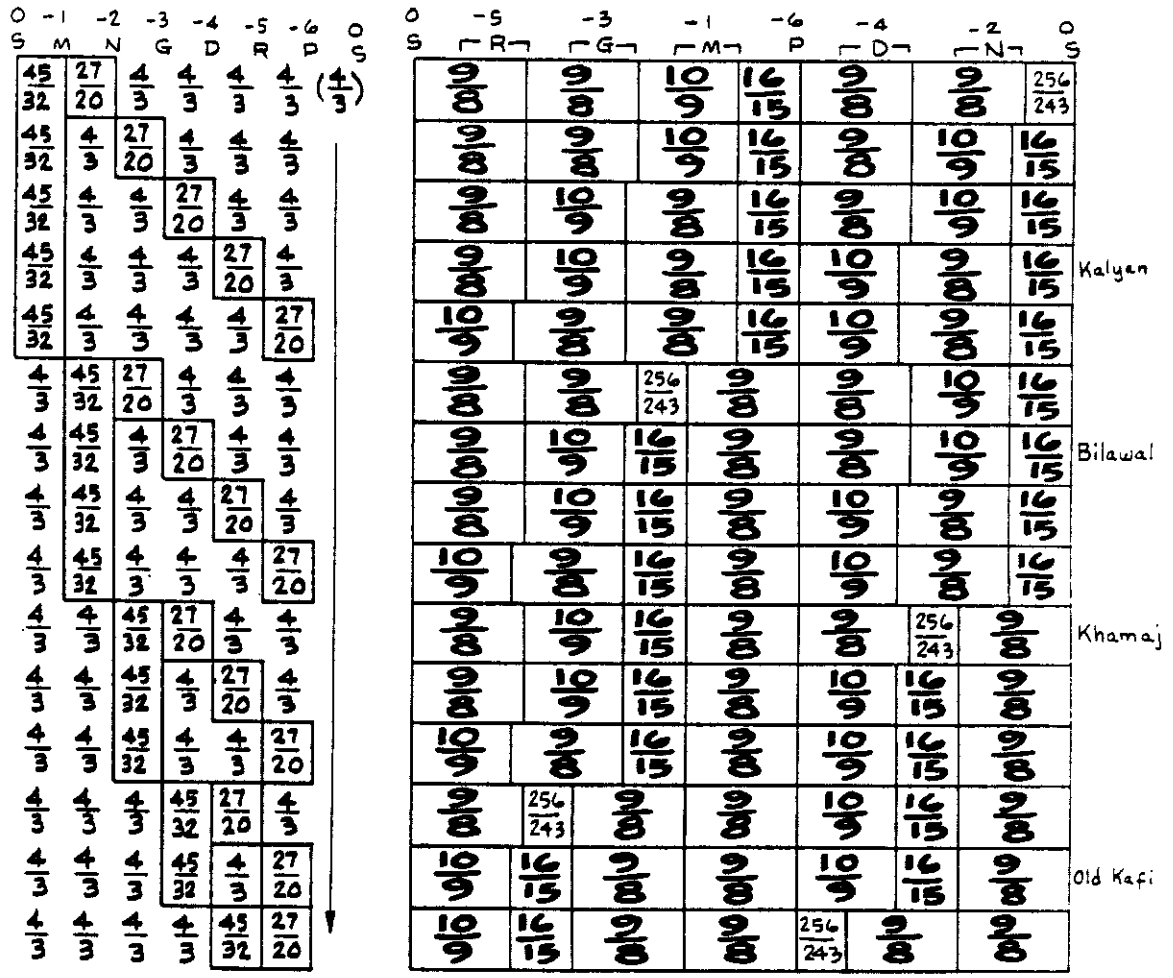
| class | | | Chain of Fourths S 0 | M -1 | N -2 | G -3 | D -4 | R -5 | P -6 | S 0 | Thats Grouped According to Class A, B, C S 0 | R -5 | G -3 | M -1 | P -6 | D -4 | N -2 | S 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | Kalyan | 1. | 4′ | 4 | 4 | 4 | 4 | 4 | (4) | | S | R | G | M | P | D | N | S |
| A | Bilawal | 2. | 4 | 4′ | 4 | 4 | 4 | 4 | | | S | R | G | M | P | D | N | S |
| | Khamaj | 3. | 4 | 4 | 4′ | 4 | 4 | 4 | | | S | R | G | M | P | D | N | S |
| | Kafi | 4. | 4 | 4 | 4 | 4′ | 4 | 4 | | | S | R | G | M | P | D | N | S |
| | Asawari | 5. | 4 | 4 | 4 | 4 | 4′ | 4 | | | S | R | G | M | P | D | N | S |
| | Bhairavi | 6. | 4 | 4 | 4 | 4 | 4 | 4′ | | | S | R | G | M | P | D | N | S |
| B | Champakali | 1. | 4′ | 4 | 4′ | 4 | 4 | 4 | (4) | | S | R | G | M′ | P | D | N | S |
| | | 2. | 4′ | 4 | 4 | 4′ | 4 | 4 | | | S | R | G | M′ | P | D | N | S |
| | | 3. | 4′ | 4 | 4 | 4 | 4′ | 4 | | | S | R | G | M′ | P | D | N | S |
| | | 4. | 4′ | 4 | 4 | 4 | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | Madhubanti | 5. | 4′ | 4 | 4 | 4′ | 4 | 4 | | | S | R | G | M′ | P | D | N | S |
| | | 6. | 4′ | 4 | 4 | 4 | 4′ | 4 | | | S | R | G | M′ | P | D | N | S |
| | Todi | 7. | 4′ | 4 | 4 | 4 | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | | 8. | 4′ | 4 | 4 | 4 | 4′ | 4 | | | S | R | G | M′ | P | D | N | S |
| | Purvi | 9. | 4′ | 4 | 4 | 4 | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | Marwa | 10. | 4′ | 4 | 4 | 4 | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | Patdeep | 11. | 4 | 4′ | 4 | 4′ | 4 | 4 | | | S | R | G | M | P | D | N | S |
| | Chandra Kauns | 12. | 4 | 4′ | 4 | 4 | 4′ | 4 | | | S | R | G | M | P | D | N | S |
| | Jogiya Todi | 13. | 4 | 4′ | 4 | 4 | 4 | 4′ | | | S | R | G | M | P | D | N | S |
| | Nat Bhairav | 14. | 4 | 4′ | 4 | 4 | 4′ | 4 | | | S | R | G | M | P | D | N | S |
| | Bhairav | 15. | 4 | 4′ | 4 | 4 | 4 | 4′ | | | S | R | G | M | P | D | N | S |
| | Anand Bhairav | 16. | 4 | 4′ | 4 | 4 | 4 | 4′ | | | S | R | G | M | P | D | N | S |
| | Mohan Kauns | 17. | 4 | 4 | 4′ | 4 | 4′ | 4 | | | S | R | G | M | P | D | N | S |
| | Basant Mukhari | 18. | 4 | 4 | 4′ | 4 | 4 | 4′ | | | S | R | G | M | P | D | N | S |
| | Ahir Bhairav | 19. | 4 | 4 | 4′ | 4 | 4 | 4′ | | | S | R | G | M | P | D | N | S |
| | Parameswari | 20. | 4 | 4 | 4 | 4′ | 4 | 4′ | | | S | R | G | M | P | D | N | S |
| C | | 1. | 4′ | 4 | 4′ | 4 | 4′ | 4 | (4) | | S | R | G | M′ | P | D | N | S |
| | | 2. | 4′ | 4 | 4′ | 4 | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | | 3. | 4′ | 4 | 4′ | 4 | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | | 4. | 4′ | 4 | 4 | 4′ | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | | 5. | 4′ | 4 | 4 | 4′ | 4 | 4′ | | | S | R | G | M′ | P | D | N | S |
| | | 6. | 4 | 4′ | 4 | 4′ | 4 | 4′ | | | S | R | G | M | P | D | N | S |

Fig 2



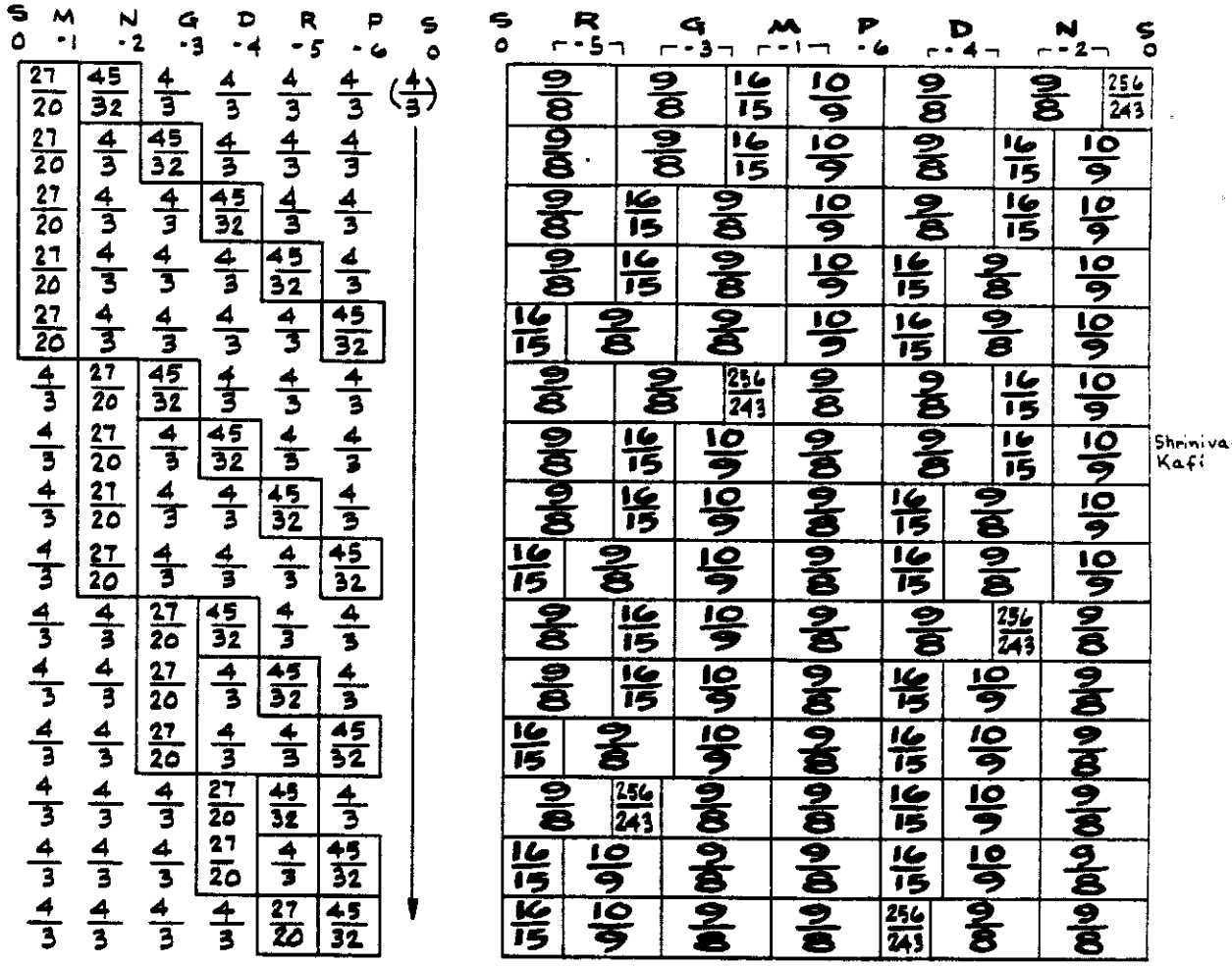

(Pythagoras $\frac{256}{243}$ $\frac{9}{8}$ $\frac{9}{8}$)

Fig 3




(Ptolemy $\frac{16}{15}$ $\frac{9}{8}$ $\frac{10}{9}$)

Fig 4




(Didymus $\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$)

Fig 5 (Pythagoras $\frac{256}{243}$ $\frac{9}{8}$ $\frac{9}{8}$)

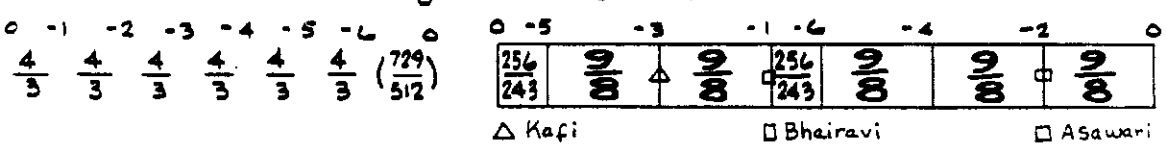



Fig 6 (Didymus/Ptolemy $\frac{16}{15}$ $\frac{9}{8}$ $\frac{10}{9}$)

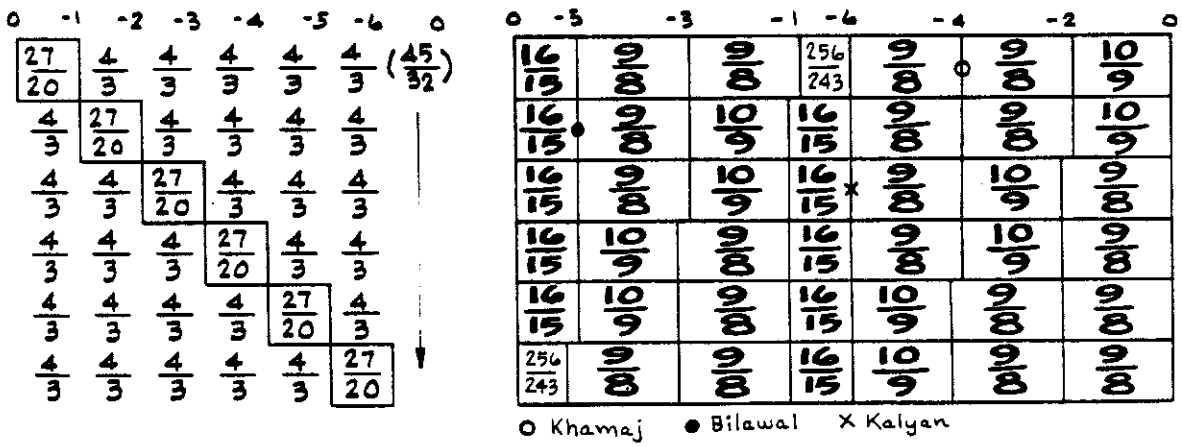
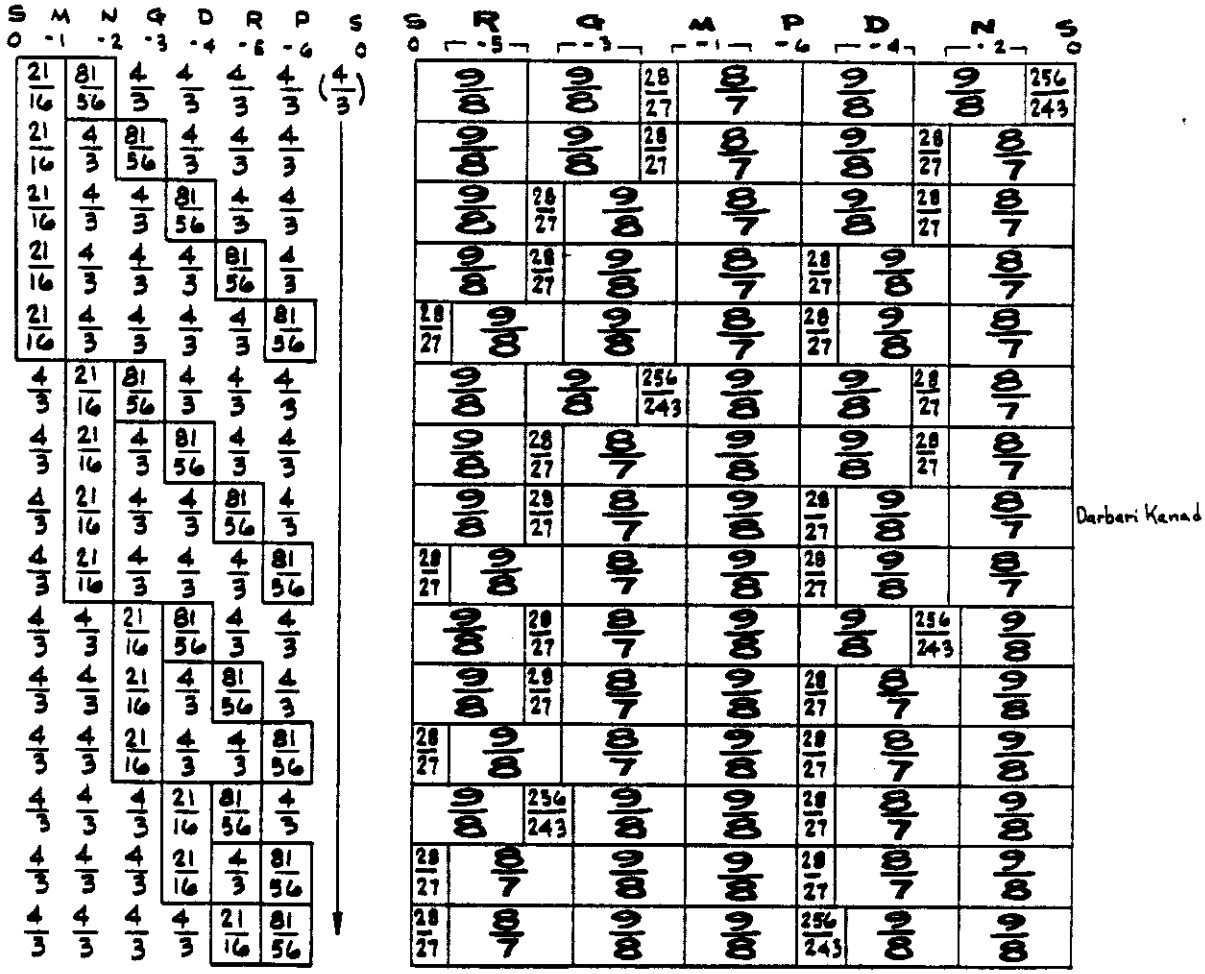
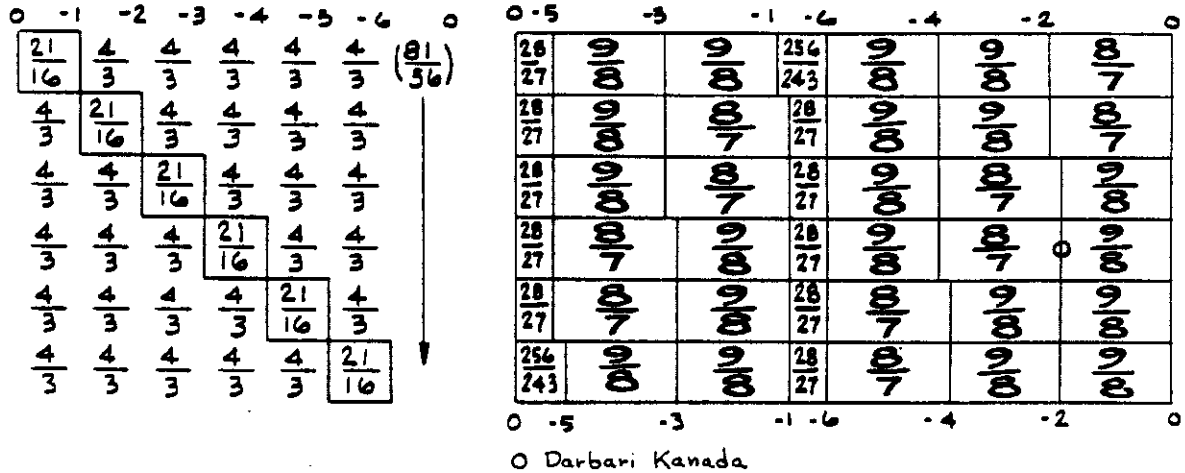


Figure 7



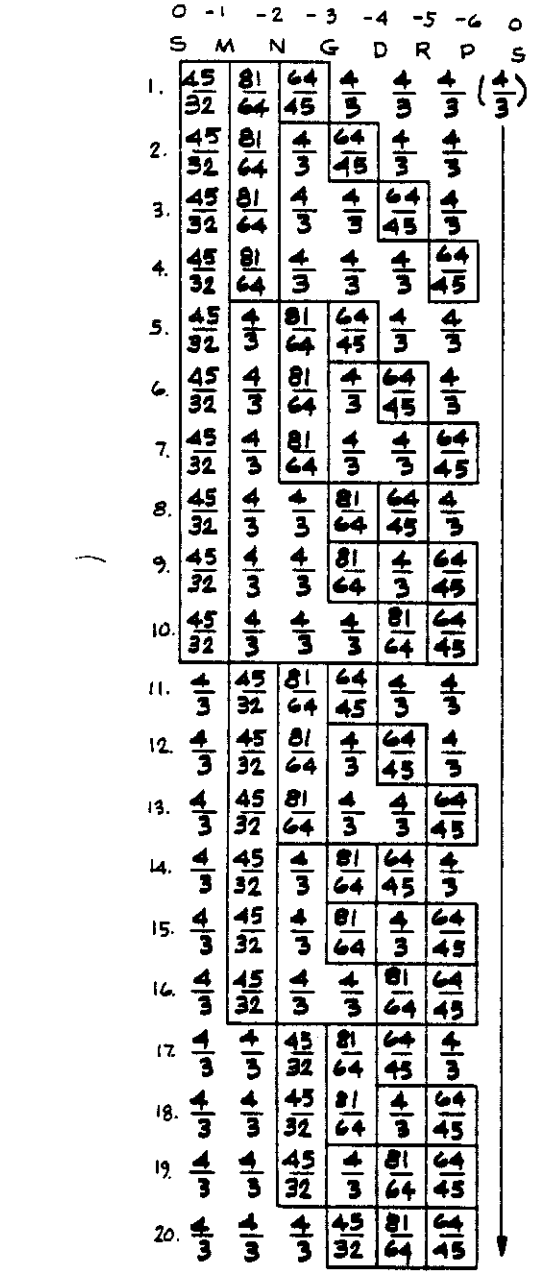

(Archytas $\frac{28}{27}$ $\frac{8}{7}$ $\frac{9}{8}$)

Figure 8




(Archytas $\frac{28}{27}$ $\frac{8}{7}$ $\frac{9}{8}$)

# Figure 9

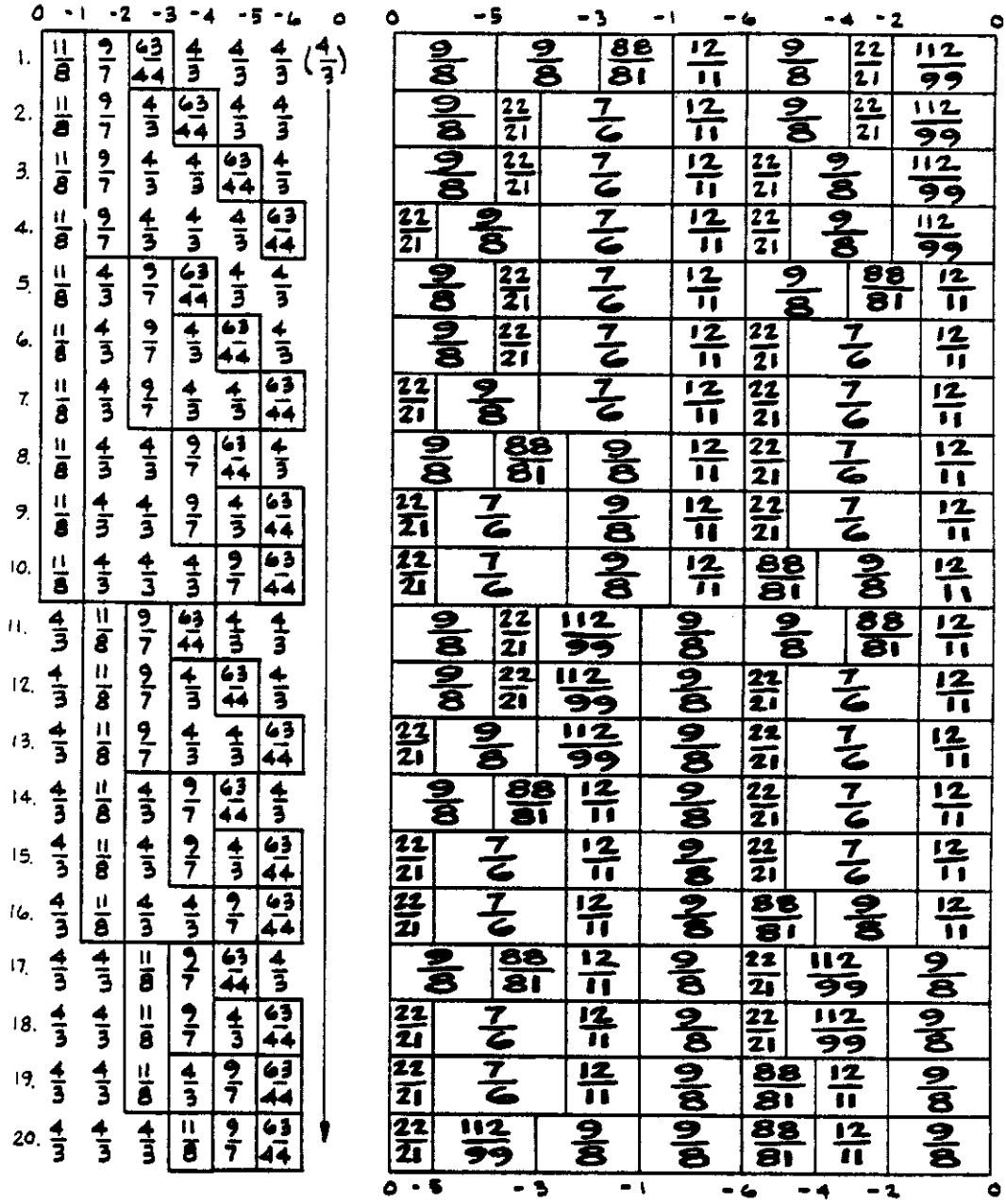



(Hawkins 16/15 135/128 32/27)


$\frac{135}{128}$ is close to $\frac{256}{243}$

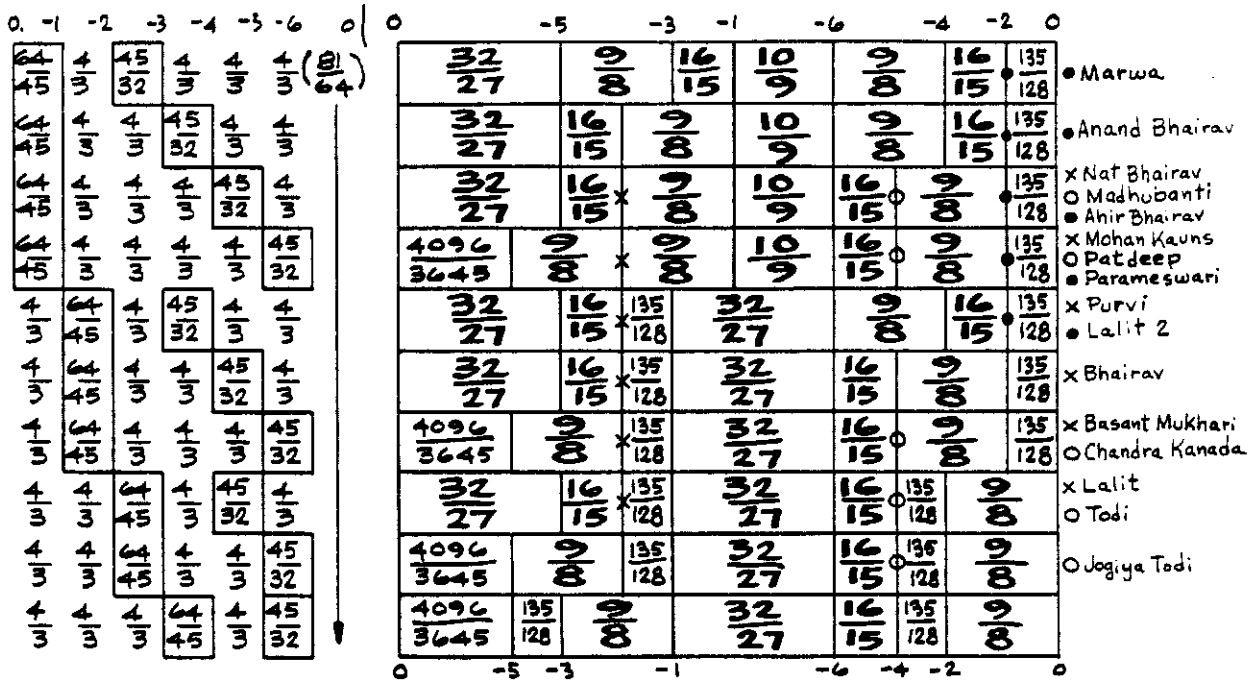
$\frac{4096}{3645}$ is close to $9/8$

Figure 10



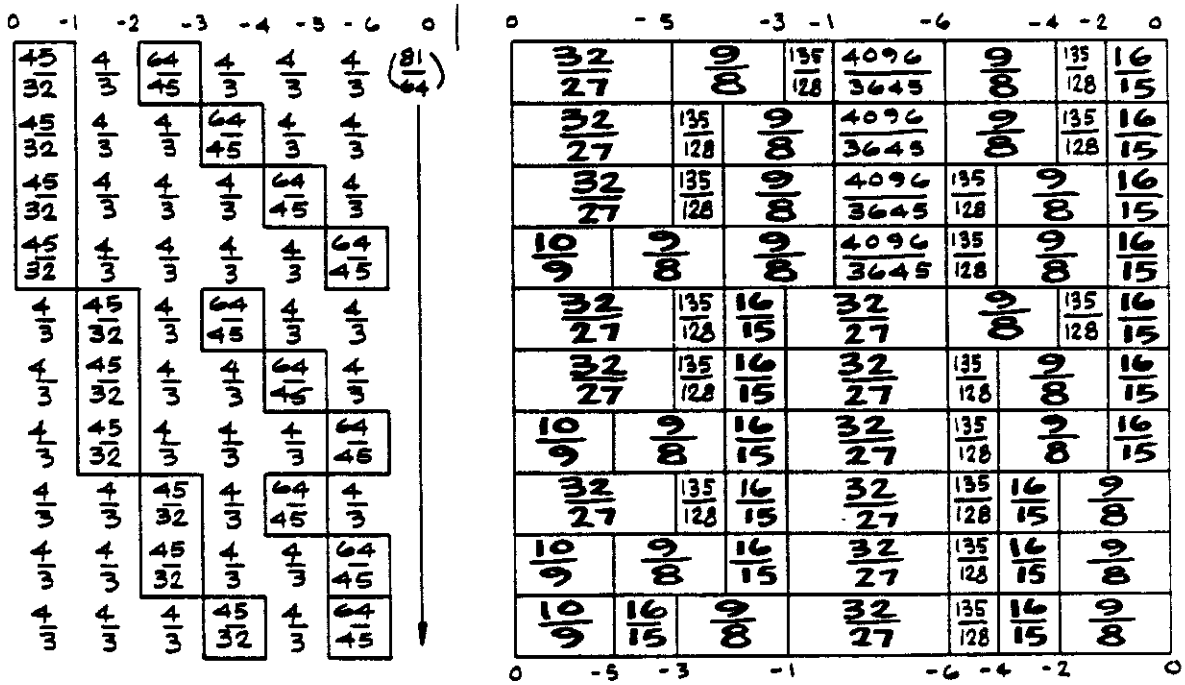

(Ptolemy $\frac{7}{6}$ $\frac{12}{11}$ $\frac{22}{21}$)

Figure IIa




(Hawkins $\frac{16}{15}$ $\frac{135}{128}$ $\frac{32}{27}$)

Figure IIb

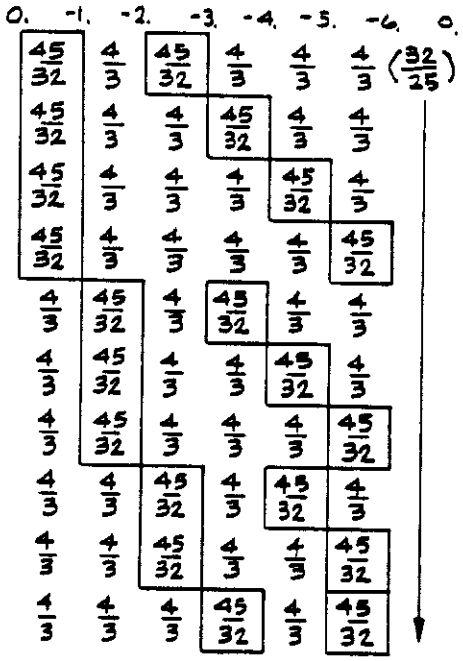


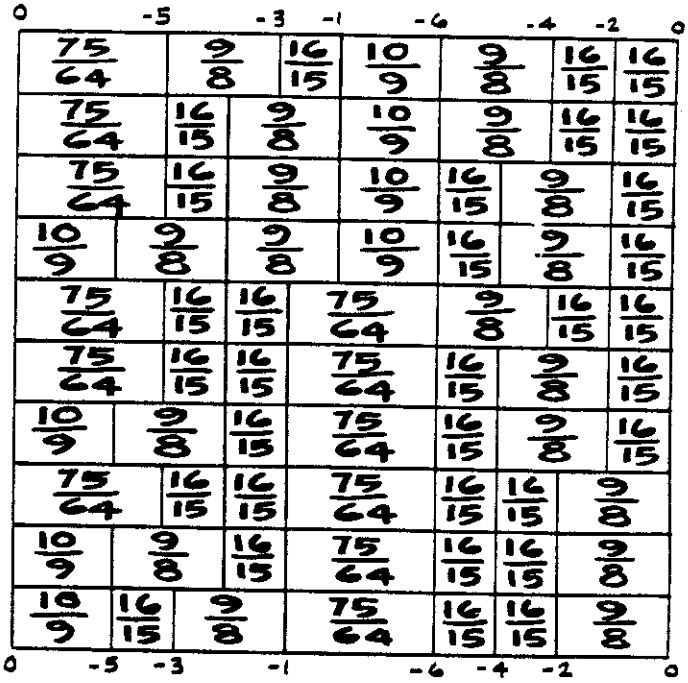

(Hawkins $\frac{135}{128}$ $\frac{16}{15}$ $\frac{32}{27}$)

Figure 12

17-Tone, Comma-Limma scale of Persia

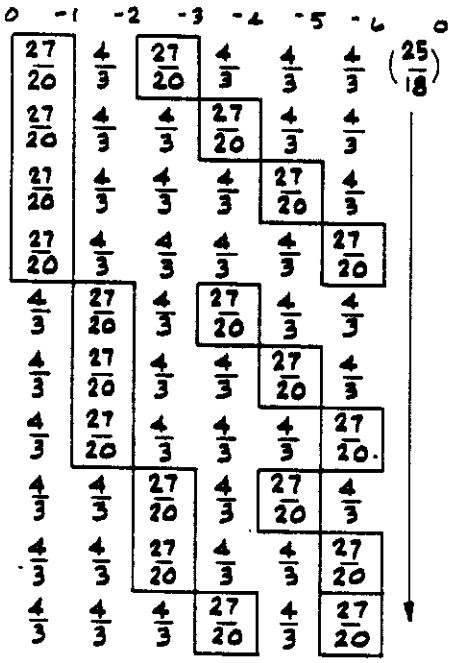
| C | C | C♯ | C | C | C♯ | C | C | C♯ | C | C♯ | C | C♯ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|

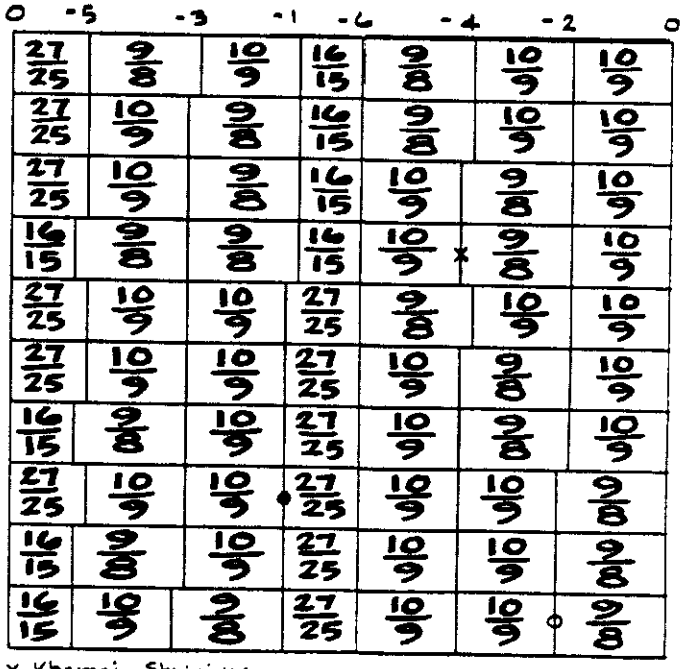



(Helmholtz 16/15 16/15 75/64)

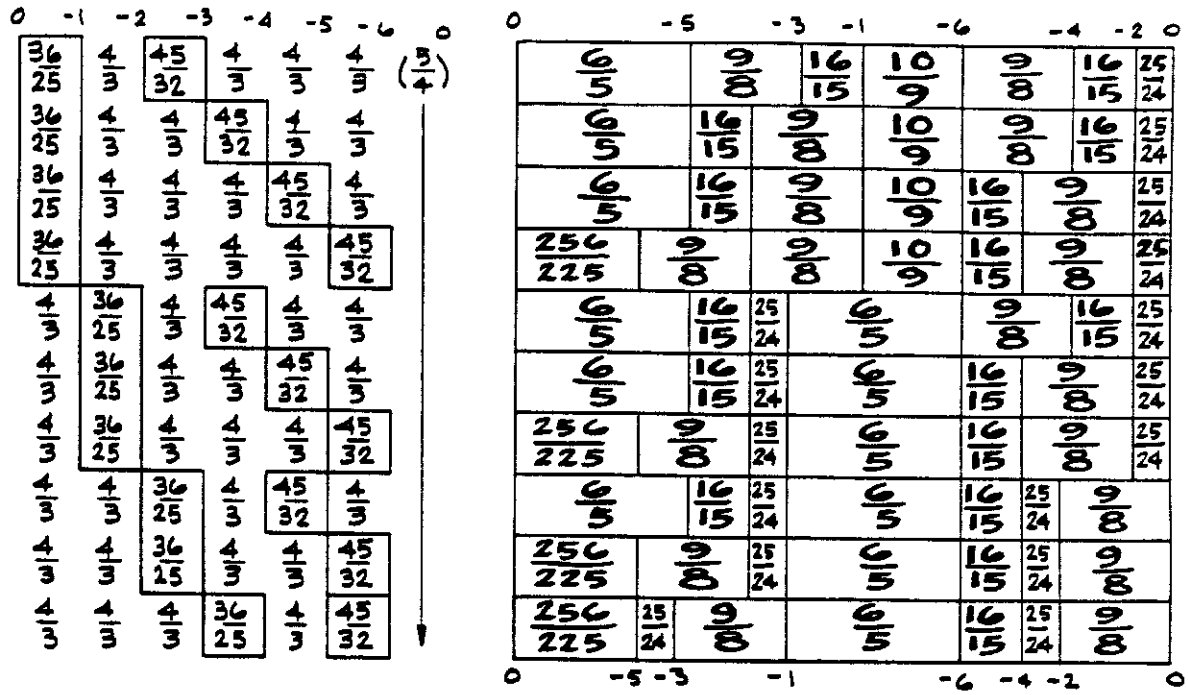
Figure 13 (Al-Farabi 10/9 10/9 27/25)



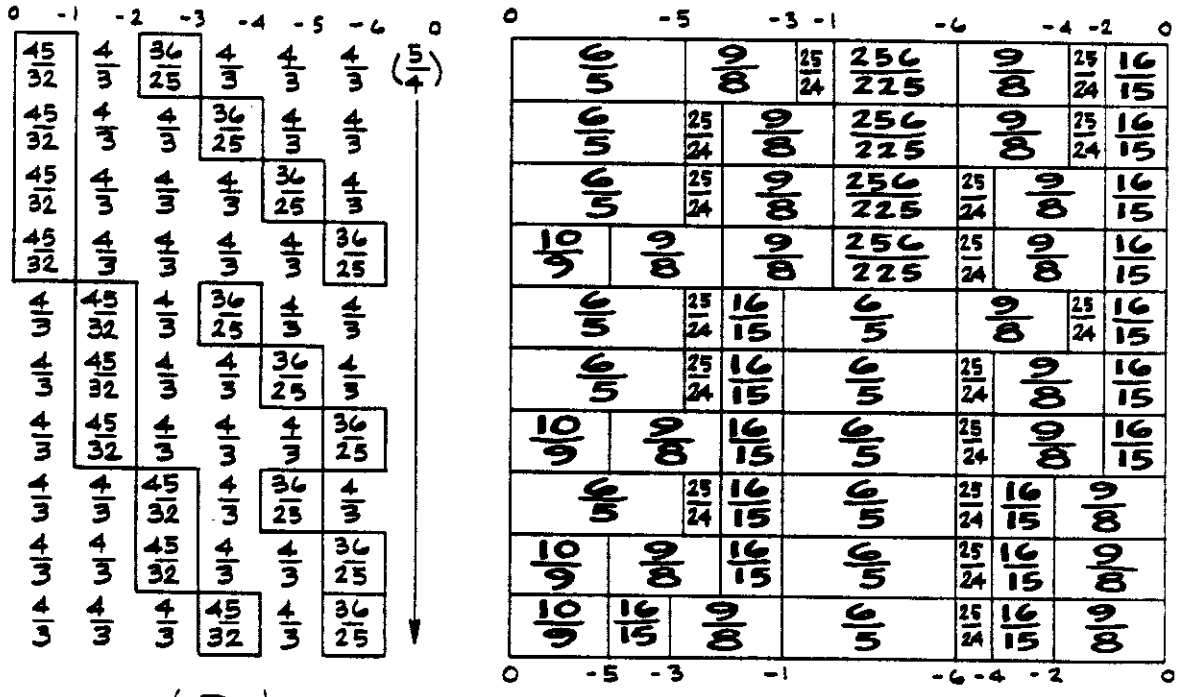

× Khamaj, Shrinivas
● Bhairavi, Shrinivas
○ Asawari, Shrinivas

Figure 14a



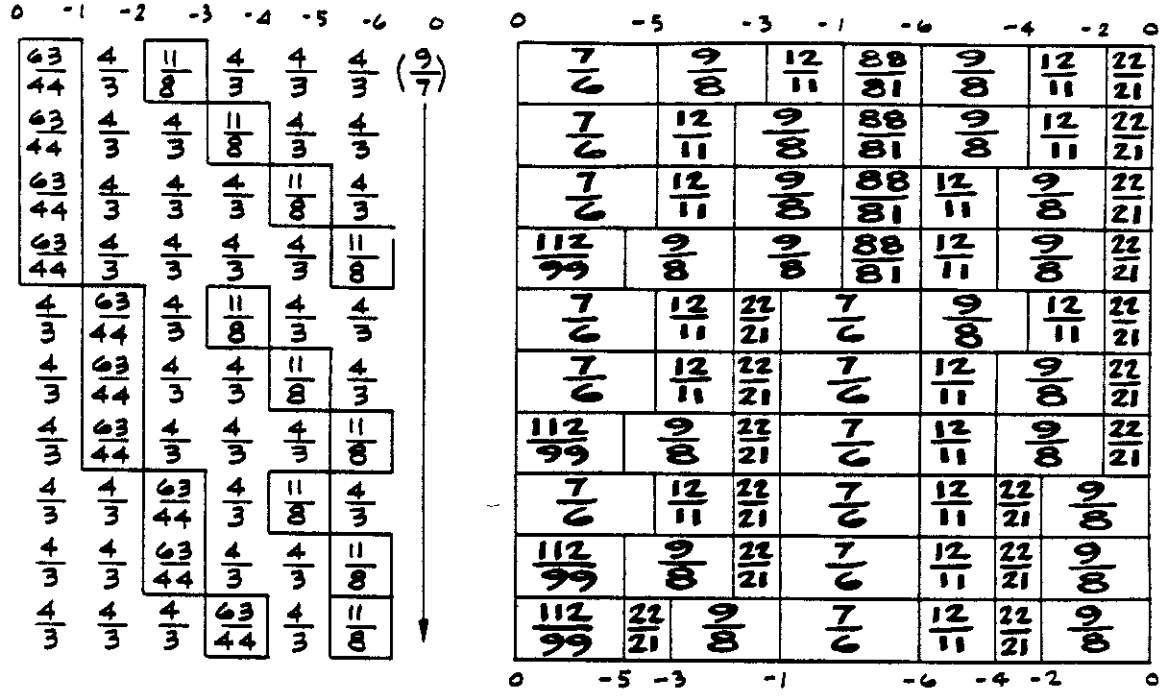

(Didymus 16/15 25/24 6/5)

Figure 14b



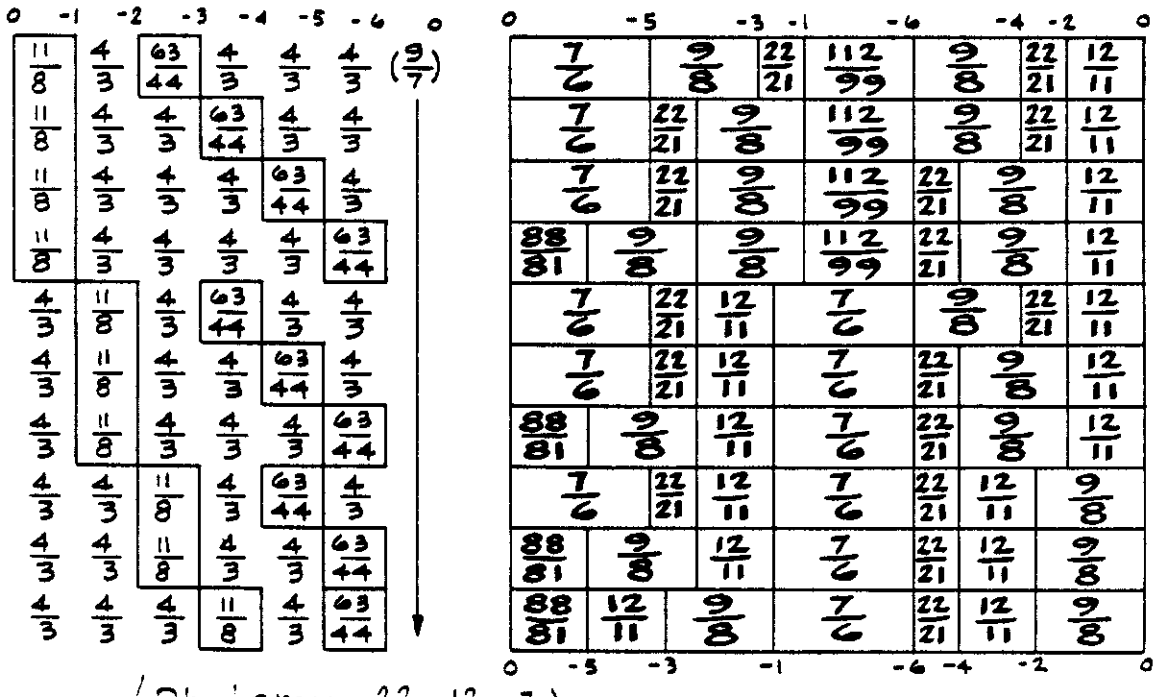

(Didymus 25/24 16/15 6/5)

Figure 15a



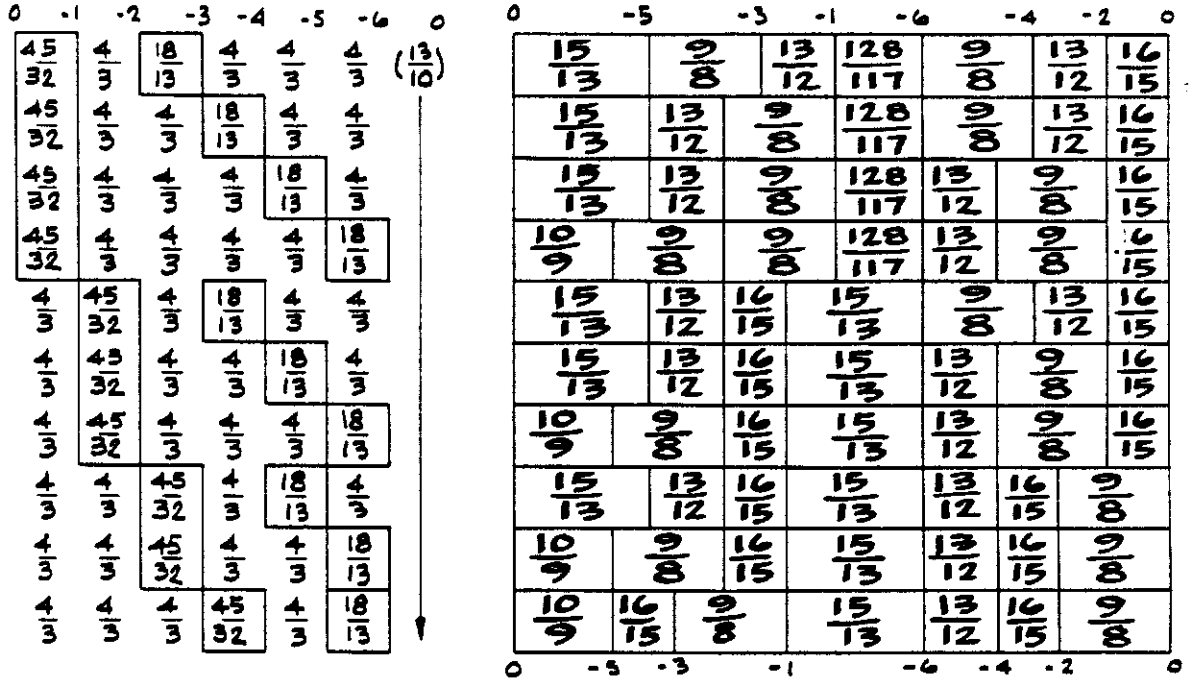

(Ptolemy 12/11 22/21 7/6)

Figure 15b



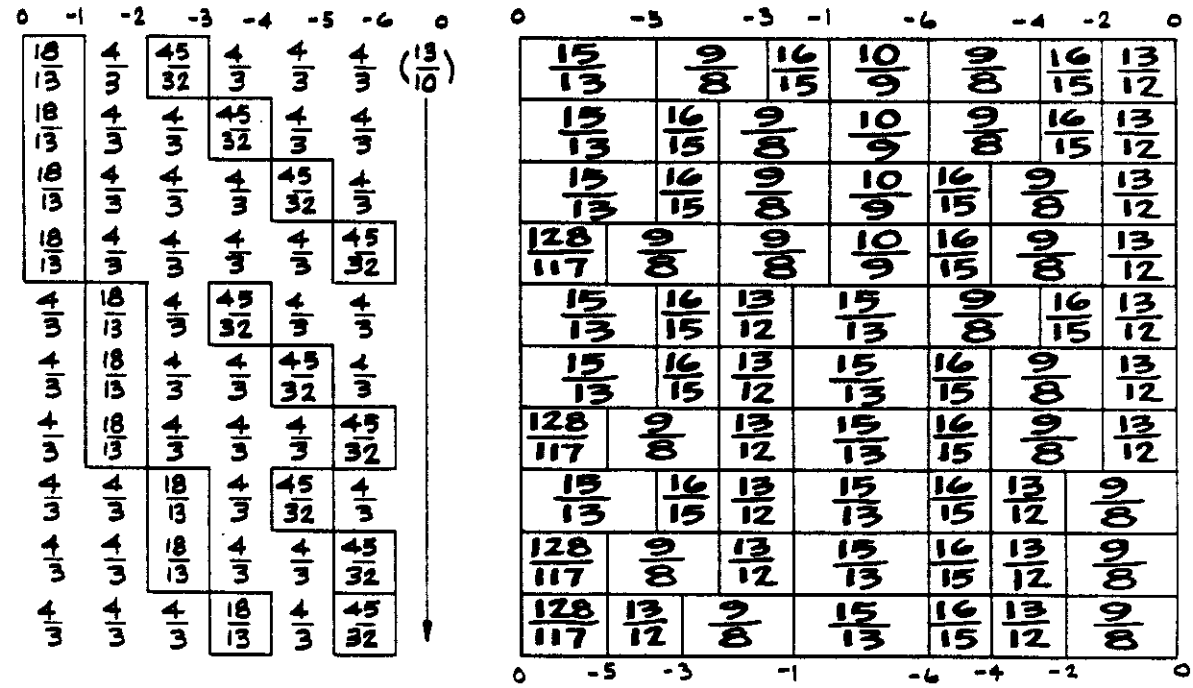

(Ptolemy 22/21 12/11 7/6)

Figure 16a



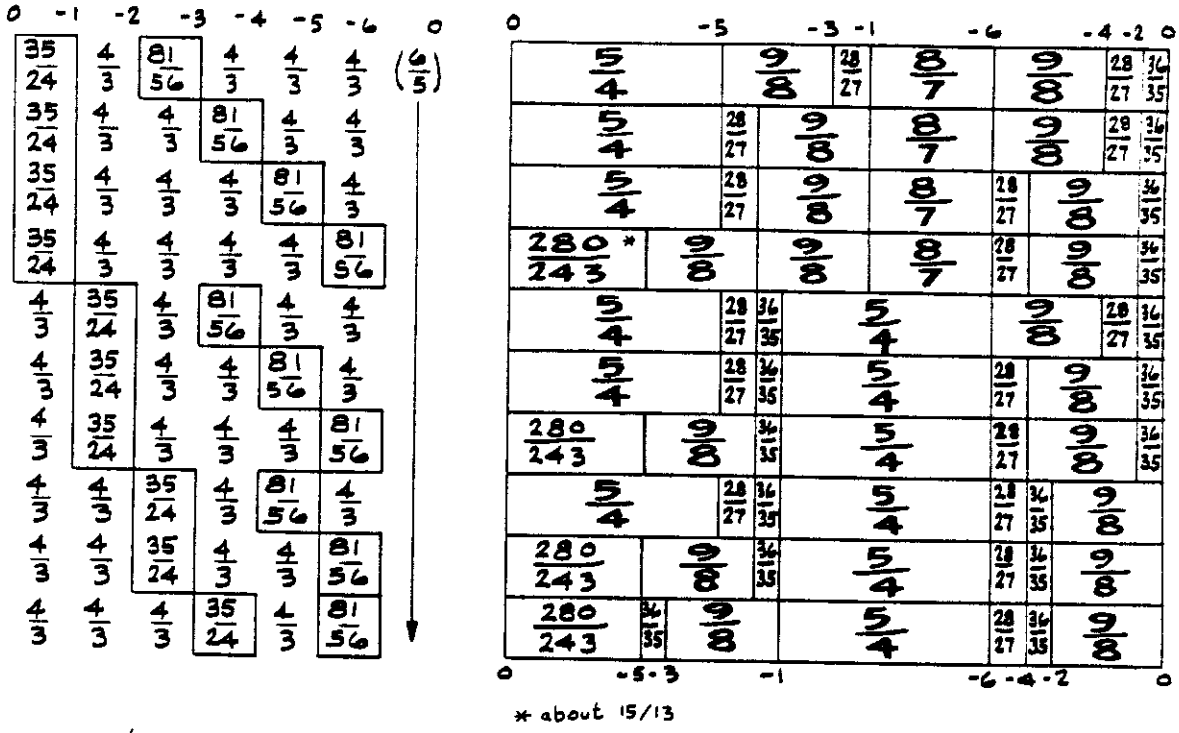

(Schlesinger 16/15 15/13 13/12)

Figure 16b




(Schlesinger 13/12 15/13 16/15)

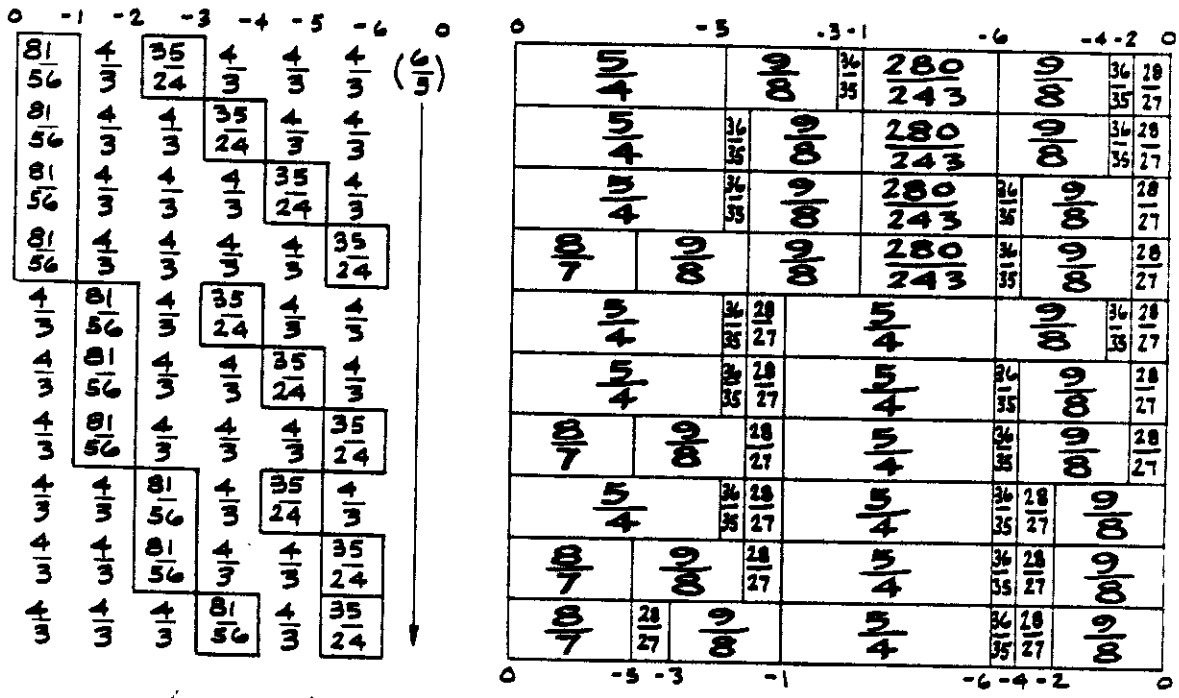
Fig. 17a




* about 15/13

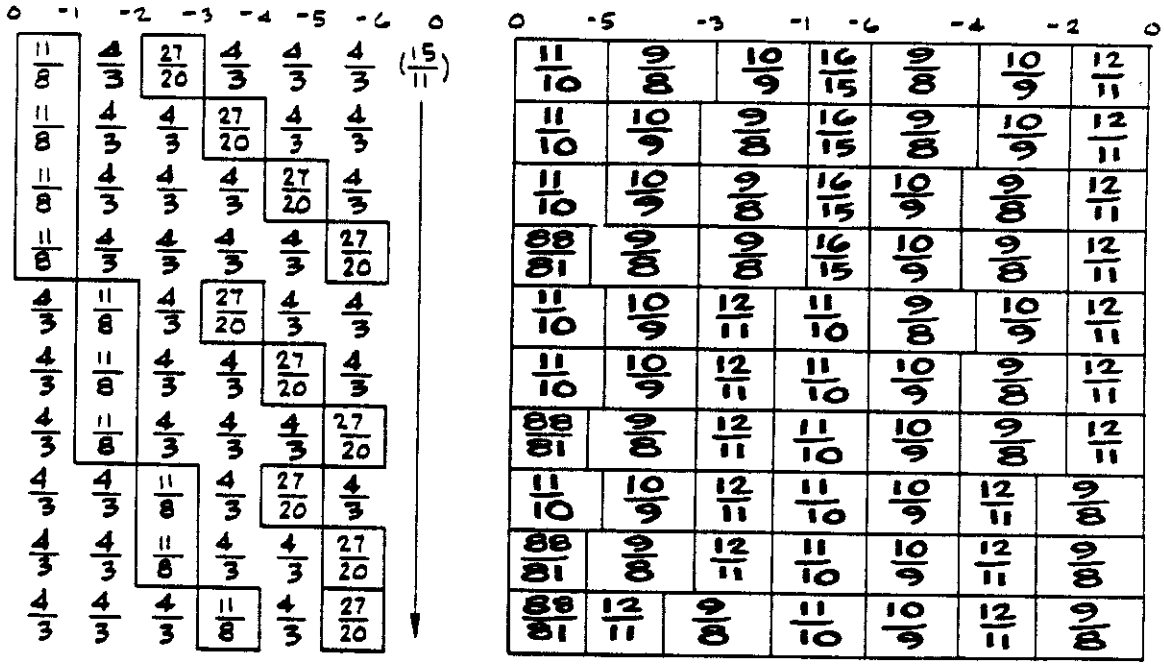
(Archytas $\frac{28}{27}$ $\frac{36}{35}$ $\frac{5}{4}$)

Figure 17 b



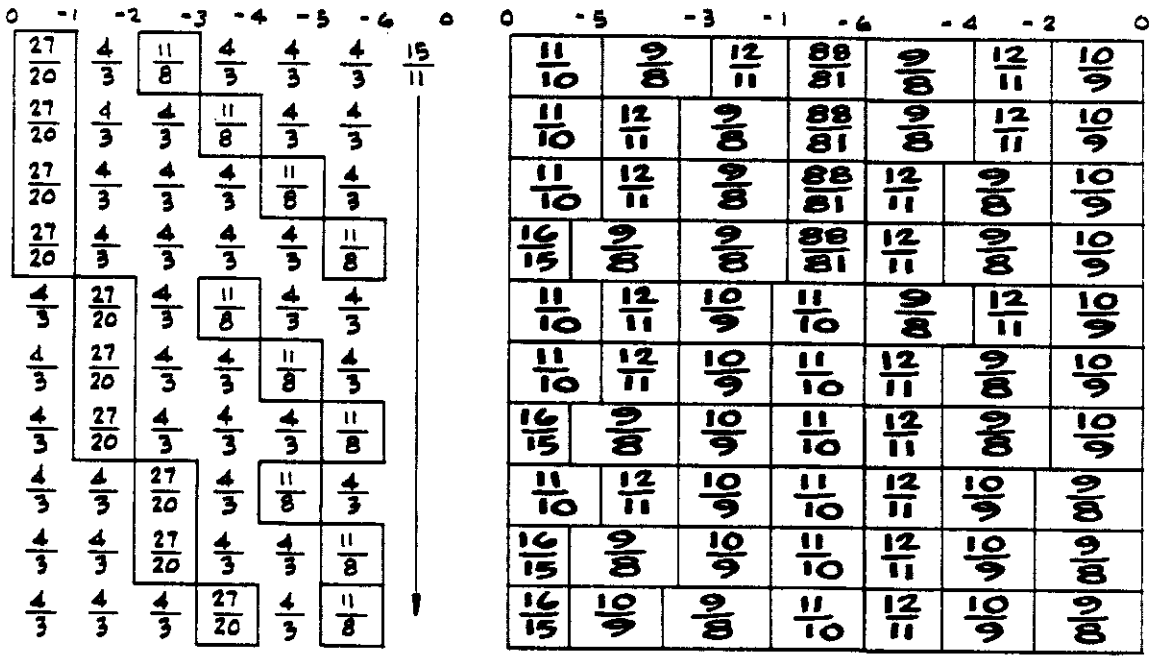

(Archytas $\frac{36}{35}$ $\frac{28}{27}$ $\frac{5}{4}$)

Figure 18a




(Ptolemy 12/11 11/10 10/9)

Fig 18b




(Ptolemy 10/9 11/10 12/11)