

22 TONE EIKOSANY TEMPLATES

This paper existed in an earlier version but is now updated with new documents that have been discovered. At first it appeared that Wilson had not solved the problem and he left it open, but in fact, he had solved it which has been proven by Praveen Ventamaramana which is included here as it represented the answer in the previous version a mere two months earlier.

Kraig Grady 30-10-21

19 April 69
 1219 Poinsettia Drive
 L. A. 90046

John,
 Hi; As it so happens, Dave Rothenberg is down here. I introduced him to the EMA last Tuesday and he told us briefly what he was doing. We are all meeting here at my place this coming Tuesday to continue his discussion.

I've found ^{four} ~~few~~ species of articulate eikosany generating hexads, in 22.

I

1x	0	1	3	12	21	6	(1 2 3 6 9 1)
3x	0	3	9	14	19	18	(3 6 5 4 1 3)
5x	0	5	15	16	17	8	(5 3 7 1 1 5)
7x	0	7	21	18	15	20	(7 8 3 2 1 1)
9x	0	9	5	20	13	10	(5 4 1 3 7 2)

II

1x	0	1	3	7	13	21	(1 2 4 6 8 1)
3x	0	3	9	21	17	19	(3 6 8 2 2 1)
5x	0	5	15	13	21	17	(5 8 2 2 4 1)
7x	0	7	21	5	3	15	(3 2 2 8 6 1)
9x	0	9	5	19	7	13	(5 2 2 4 6 3)

III

1x	0	1	3	11	17	21	(1 2 8 6 4 1)
3x	0	3	9	11	7	19	(3 4 2 2 8 3)
5x	0	5	15	11	19	17	(5 6 4 2 2 3)
7x	0	7	21	11	9	15	(7 2 2 2 1 4 6 1)
9x	0	9	5	11	21	13	(5 4 2 2 8 1)

!

IV

1x	0	1	3	5	9	15	(1 2 2 4 6 7)
3x	0	3	9	15	5	1	(1 2 2 4 6 7)
5x	0	5	15	3	1	9	(1 2 2 4 6 7)
7x	0	7	21	13	19	17	(7 6 4 2 2 1)
9x	0	9	5	1	15	3	(1 2 2 4 6 7)

By juggling and superposing the hexads in each specie a notable homogeneity can be observed. This homogeneity reaches its maximum in group IV where, quite surprisingly, the transforms are identical in melodic sequence (or reciprocal in the case of 7x).

This could very well represent all the articulate eikosany in 22, but it will be a very tedious job proving it. On the other hand it is not unthinkable that I have slipped up on a whole set.

The 16 Articulate
22 tone Eikosany Templates
-Praveen Venkataramana-

P1 [0, 1, 2, 4, 7, 13] = 1 1 2 3 6 9

P2 [0, 3, 5, 6, 7, 14] = 3 2 1 1 7 8

P3 [0, 1, 2, 4, 8, 14] = 1 1 2 4 6 8

P4 [0, 2, 3, 4, 8, 14] = 2 1 1 4 6 8

P5 [0, 2, 4, 5, 8, 14] = 2 2 1 3 6 8

P6 [0, 2, 4, 7, 8, 14] = 2 2 3 1 6 8

P7 [0, 2, 4, 8, 9, 14] = 2 2 4 1 5 8

P8 [0, 2, 4, 8, 11, 14] = 2 2 4 3 3 8

Wilson 1-5-7-9-11-15

P9 [0, 1, 6, 10, 12, 14] = 1 5 4 2 2 8

P10 [0, 1, 3, 5, 9, 15] = 1 2 2 4 6 7

P11 [0, 1, 2, 7, 12, 15] = 1 1 5 5 3 7

P12 [0, 2, 7, 11, 12, 15] = 2 5 4 1 3 7

P13 [0, 1, 7, 11, 13, 15] = 1 6 4 2 2 7

P14 [0, 3, 6, 7, 11, 16] = 3 3 1 4 5 6

P15 [0, 4, 6, 8, 11, 16] = 4 2 2 3 5 6

P16 [0, 3, 8, 10, 12, 16] = 3 5 2 2 4 6

Wilson 1-3-7-9-11-15

P1 [0, 1, 2, 4, 7, 13]x3= [0, 3, 6, 12, 21, 17]sort [0, 3, 6, 12, 17, 21] 3 3 6 5 4 1 =P14	P2 [0, 3, 5, 6, 7, 14] x3 [0, 9, 15, 18, 21, 20]sort [0, 9, 15, 18, 20, 21] 9 6 3 2 1 1 =P1	P3 [0, 1, 2, 4, 8, 14]x3 [0, 3, 6, 12, 2, 20]sort [0, 2, 3, 6, 12, 20] 2 1 3 6 8 2 =P5	P4 [0, 2, 3, 4, 8, 14]x3 [0, 6, 9, 12, 2, 20]sort [0, 2, 6, 9, 12, 20] 2 4 3 3 8 2 =P8	P5~ [0, 2, 4, 5, 8, 14]x3 [0, 6, 12, 15, 2, 20]sort [0, 2, 6, 12, 15, 20] 2 4 6 3 5 2 =P16	P6 [0, 2, 4, 7, 8, 14]x3 [0, 6, 12, 21, 2, 20]sort [0, 2, 6, 12, 20, 21] 2 4 6 8 1 1 =P3	P7 [0, 2, 4, 8, 9, 14]x3 [0, 6, 12, 2, 5, 20]sort [0, 2, 5, 6, 12, 20] 2 3 1 6 8 2 =P7	P8 [0, 2, 4, 8, 11, 14]x3 [0, 6, 12, 2, 11, 20]sort [0, 2, 6, 11, 12, 20] 2 4 5 1 8 2 =P9
[0, 1, 2, 4, 7, 13] x5= [0, 5, 10, 20, 13, 21] sort [0, 5, 10, 13, 20, 21] 5 5 3 7 1 1 =P11	[0, 3, 5, 6, 7, 14] x5 [0, 15, 3, 8, 13, 4]sort [0, 3, 4, 8, 13, 15] 3 1 4 5 2 7 =P12	[0, 1, 2, 4, 8, 14]x5 [0, 5, 10, 20, 18, 4]sort [0, 4, 5, 10, 18, 20] 4 1 5 8 2 2 =P7	[0, 2, 3, 4, 8, 14]x5 [0, 10, 15, 20, 18, 4]sort [0, 4, 10, 15, 18, 20] 4 6 5 3 2 2 =P15	[0, 2, 4, 5, 8, 14]x5 [0, 10, 20, 3, 18, 4]sort [0, 3, 4, 10, 18, 20] 3 1 6 8 2 2 =P6	[0, 2, 4, 7, 8, 14]x5 [0, 10, 20, 13, 18, 4]sort [0, 4, 10, 13, 18, 20] 4 6 3 5 2 2 =P16	[0, 2, 4, 8, 9, 14]x5 [0, 10, 20, 18, 1, 4]sort [0, 1, 4, 10, 18, 20] 1 3 6 8 2 2 =P5	[0, 2, 4, 8, 11, 14]x5 [0, 10, 20, 18, 11, 4]sort [0, 4, 10, 11, 18, 20] 4 6 1 7 2 2 =P13
[0, 1, 2, 4, 7, 13] x7= [0, 7, 14, 6, 5, 3]sort [0, 3, 5, 6, 7, 14] 3 2 1 1 7 8 =P2	[0, 3, 5, 6, 7, 14] x7 [0, 21, 13, 20, 5, 10]sort [0, 5, 10, 13, 20, 21] 5 5 3 7 1 1 =P11	[0, 1, 2, 4, 8, 14]x7 [0, 7, 14, 6, 12, 10]sort [0, 6, 7, 10, 12, 14] 6 1 3 2 2 8 =P6	[0, 2, 3, 4, 8, 14]x7 [0, 14, 21, 6, 12, 10]sort [0, 6, 10, 12, 14, 21] 6 4 2 2 7 1 =P13	[0, 2, 4, 5, 8, 14]x7 [0, 14, 6, 13, 12, 10]sort [0, 6, 10, 12, 13, 14] 6 4 2 1 1 8 =P3	[0, 2, 4, 7, 8, 14]x7 [0, 14, 6, 5, 12, 10]sort [0, 5, 6, 10, 12, 14] 5 1 4 2 2 8 =P7	[0, 2, 4, 8, 9, 14]x7 [0, 14, 6, 12, 19, 10]sort [0, 6, 10, 12, 14, 19] 6 4 2 2 5 3 =P16	[0, 2, 4, 8, 11, 14]x7 [0, 14, 6, 12, 11, 10]sort [0, 6, 10, 11, 12, 14] 6 4 1 1 2 8 =P4
[0, 1, 2, 4, 7, 13]x9= [0, 9, 18, 14, 19, 7]sort [0, 7, 9, 14, 18, 19] 7 2 5 4 1 3 =P12	[0, 3, 5, 6, 7, 14] x9 [0, 5, 1, 10, 19, 16]sort [0, 1, 5, 10, 16, 19] 1 4 5 6 3 3 =P14	[0, 1, 2, 4, 8, 14]x9 [0, 9, 18, 14, 6, 16]sort [0, 6, 9, 14, 16, 18] 6 3 5 2 2 4 =P16	[0, 2, 3, 4, 8, 14]x9 [0, 18, 5, 14, 6, 16]sort [0, 5, 6, 14, 16, 18] 5 1 8 2 2 4 =P9	[0, 2, 4, 5, 8, 14]x9 [0, 18, 14, 1, 6, 16]sort [0, 1, 6, 14, 16, 18] 1 5 8 2 2 4 =P7	[0, 2, 4, 7, 8, 14]x9 [0, 18, 14, 19, 6, 16]sort [0, 6, 14, 16, 18, 19] 6 8 2 2 1 3 =P5	[0, 2, 4, 8, 9, 14]x9 [0, 18, 14, 6, 15, 16]sort [0, 6, 14, 15, 16, 18] 6 8 1 1 2 4 =P3	[0, 2, 4, 8, 11, 14]x9 [0, 18, 14, 6, 11, 16]sort [0, 6, 11, 14, 16, 18] 6 5 3 2 2 4 =P15
P9 [0, 1, 6, 10, 12, 14]x3 [0, 3, 18, 8, 14, 20]sort [0, 3, 8, 14, 18, 20] 3 5 6 4 2 2 =P15	P10 [0, 1, 3, 5, 9, 15]x3 [0, 3, 9, 15, 5, 1]sort [0, 1, 3, 5, 9, 15] 1 2 2 4 6 7 =P10	P11 [0, 1, 2, 7, 12, 15]x3 [0, 3, 6, 21, 14, 1]sort [0, 1, 3, 6, 14, 21] 1 2 3 8 7 1 =P2	P12 [0, 2, 7, 11, 12, 15]x3 [0, 6, 21, 11, 14, 1]sort [0, 1, 6, 11, 14, 21] 1 5 5 3 7 1 =P11	P13 [0, 1, 7, 11, 13, 15]x3 [0, 3, 21, 11, 17, 1]sort [0, 1, 3, 11, 17, 21] 1 2 8 6 4 1 =P4	P14 [0, 3, 6, 7, 11, 16]x3 [0, 9, 18, 21, 11, 4] [0, 4, 9, 11, 18, 21] 4 5 2 7 3 1 =P12	P15 [0, 4, 6, 8, 11, 16]x3 [0, 12, 18, 2, 11, 4]sort [0, 2, 4, 11, 12, 18] 2 2 7 1 6 4 =P13	P16 [0, 3, 8, 10, 12, 16]x3 [0, 9, 2, 8, 14, 4]sort [0, 2, 4, 8, 9, 14] 2 2 4 1 5 8 =P7
[0, 1, 6, 10, 12, 14]x5 [0, 5, 8, 6, 16, 4]sort [0, 4, 5, 6, 8, 16] 4 1 1 2 8 6 =P4	[0, 1, 3, 5, 9, 15]x5 [0, 5, 15, 3, 1, 9]sort [0, 1, 3, 5, 9, 15] 1 2 2 4 6 7 =P10	[0, 1, 2, 7, 12, 15]x5 [0, 5, 10, 13, 16, 9]sort [0, 5, 9, 10, 13, 16] 5 4 1 3 3 6 =P14	[0, 2, 7, 11, 12, 15]x5 [0, 10, 13, 11, 16, 9]sort [0, 9, 10, 11, 13, 16] 9 1 1 2 3 6 =P1	[0, 1, 7, 11, 13, 15]x5 [0, 5, 13, 11, 21, 9]sort [0, 5, 9, 11, 13, 21] 5 4 2 2 8 1 =P9	[0, 3, 6, 7, 11, 16]x5 [0, 15, 8, 13, 11, 14]sort [0, 8, 11, 13, 14, 15] 8 3 2 1 1 7 =P2	[0, 4, 6, 8, 11, 16]x5 [0, 20, 8, 18, 11, 14]sort [0, 8, 11, 14, 18, 20] 8 3 3 4 2 2 =P8	[0, 3, 8, 10, 12, 16]x5 [0, 15, 18, 6, 16, 14]sort [0, 6, 14, 15, 16, 18] 6 8 1 1 2 4 =P3
[0, 1, 6, 10, 12, 14]x7 [0, 7, 20, 4, 18, 10]sort [0, 4, 7, 10, 18, 20] 4 3 3 8 2 2 =P8	[0, 1, 3, 5, 9, 15]x7 [0, 7, 21, 13, 19, 17]sort : [0, 7, 13, 17, 19, 21] 7 6 4 2 2 1 =P10	[0, 1, 2, 7, 12, 15]x7 [0, 7, 14, 5, 18, 17]sort [0, 5, 7, 14, 17, 18] 5 2 7 3 1 4 =P12	[0, 2, 7, 11, 12, 15]x7 [0, 14, 5, 11, 18, 17]sort [0, 5, 11, 14, 17, 18] 5 6 3 3 1 4 =P14	[0, 1, 7, 11, 13, 15]x7 [0, 7, 5, 11, 3, 17]sort [0, 3, 5, 7, 11, 17] 3 2 2 4 6 5 =P15	[0, 3, 6, 7, 11, 16]x7 [0, 21, 20, 5, 11, 2]sort [0, 2, 5, 11, 20, 21] 2 3 6 9 1 1 =P1	[0, 4, 6, 8, 11, 16]x7 [0, 6, 20, 12, 11, 2]sort [0, 2, 6, 11, 12, 20] 2 4 5 1 8 2 =P9	[0, 3, 8, 10, 12, 16]x7 [0, 21, 12, 4, 18, 2]sort [0, 2, 4, 12, 18, 21] 2 2 8 6 3 1 =P5
[0, 1, 6, 10, 12, 14]x9 [0, 9, 10, 2, 20, 16]sort [0, 2, 9, 10, 16, 20] 2 7 1 6 4 2 =P13	[0, 1, 3, 5, 9, 15]x9 [0, 9, 5, 1, 15, 3]sort [0, 1, 3, 5, 9, 15] 1 2 2 4 6 7 =P10	[0, 1, 2, 7, 12, 15]x9 [0, 9, 18, 19, 20, 3]sort [0, 3, 9, 18, 19, 20] 3 6 9 1 1 2 =P1	[0, 2, 7, 11, 12, 15]x9 [0, 18, 19, 11, 20, 3]sort [0, 3, 11, 18, 19, 20] 3 8 7 1 1 2 =P2	[0, 1, 7, 11, 13, 15]x9 [0, 9, 19, 11, 7, 3]sort [0, 3, 7, 9, 11, 19] 3 4 2 2 8 3 =P8	[0, 3, 6, 7, 11, 16]x9 [0, 5, 10, 19, 11, 12]sort [0, 5, 10, 11, 12, 19] 5 5 1 1 7 3 =P11	[0, 4, 6, 8, 11, 16]x9 [0, 14, 10, 6, 11, 12]sort [0, 6, 10, 11, 12, 14] 6 4 1 1 2 8 =P4	[0, 3, 8, 10, 12, 16]x9 [0, 5, 6, 2, 20, 12]sort [0, 2, 5, 6, 12, 20] 2 3 1 6 8 2 =P6

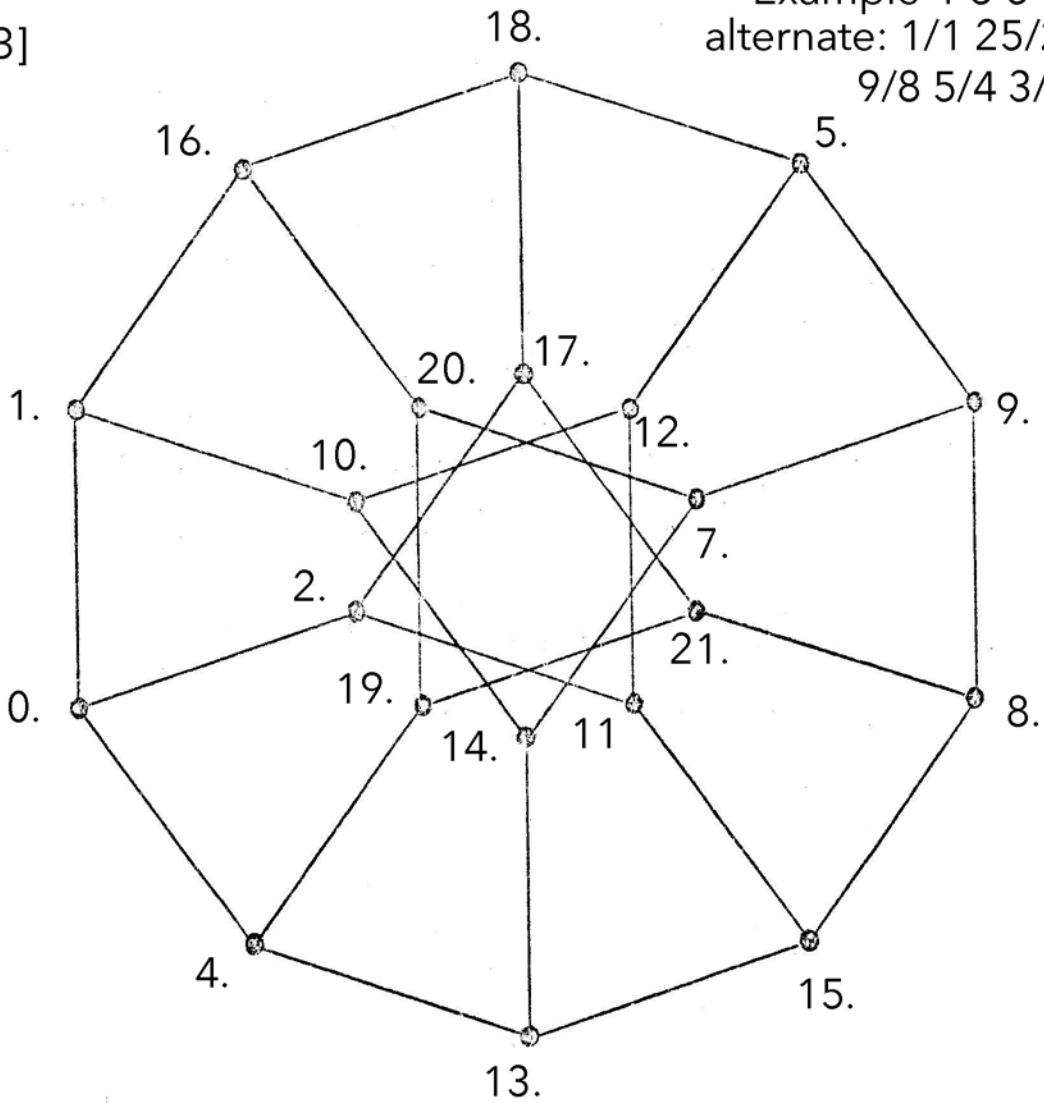
22 tone Eikosany inter-relationships by multiplication

A B C D E F E I K O S A N Y

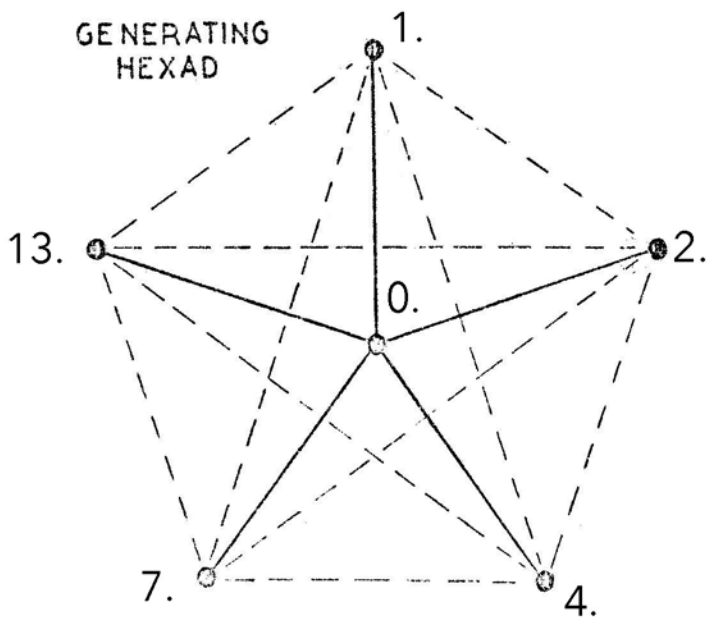
P1
 [0, 1, 2, 4, 7, 13]
 = 1 1 2 3 6 9

3 and 6
 to be filled

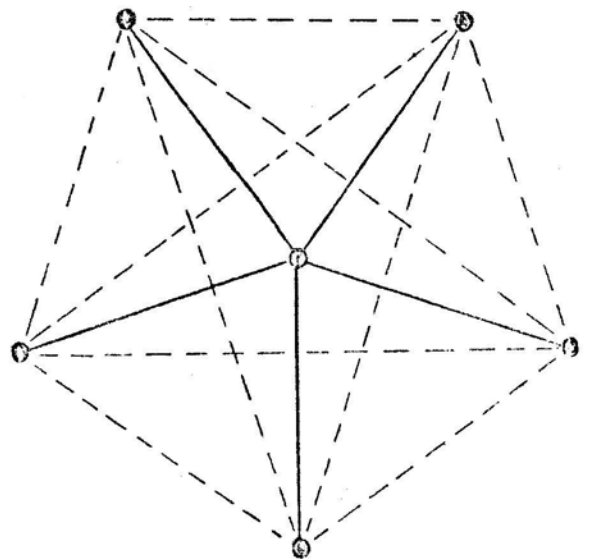
Example-1 3 5 9 17 33
 alternate: 1/1 25/24 16/15
 9/8 5/4 3/2



GENERATING
 HEXAD



RECIPROCAL

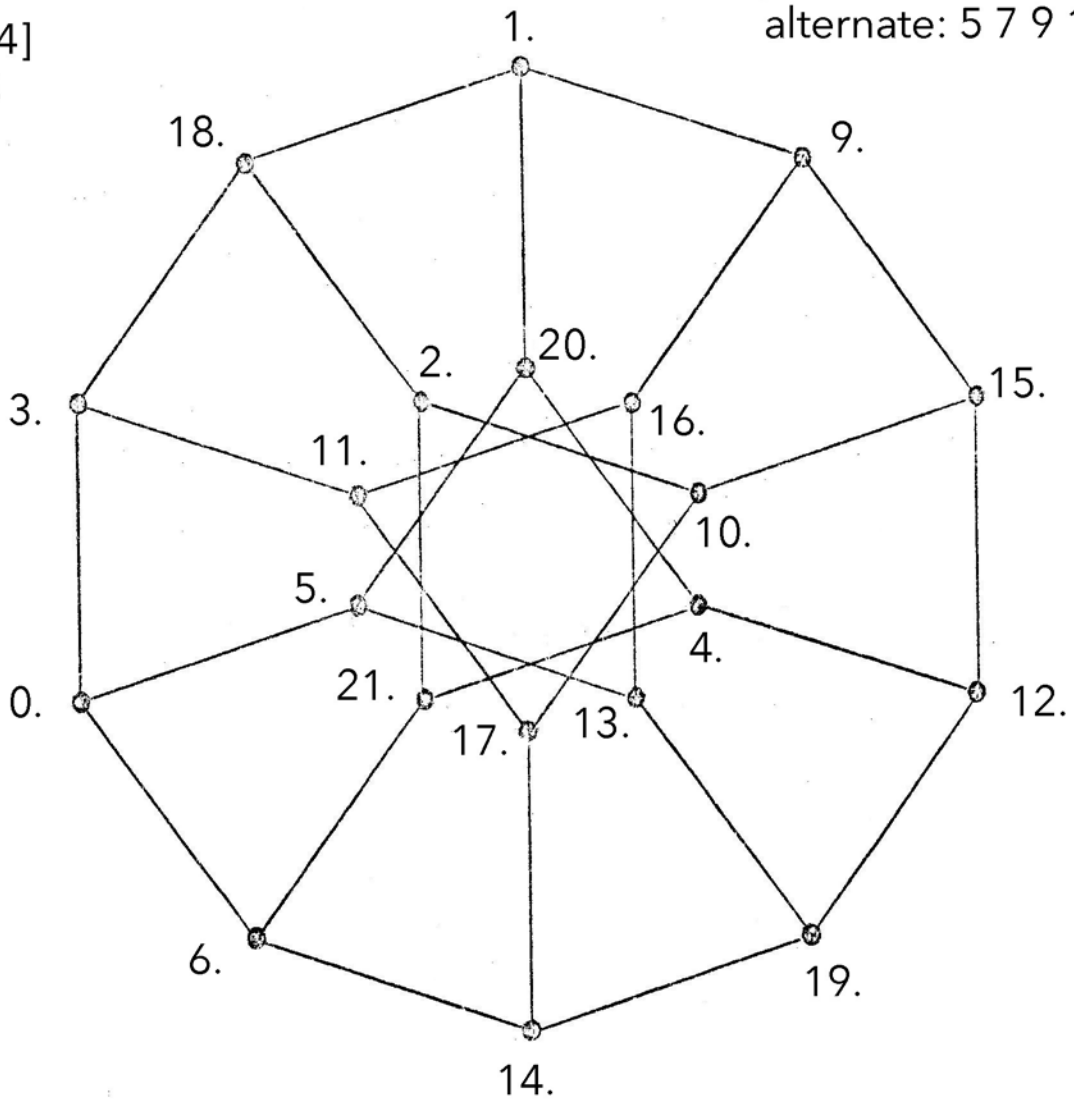


A B C D E F E I K O S A N Y

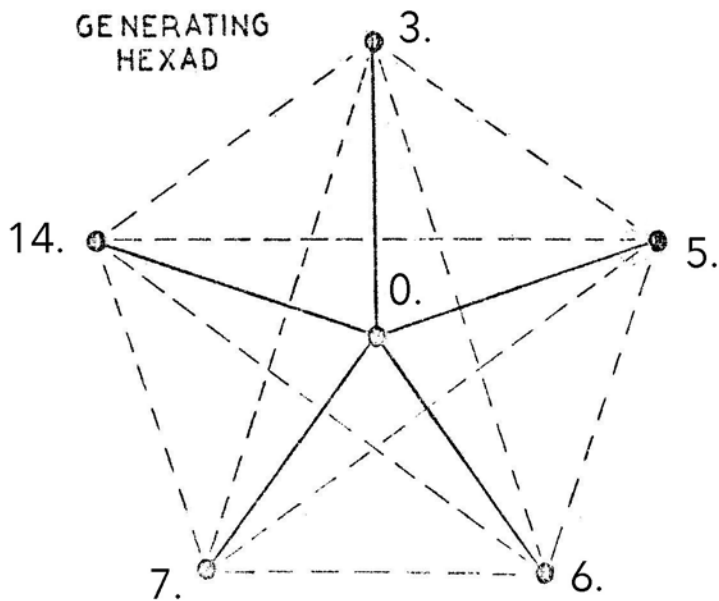
P2
 [0, 3, 5, 6, 7, 14]
 = 3 2 1 1 7 8

Example 5 7 9 11 7/3 11/5
 alternate: 5 7 9 11 21 45

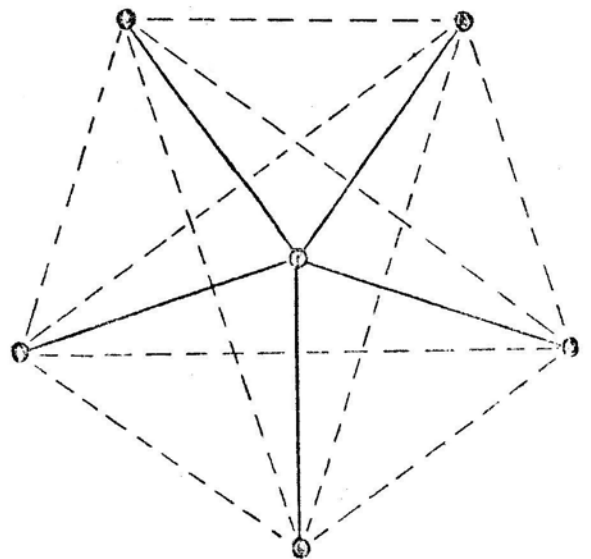
7 and 8
 to be filled



GENERATING
 HEXAD



RECIPROCAL

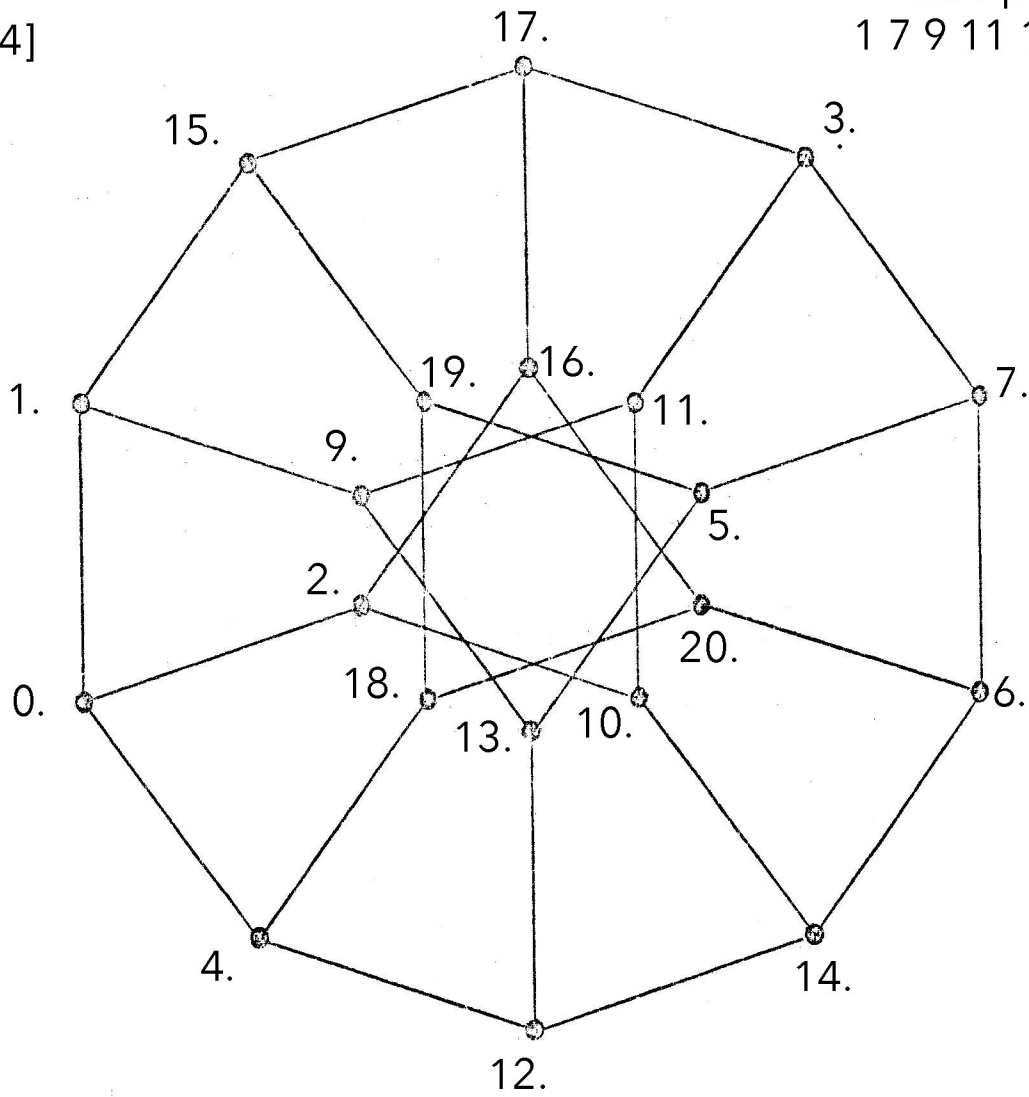


A B C D E F EIKOSANY

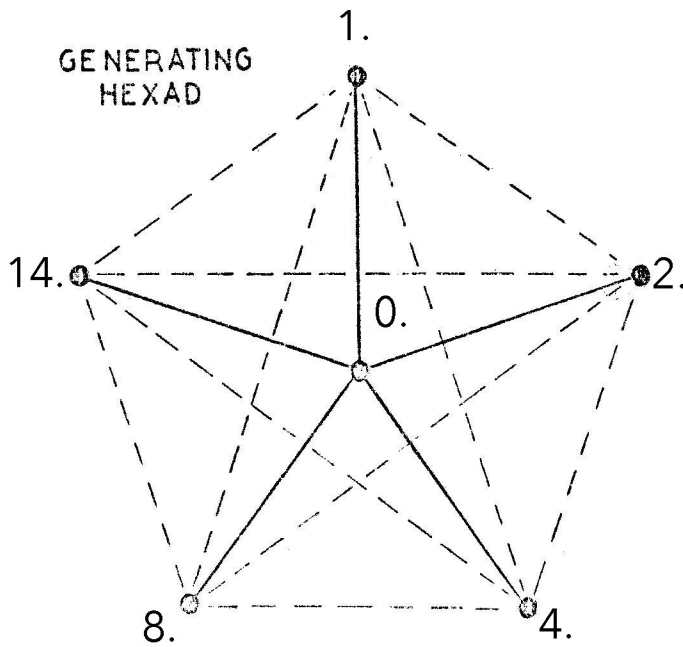
P3
 [0, 1, 2, 4, 8, 14]
 = 1 1 2 4 6 8

8 and 21
 to be filled

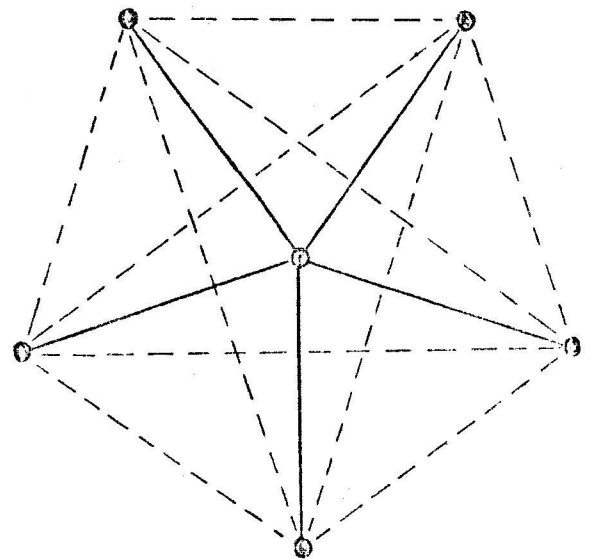
Example
 1 7 9 11 15 11/3



GENERATING
 HEXAD



RECIPROCAL

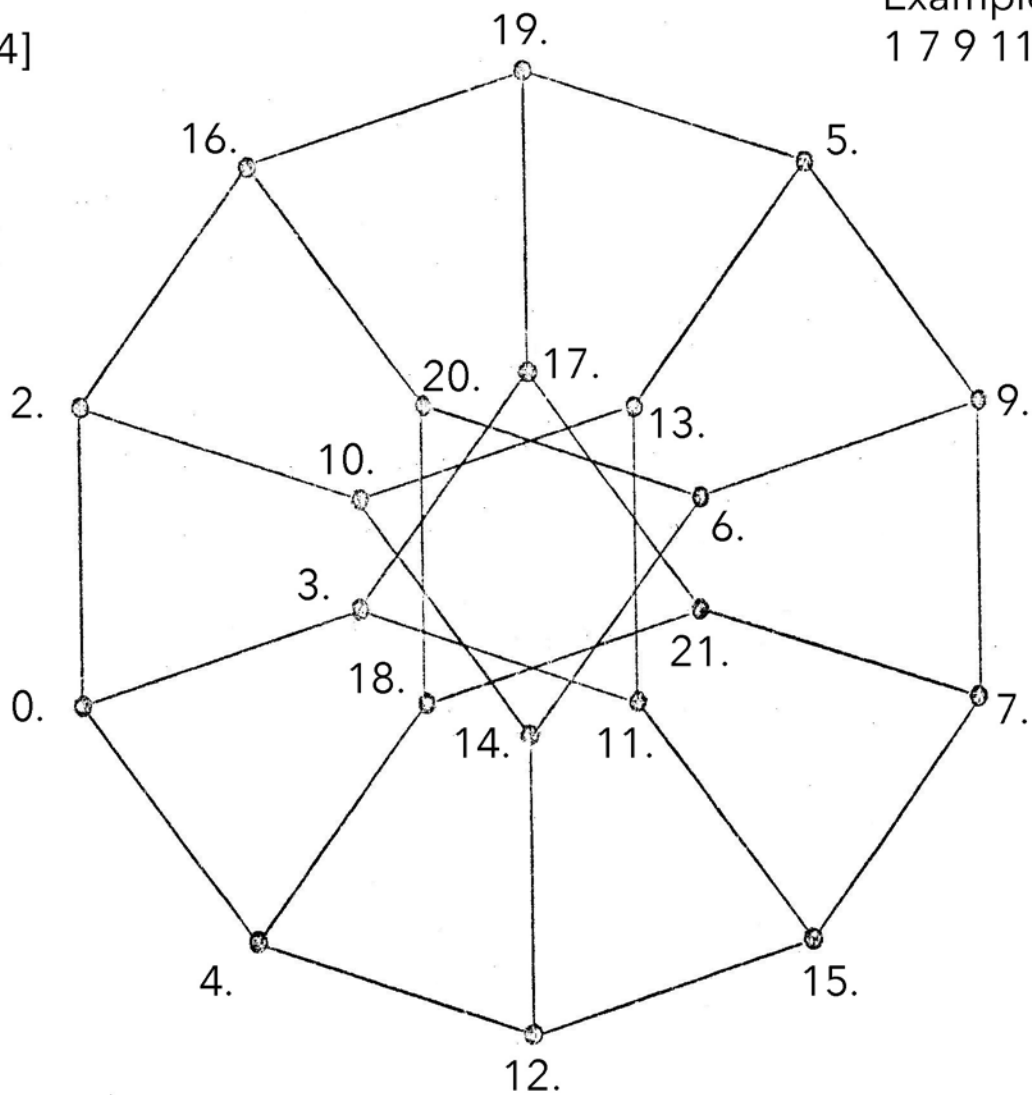


A B C D E F EIKOSANY

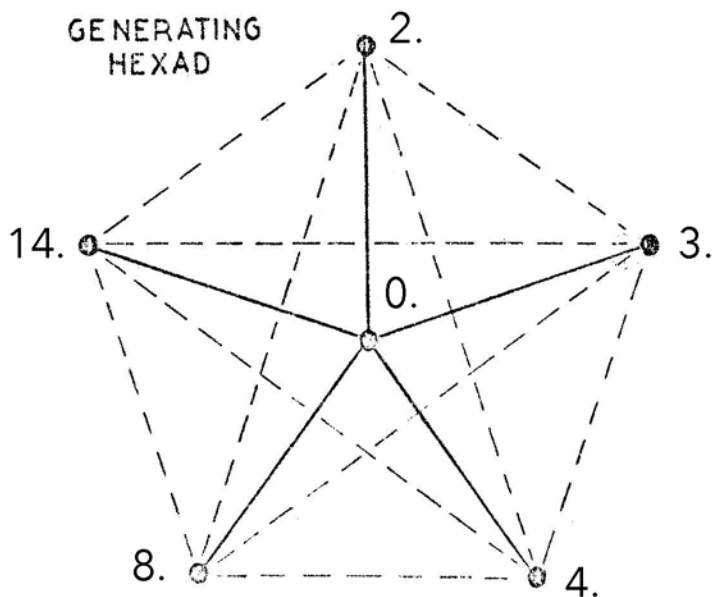
P4
 [0, 2, 3, 4, 8, 14]
 = 2 1 1 4 6 8

1 and 8
 to be added

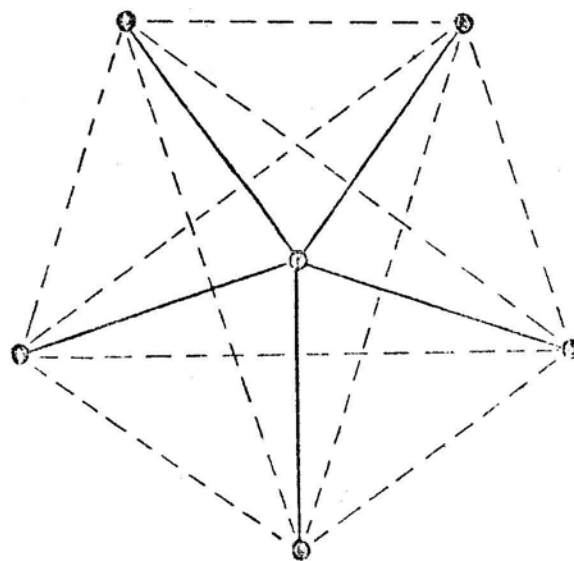
Example
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GENERATING
 HEXAD



RECIPROCAL

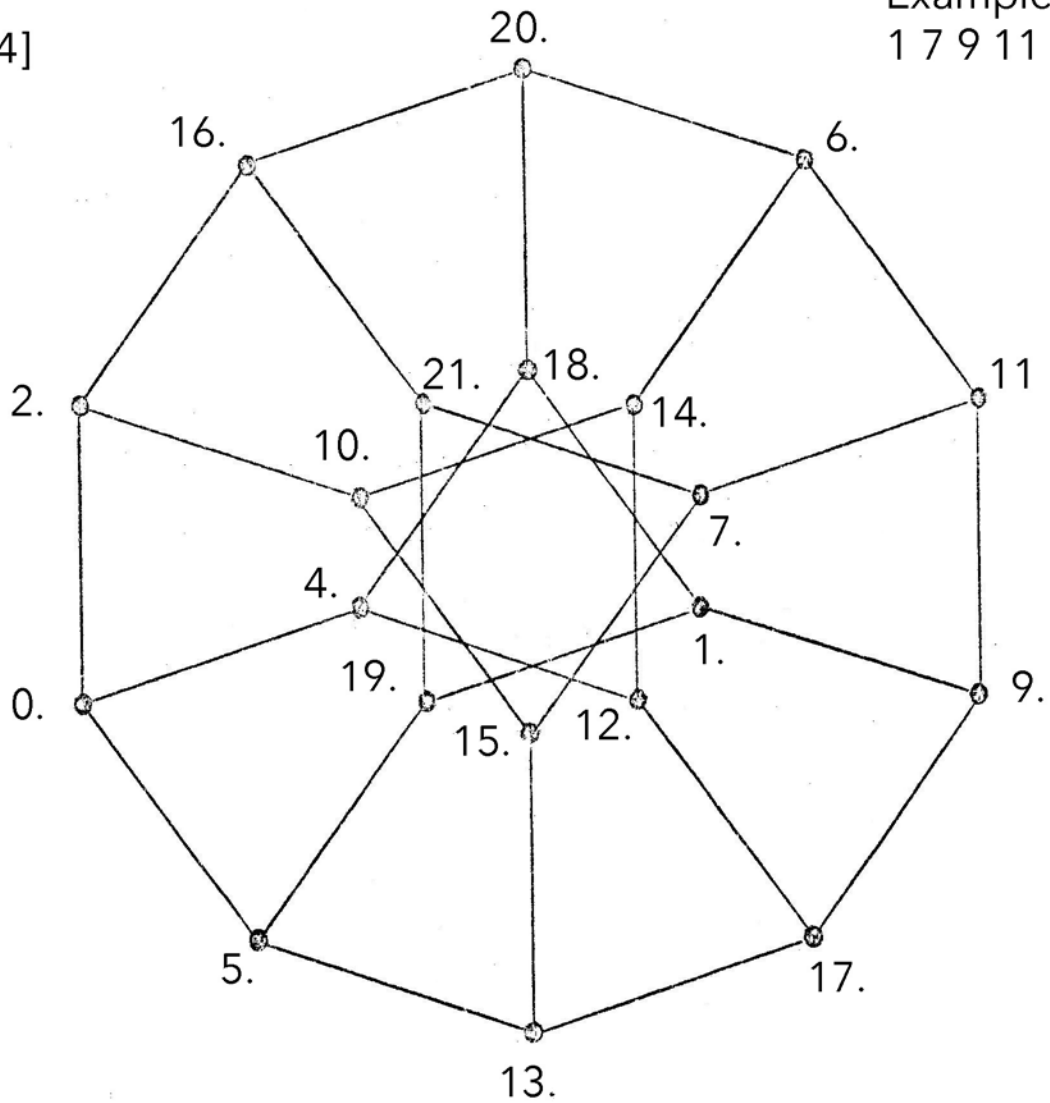


A B C D E F E I K O S A N Y

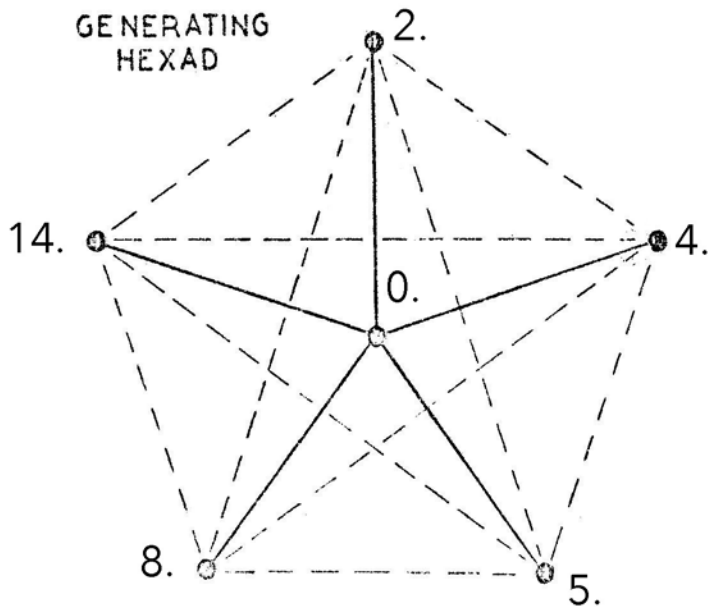
P5
 [0, 2, 4, 5, 8, 14]
 = 2 2 1 3 6 8

Example
 1 7 9 11 15 33

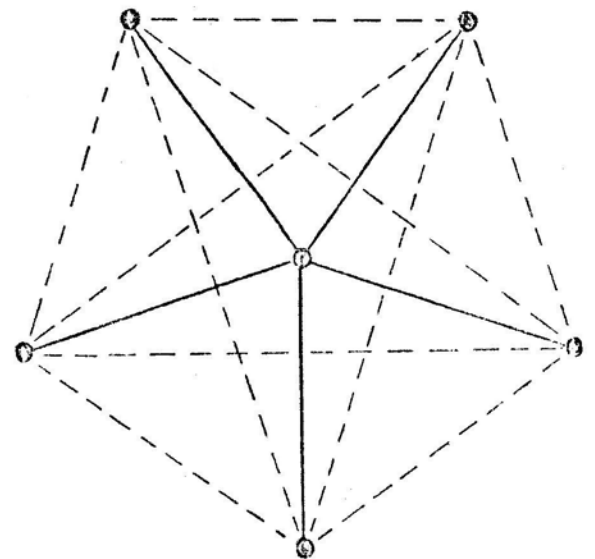
3. and 8
 to be filled



GENERATING
 HEXAD



RECIPROCAL

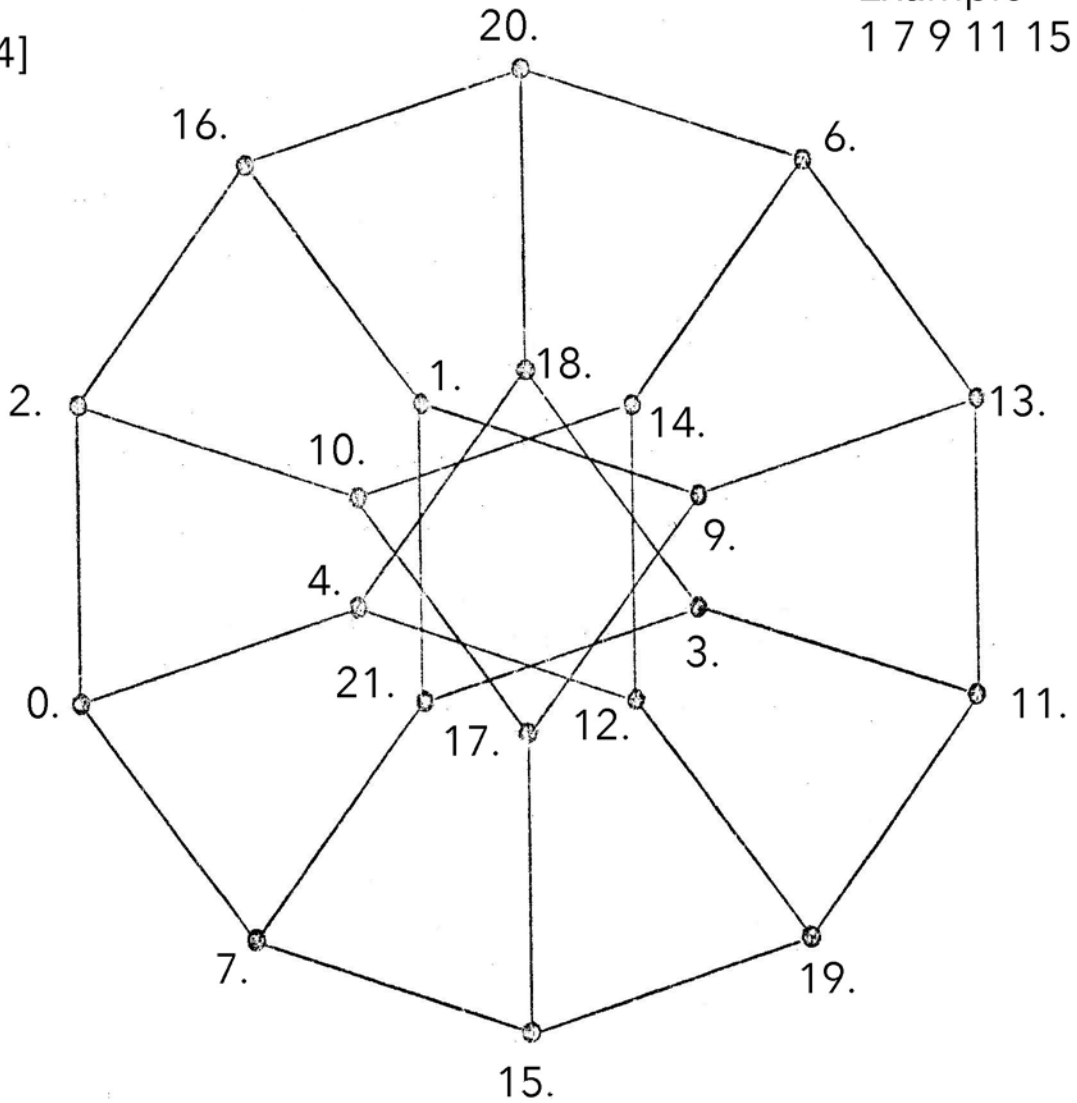


A B C D E F E I K O S A N Y

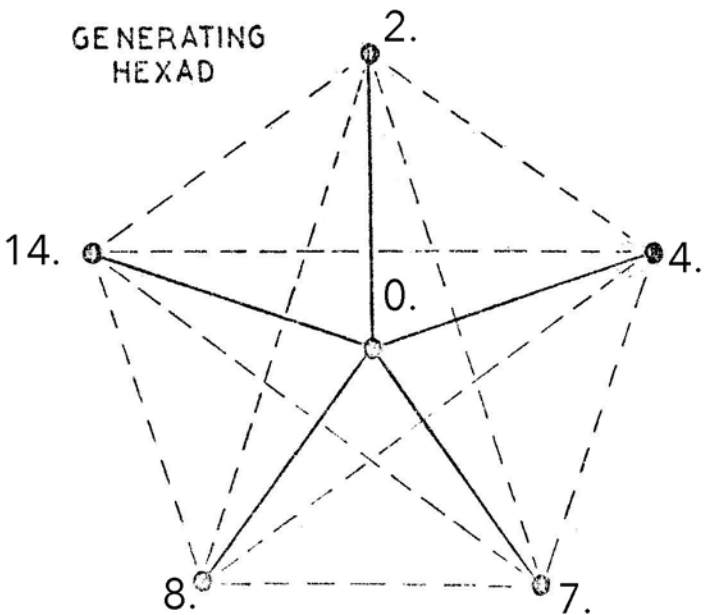
P6
 [0, 2, 4, 7, 8, 14]
 = 2 2 3 1 6 8

Example
 1 7 9 11 15 35

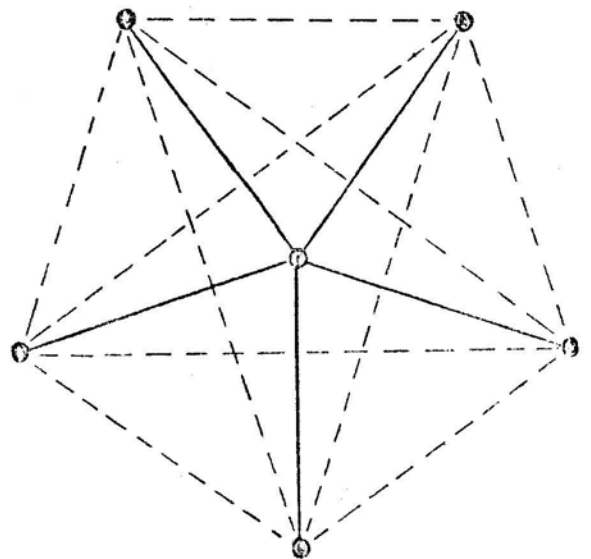
5 and 8
 tp be filled



GENERATING
 HEXAD

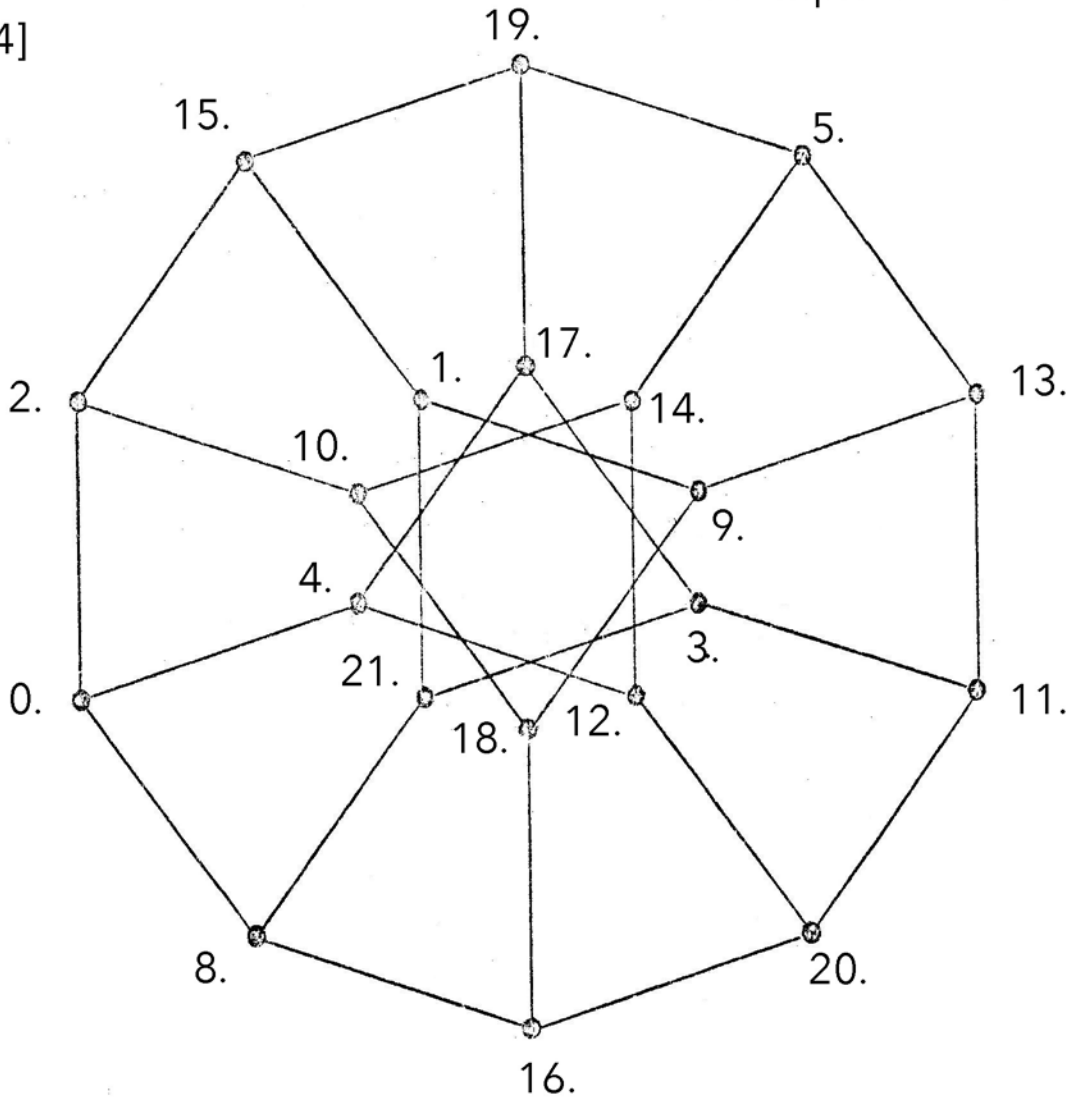


RECIPROCAL

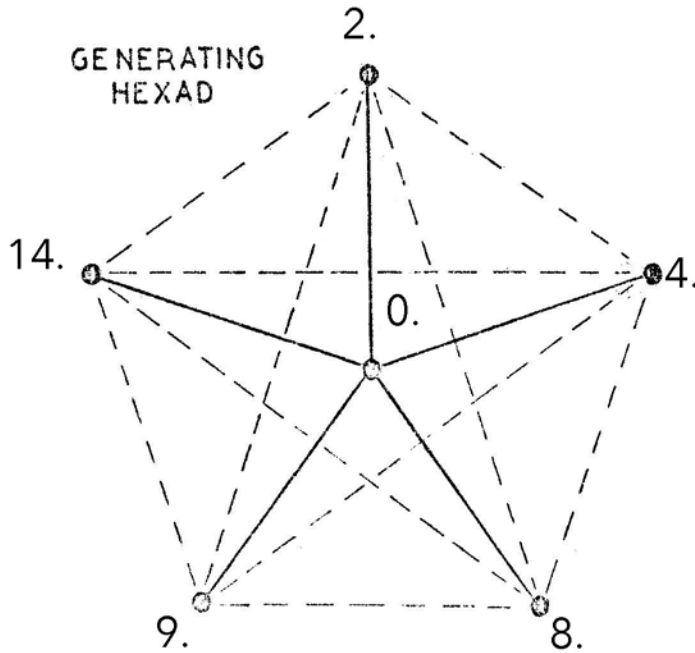


P7
 [0, 2, 4, 8, 9, 14]
 = 2 2 4 1 5 8

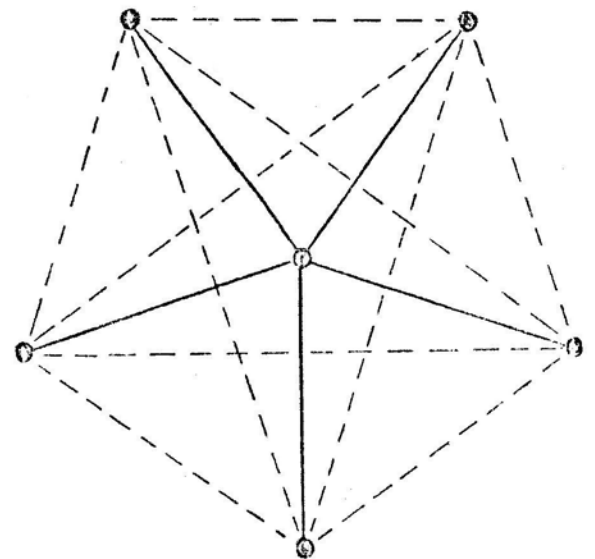
6. and 7..
 to be filled



GENERATING
 HEXAD



RECIPROCAL

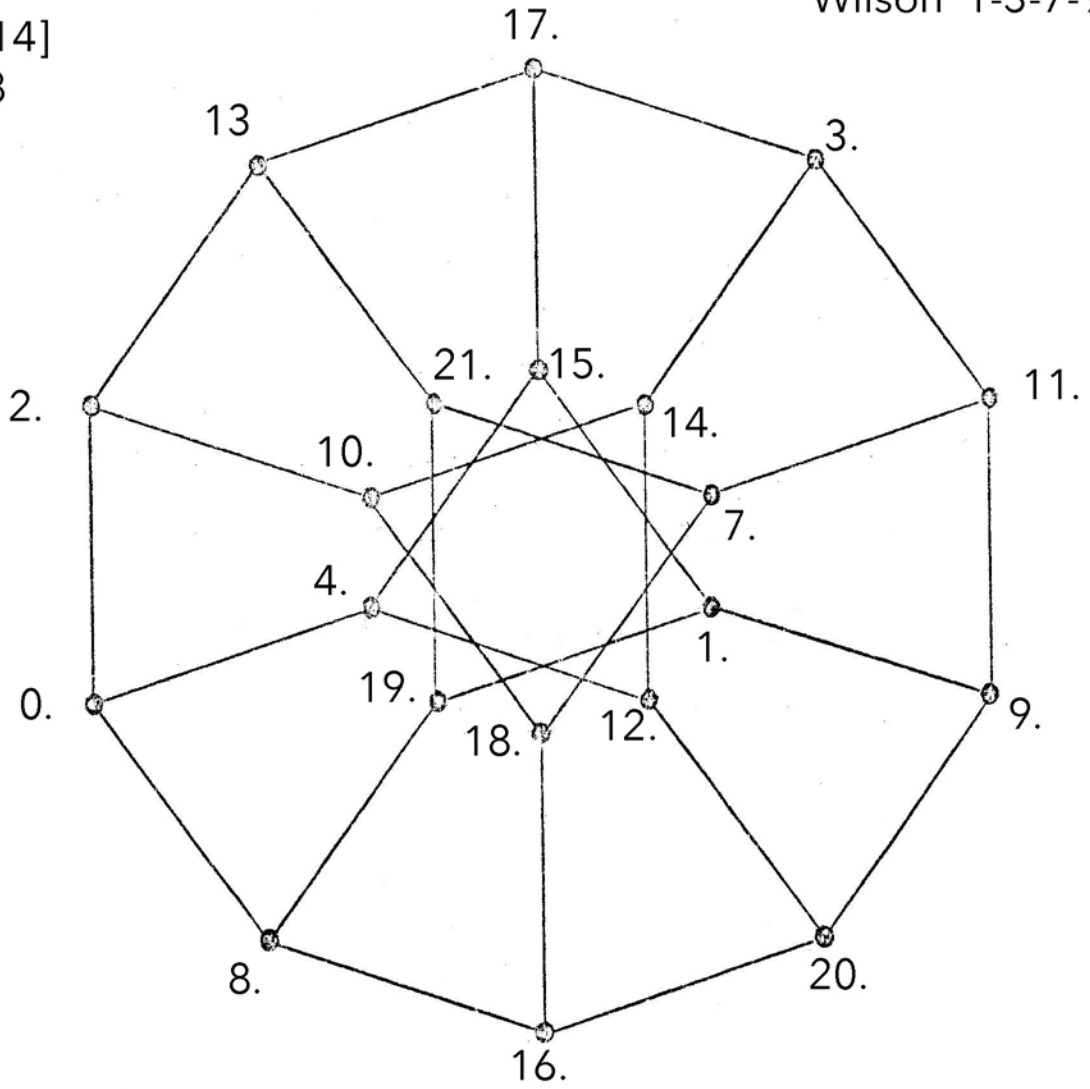


A B C D E F EIKOSANY

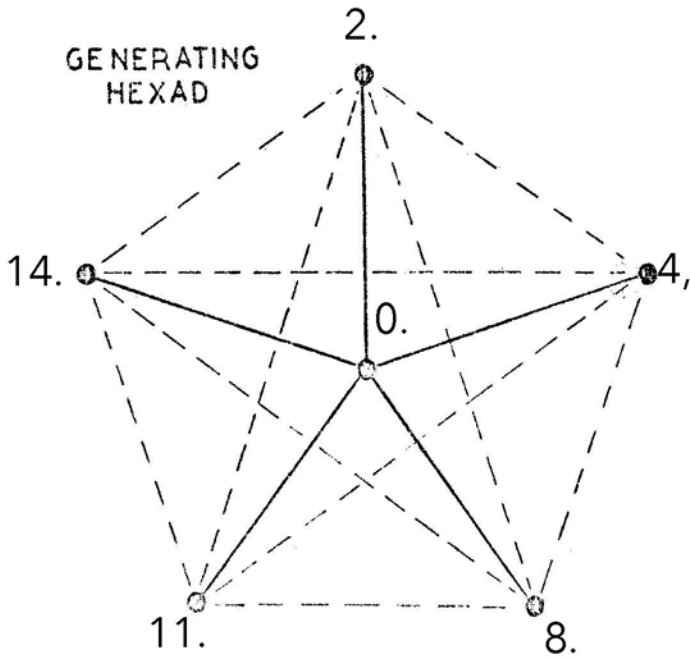
P8
 [0, 2, 4, 8, 11, 14]
 = 2 2 4 3 3 8

Wilson 1-3-7-9-11-15

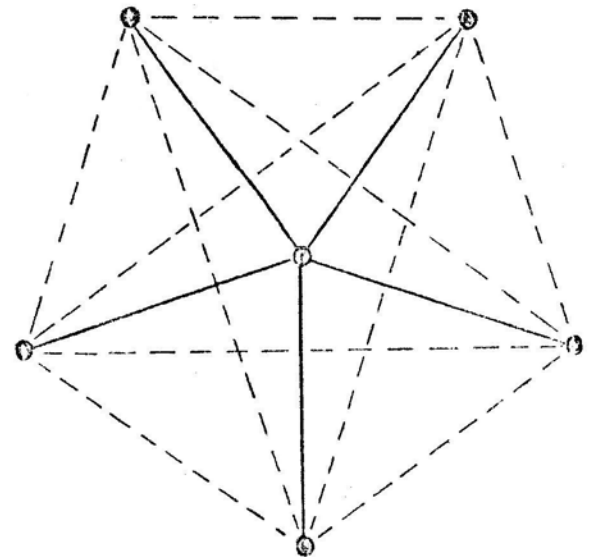
5 and 6
 to be filled



GENERATING
 HEXAD

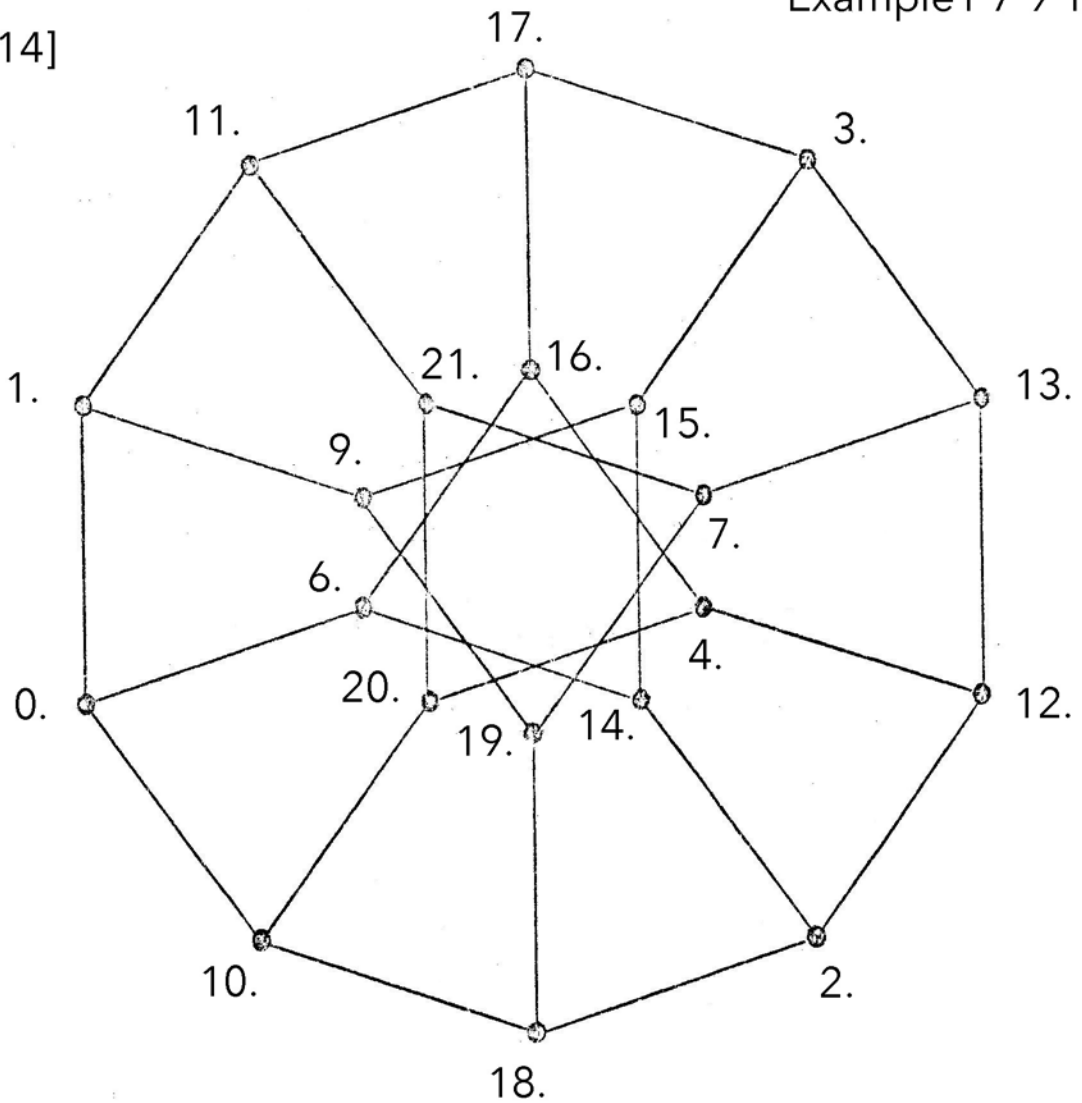


RECIPROCAL

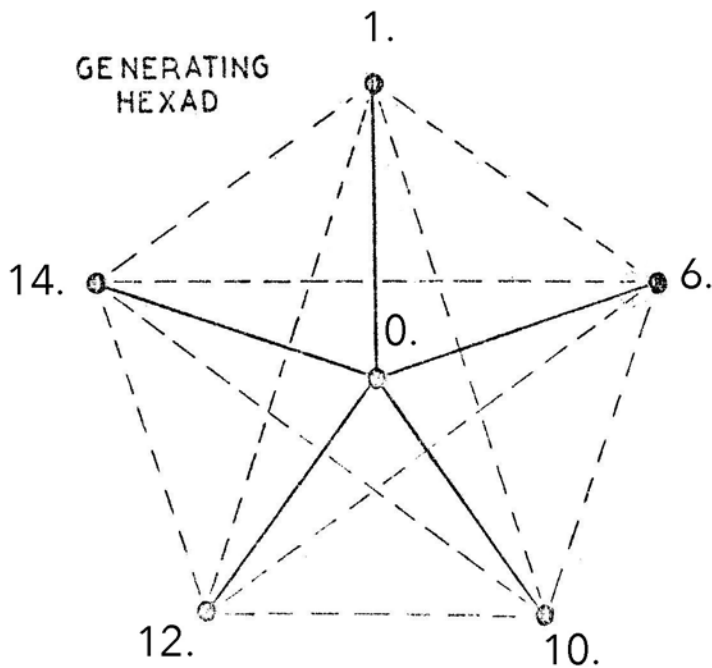


P9
 [0, 1, 6, 10, 12, 14]
 = 1 5 4 2 2 8

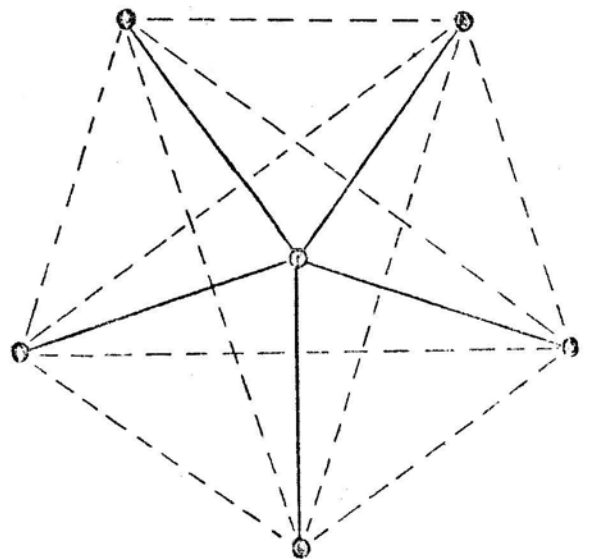
5 and 8
 to be filled



GENERATING
 HEXAD



RECIPROCAL

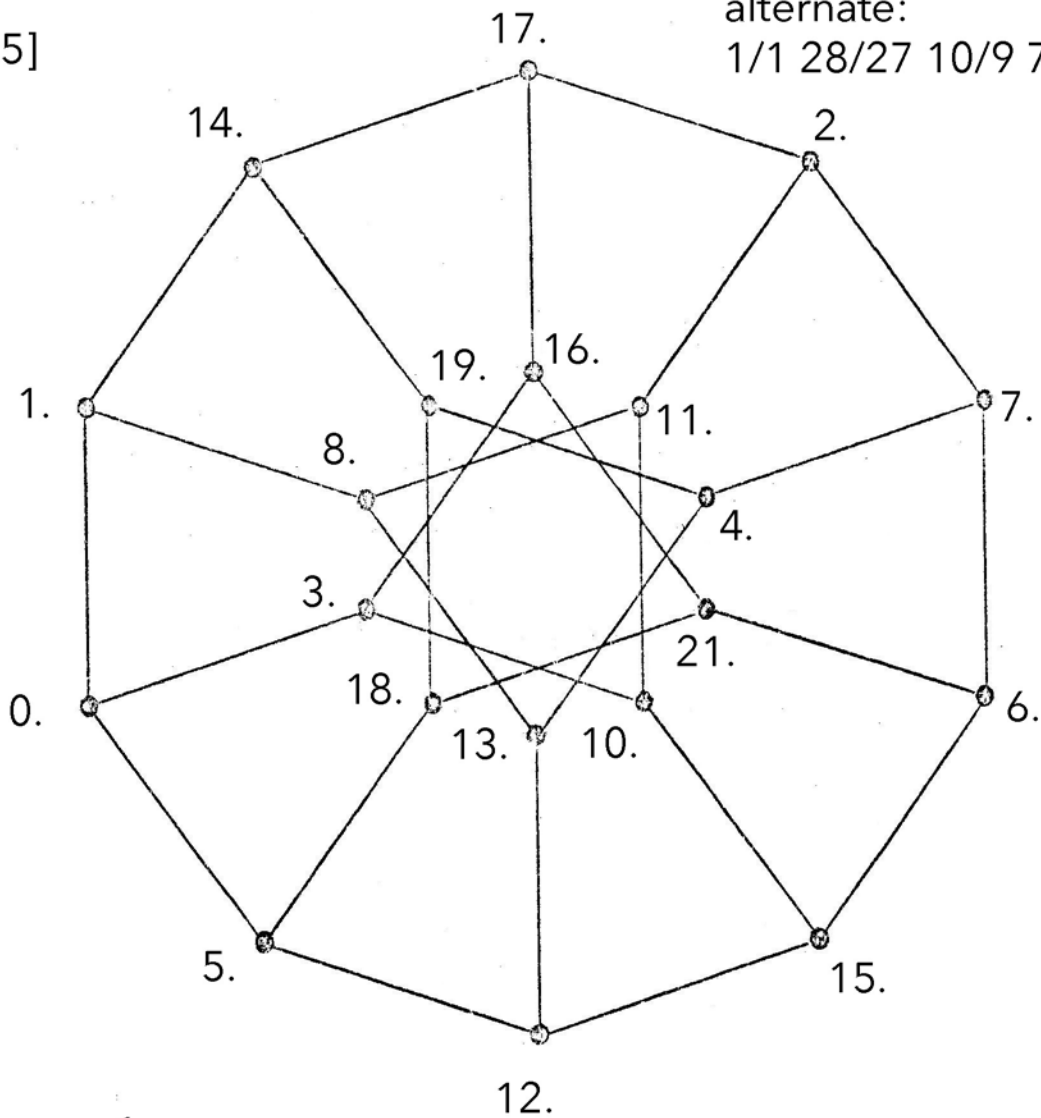


A B C D E F EIKOSANY

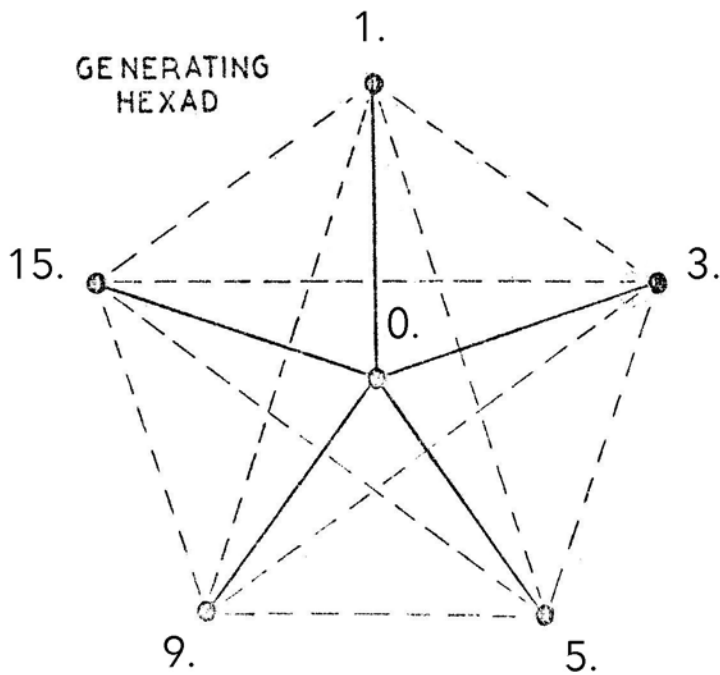
Example 1 7 9 11 15 27
 alternate:
 1/1 28/27 10/9 7/6 4/3 8/5

P10
 [0, 1, 3, 5, 9, 15]
 = 1 2 2 4 6 7

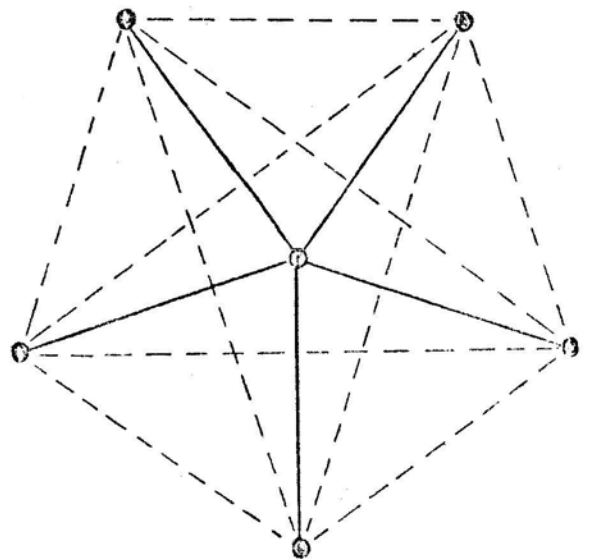
9 and 20.
 to be filled



GENERATING
 HEXAD



RECIPROCAL

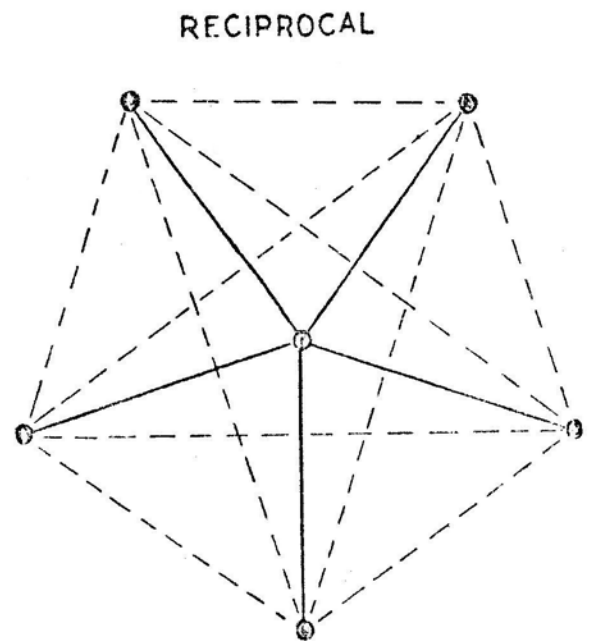
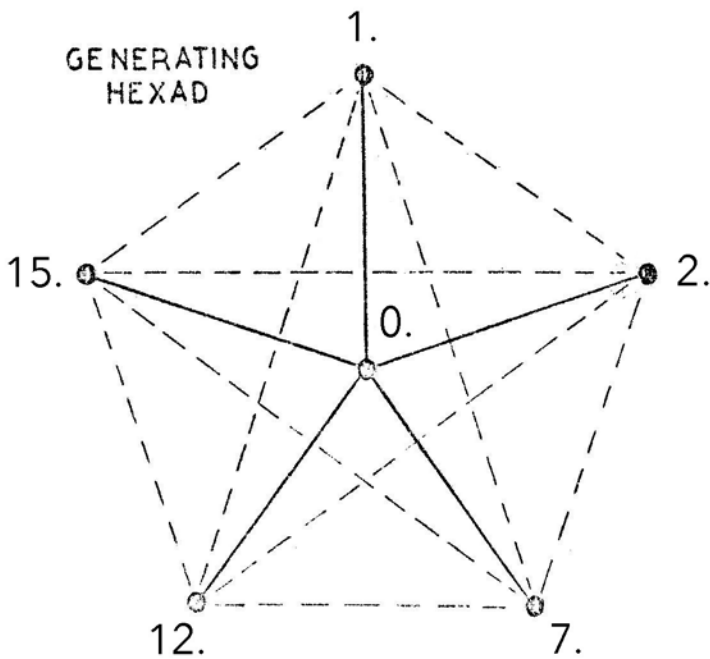
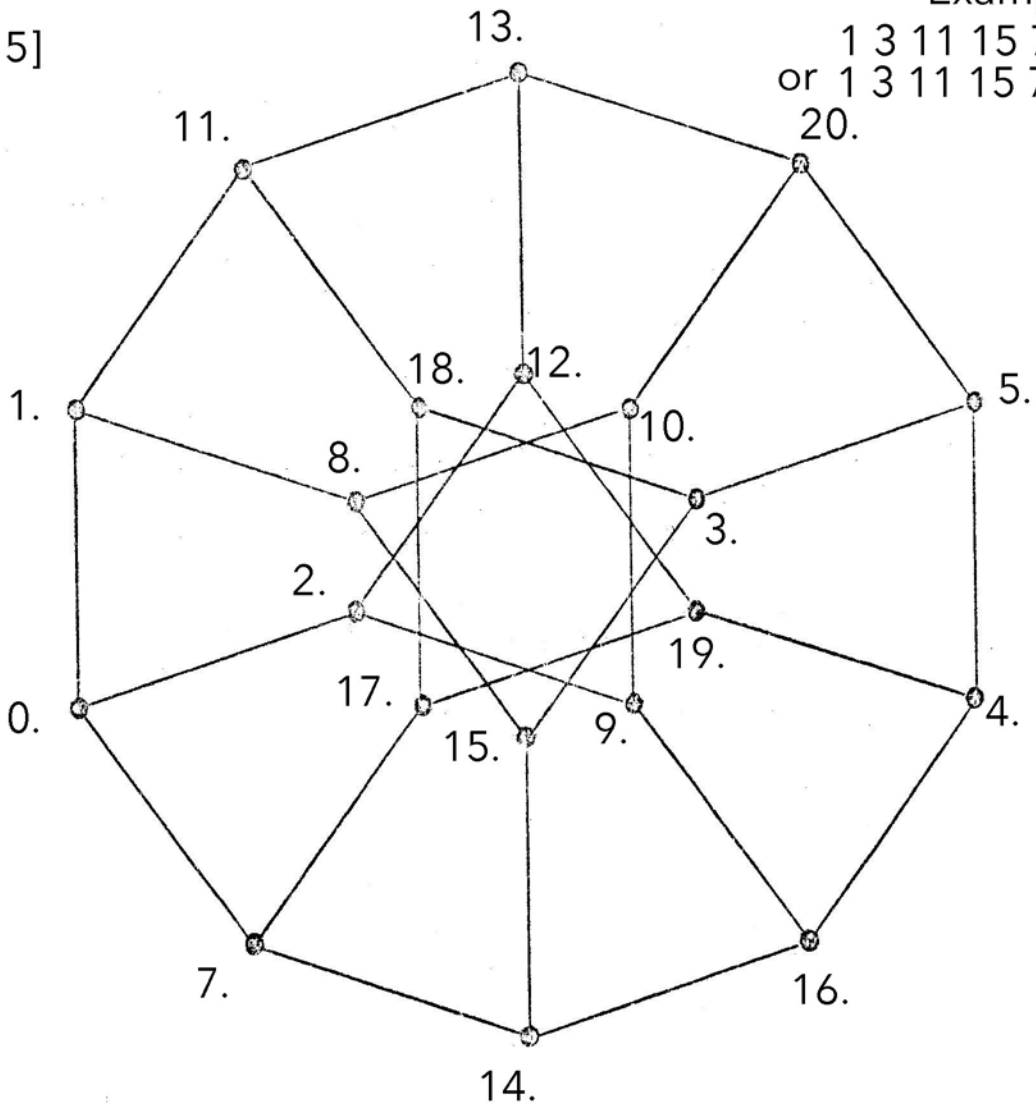


A B C D E F E I K O S A N Y

P11
 [0, 1, 2, 7, 12, 15]
 = 1 1 5 5 3 7

6 and 21
 to be filled

Example
 1 3 11 15 7/3 35/9
 or 1 3 11 15 7/3 21/11

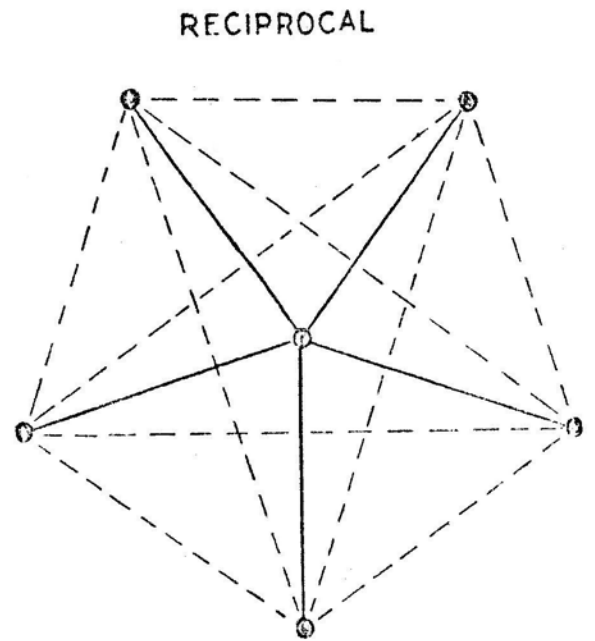
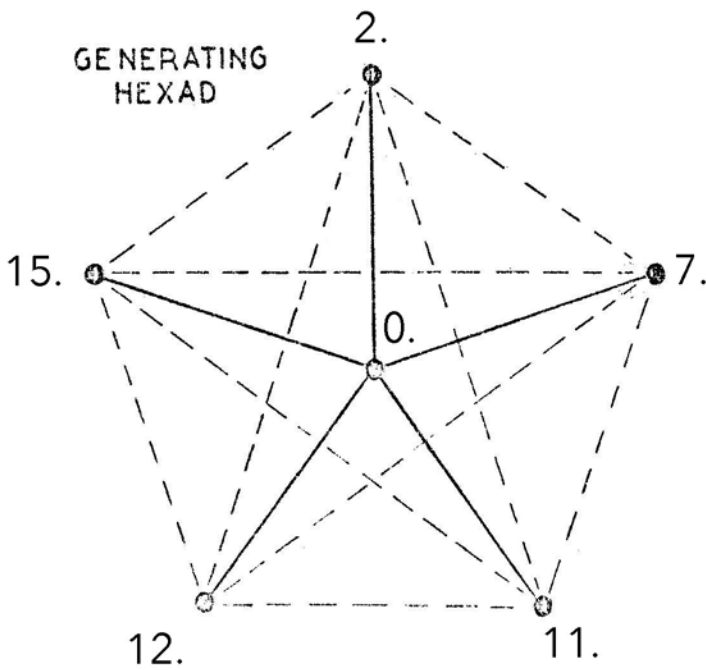
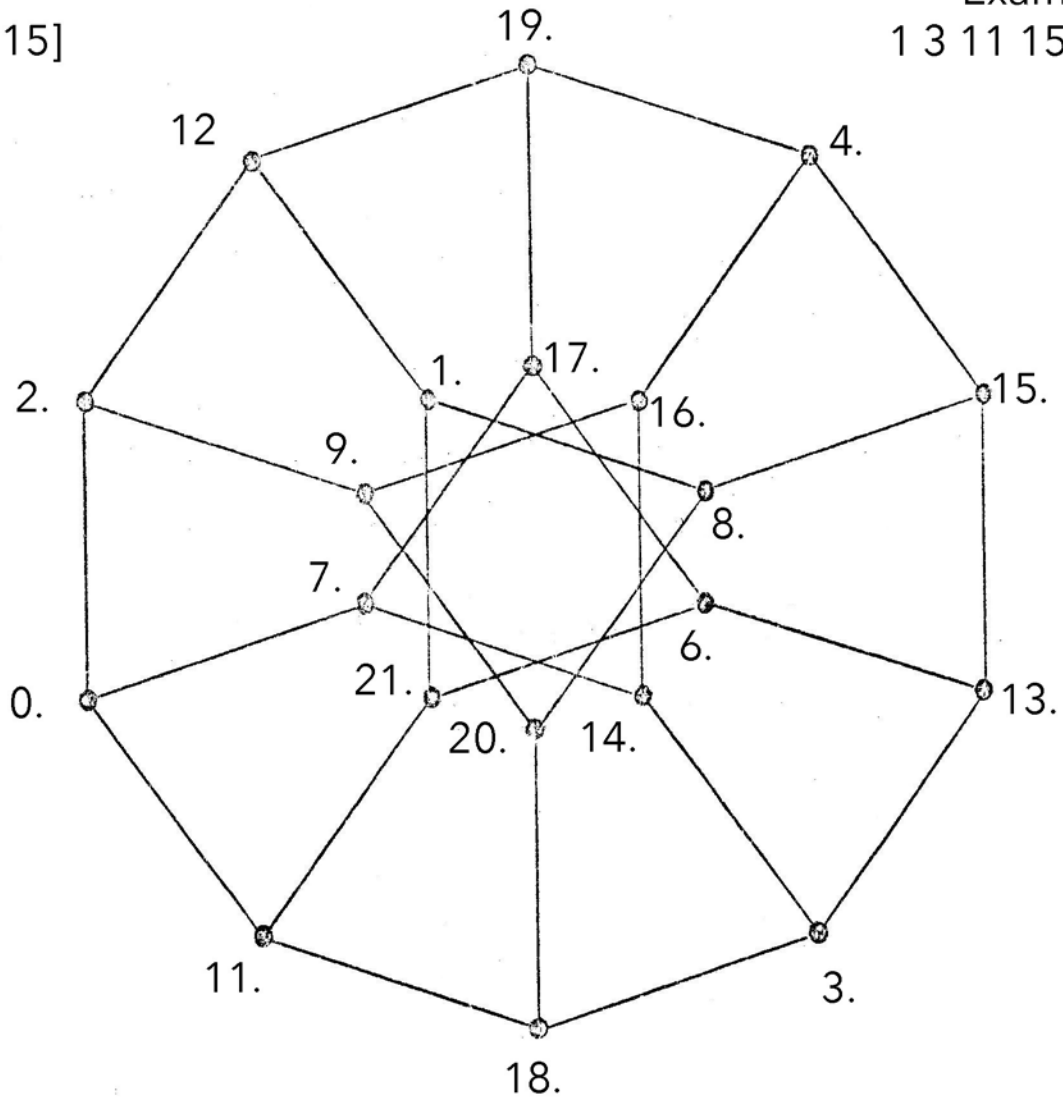


A B C D E F EIKOSANY

P12
 [0, 2, 7, 11, 12, 15]
 = 2 5 4 1 3 7

5. and 10.
 to be added

Example
 1 3 11 15 21 7/3

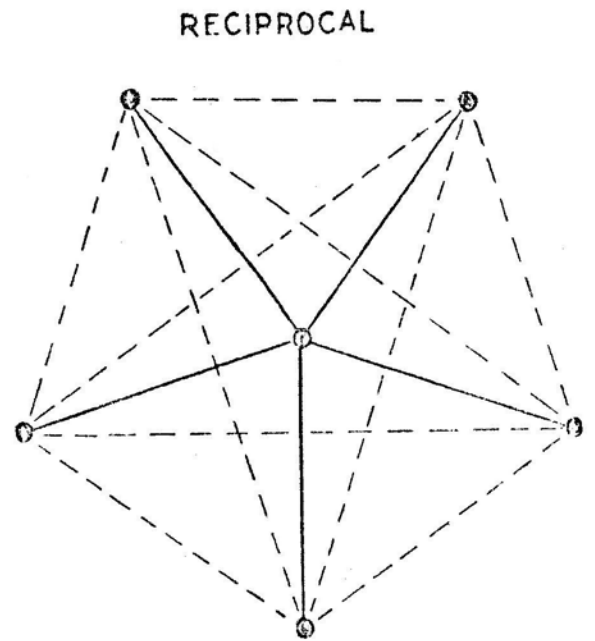
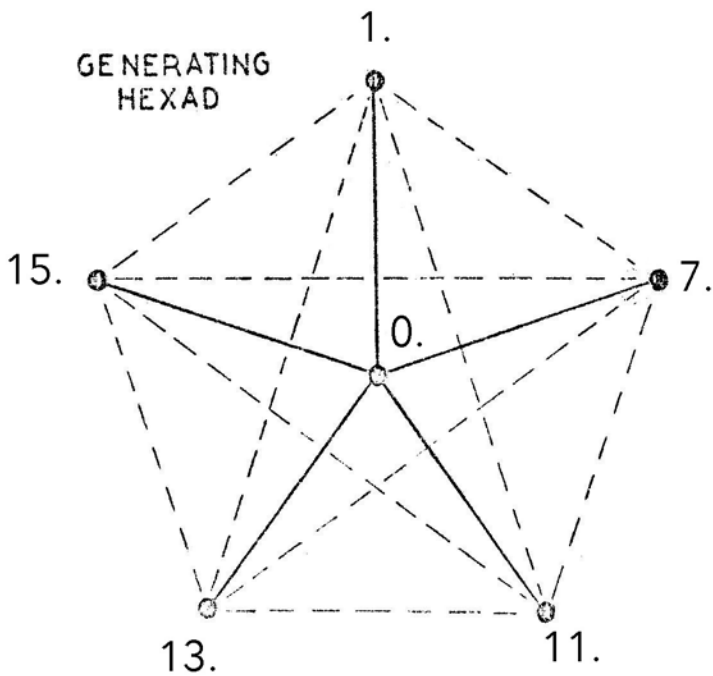
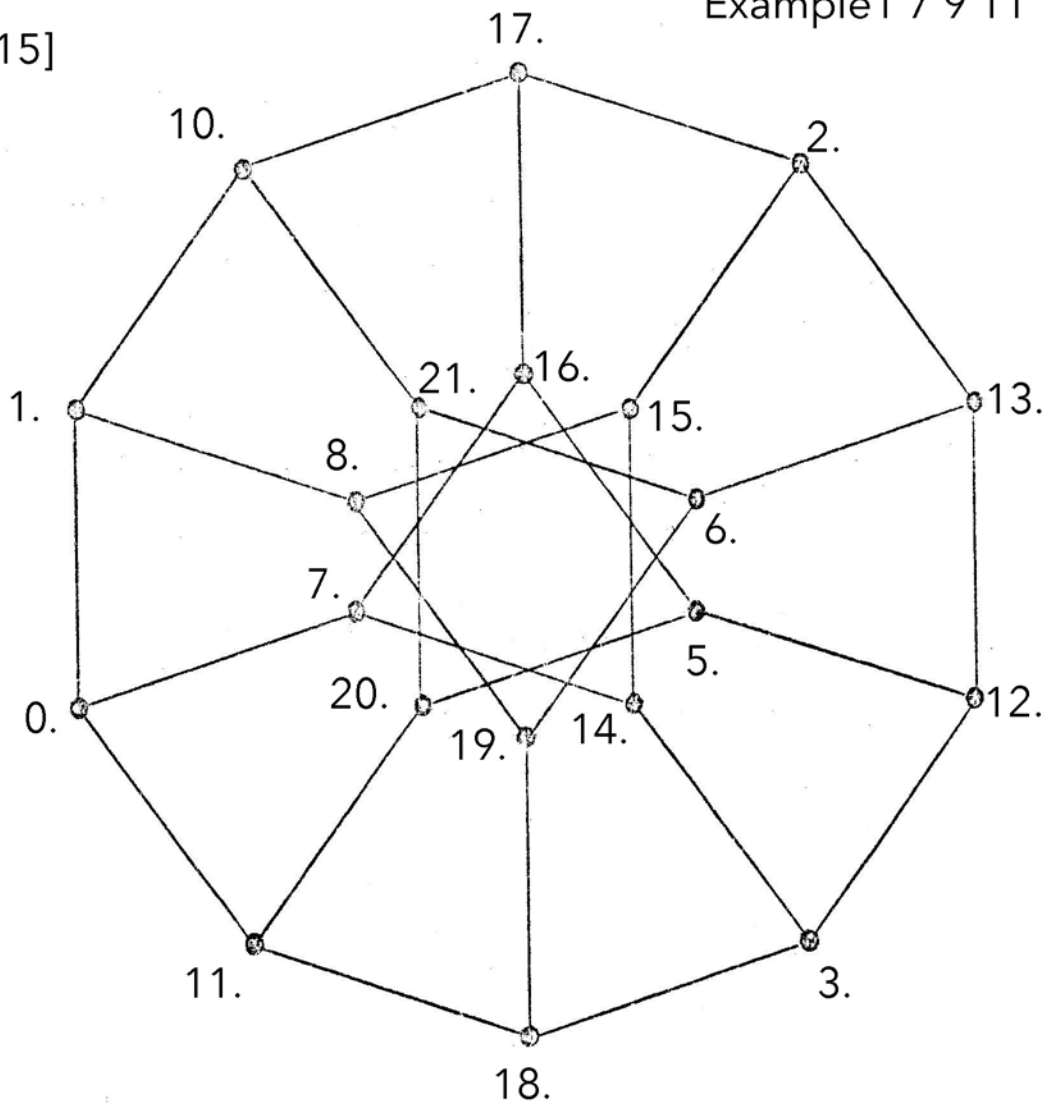


A B C D E F E I K O S A N Y

P13
 [0, 1, 7, 11, 13, 15]
 = 1 6 4 2 2 7

Example 1 7 9 11 15 45

4. and 9.
 to be added

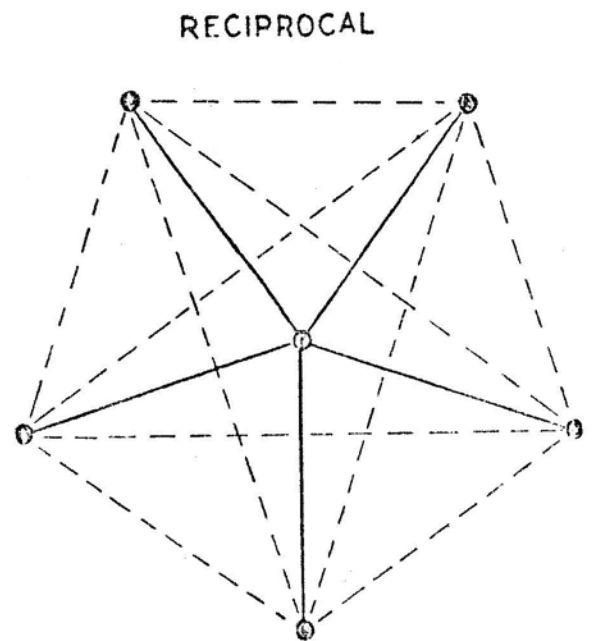
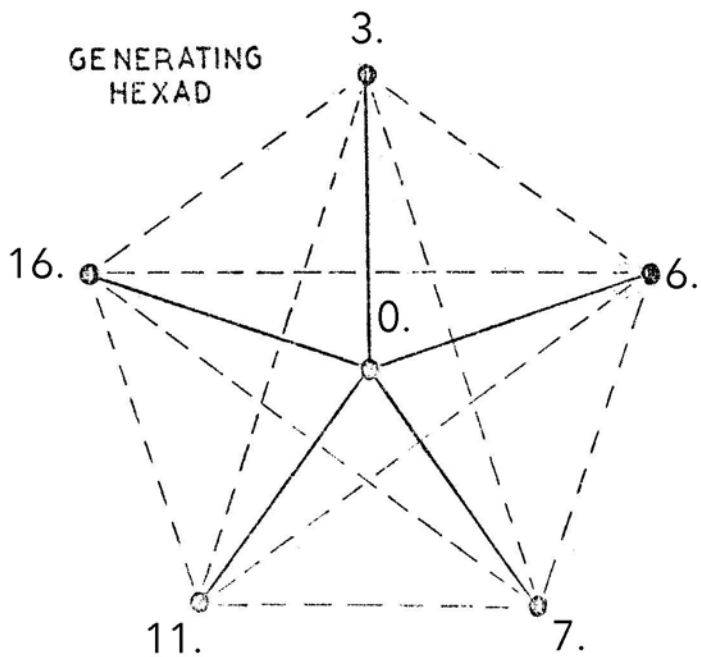
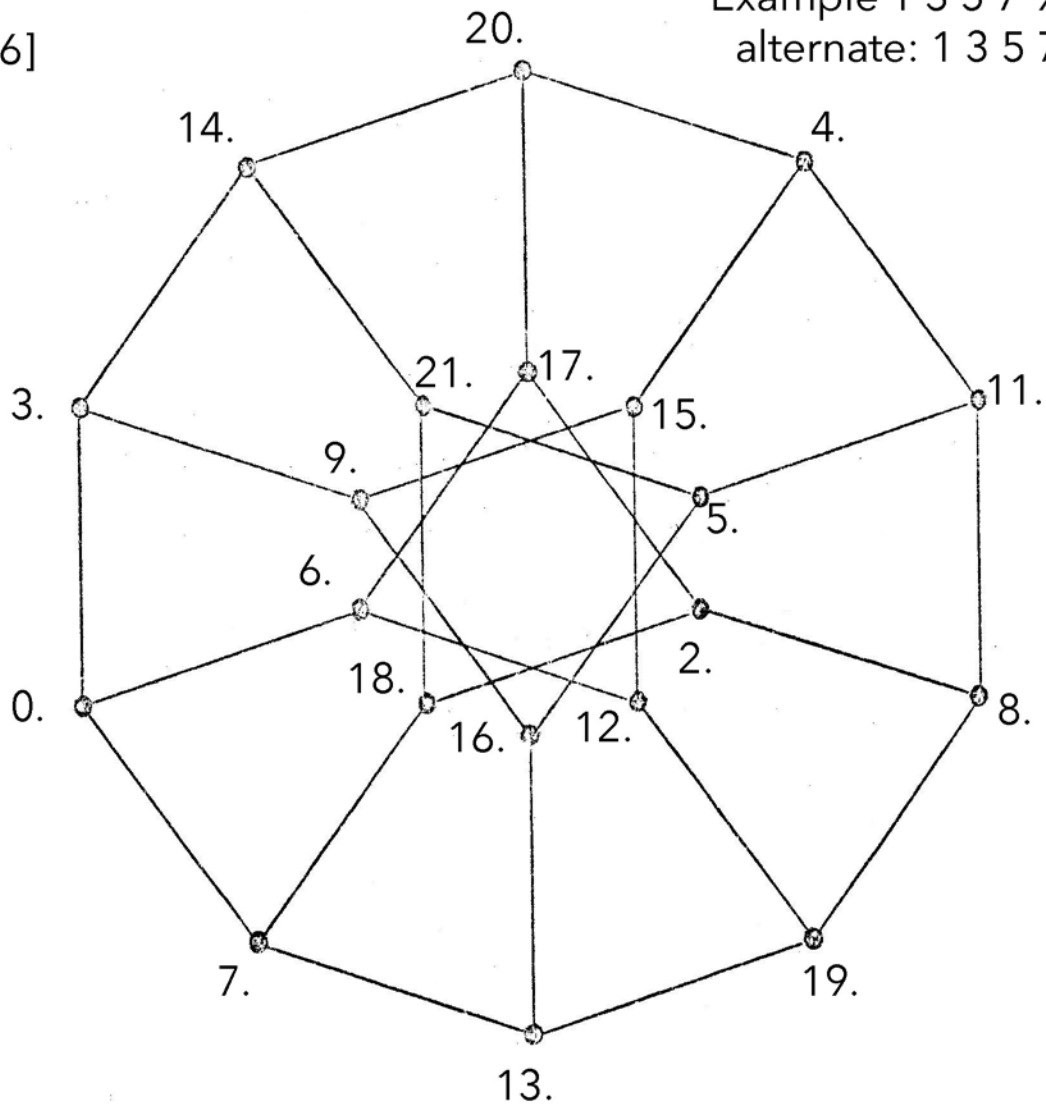


A B C D E F E I K O S A N Y

P14
 [0, 3, 6, 7, 11, 16]
 = 3 3 1 4 5 6

1 and 10.
 to be added

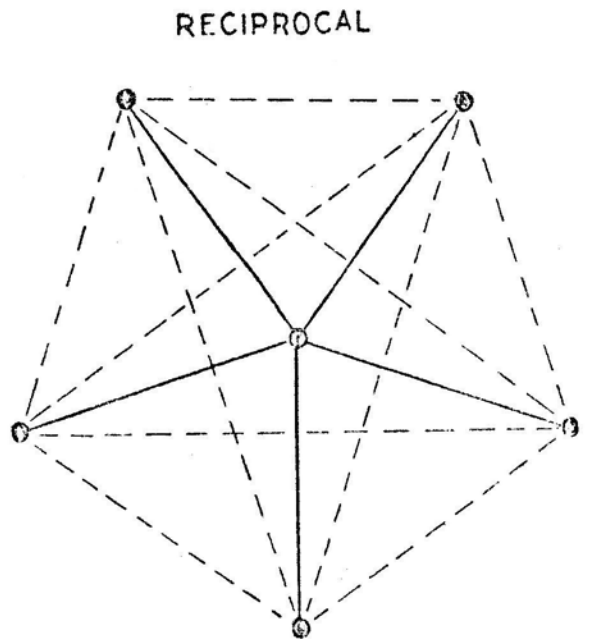
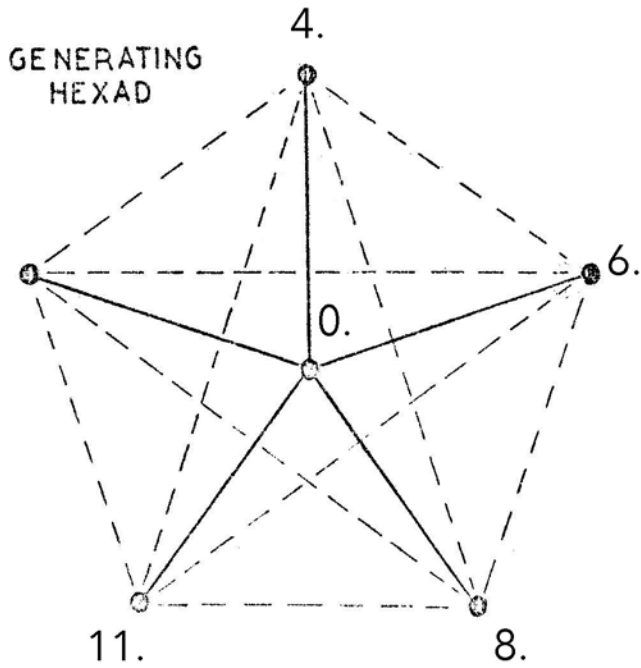
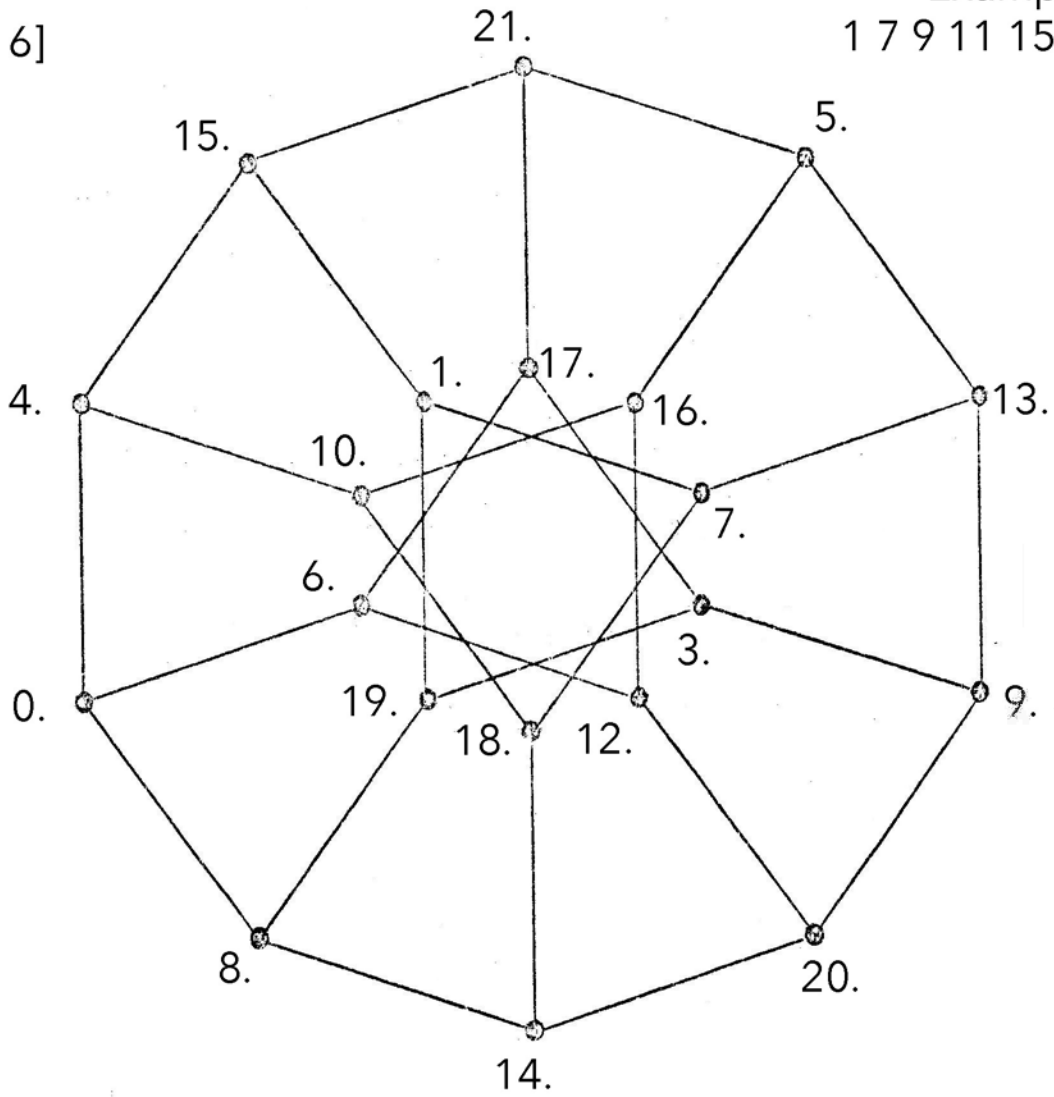
Example 1 3 5 7 9 45/11
 alternate: 1 3 5 7 9 33



P15
 [0, 4, 6, 8, 11, 16]
 =4 2 2 3 5 6

2. and 11.
 to be added

Example
 1 7 9 11 15 35/11

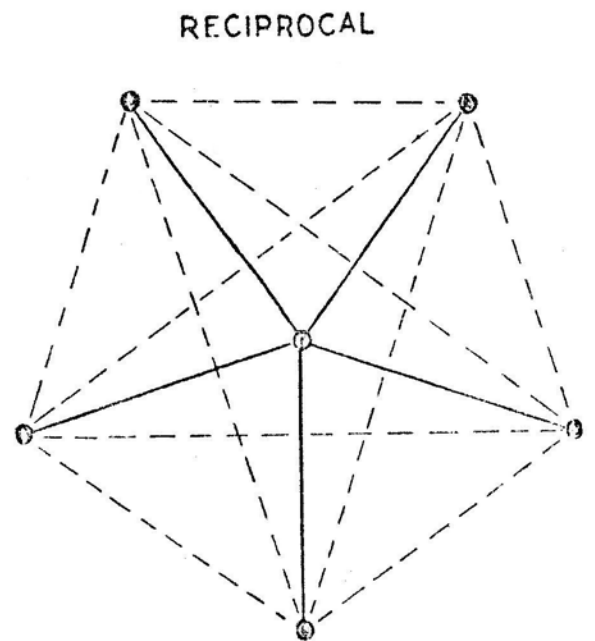
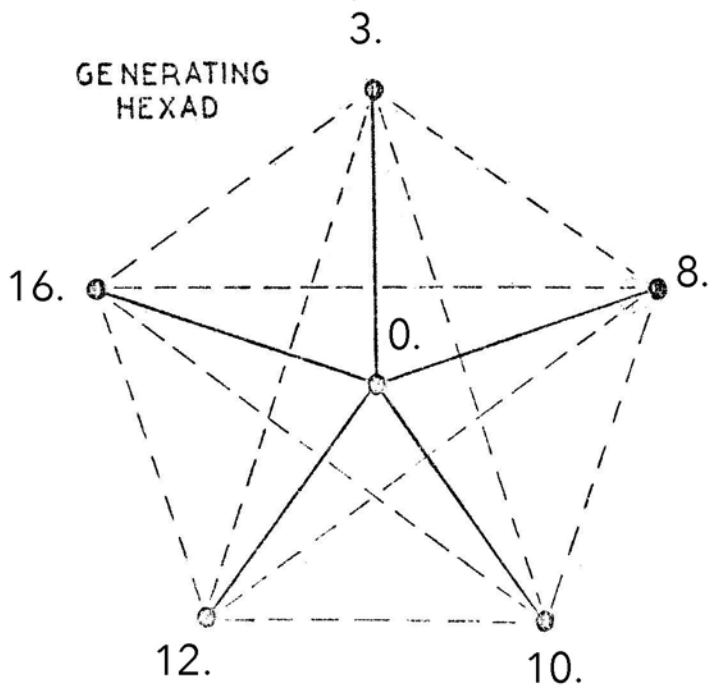
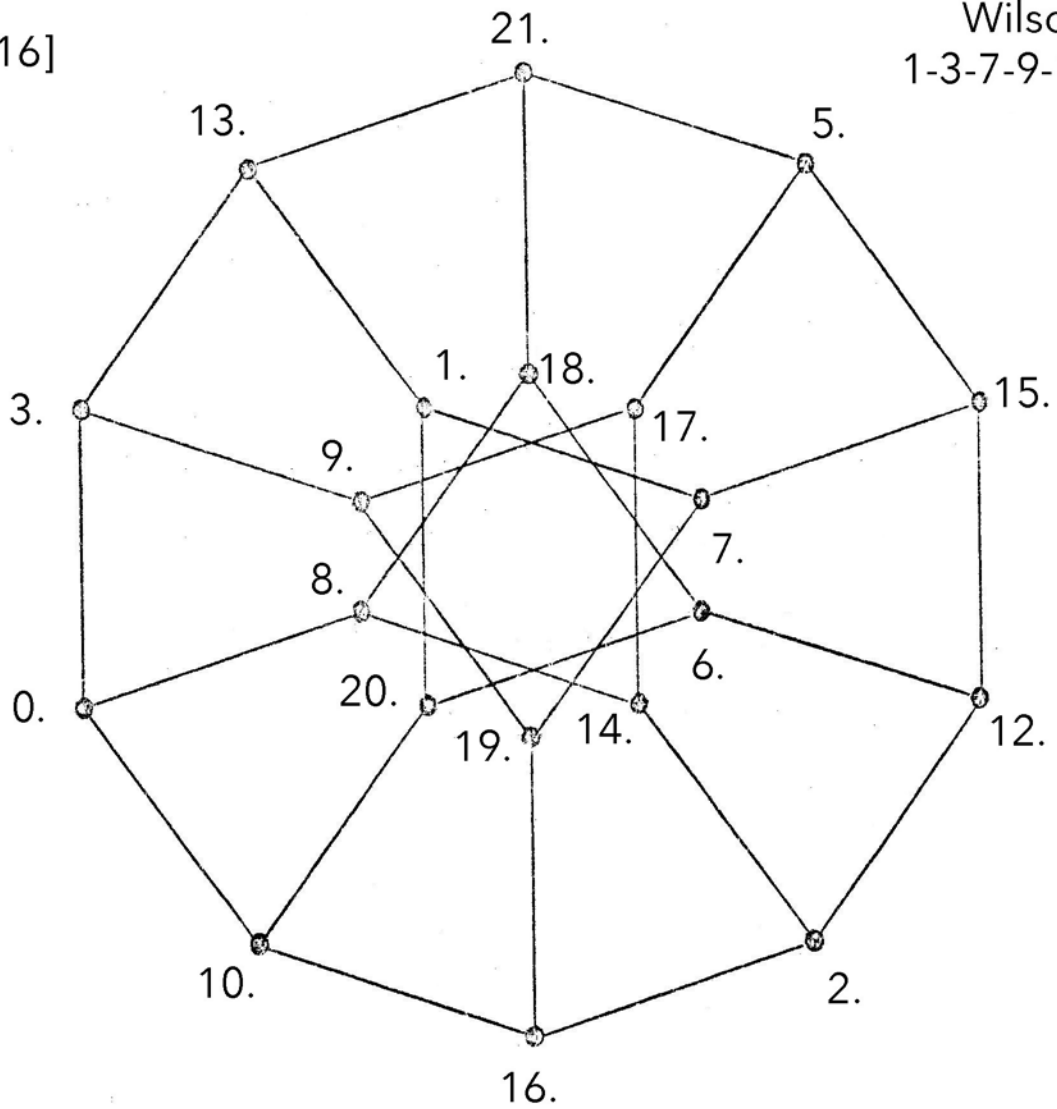


A B C D E F EIKOSANY

P16
 [0, 3, 8, 10, 12, 16]
 =3 5 2 2 4 6

Wilson
 1-3-7-9-11-15

4 and 11
 to be added



Examples in terms of scala

P1: 1/1 33/32 17/16 [12/11] 9/8 187/160 [6171/5120] 99/80 51/40 1683/1280 11/8
17/12 187/128 3/2 99/64 51/32 33/20 17/10 561/320 9/5 297/160 153/80 2/1, ABC: 1/1

P2: 1/1 45/44 297/280 11/10 9/8 33/28 135/112 5/4 [9/7] [21/16] 27/20
7/5 63/44 3/2 135/88 35/22 33/20 27/16 7/4 9/5 15/8 27/14 2/1, ABC: 135/88

P3: 1/1 45/44 15/14 [495/448] 9/8 405/352 135/112 27/22 9/7 405/308 11/8 45/32
165/112 3/2 11/7 45/28 [18/11] 27/16 99/56 405/224 15/8 27/14 2/1, ABC: 27/22

P4: 1/1 33/32 15/14 11/10 55/48 847/720 11/9 121/96 55/42 121/90 11/8 [10/7]
165/112 121/80 11/7 77/48 5/3 77/45 [847/480] 11/6 15/8 77/40 2/1, ABC: 1/1

P5: 1/1 56/55 21/20 12/11 9/8 7/6 6/5 56/45 77/60 4/3 11/8 7/5 22/15 3/2 [14/9] 8/5
33/20 56/33 7/4 [9/5] 28/15 21/11 2/1, ABC: 56/55

P6: 1/1 22/21 77/72 11/10 55/48 33/28 11/9 44/35 [9/7] 4/3 11/8 [10/7] 35/24 3/2 14/9
8/5 5/3 12/7 7/4 11/6 15/8 88/45 2/1, ABC: 8/5

P7: 1/1 33/32 77/72 495/448 55/48 33/28 11/9 5/4 [165/128] [4/3] 11/8 45/32 35/24
3/2 14/9 45/28 5/3 55/32 7/4 11/6 15/8 55/28 2/1, ABC: 3/2

P8: 1/1 22/21 [15/14] [11/10] 9/8 33/28 6/5 5/4 9/7 4/3 11/8 10/7 22/15 3/2 11/7 45/28
33/20 12/7 99/56 11/6 66/35 55/28 2/1, ABC: 4/3

P9: 1/1 22/21 21/20 11/10 9/8 33/28 6/5 44/35 77/60 4/3 11/8 [7/5] 22/15 3/2 [11/7]
8/5 33/20 12/7 7/4 11/6 28/15 88/45 2/1, ABC: 8/5

P10: 1/1 28/27 [35/33] 10/9 9/8 7/6 105/88 5/4 14/11 4/3 15/11 140/99 63/44 [3/2]
135/88 35/22 18/11 56/33 7/4 20/11 15/8 35/18 2/1, ABC: 56/33

P11: 1/1 33/32 77/72 11/10 9/8 7/6 6/5 5/4 [165/128] 297/224 11/8 99/70 35/24 3/2
14/9 [8/5] 33/20 55/32 99/56 11/6 15/8 27/14 2/1, ABC: 99/70

P12: 1/1 33/32 16/15 11/10 9/8 33/28 6/5 44/35 77/60 4/3 [11/8] 7/5 22/15 3/2 11/7
[8/5] 33/20 12/7 7/4 11/6 28/15 88/45 2/1, ABC: 44/35

P13: 1/1 33/32 77/72 35/32 154/135 7/6 11/9 5/4 77/60 21/16 11/8 45/32 22/15 3/2
14/9 77/48 [5/3] 55/32 7/4 11/6 28/15 [77/40] 2/1, ABC: 28/15

P14: 1/1 [33/32] 77/72 11/10 9/8 7/6 11/9 5/4 77/60 21/16 11/8 77/54 35/24 3/2
[14/9] 77/48 5/3 77/45 7/4 11/6 15/8 77/40 2/1, ABC: 11/9

P15: 1/1 25/24 15/14 10/9 55/48 [33/28] 11/9 5/4 55/42 175/132 11/8 140/99
165/112 50/33 11/7 35/22 5/3 75/44 [175/99] 20/11 15/8 35/18 2/1, ABC: 1/1

P16: 1/1 33/32 15/14 495/448 9/8 33/28 135/112 5/4 9/7 [297/224] 11/8 45/32 165/112 3/2
11/7 45/28 [5/3] 55/32 99/56 11/6 15/8 55/28 2/1

20 11 1960
1219 Poinsettia Drive
L.A. Calif 90046

Hi, John,

For the purpose of generating eikosanyx, the hexad, its modal inversions (rotations) and reciprocals (reflections) are all equivalent. These rotations & reflections are most apparent when the hexad is described as the sequence of intervals from tone to tone. To illustrate the hexad species, 1, 2, 3, 4, 5, 7 has the rotations & reflections as shown;

	Rotations						Reflections						
	1	2	3	4	5	7	7	5	4	3	2	1	
	2	3	4	5	7	1	1	7	5	4	3	2	
Rotations	3	4	5	7	1	2	2	1	7	5	4	3	A
	4	5	7	1	2	3	3	2	1	7	5	4	
	5	7	1	2	3	4	4	3	2	1	7	5	
	7	1	2	3	4	5	5	4	3	2	1	7	

These, when related to tone gens give the following hexads respectively:

0	1	3	6	10	15	0	7	12	16	19	21	
0	2	5	9	14	21	0	1	8	13	17	20	
0	3	7	12	19	20	0	2	3	10	15	19	B
0	4	9	16	17	19	0	3	5	6	13	18	
0	5	12	13	15	18	0	4	7	9	10	17	
0	7	8	10	13	17	0	5	9	12	14	15	

These hexads generate a dodecuplet set of eikosanyx, identical except for the key they happen to be in. Systematically excluding all but the first (or one) hexany as generators guarantees but a single occurrence of that eikosany species.

What is needed first, then, is a tabulation of all the "unique" hexads (sans rotations or reflections) to occur in a given system, (In this case 22).

This can be done in the following way:

Establish how many combinations of 6 numbers will equal 22. Duplication is avoided by utilizing the technique illustrated in Fig. I (which shows all combinations barring errors on my part) encl. If the intervals in sequence are represented by A, B, C, D, E, F, then B equals/is larger than A; C equals or is larger than B; D equals or is larger than C; E equals or is larger than D; F equals or is larger than E.

Each of these combinations produces, by permutations, a hexadic family of 720 hexads. However, since we want to avoid rotations, and, since this can be done by holding "A" in first position, we are concerned with 120 hexads. This number is cut in half when we exclude the reflections. In making further checks the computer has only to refer to other members in this family of 60, Fig II. (encl.). Within this group of 60, when any of the values for A, B, C, D, E, F are identical there will be duplications, rotations, or reflections. These, also, must be filtered out. Example: The 59 remaining permutations of the Master Hexad 1,1,1,1,1,7 will all be duplications; All permutations of Master Hexad 1,2,3,4,5,7 will be unique. This final filtering is best done by computer.

We are left, now, with somewhat over 4,000 unique hexads, a guess. (Multiplied the number of relevant permutations (60) by the number of "Master Hexads" (135) and divided by 2)

Designating the unique hexads by the interval sequence, now, $A_1, B_1, C_1, D_1, E_1, F_1$ and the tone sequence as

G H J K L M we see that:

$$G = 0$$

$$H = A_1$$

$$J = A_1 + B_1$$

$$K = A_1 + B_1 + C_1$$

$$L = A_1 + B_1 + C_1 + D_1$$

$$M = A_1 + B_1 + C_1 + D_1 + E_1$$

Constructing the eikosony and identifying its members as $1e$ thru $20e$ we get:

G	H	J	K	L	M	
G	H	J				$1e = 2A_1 + B_1$
G	H		K			$2e = 2A_1 + B_1 + C_1$
G	H			L		$3e = 2A_1 + B_1 + C_1 + D_1$
G	H				M	$4e = 2A_1 + B_1 + C_1 + D_1 + E_1$
G		J	K			$5e = 2A_1 + 2B_1 + C_1$
G		J		L		$6e = 2A_1 + 2B_1 + C_1 + D_1$
G		J			M	$7e = 2A_1 + 2B_1 + C_1 + D_1 + E_1$
G			K	L		$8e = 2A_1 + 2B_1 + 2C_1 + D_1$
G			K		M	$9e = 2A_1 + 2B_1 + 2C_1 + D_1 + E_1$
G				L	M	$10e = 2A_1 + 2B_1 + 2C_1 + 2D_1 + E_1$
	H	J	K			$11e = 3A_1 + 2B_1 + C_1$
	H	J		L		$12e = 3A_1 + 2B_1 + C_1 + D_1$
	H	J			M	$13e = 3A_1 + 2B_1 + C_1 + D_1 + E_1$
	H		K	L		$14e = 3A_1 + 2B_1 + 2C_1 + D_1$
	H		K		M	$15e = 3A_1 + 2B_1 + 2C_1 + D_1 + E_1$
	H			L	M	$16e = 3A_1 + 2B_1 + 2C_1 + 2D_1 + E_1$
		J	K	L		$17e = 3A_1 + 3B_1 + 2C_1 + D_1$
		J	K		M	$18e = 3A_1 + 3B_1 + 2C_1 + D_1 + E_1$
		J		L	M	$19e = 3A_1 + 3B_1 + 2C_1 + 2D_1 + E_1$
			K	L	M	$20e = 3A_1 + 3B_1 + 3C_1 + 2D_1 + E_1$

The results may be entered into the eikosany construction Fig III (encl.) in the sequence shown.

At this point we have derived a full set of "unique" hexads and their derivative eikosany. From this point of view, all valid information. Now, to facilitate value judgements, we need some indication as to degree of articulation. The hexad, 2 2 5 3 6 4 generates an eikosany, fully articulate, of 20 different tones. On the other hand, the hexad 1 4 4 5 4 4 generates an eikosany, minimally articulate, of 10 different tones. (It, however, we rationalize this hexad as a pythagorean series, the doublings in the eikosany are acoustically equivalent "fold-back" and there is no good reason for devaluing it.) Since each allows for the realization of a fantasy impossible in the other, they should both be printed, paper permitting; the number of different tones (from 10 (?) to 20) should be given with them. It would seem that unless the print-out is reduced to 1 hexad & its eikosany & articulation level (Fig IV) per line we will run into a paper problem, and forfeit easy reproduction. If it is impractical to print 1 hexad & eikosany per line, perhaps all but the 20-tone eikosany should be eliminated before printing. This leaves only the fully articulate eikosany.

This technique can be used, with the appropriate variations, to generate the unique Tetrad-Hexany spectrum and the unique Octad-Hebdomekontany spectrum.

A	B	C	D	E	F
1	1	1	1	1	17
1	1	1	1	2	16
1	1	1	1	3	15
1	1	1	1	4	14
1	1	1	1	5	13
1	1	1	1	6	12
1	1	1	1	7	11
1	1	1	1	8	10
1	1	1	1	9	9
1	1	1	2	2	15
1	1	1	2	3	14
1	1	1	2	4	13
1	1	1	2	5	12
1	1	1	2	6	11
1	1	1	2	7	10
1	1	1	2	8	9
1	1	1	3	3	13
1	1	1	3	4	12
1	1	1	3	5	11
1	1	1	3	6	10
1	1	1	3	7	9
1	1	1	3	8	8
1	1	1	4	4	11
1	1	1	4	5	10
1	1	1	4	6	9
1	1	1	4	7	8
1	1	1	5	5	9
1	1	1	5	6	8
1	1	1	5	7	7
1	1	1	6	6	7
1	1	2	2	2	14
1	1	2	2	3	13
1	1	2	2	4	12
1	1	2	2	5	11
1	1	2	2	6	10
1	1	2	2	7	9
1	1	2	2	8	8
1	1	2	3	3	12
1	1	2	3	4	11
1	1	2	3	5	10
1	1	2	3	6	9
1	1	2	3	7	8
1	1	2	4	4	10
1	1	2	4	5	9
1	1	2	4	6	8
1	1	2	4	7	7
1	1	2	5	5	8
1	1	2	5	6	7
1	1	2	6	6	6

A	B	C	D	E	F
1	1	3	3	3	11
1	1	3	3	4	10
1	1	3	3	5	9
1	1	3	3	6	8
1	1	3	3	7	7
1	1	3	4	4	9
1	1	3	4	5	8
1	1	3	4	6	7
1	1	3	5	5	7
1	1	3	5	6	6
1	1	4	4	4	8
1	1	4	4	5	7
1	1	4	4	6	6
1	1	4	5	5	6
1	1	5	5	5	5
1	2	2	2	2	13
1	2	2	2	3	12
1	2	2	2	4	11
1	2	2	2	5	10
1	2	2	2	6	9
1	2	2	2	7	8
1	2	2	3	3	11
1	2	2	3	4	10
1	2	2	3	5	9
1	2	2	3	6	8
1	2	2	3	7	7
1	2	2	4	4	9
1	2	2	4	5	8
1	2	2	4	6	7
1	2	2	5	5	7
1	2	2	5	6	6
1	2	3	3	3	10
1	2	3	3	4	9
1	2	3	3	5	8
1	2	3	3	6	7
1	2	3	4	4	8
1	2	3	4	5	7
1	2	3	4	6	6
1	2	3	5	5	6
1	2	4	4	4	7
1	2	4	4	5	6
1	3	3	3	3	9
1	3	3	3	4	8
1	3	3	3	5	7
1	3	3	3	6	6
1	3	3	4	4	7
1	3	3	4	5	6
1	3	3	5	5	5
1	3	4	4	4	6

A	B	C	D	E	F
1	3	4	4	5	5
1	4	4	4	4	5
2	2	2	2	2	12
2	2	2	2	3	11
2	2	2	2	4	10
2	2	2	2	5	9
2	2	2	2	6	8
2	2	2	2	7	7
2	2	2	3	3	10
2	2	2	3	4	9
2	2	2	3	5	8
2	2	2	3	6	7
2	2	2	4	4	8
2	2	2	4	5	7
2	2	2	4	6	6
2	2	2	5	5	6
2	2	3	3	3	9
2	2	3	3	4	8
2	2	3	3	5	7
2	2	3	3	6	6
2	2	3	4	4	7
2	2	3	4	5	6
2	2	3	5	5	5
2	2	4	4	4	6
2	2	4	4	5	5
2	3	3	3	3	8
2	3	3	3	4	7
2	3	3	3	5	6
2	3	3	4	4	6
2	3	3	4	5	5
2	3	4	4	4	5
2	4	4	4	4	4
3	3	3	3	3	7
3	3	3	3	4	6
3	3	3	3	5	5
3	3	3	4	4	5
3	3	4	4	4	4

Fig I
"Master Hexads"

1. A B C D E F
 2. A B C D F E
 3. A B C E D F
 4. A B C E F D
 5. A B C F D E
 6. A B C F E D
 7. A B D C E F
 8. A B D C F E
 9. A B D E C F
 10. A B D E F C
 11. A B D F C E
 12. A B D F E C
 13. A B E C D F
 14. A B E C F D
 15. A B E D C F
 16. A B E D F C
 17. A B E F C D
 18. A B E F D C
 19. A B F C D E
 20. A B F C E D
 21. A B F D C E
 22. A B F D E C
 23. A B F E C D
 24. A B F E D C
 25. A C B D E F
 26. A C B D F E
 27. A C B E D F
 28. A C B E F D
 29. A C B F D E
 30. A C B F E D
 31. A C D B E F
 32. A C D B F E
 33. A C D E B F
~~A C D E F B~~ 24.
 34. A C D F B E
~~A C D F E B~~ 18.
 35. A C E B D F
 36. A C E B F D
 37. A C E D B F
~~A C E D F B~~ 22.
 38. A C E F B D
~~A C E F D B~~ 12.
 39. A C F B D E
 40. A C F B E D
 41. A C F D B E
~~A C F D E B~~ 16.
 42. A C F E B D
~~A C F E D B~~ 10.

43. A D B C E F
 44. A D B C F E
 45. A D B E C F
~~A D B E F C~~ 42.
 46. A D B F C E
~~A D B F E C~~ 38.
 47. A D C B E F
 48. A D C B F E
 49. A D C E B F
~~A D C E F B~~ 23.
 50. A D C F B E
~~A D C F E B~~ 17.
 51. A D E B C F
 A D E B F C 40.
 52. A D E C B F
~~A D E C F B~~ 20.
~~A D E F B C~~ 30.
~~A D E F C B~~ 6.
 53. A D F B C E
~~A D F B E C~~ 36.
 34. A D F C B E
~~A D F C E B~~ 14.
~~A D F E B C~~ 28.
~~A D F E C B~~ 4.
 55. A E B C D F
~~A E B C F D~~ 54.
 56. A E B D C F
~~A E B D F C~~ 41.
~~A E B F C D~~ 50.
~~A E B F D C~~ 34.
 57. A E C B D F
~~A E C B F D~~ 53.
 58. A E C D B F
~~A E C D F B~~ 21.
~~A E C F B D~~ 46.
~~A E C F D B~~ 11.
 59. A E D B C F
~~A E D B F C~~ 39.
 60. A E D C B F
~~A E D C F B~~ 19.
~~A E D F B C~~ 29.
~~A E D F C B~~ 5.
~~A E F B C D~~ 48.
~~A E F B D C~~ 32.
~~A E F C B D~~ 45.
~~A E F C D B~~ 8.
~~A E F D B C~~ 26.
~~A E F D C B~~ 2.

~~A F B C D E~~ 60
~~A F B C E D~~ 52
~~A F B D C E~~ 58
~~A F B D E C~~ 37.
~~A F B E C D~~ 49
~~A F B E D C~~ 33
~~A F C B D E~~ 59
~~A F C B E D~~ 51
~~A F C D B E~~ 54
~~A F C D E B~~ 15.
~~A F C E B D~~ 45
~~A F C E D B~~ 9.
~~A F D B C E~~ 57
~~A F D B E C~~ 35
~~A F D C B E~~ 55
~~A F D C E B~~ 13.
~~A F D E B C~~ 27.
~~A F D E C B~~ 3.
~~A F E B C D~~ 47
~~A F E B D C~~ 31.
~~A F E C B D~~ 43.
~~A F E C D B~~ 7.
~~A F E D B C~~ 25.
~~A F E D C B~~ 1.

Fig II

The 60 permutations of the "Master Hexads"

Fig III

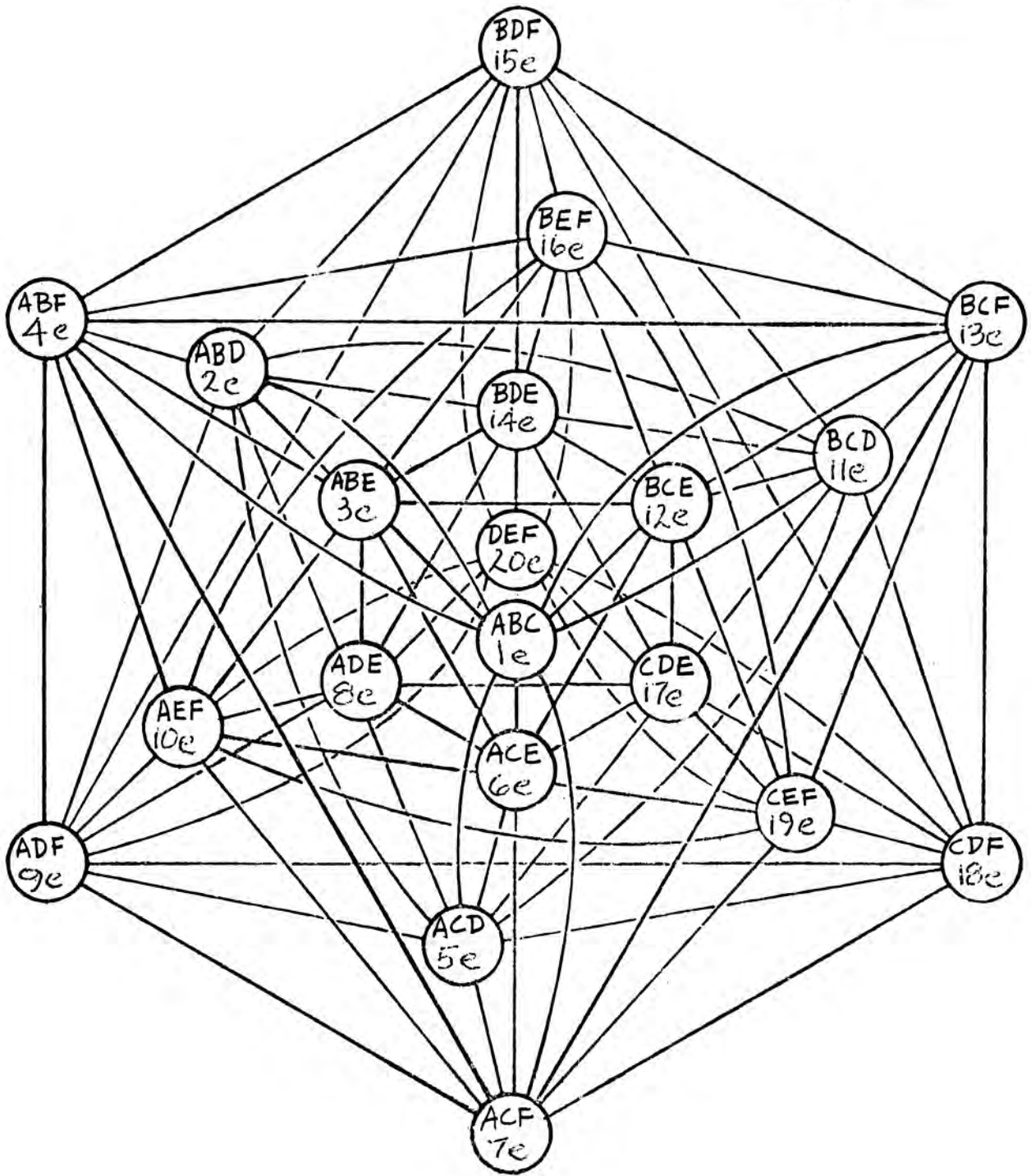


Fig IV

In the print-up the intervalic sequence A, B, C, D, E, F, may be shortened to ABCDEF. The Eikosany members may be shortened from 1e 2e 3e 4e etc to 1 2 3 4 etc. A print-up in this format would allow for reproduction on $8\frac{1}{2} \times 11$:

HEXAD						EIKOSANY MEMBERS															NUMBER OF dif. tones					
A	B	C	D	E	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	NO
1	1	1	1	1	17	3	4	5	6	5	6	7	7	8	9	6	7	8	8	9	10	9	10	11	12	10
1	2	3	4	5	7	4	7	11	16	9	13	19	16	21	3	10	14	19	17	0	4	19	2	6	9	16

etc.