

THE 12+7+12=31 GEOMETRY
APPLIED TO 3 PERCUSSION KEYBOARDS

© 1975 by Erv Wilson
1219 Poinsettia Dr. L.A. CA 90046

Because this geometry is not suited to manual keyboard design its usefulness for the percussion keyboard may be overlooked. In malletted instruments, such as vibes, marimba, and gamelan it provides a pattern that is easy to perform upon and one that is readily understood by players with conventional training. The tuning is by meantone Fifths ($\frac{1}{4}$ comma small) or by the 31 equal-tempered Fifth of 18/31 Octave. The tuning series begins at F $\sharp$ in the near rank, passing thru all 12 members of the near rank it then proceeds to the middle rank where it traverses all 7 members, and then continues thru all 12 members of the far rank, ending on B $\flat$. Thus:

start	
F $\sharp$ C $\sharp$ G $\sharp$ D $\sharp$ A $\sharp$ E $\sharp$ B $\sharp$ G $\flat$ D $\flat$ A $\flat$ E $\flat$ B $\flat$	NEAR RANK
F C G D A E B	MIDDLE RANK
F $\sharp$ C $\sharp$ G $\sharp$ D $\sharp$ A $\sharp$ F $\flat$ C $\flat$ G $\flat$ D $\flat$ A $\flat$ E $\flat$ B $\flat$	FAR RANK
END	

and according to the accompanying diagrams.

THE CHROMASONG 31 is suited for 'vibes' and other metallophones made of flat bars having a long ring time, and where, therefore, damping is required. This is the classic foot-damper, and it damps the back 2 ranks with the principal damping bar. An auxiliary bar damps the front rank only. The shown diagram is a reduction of a layout for an instrument now in construction.

THE 31-TONE TUBULONGS is suited for metal, long tubes. A 2-Octave set has been cut from aluminum tubing 5/8" I.D. with a 1/16" wall thickness (of the type found quite inexpensively at the hardware store, used for electrical conduit). 1" round brass tubing with 1/32" wall thickness has an exquisitely lovely tone and a prolonged ring time, and I will use these in a three-Octave set. 1" square brass tubing is also an interesting material, but tends to go flat if hit too enthusiastically. Perhaps one should experiment also with polycarbonate (plastic) tubing.

THE 31-TONE MARIMBA is suited for xylophones, also, and for metallophones where damping is not required. This is a good pattern for 'song-bells', and where resonating troughs are used for each of the three ranks, would make a compact instrument for classroom use. I have not begun construction on any instrument using this configuration, but do look forward to seeing any of these instruments realized.

A possible source of these and other new percussion instruments may be Wm. Dreiman/Bill Marimba (Good Vibes Malletworks, Inc., 407 Dover Road, Rockville, Maryland 20850) who has expressed an interest in building these kinds of instruments.

A second possible source of these instruments may be Hackleman/Wilson Clavichord Works, 1219 Poinsettia Drive, Los Angeles, CA 90046.

THE 31-TONE SCALE, MEANTONE CYCLE
ITS MUSICAL RESOURCES AND PRACTICAL APPLICATION

Prepared by Ervin M. Wilson

Various historical instruments have been constructed, providing this very important temperament with a more or less effective keyboard. The keyboard diagram, Fig. I, provides a meaningfully organized arrangement of 31 digitals and optimum manual accessibility.

The musical advantage of the meantone cycle over the 12-tone cycle, for the classical idiom, is represented in Figure II.

The application of the 31-tone scale to significant melodic and harmonic developments is demonstrated in Figure III.

Traditional notation, in fact, invented for meantone and Pythagorean tunings, applies directly to this keyboard. The diagrammatic notation, Fig. IV, is very useful and easy to learn. Figure V is merely a sheet of notation paper for this scheme.

The 31-tone system, meantone cycle, will have wide appeal for the reasons listed below:

1. Simple and eloquent aural preference.
2. The wide acceptance of meantone tuning in Europe for hundreds of years. The vast body of musical literature conceived in this tuning.
3. Superior orchestral and choral accompaniment, for the concert hall or the church choir.
4. The ease with which the musician may transfer his past learning from a 12-tone context to a 31-tone context.
5. The logical extension of significant melodic and harmonic possibility; this, in reconciliation with an accepted musical idiom, and in a single organized tonal system.
6. Greatly increased correspondence to historical and ethnic scales, including persistent Jazz inflection.
7. A stimulating medium for the composer, the experimentalist, and the everyday musical adventurer.
8. Obvious application to the eerie and special musical effects for the cinema, television, and radio.
9. Undiluted novelty.

Published May 24, 1961 by Ervin M. Wilson, 1832 purdue, Los Angeles 25, California. Additional copies: 25 ¢

Copyright 1961 by Ervin M. Wilson

The keyboard is a symmetrical arrangement of 31 digitals per octave, based upon the familiar pattern, and systematically tuned to 31-tone equal temperament.

While the 31-tone scale is thoroughly justified as the cyclic fulfillment of meantone temperament (all 31 intervals could be found on Handel's organ), it is a most fortunate and happy coincidence that the harmonic series is also approximated in this division with exceptional efficiency. It is, in fact, the first division of the octave to provide a satisfactory approximation of the harmonic series, 1 thru 16. Although the 13th is somewhat sharp, especially in relation to the 9th, it is not unbearably so. The series, 1 thru 12 is suited for harmonic synthesis; 1 thru 16 for melodic and general harmonic purposes; 1 thru 26 for chromatic melody & complex harmonic purposes (the 17th and 19th being quite sharp). Thus, good harmonic and melodic results are obtained by expanding the principle of the Major triad, harmonics 4, 5, & 6. Neither the 12-tone nor the 24-tone system provides even a suitable 7th harmonic!

By virtue of equal division the equivalent subharmonic series is also obtained. It so happens that the subharmonic series is the basis of a vast system of scales and modes derived from the equally divided string or the equally divided column of air (folk flutes, etc). These scales originated with the first cave dwellers who began boring equally spaced holes in their bone whistles; they are still found almost anywhere on Earth. (Ref: The Greek Aulos, Kathleen Schlesinger) Of the principle tones, $\frac{1}{16}$ thru $\frac{1}{6}$, the 12-tone scale systematically censors the $\frac{1}{14}$ th, $\frac{1}{13}$ th, $\frac{1}{11}$ th, & the $\frac{1}{7}$ th. In the 31-tone system we are privileged to borrow not only the melodies, but also the beautiful scales of neighbors; from Peru, Turkey, Japan, Yugoslavia, Africa, Java, ---

In addition, the pentatonic, diatonic, chromatic, & enharmonic, (19-tone) species are inherent in the 31-tone division. These are provided, in this keyboard with logical and contiguous fingerings, and greater fingering homogeneity than is possible on the 12-tone keyboard.

PITCHES OF THE 31-TONE SCALE
HUYGHEN'S MEANTONE CYCLE
TO THE 31ST ROOT OF 2 (1.0226114357)
BASE A 440

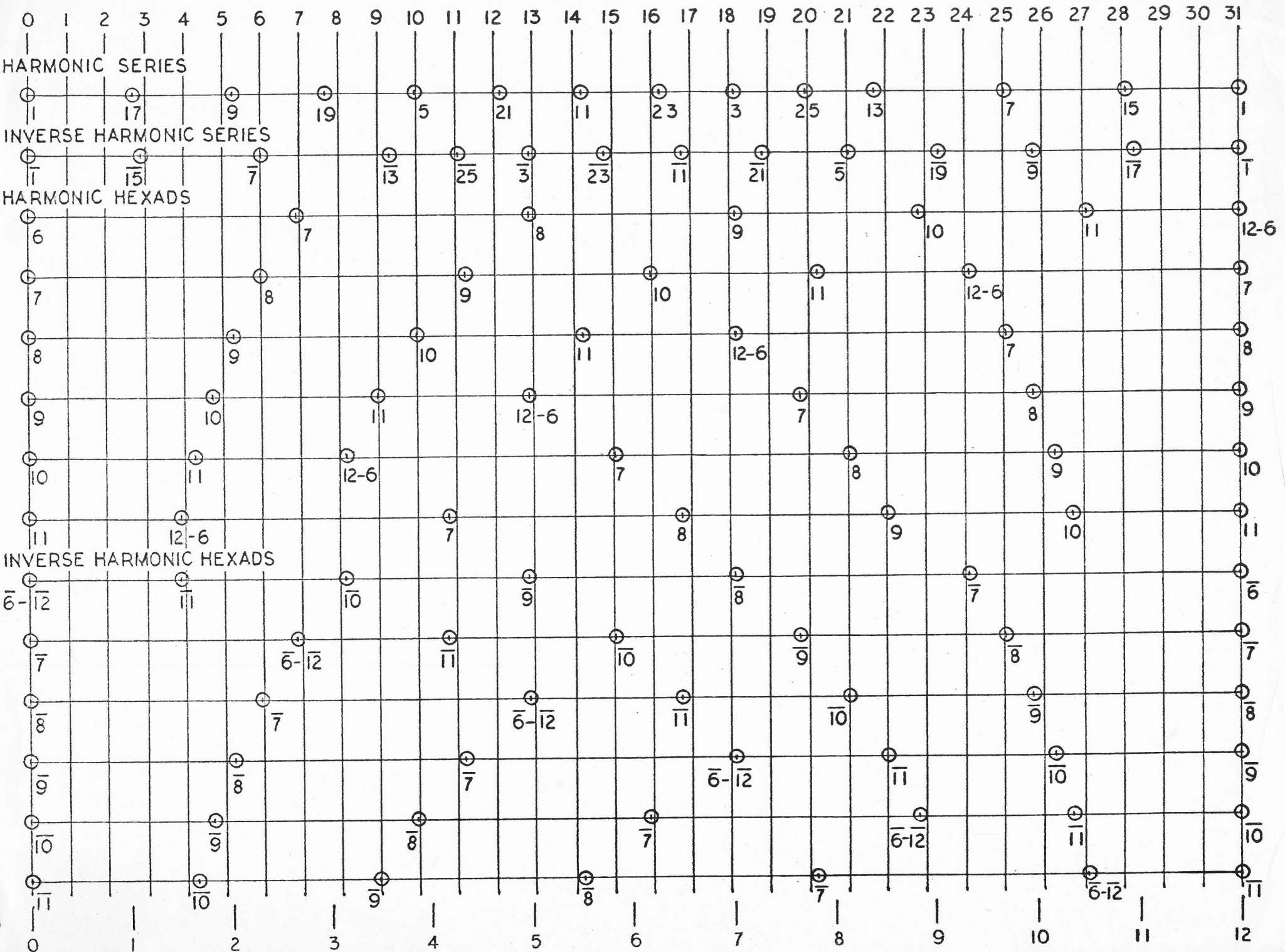
Issued by Ervin M. Wilson Jan 31, 1963

0. A	13.75	27.50	55.00	110.00	220.00	440.00	880.00	1760.00	3520.00	7040.00
1. A#	14.06	28.12	56.24	112.49	224.97	449.95	899.90	1799.80	3599.59	7199.18
2. A#	14.38	28.76	57.52	115.03	230.06	460.12	920.25	1840.49	3680.98	7361.97
3. Bb	14.70	29.41	58.82	117.63	235.26	470.53	941.05	1882.11	3764.22	7528.43
4. Bb	15.04	30.07	60.14	120.29	240.58	481.17	962.33	1924.66	3849.33	7698.66
5. B	15.38	30.75	61.50	123.01	246.02	492.05	984.09	1968.18	3936.37	7872.74
6. B#	15.72	31.45	62.90	125.79	251.59	503.17	1006.34	2012.69	4025.38	8050.75
7. Cb	16.08	32.16	64.32	128.64	257.27	514.55	1029.10	2058.20	4116.40	8232.79
8. C	16.44	32.89	65.77	131.55	263.09	526.18	1052.37	2104.74	4209.47	8418.95
9. C#	16.82	33.63	67.26	134.52	269.04	538.08	1076.16	2152.33	4304.66	8609.31
10 C#	17.20	34.39	68.78	137.56	275.12	550.25	1100.50	2201.00	4401.99	8803.98
11 Db	17.58	35.17	70.34	140.67	281.34	562.69	1125.38	2250.76	4501.52	9003.05
12 D#	17.98	35.96	71.93	143.85	287.71	575.41	1150.83	2301.66	4603.31	9206.62
13 D	18.39	36.78	73.55	147.11	294.21	588.42	1176.85	2353.70	4707.40	9414.80
14 D#	18.80	37.61	75.22	150.43	300.86	601.73	1203.46	2406.92	4813.84	9627.68
15 D#	19.23	38.46	76.92	153.83	307.67	615.34	1230.67	2461.34	4922.69	9845.38
16 Eb	19.66	39.33	78.66	157.31	314.62	629.25	1258.50	2517.00	5034.00	10067.99
17 Eb	20.11	40.22	80.43	160.87	321.74	643.48	1286.96	2573.91	5147.82	10295.65
18 E	20.56	41.13	82.25	164.51	329.01	658.03	1316.06	2632.11	5264.22	10528.44
19 E#	21.03	42.06	84.11	168.23	336.45	672.91	1345.81	2691.63	5383.25	10766.51
20 Fb	21.50	43.01	86.02	172.03	344.06	688.12	1376.24	2752.49	5504.98	11009.96
21 F	21.99	43.98	87.96	175.92	351.84	703.68	1407.36	2814.73	5629.45	11258.91
22 F#	22.49	44.97	89.95	179.90	359.80	719.59	1439.18	2878.37	5756.74	11513.49
23 F#	23.00	45.99	91.98	183.96	367.93	735.86	1471.73	2943.46	5886.91	11773.82
24 Gb	23.52	47.03	94.06	188.12	376.25	752.50	1505.00	3010.01	6020.02	12040.04
25 Gr	24.05	48.09	96.19	192.38	384.76	769.52	1539.04	3078.07	6156.14	12312.29
26 G	24.59	49.18	98.36	196.73	393.46	786.92	1573.84	3147.67	6295.34	12590.69
27 G#	25.15	50.29	100.59	201.18	402.36	804.71	1609.42	3218.84	6437.69	12875.38
28 G#	25.72	51.43	102.86	205.73	411.45	822.91	1645.81	3291.63	6583.26	13166.51
29 Ab	26.30	52.59	105.19	210.38	420.76	841.51	1683.03	3366.06	6732.11	13464.22
30 Ab	26.89	53.78	107.57	215.14	430.27	860.54	1721.08	3442.17	6884.34	13768.67
31 A	27.50	55.00	110.00	220.00	440.00	880.00	1760.00	3520.00	7040.00	14080.00

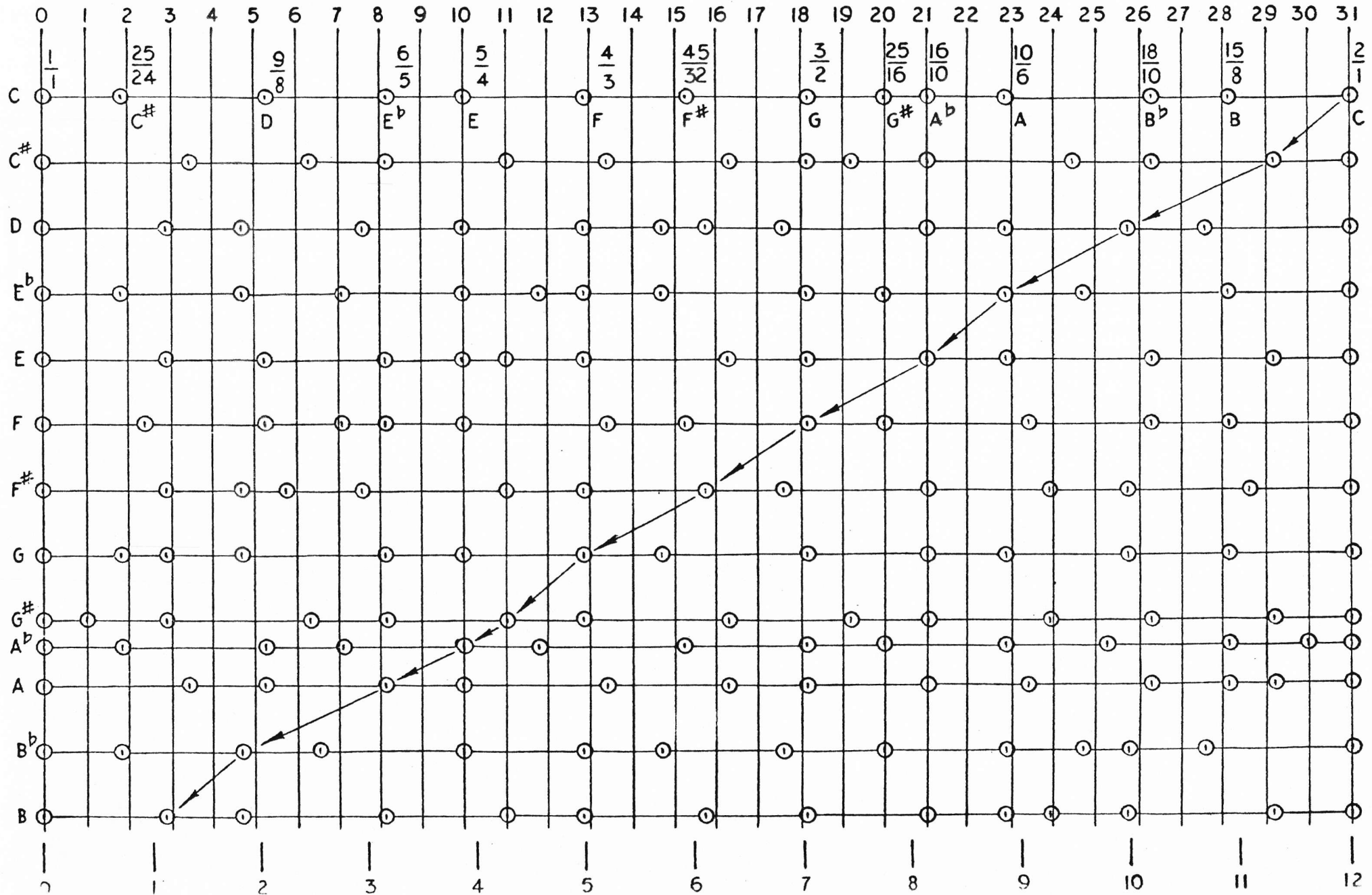
units	i	MVu/d	p	cents	log
0/31	1	0	0	0	0
1	1.0204165	+19	N	34.989468c.	.0087774
1	1.024	-12	S	41.05903c.	.0103000
2	1.04125	+38	N	69.980133c.	.0175551
2	1.0449066	+7	NbE	76.048897c.	.0190775
2	1.048576	-24	S	82.117661c.	.0205999
3	1.06624	+26	NbE	111.03876c.	.0278550
3	1.0699844	-5	SSE	117.10075c.	.0293757
3	1.0737417	-36	S	123.17629c.	.0308998
4	1.0918298	+14	NbE	152.09779c.	.0381550
4	1.095664	-17	SbE	158.16655c.	.0396774
5	1.1141215	+33	NbE	187.08806c.	.0469326
5	1.1180339	+2	NEbE	193.15682c.	.0484550
5	1.1219599	-29	S	199.22519c.	.0499773
6	1.1408605	+21	NbE	228.14709c.	.0572326
6	1.1448666	-10	SSE	234.21545c.	.0587549
7	1.1682412	+9	NEbN	269.20572c.	.0675325
7	1.1723433	-22	SbE	275.27408c.	.0690548
8	1.1920928	+28	NbE	304.19598c.	.0763101
8	1.196279	-3	SEbE	310.26475c.	.0778325
8	1.2004794	-34	SbE	316.33271c.	.0793547
9	1.2207031	+16	NNE	345.25462c.	.0866100
9	1.2249896	-15	SSE	351.32338c.	.0881324
10	1.2456255	+35	NbE	380.24448c.	.0953875
10	1.25	+4	NEbE	386.31365c.	.0969100
10	1.2543892	-27	SbE	392.38201c.	.0984323
11	1.2755206	+23	NNE	421.30351c.	.1056875
11	1.28	-8	SE	427.37268c.	.1072100
11	1.2844944	-39	SbE	433.44064c.	.1087322
12	1.3061332	+11	NE	462.36254c.	.1159875
12	1.31072	-20	SSE	468.43131c.	.1175099

units	i	MVu/d	p	cents	log
13	1.3328	+30	NNE	497.35241c.	.1247650
13	1.3374806	-1	EbS	503.42157c.	.1262875
13	1.3421772	-32	SbE	509.48994c.	.1278098
14	1.3647872	+18	NEbN	538.41144c.	.1350650
14	1.36958	-13	SEbS	544.48021c.	.1365874
15	1.3926518	+37	NNE	573.40171c.	.1438426
15	1.3975423	+6	NEbE	579.47047c.	.1453650
15	1.4024499	-25	SSE	585.53924c.	.1468874
16	1.4260756	+25	NEbN	614.46074c.	.1541426
16	1.4310834	-6	SEbE	620.5295c.	.1556650
16	1.4361086	-37	SbE	626.59787c.	.1571873
17	1.4603015	+13	NE	655.51977c.	.1644426
17	1.4654292	-18	SEbS	661.58773c.	.1659648
18	1.490116	+32	NNE	690.62604c.	.1732493
18	1.4953488	+1	E	696.5784c.	.1747425
18	1.5005993	-30	SSE	702.64637c.	.1762647
19	1.5258788	+20	NEbN	731.56867c.	.1835201
19	1.531237	-11	SEbE	737.63703c.	.1850424
20	1.5570318	+39	NNE	766.55813c.	.1922975
20	1.5625	+8	NEbE	772.6273c.	.1938200
20	1.5679865	-23	SEbS	778.69606c.	.1953424
21	1.5944007	+27	NEbN	807.61716c.	.2025975
21	1.6	-4	ESE	813.68633c.	.2041200
21	1.605618	-35	SSE	819.75469c.	.2056423
22	1.6326667	+15	NE	848.67619c.	.2128975
22	1.6384	-16	SE	854.74496c.	.2144199
23	1.666	+34	NNE	883.66606c.	.2216750
23	1.6718507	+3	EbN	889.73522c.	.2231975
23	1.6777216	-28	SSE	895.80399c.	.2247199

units	i	MVu/d	p	cents	log
24	1.705984	+22	NE	924.72469c.	.2319749
24	1.7119751	-9	SEbE	930.79425c.	.2334975
25	1.746928	+10	ENE	965.78412c.	.2422750
25	1.7530624	-21	SEbS	971.85289c.	.2437974
26	1.7825945	+29	NEbN	1000.7743c.	.2510526
26	1.7888543	-2	EbS	1006.8431c.	.2525750
26	1.7951358	-33	SSE	1012.9115c.	.2540973
27	1.8253768	+17	NEbE	1041.833c.	.2613525
27	1.8317866	-14	SEbE	1047.9017c.	.2628749
28	1.862645	+36	NEbN	1076.8232c.	.2701301
28	1.8691859	+5	EbN	1082.892c.	.2716525
28	1.8757492	-26	SEbS	1088.9604c.	.2731748
29	1.9073485	+24	NE	1117.8823c.	.2804301
29	1.9140463	-7	ESE	1123.9506c.	.2819524
29	1.920767	-38	SSE	1130.0186c.	.2834746
30	1.953125	+12	ENE	1158.9413c.	.2907301
30	1.9599833	-19	SE	1165.0097c.	.2922524
31	1.9930008	+31	NEbN	1193.9308c.	.2995075
31	2	0	EbS	1200c.	.3010300
31	2.0070226	-31	SEbS	1206.0683c.	.3025523
0/31	1.0035113	-31	SbW	6.0683651c.	.0015223



JUST INTERVALS IN CLASSICAL CONTEXT COMPARED WITH 31-TONE EQUAL TEMPERAMENT
(12-TONE TEMP. REF. AT BOTTOM OF PAGE)



C C[♯] C[♯] D^b D^r D D[♯] D[♯] E^b E^r E E[♯] E^r F F[♯] F[♯] G^b G^r G G[♯] G[♯] A^b A^r A A[♯] A[♯] B^b B^r B B[♯] C^r C

ROOT
POSITION

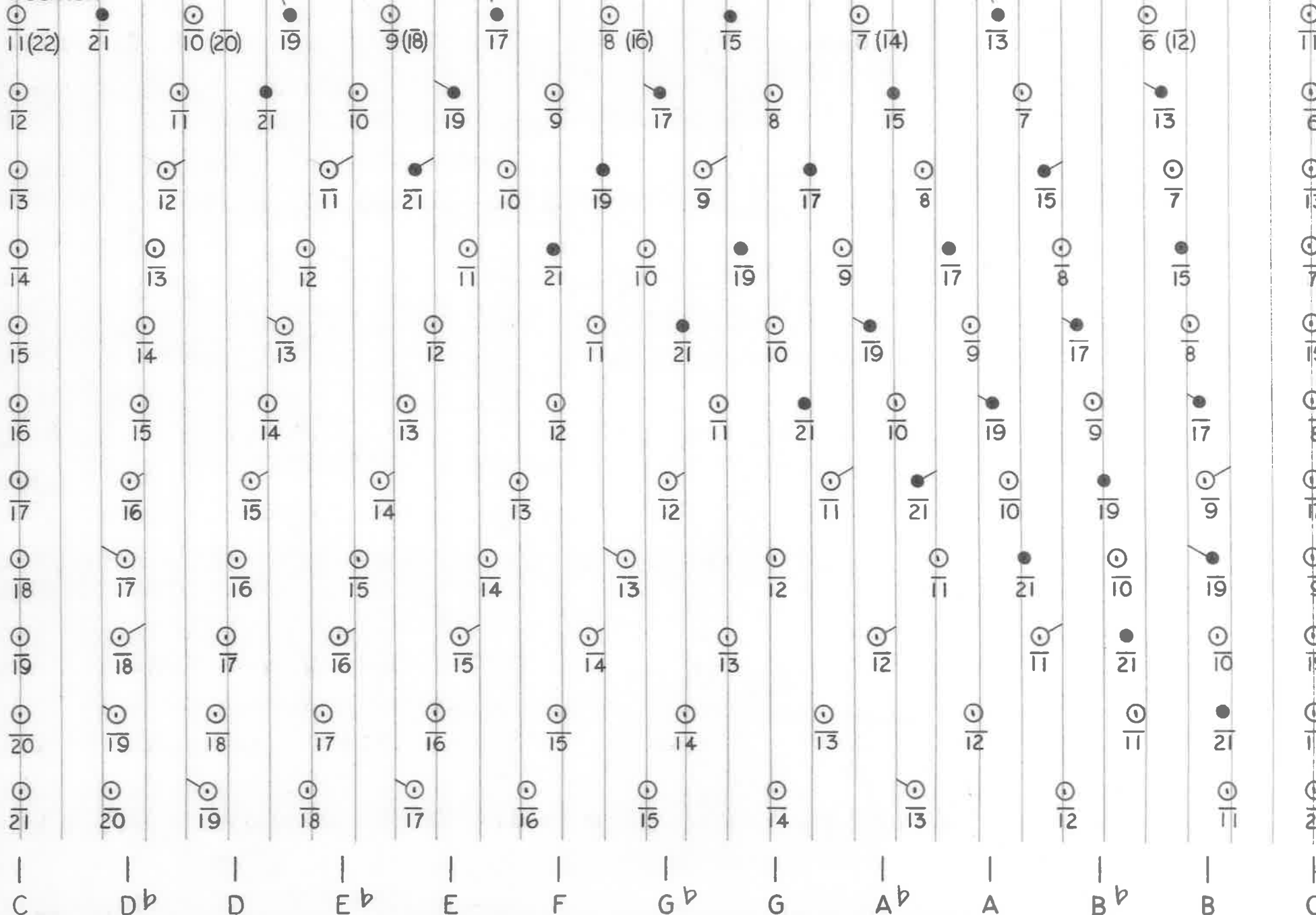
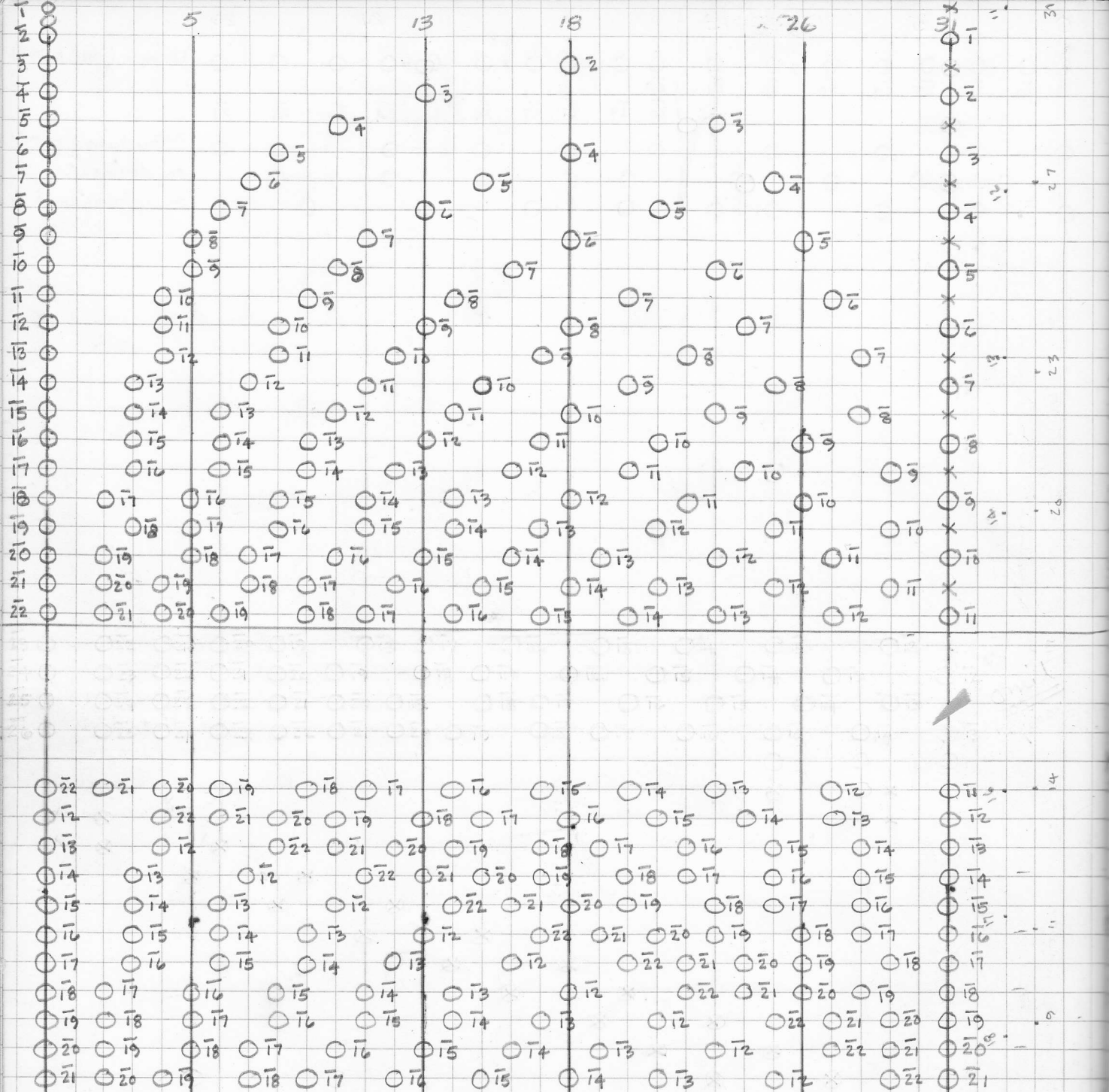


FIG VII
INVERSIONS (MODES) OF THE UNDECUPLE SUBHARMONIC (EQUI-SPACED FLUTE) SCALE



The Spurious condensation of our musical heritage into 12 equal tones has deprived us of the ratios of 11 and 13

The Moments-of-Symmetry of 31
©1989 by Ervin McD. Wilson

Generating interval; 15/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

[illegible]

Generating Interval; 14/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

																		31	1
14									17									2	
14									14									3	
11									11									5	
8									8									7	
5									5									9	
2									2									11	
2									2									20	
																		(31)	

Generating Interval; 13/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{6}$

																		31		1								
																		13	18	2								
																		13	13	5	3							
																		8	5	8	5	5	5	5	7			
																		3	5	5	3	5	5	5	12			
																		3	3	2	3	2	3	3	2	3	2	19
1	2	1	2	2	1	2	2	1	2	1	2	2	1	2	2	1	2	2	(31)									

Generating Interval; 12/31

0 13 8 3 16 11 6 1 14 9 4 17 12 7 2 15 10 5 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

[illegible]

Generating Interval; 11/31

0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60 63 66 69 72 75 78 81 84 87 90 93 96 99 100

						31												1
11												20						2
11						11						9						3
2	9					2	9					9					5	
2	2	7				2	2	7				2	7				8	
2	2	2	5			2	2	2	5			2	2	5			11	
2	2	2	2	3		2	2	2	2	3		2	2	2	3		14	
2	2	2	2	2	1	2	2	2	2	2	1	2	2	2	2	1	17	
																	(31)	

Generating Interval; 10/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

															31																														1																														
10																														21																														2															
10															10																														11																														3
10															10																														10															1															4
9															1															9															1															7															
8															1															8															1															10															
7															1															7															1															13															
6															1															6															1															16															
5															1															5															1															19															
4															1															4															1															22															
3															1															3															1															25															
2															1															2															1															28															
																																																												(31)															

(31)

Generating Interval; 9/31

0 7 14 4 11 1 8 15 5 12 2 9 16 6 13 3 10 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

31																														1
9										22																				2
9										9								13												3
9										9								9												4
5										4								5												7
4										4								4												10
3										3								3												17
2										2								2												22

(31)

Generating Interval; 8/31

0 4 8 12 1 5 9 13 2 6 10 14 3 7 11 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

31																														1
8										23																				2
8										8								15												3
8										8								8												4
7										7								7												7
6										6								6												11
5										5								5												15
4										4								4												19
3										3								3												24
2										2								2												27

(31)

Generating Interval; $7/31$

0 9 18 5 14 1 10 19 6 15 2 11 20 7 16 3 12 21 8 17 4 13 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{10}$

[illegible]

Generating Interval; 6/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

				31							
6						25					
6		6				19					
6		6		6				13			
6		6		6		6				7	
6		6		6		6		6		1	
5		1		5		1		5		1	
4		1 1		4		1 1		4		1 1	
3		1 1 1		3		1 1 1		3		1 1 1	
2		1 1 1 1		2		1 1 1 1		2		1 1 1 1	

Generating Interval; 5/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

31												1
5	26										2	
5	5	21								3		
5	5	5	16							4		
5	5	5	5	11						5		
5	5	5	5	5	5	6				6		
5	5	5	5	5	5	5	5			7		
4	1	4	1	4	1	4	1	4	1	4	1 1	13
3	1 1	3	1 1	3	1 1	3	1 1	3	1 1	3	1 1 1	19
2	1 1 1	2	1 1 1	2	1 1 1	2	1 1 1	2	1 1 1	2	1 1 1 1	25
												(31)

Generating Interval; 4/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

															31						1	
4															27						2	
4		4													23						3	
4		4		4												19						4
4		4		4		4										15						5
4		4		4		4		4							11						6	
4		4		4		4		4		4		4		7				7				
4		4		4		4		4		4		4		4		3		8				
1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	15				
1	1	2	1	1	2	1	1	2	1	1	2	1	1	2	1	1	2	23				
																		(31)				

Generating Interval; 3/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{3}{10}$

[illegible]

Generating Interval; 2/31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{10}$

[illegible]

Generating Interval; $1/31$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 $\frac{31}{0}$

[illegible]

Address:

ERV WILSON
844 N. AVE 65
LOS Angeles, CA 90042
Phone: (213) 256-2624

[illegible]

[illegible]

1		2		3		4	1	1
1		2		3		4	1	2
1		2		3		4	1	3
1		2		3		4	1	4
1	5	2	6	3	7	4	1	7
1 8	5	2 9	6	3 10	7	4	1	10
1 8 15	5 12	2 9 16	6 13	3 10 17	7 14	4 11	1	17
1 8 15 22	5 12 19	2 9 16 23	6 13 20	3 10 17 24	7 14 21	4 11 18	1	24

1	2	3	4	1	1
1	2	3	4	1	2
1	2	3	4	1	3
1	2	3	4	1	4
1 5	2 6	3 7	4	1	7
1 5 9	2 6 10	3 7 11	4 8	1	11
1 5 9 13	2 6 10 14	3 7 11 15	4 8 12	1	15
1 5 9 13 17	2 6 10 14 18	3 7 11 15 19	4 8 12 16	1	19
1 5 9 13 17 21	2 6 10 14 18 22	3 7 11 15 19 23	4 8 12 16 20	1	23
1 5 9 13 17 21 25	2 6 10 14 18 22 26	3 7 11 15 19 23 27	4 8 12 16 20 24	1	27

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

1																																1	1
1	2																														1	2	
1	2	3																													1	3	
1	2	3	4																											1	4		
1	2	3	4	5																										1	5		
1	2	3	4	5	6																									1	6		
1	2	3	4	5	6	7																								1	7		
1	2	3	4	5	6	7	8																							1	8		
1	2	3	4	5	6	7	8	9																						1	9		
1	2	3	4	5	6	7	8	9	10																					1	10		
1	2	3	4	5	6	7	8	9	10	11																				1	11		
1	12	2	13	3	14	4	15	5	16	6	17	7	18	8	19	9	20	10	21	11	1									1	12		

1																															1	1
1	2																														1	2
1	2	3																													1	3
1	2	3	4																												1	4
1	2	3	4	5																											1	5
1	2	3	4	5	6																										1	6
1	2	3	4	5	6	7																									1	7
1	2	3	4	5	6	7	8																								1	8
1	2	3	4	5	6	7	8	9																							1	9
1	2	3	4	5	6	7	8	9	10																						1	10
1	2	3	4	5	6	7	8	9	10	11																					1	11
1	2	3	4	5	6	7	8	9	10	11	12																				1	12
1	2	3	4	5	6	7	8	9	10	11	12	13																			1	13
1	2	3	4	5	6	7	8	9	10	11	12	13	14																		1	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15																	1	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16																1	16

1																															1	1
1	2																														1	2
1	2	3																													1	3
etc																																etc
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	1	30	

A TABULATION OF TETRACHORDS IN THE 31-TONE SCALE

©1976 by Erv Wilson

Key

0 1 2 3 4 5 6 7 8 9 10 11 12 13

0		2											
0			3										
0				4									
0					5								
0						6							
0							7						
0		2		4									
0		2			5								
0		2				6							
0		2					7						
0			3			6							
0			3				7						
0			3					8					
0				4				8					

13		1	1	11
13		1	2	10
13		1	3	9
13		1	4	8
13		1	5	7
13		1	6	6
13		2	2	9
13		2	3	8
13		2	4	7
13		2	5	6
13		3	3	7
13		3	4	6
13		3	5	5
13		4	4	5

0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 1 2 13

0 1 12 13

0 11 12 13

1 1 11

1 11 1

11 1 1

0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 1 3 13

0 1 11 13

0 10 11 13

0 10 12 13

0 2 12 13

0 2 3 13

1 2 10

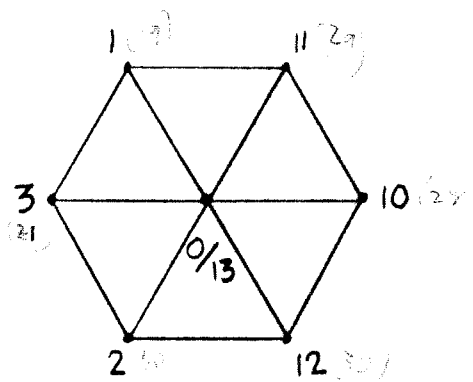
1 10 2

10 1 2

10 2 1

2 10 1

2 1 10



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 1 4 13

0 1 10 13

0 9 10 13

0 9 12 13

0 3 12 13

0 3 4 13

1 3 9

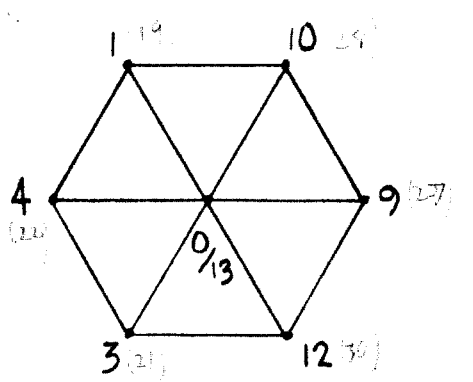
1 9 3

9 1 3

9 3 1

3 9 1

3 1 9



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 1 5 13

0 1 9 13

0 8 9 13

0 8 12 13

0 4 12 13

0 4 5 13

1 4 8

X

1 8 4

X

8 1 4

X

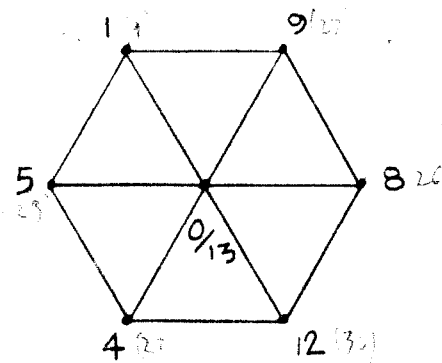
8 4 1

X

4 8 1

X

4 1 8



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 1 6 13

0 1 8 13

0 7 8 13

0 7 12 13

0 5 12 13

0 5 6 13

1 5 7

X

1 7 5

X

7 1 5

X

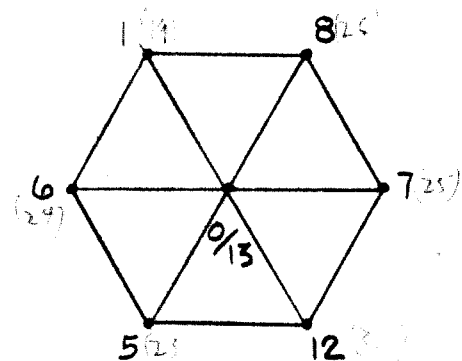
7 5 1

X

5 7 1

X

5 1 7



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 1 7 13

0 6 7 13

0 6 12 13

1 6 6

X

6 1 6

X

6 6 1

0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 2 4 13

0 2 11 13

0 9 11 13

2 2 9

X
2 9 2

X
9 2 2

0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 2 5 13

0 2 10 13

0 8 10 13

0 8 11 13

0 3 11 13

0 3 5 13

2 3 8

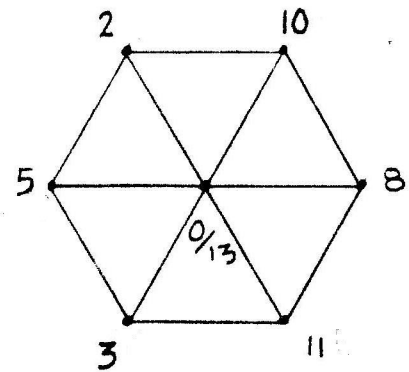
X
2 8 3

X
8 2 3

X
8 3 2

X
3 8 2

X
3 2 8



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 2 6 13

0 2 9 13

0 7 9 13

0 7 11 13

0 4 11 13

0 4 6 13

2 4 7

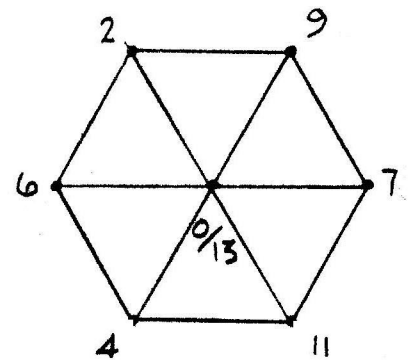
X
2 7 4

X
7 2 4

X
7 4 2

X
4 7 2

X
4 2 7



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 2 7 13

0 2 8 13

0 6 8 13

0 6 11 13

0 5 11 13

0 5 7 13

2 5 6

X

2 6 5

X

6 2 5

X

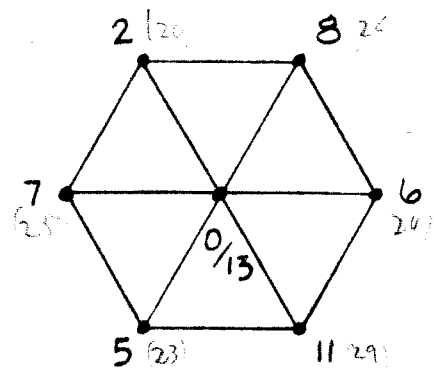
6 5 2

X

5 6 2

X

5 2 6



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 3 6 13

0 3 10 13

0 7 10 13

3 3 7

X

3 7 3

X

7 3 3

0 1 2 3 4 5 6 7 8 9 10 11 12 13

0 3 7 13

0 3 9 13

0 6 9 13

0 6 10 13

0 4 10 13

0 4 7 13

3 4 6

X

3 6 4

X

6 3 4

X

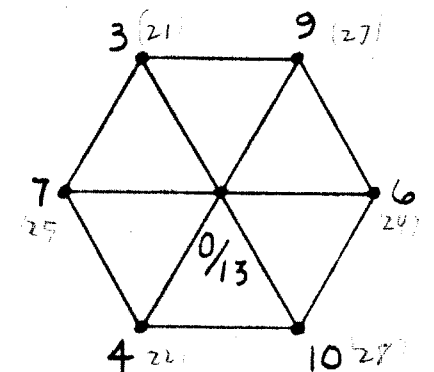
6 4 3

X

4 6 3

X

4 3 6



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0		3					8					13	
0				5			8					13	
0				5					10			13	

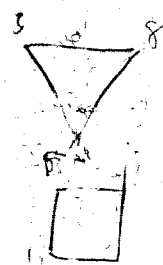
3 5 5

X

5 3 5

X

5 5 3



0 1 2 3 4 5 6 7 8 9 10 11 12 13

0			4				8					13	
0			4				9					13	
0				5			9					13	

4 4 5

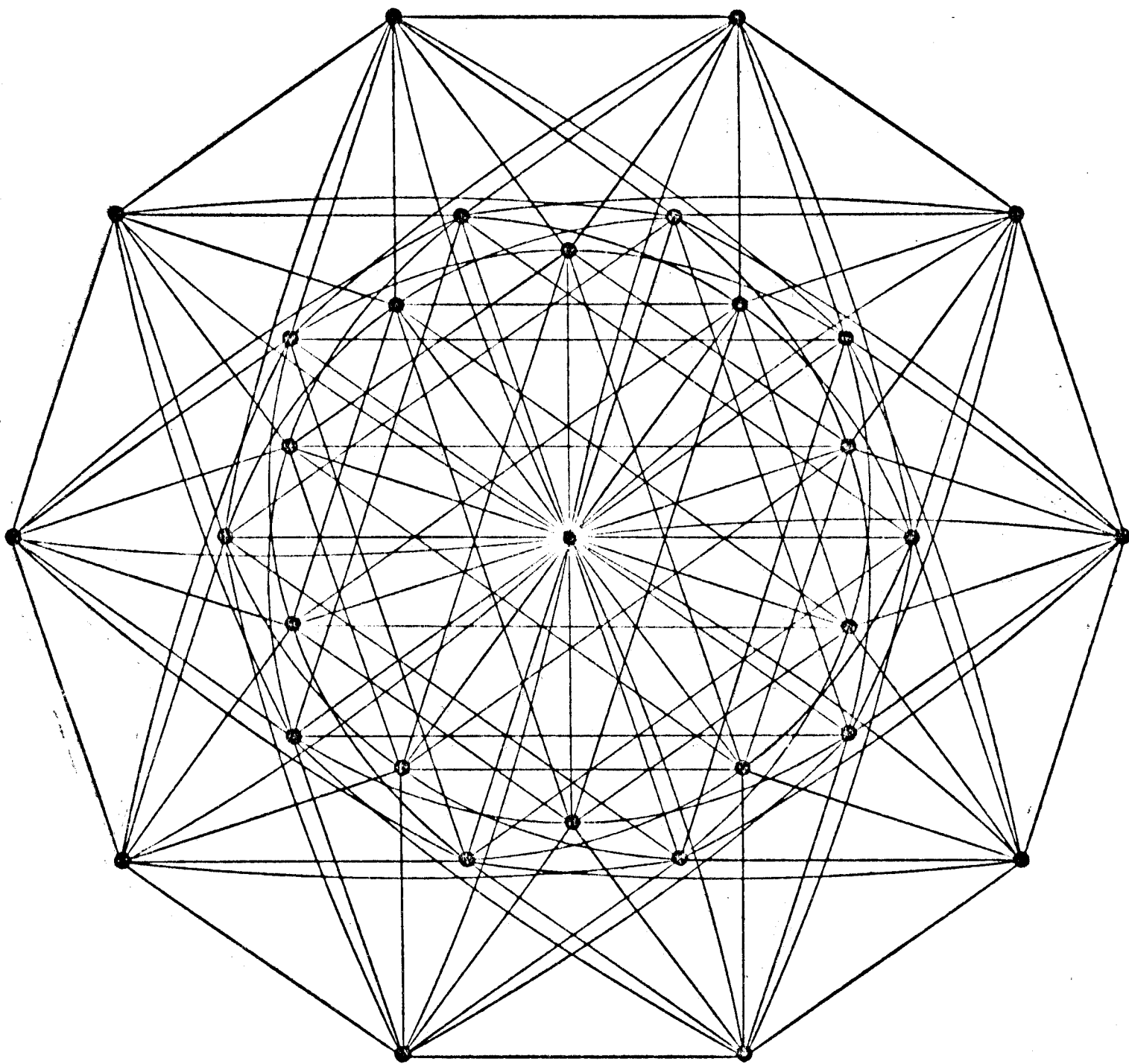
X

4 5 4

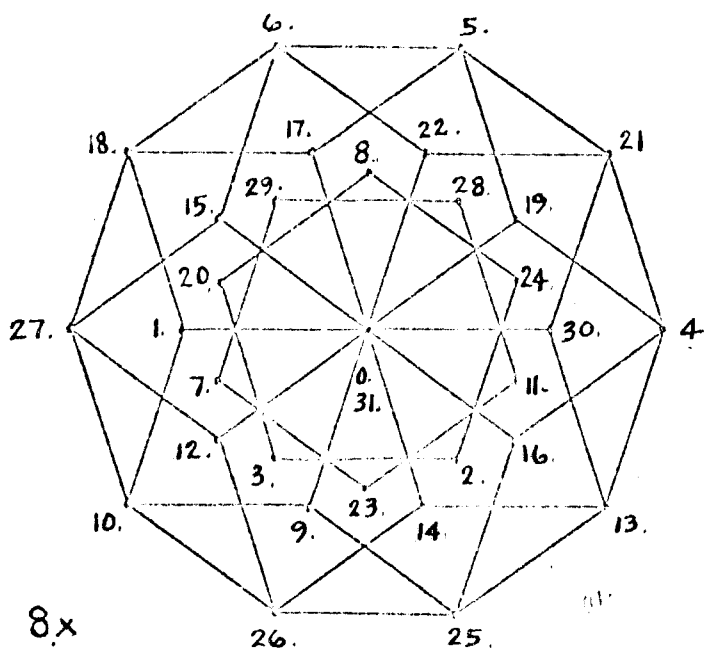
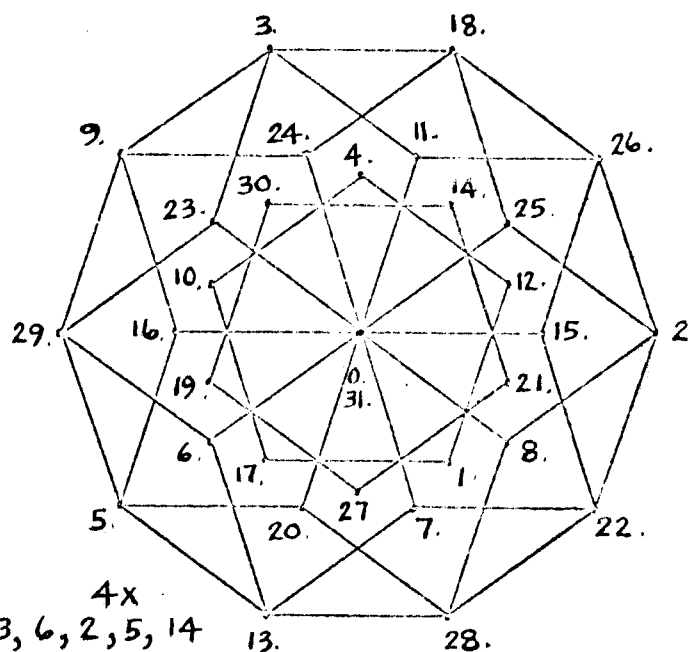
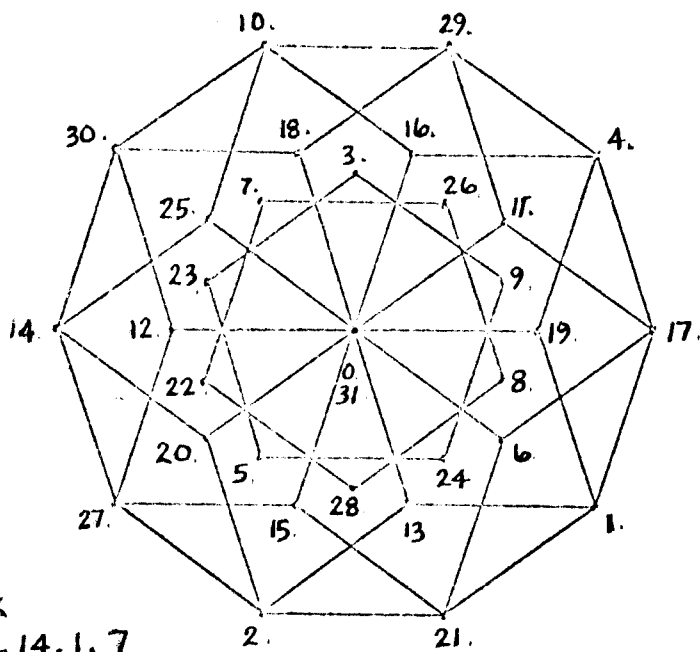
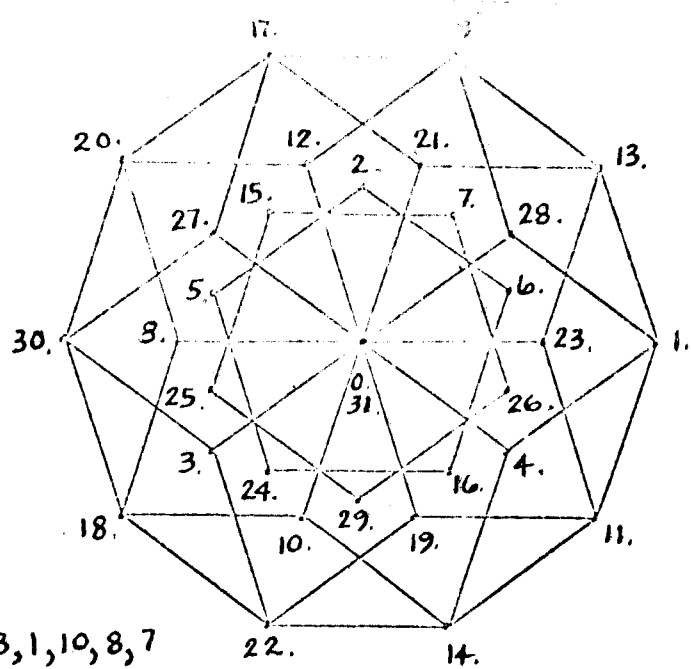
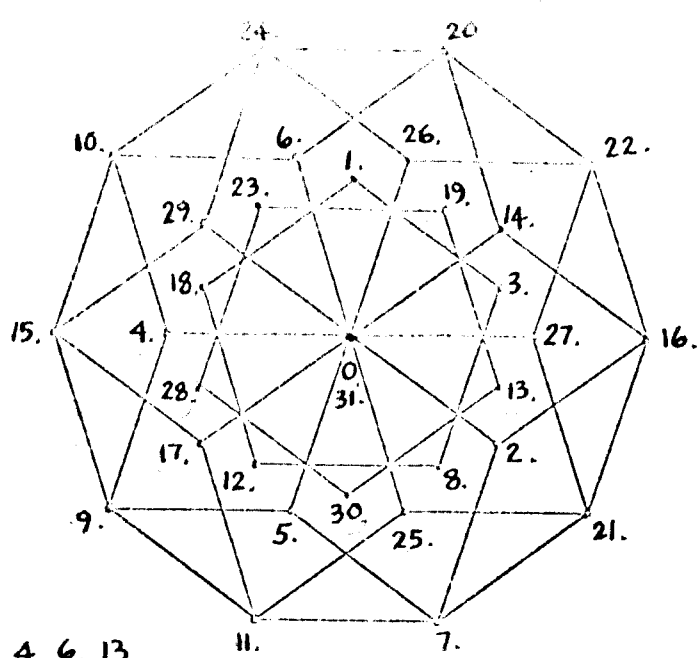
X

5 4 4





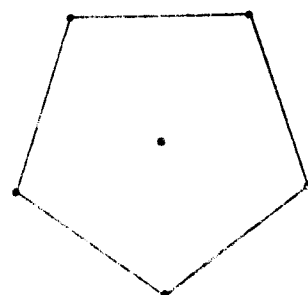
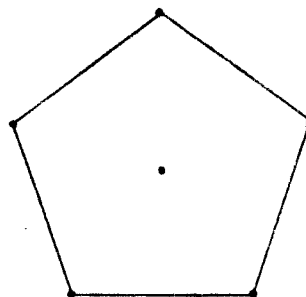
The All-Interval Sets & Cross-Sets
of 31
©1975 by Erv Wilson

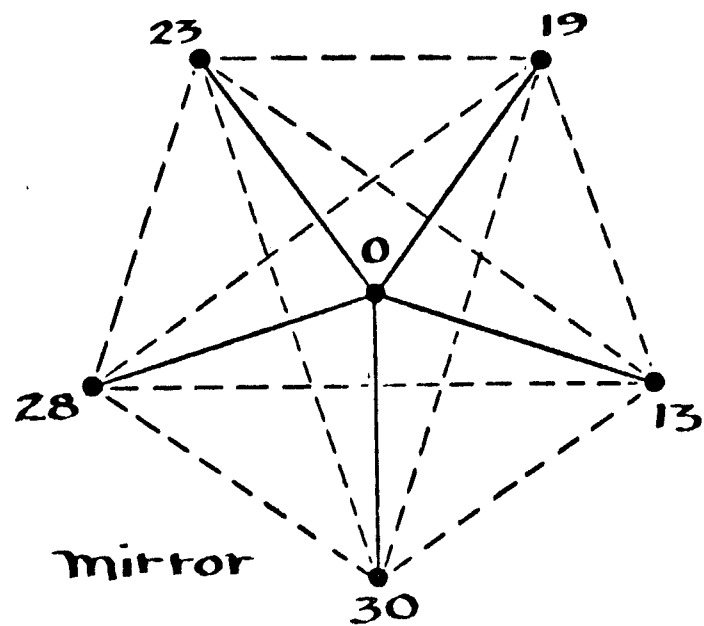
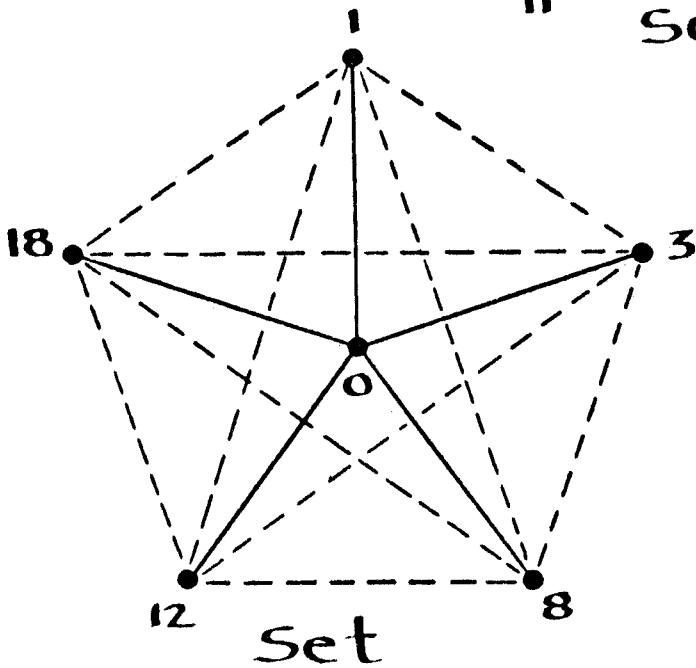
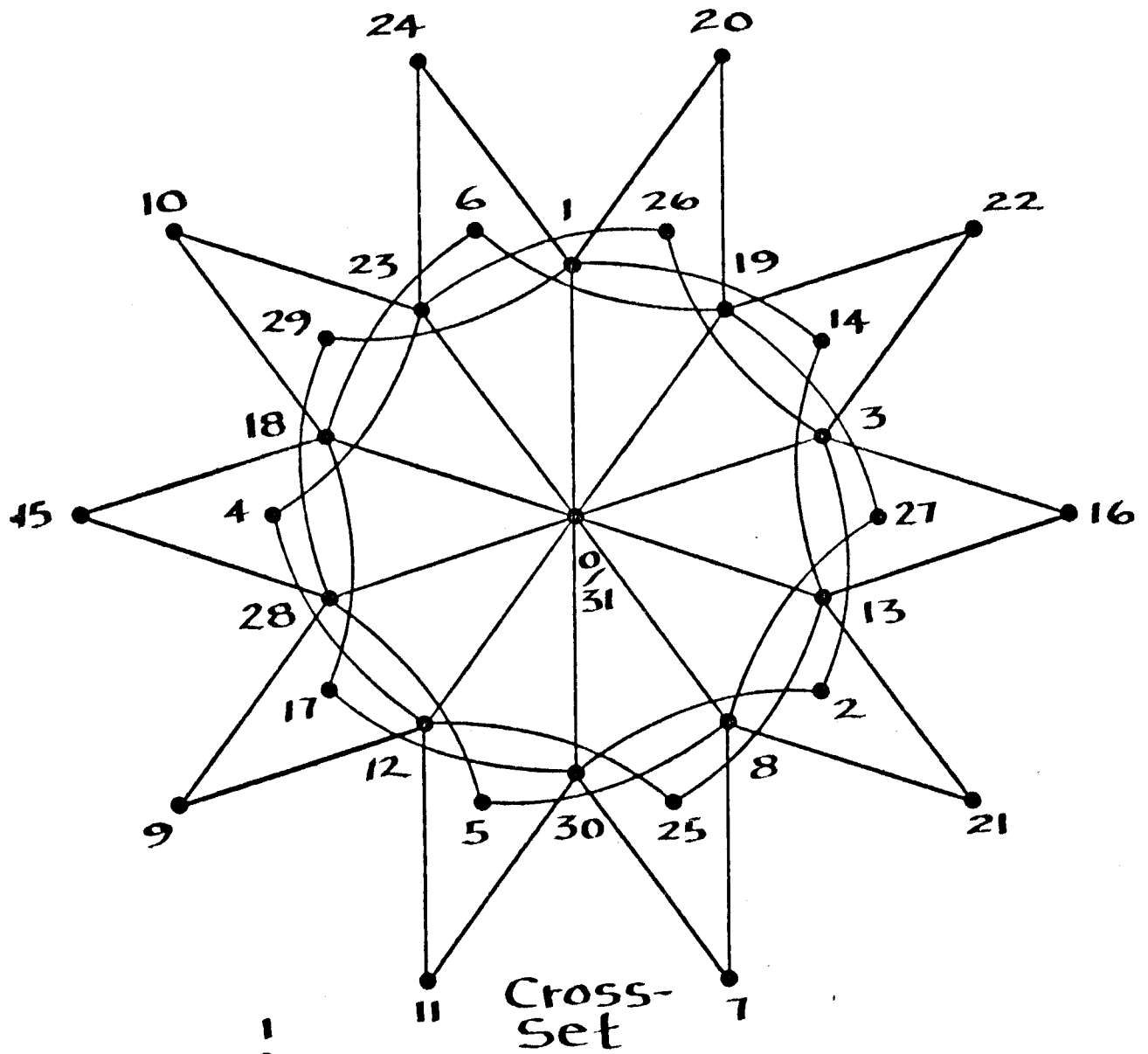


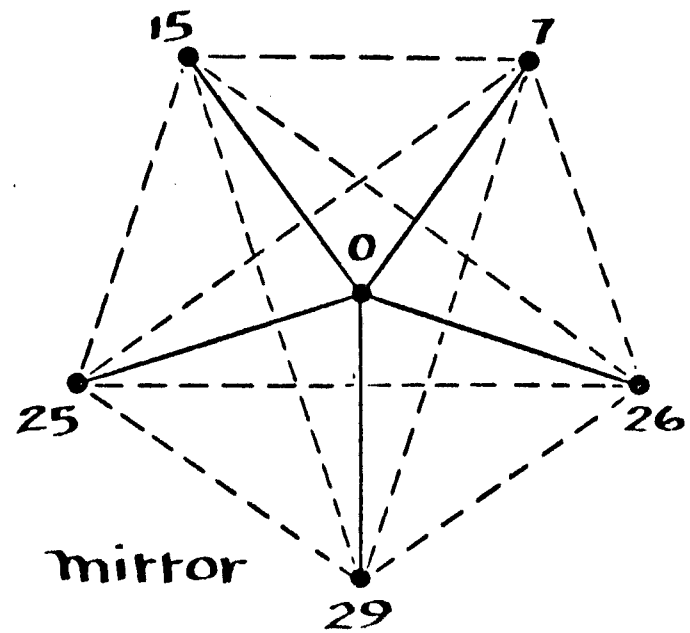
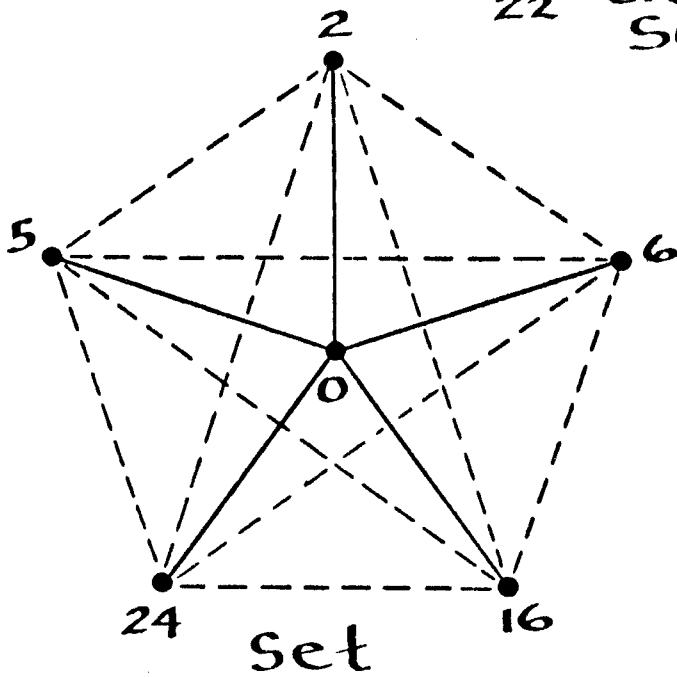
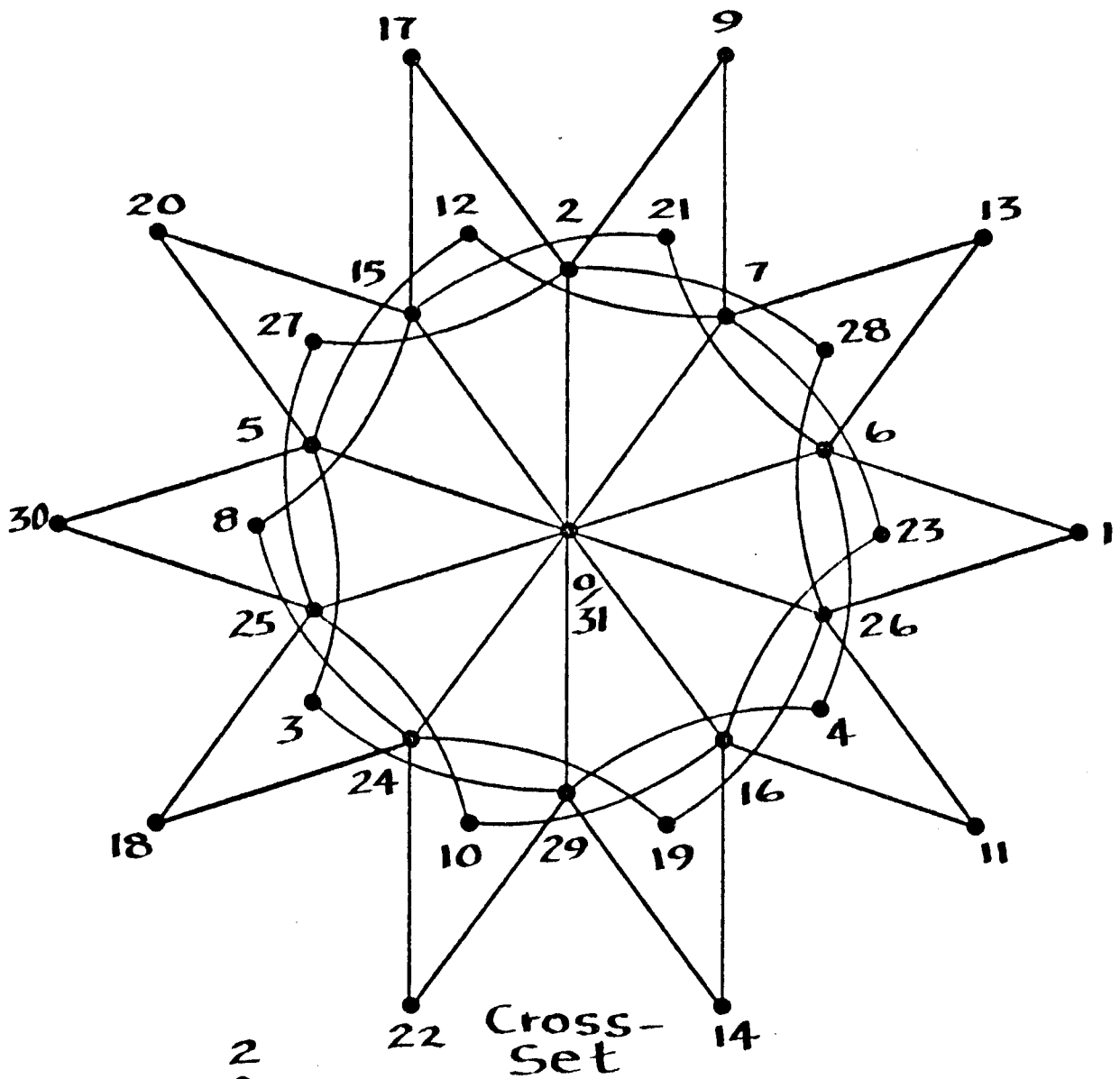
The All-Interval Cross-Sets
of 31, (mapped over a centered-
pentagon lattice)

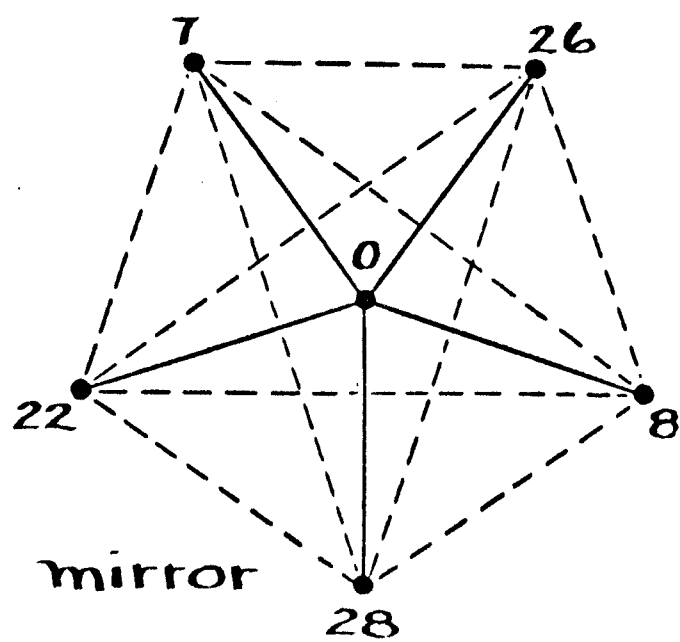
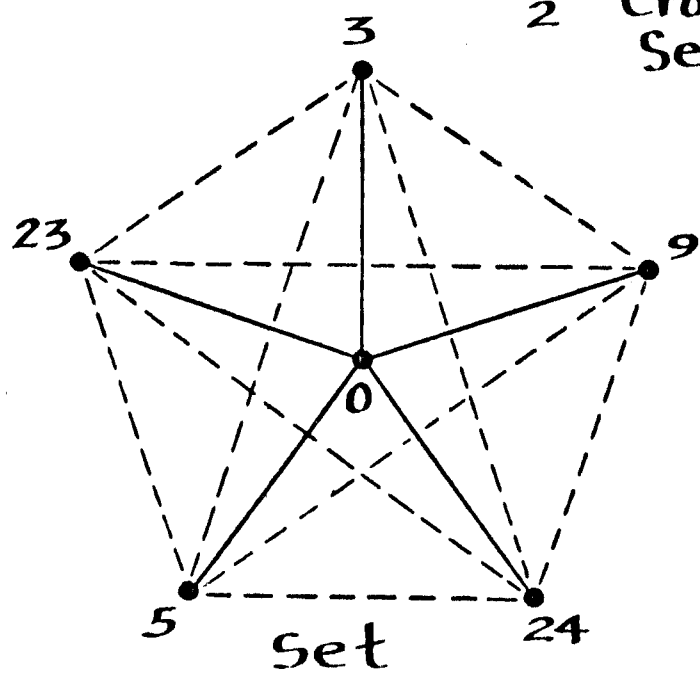
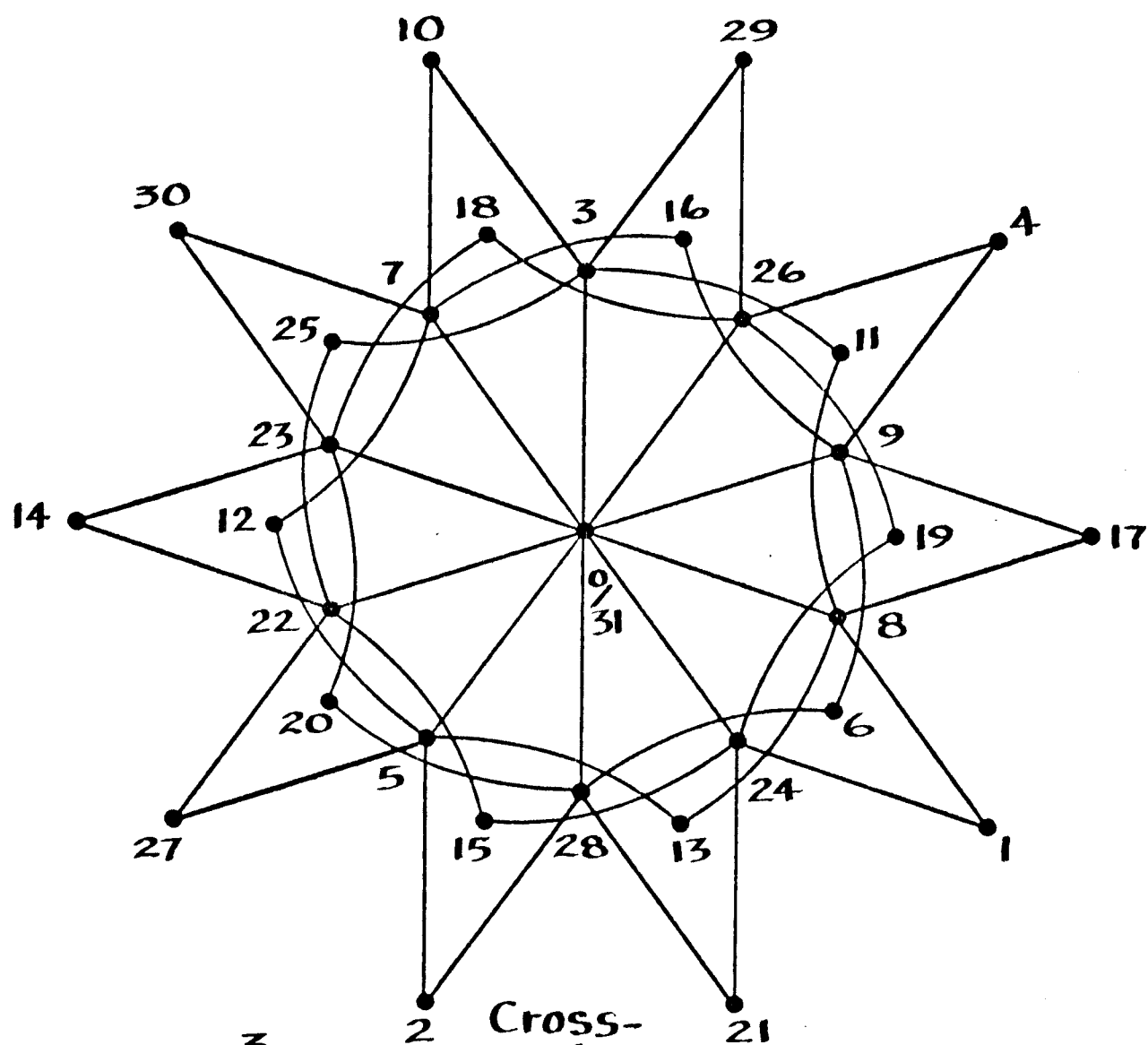
© 1975 by Erv Wilson

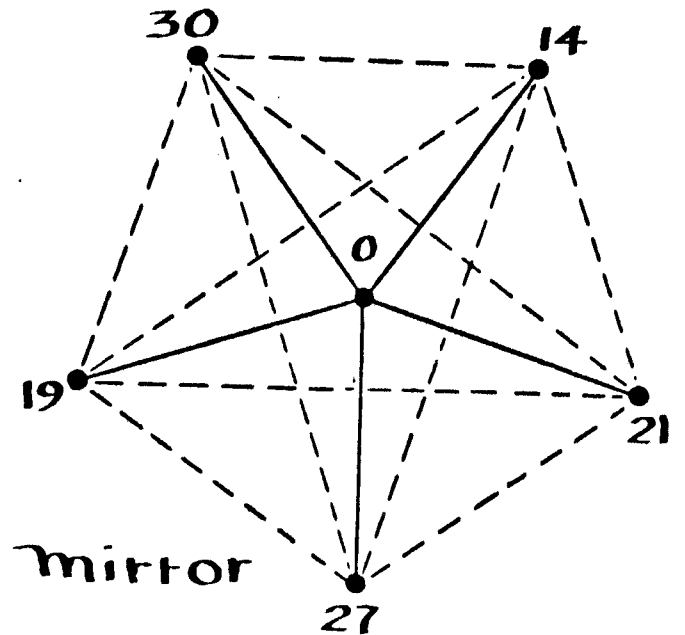
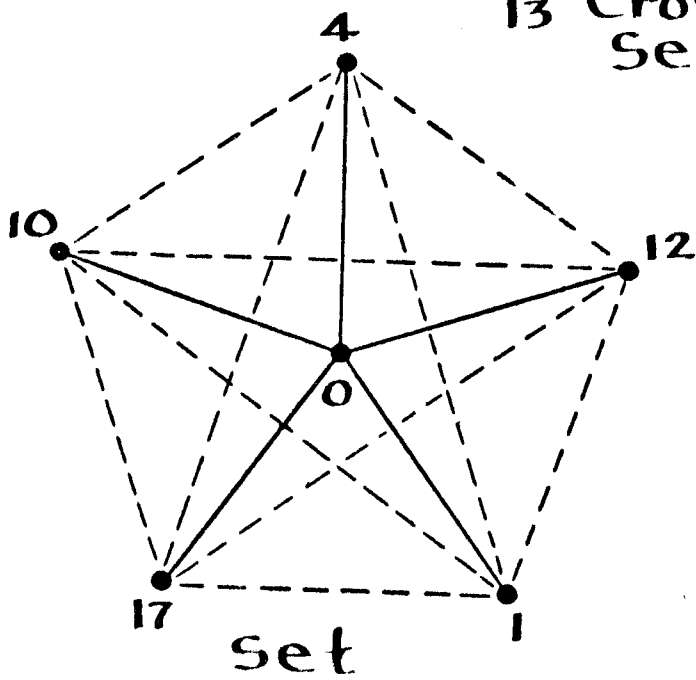
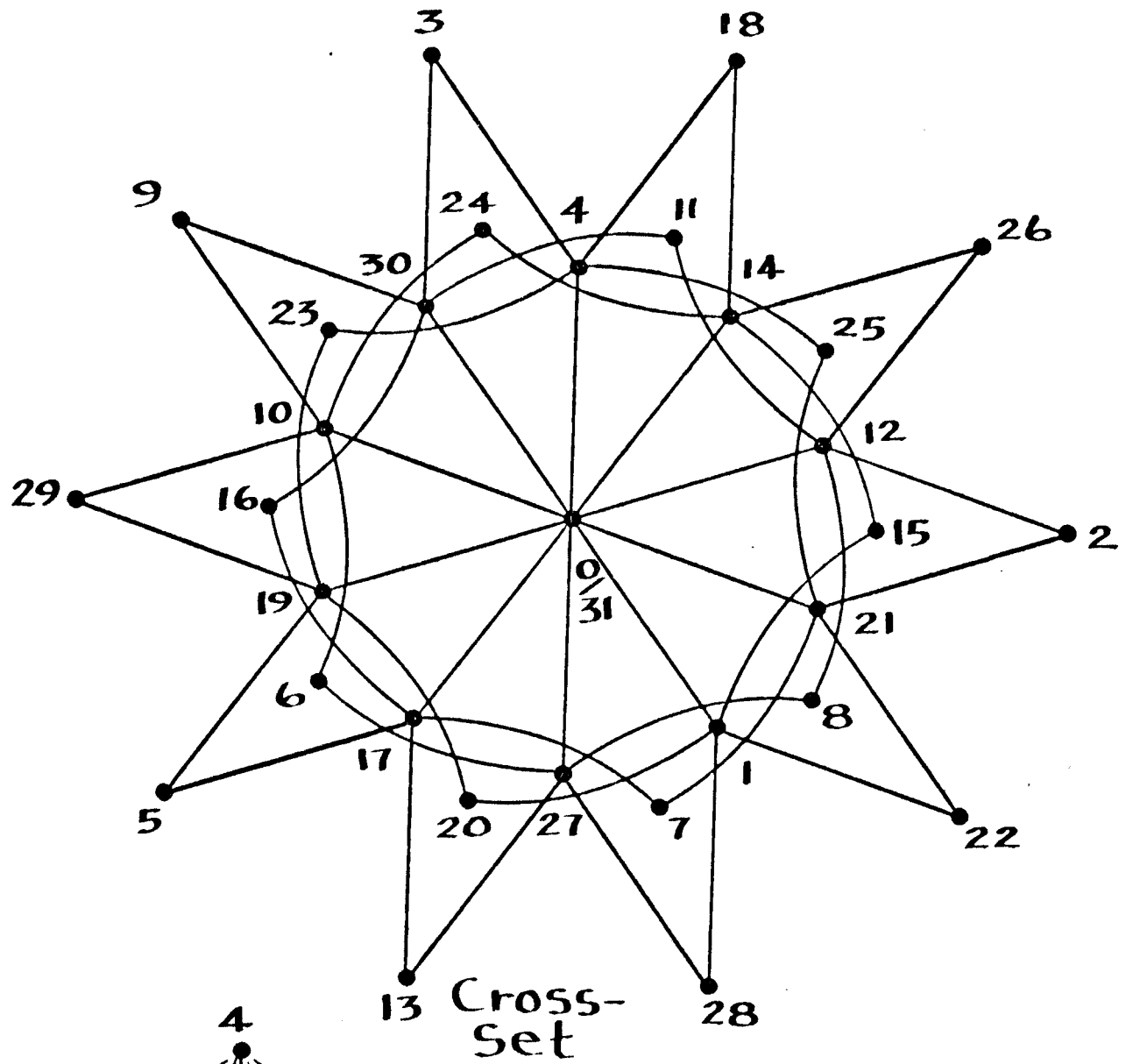
Ref: "The Crystalline Congruences"
by Walter O'Connell

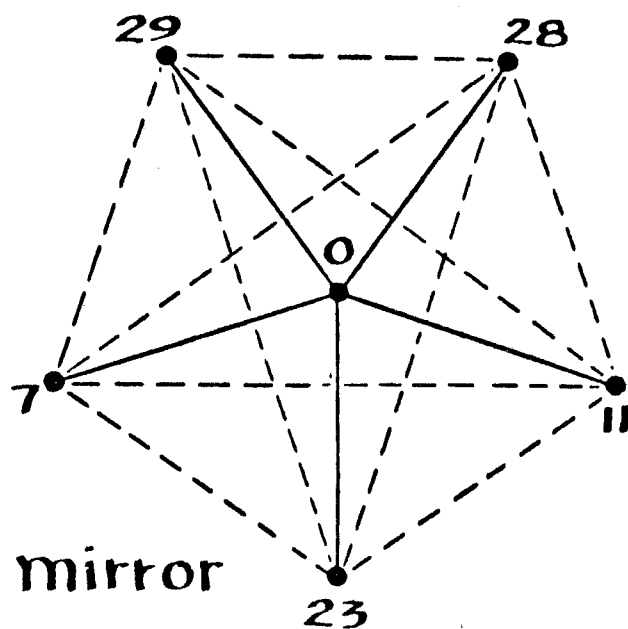
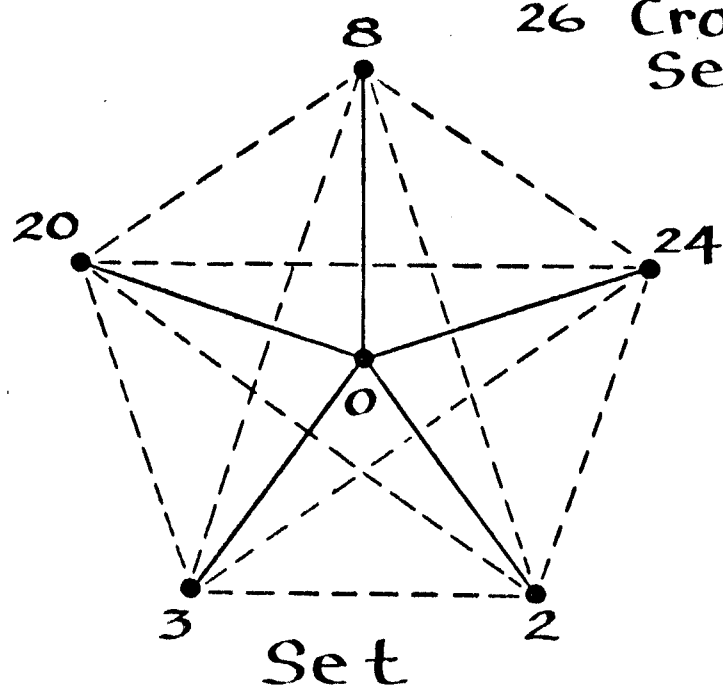
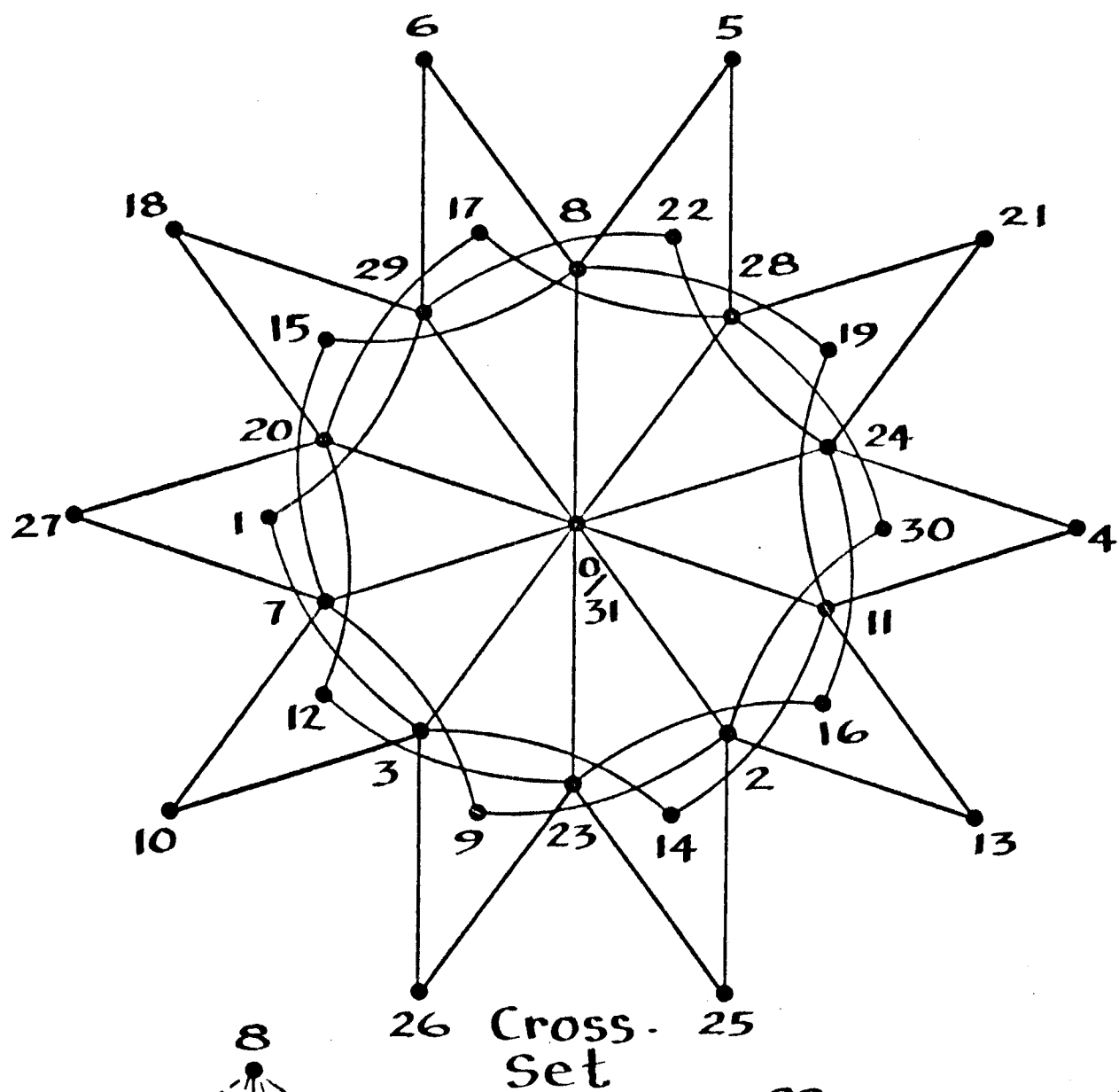






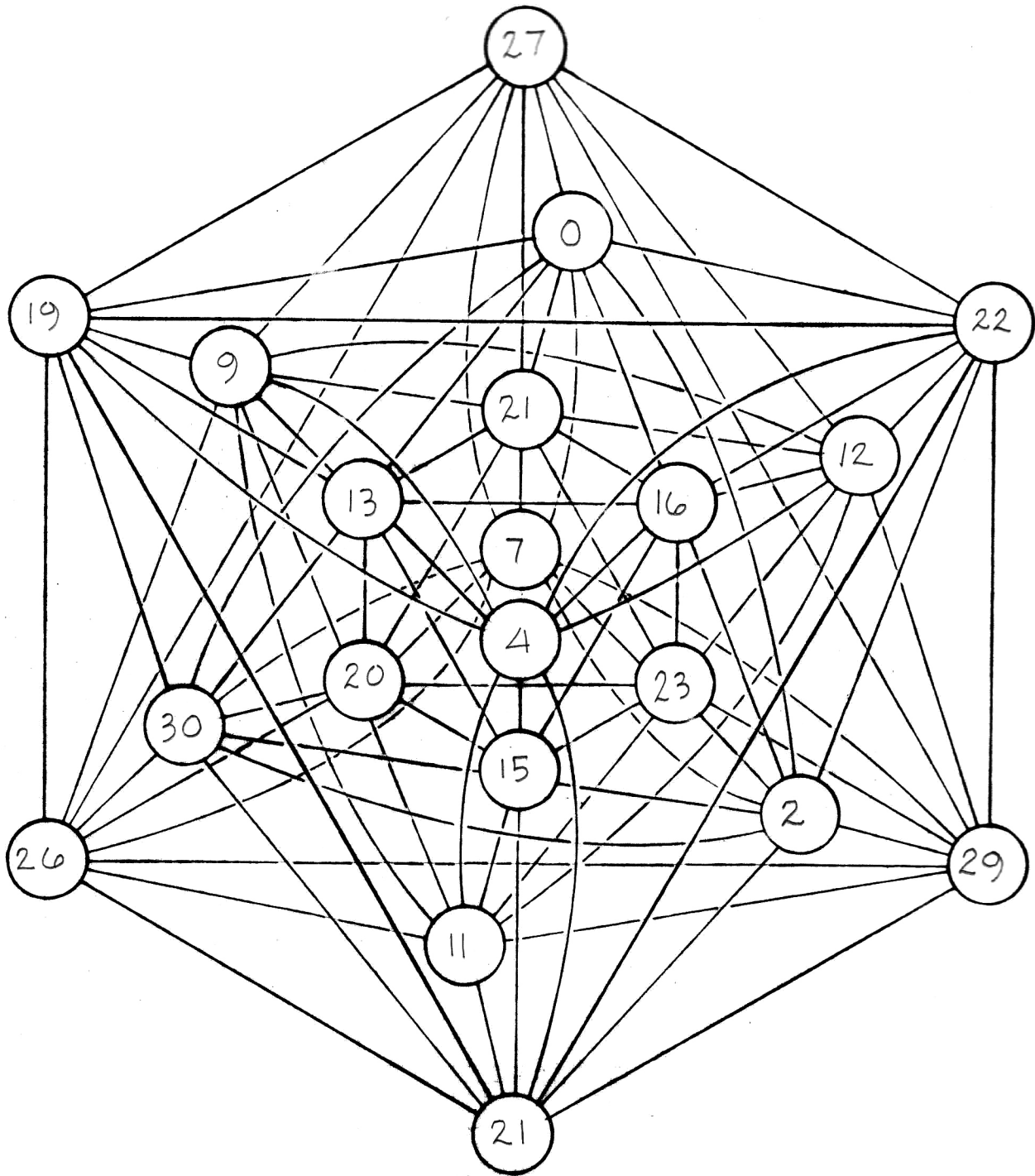




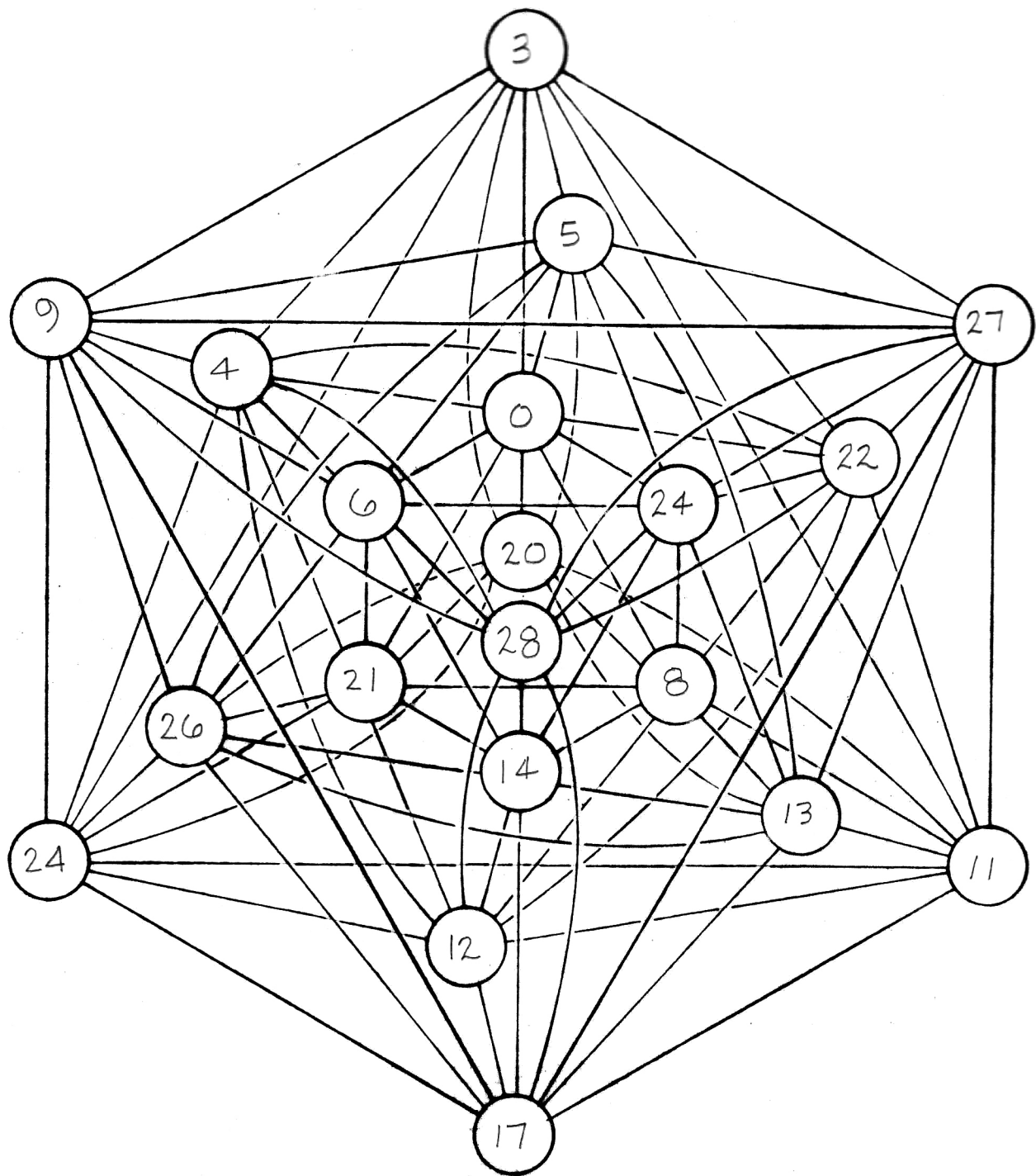


31T Eikosany on all-interval set

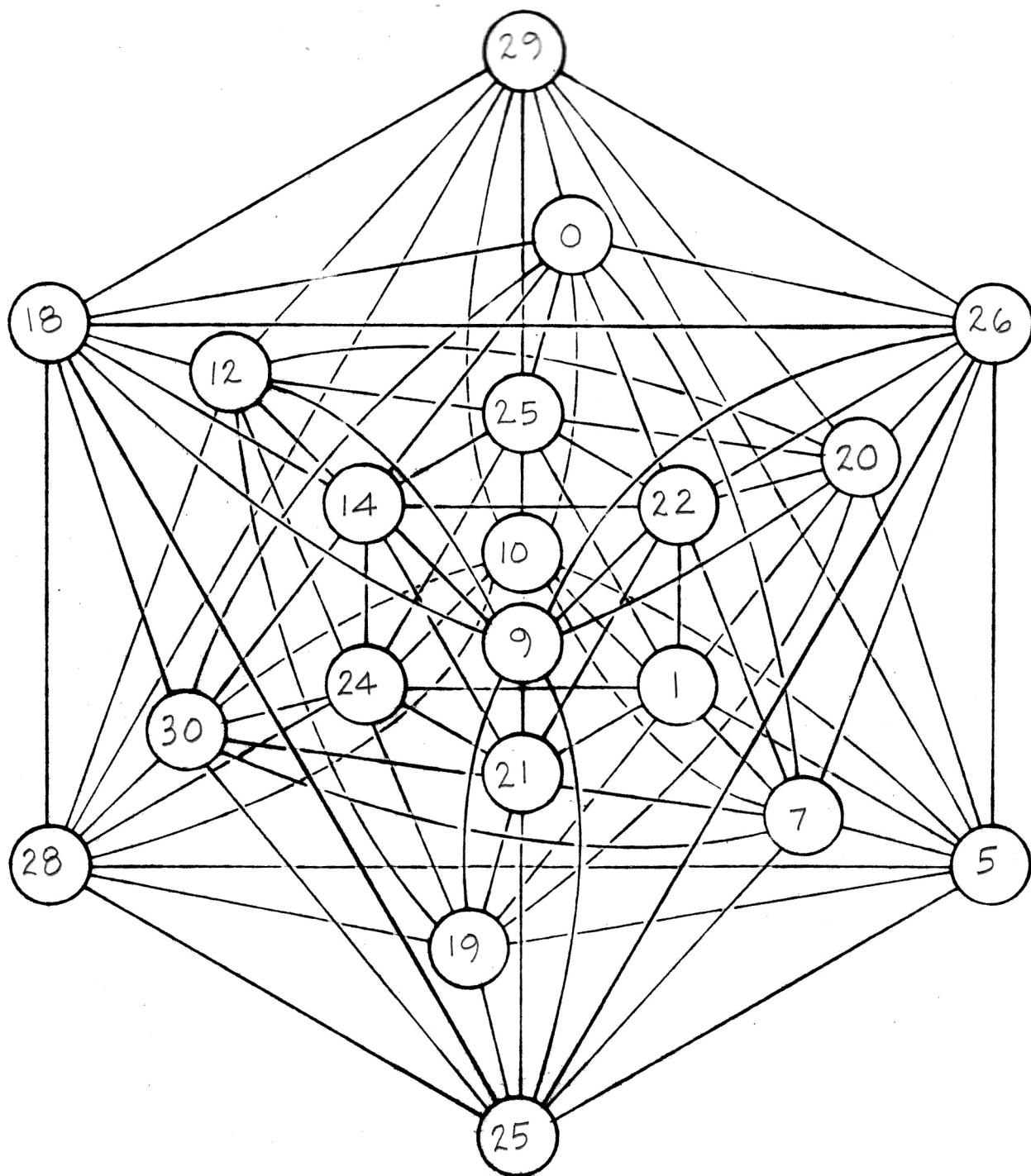
$\frac{1 \ 2 \ 5 \ 4 \ 6 \ 13}{0. \ 1. \ 3. \ 8. \ 12. \ 18}$



31T Eikosany on all-interval set $\frac{10\ 8\ 7\ 2\ 3\ 1}{0,\ 10,\ 18,\ 25,\ 27,\ 30}.$

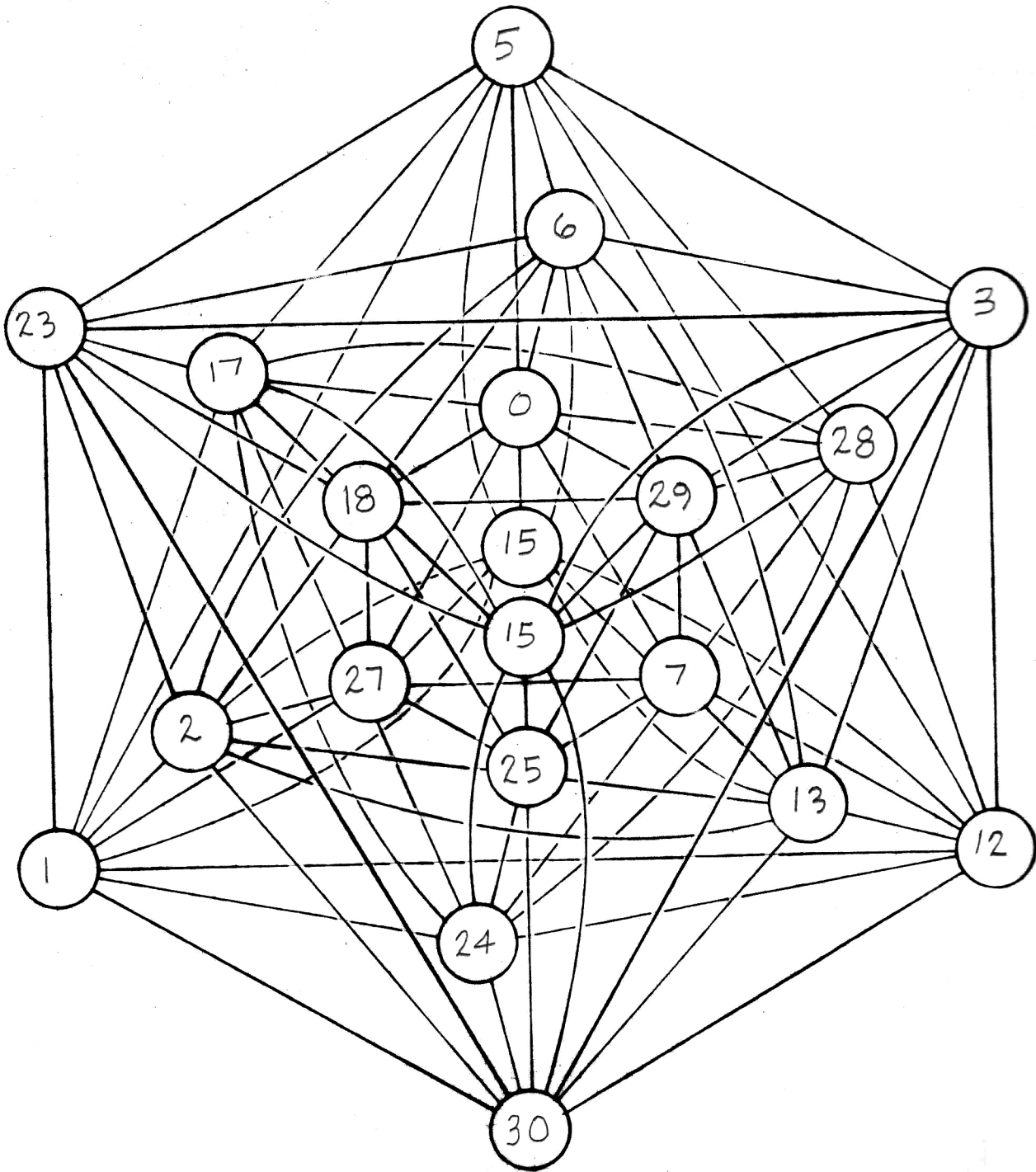


31T Eikosany on all-interval set $\frac{1 \ 7 \ 3 \ 2 \ 4 \ 14}{0. \ 1. \ 8. \ 11. \ 13. \ 17.}$

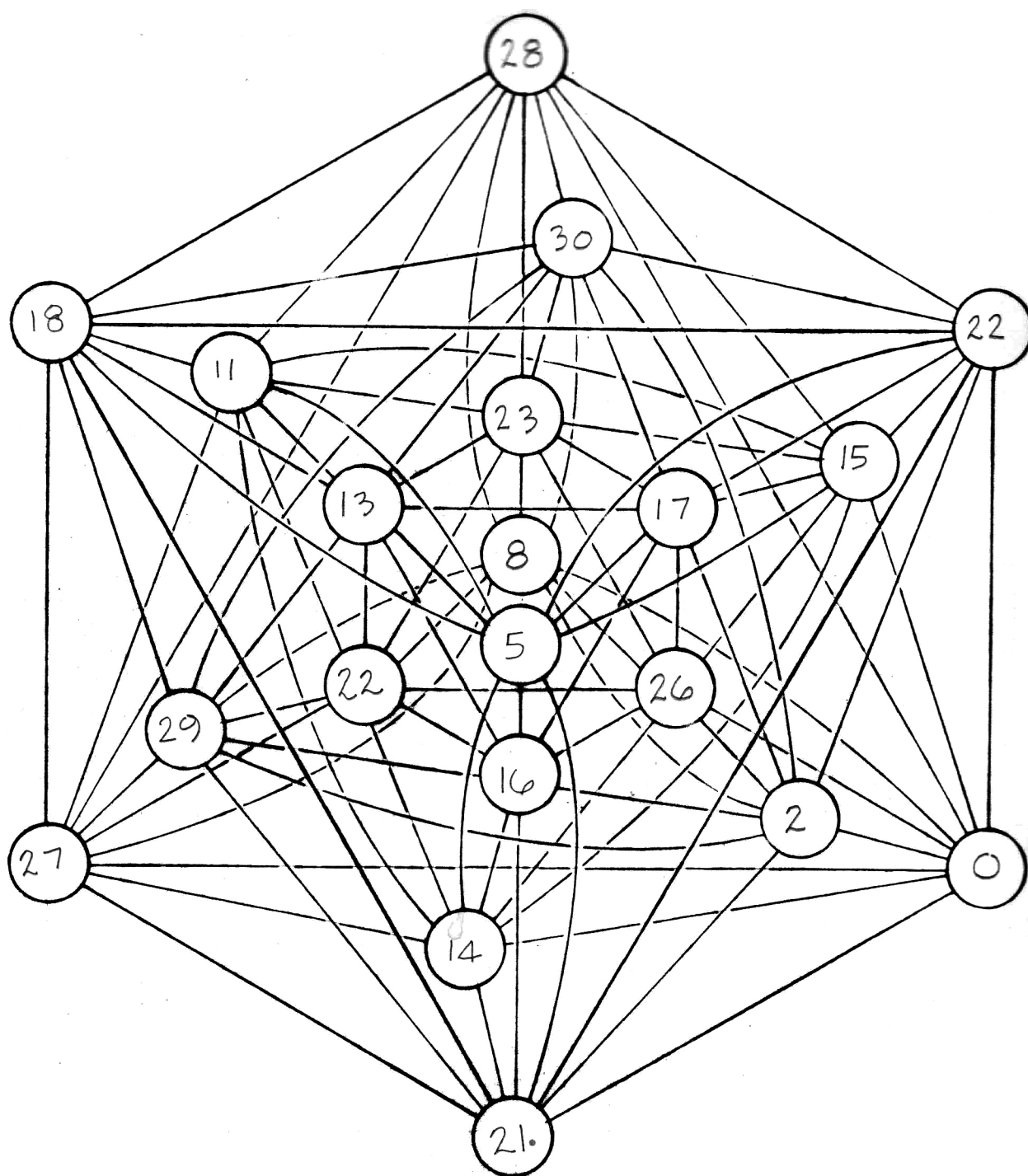


31T Eikosany on all-interval set

$\begin{array}{cccccc} 4 & 7 & 2 & 1 & 5 & 12 \\ \hline 0, & 4, & 11, & 13, & 14, & 19, \end{array}$

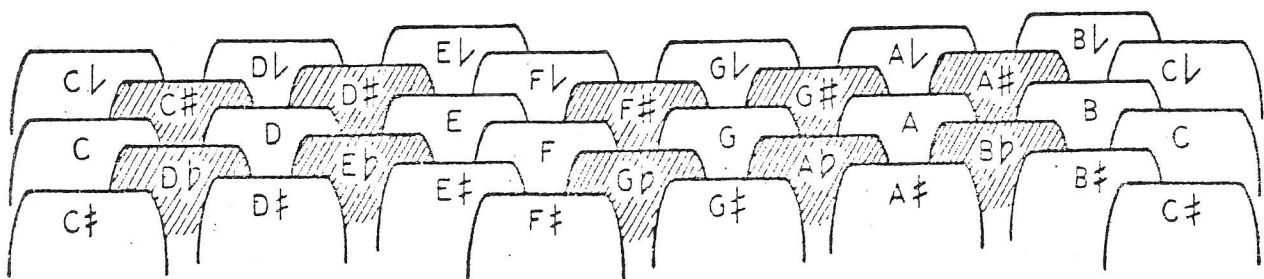
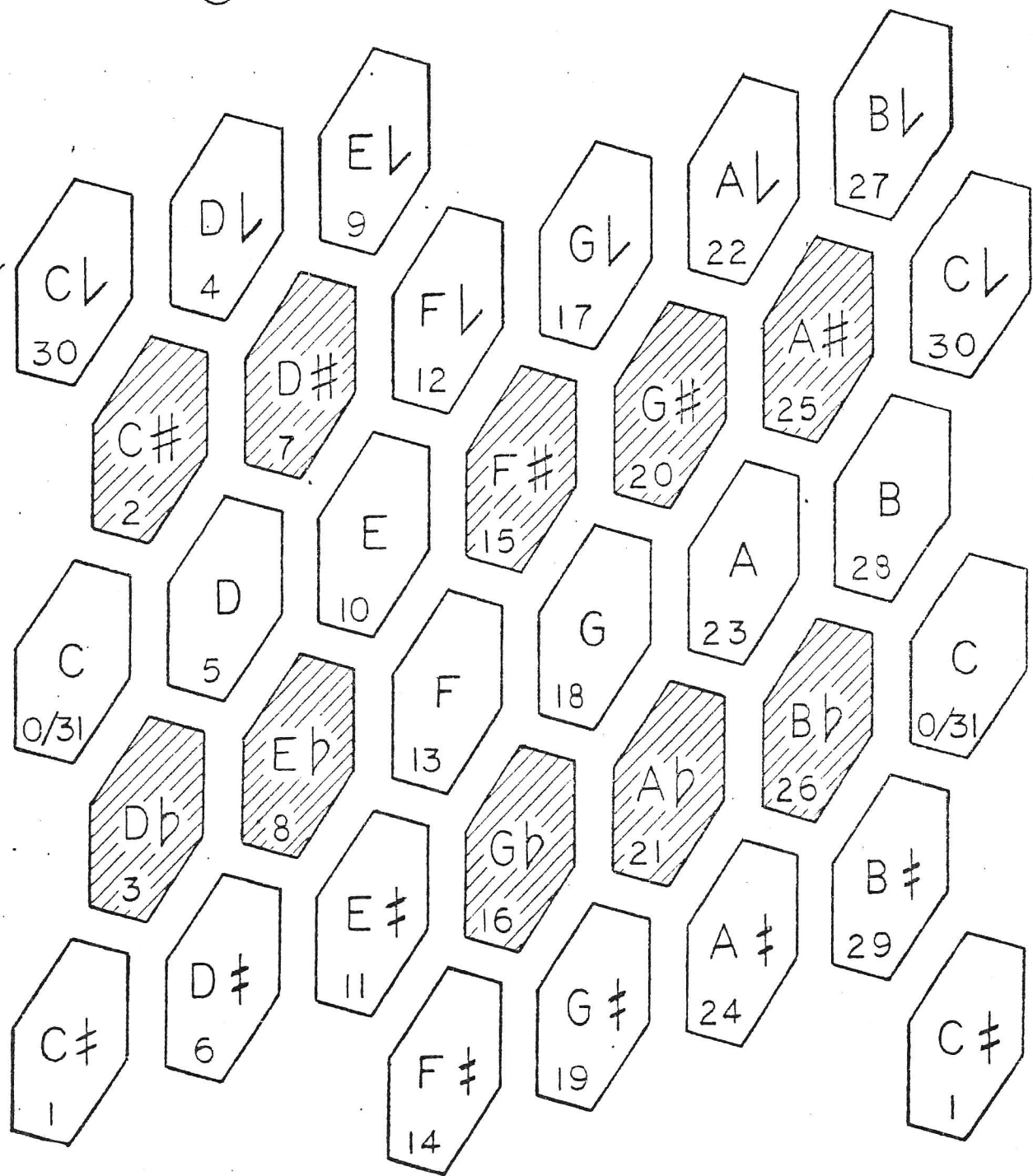


31T Eikosany on all-interval set $\frac{1\ 3\ 4\ 2\ 5\ 14.}{0.\ 1.\ 4.\ 10.\ 12.\ 17.}:$



Variation on a Keyboard by R. H. M. Bosanquet (60:24)
 Showing the 31-tone Scale

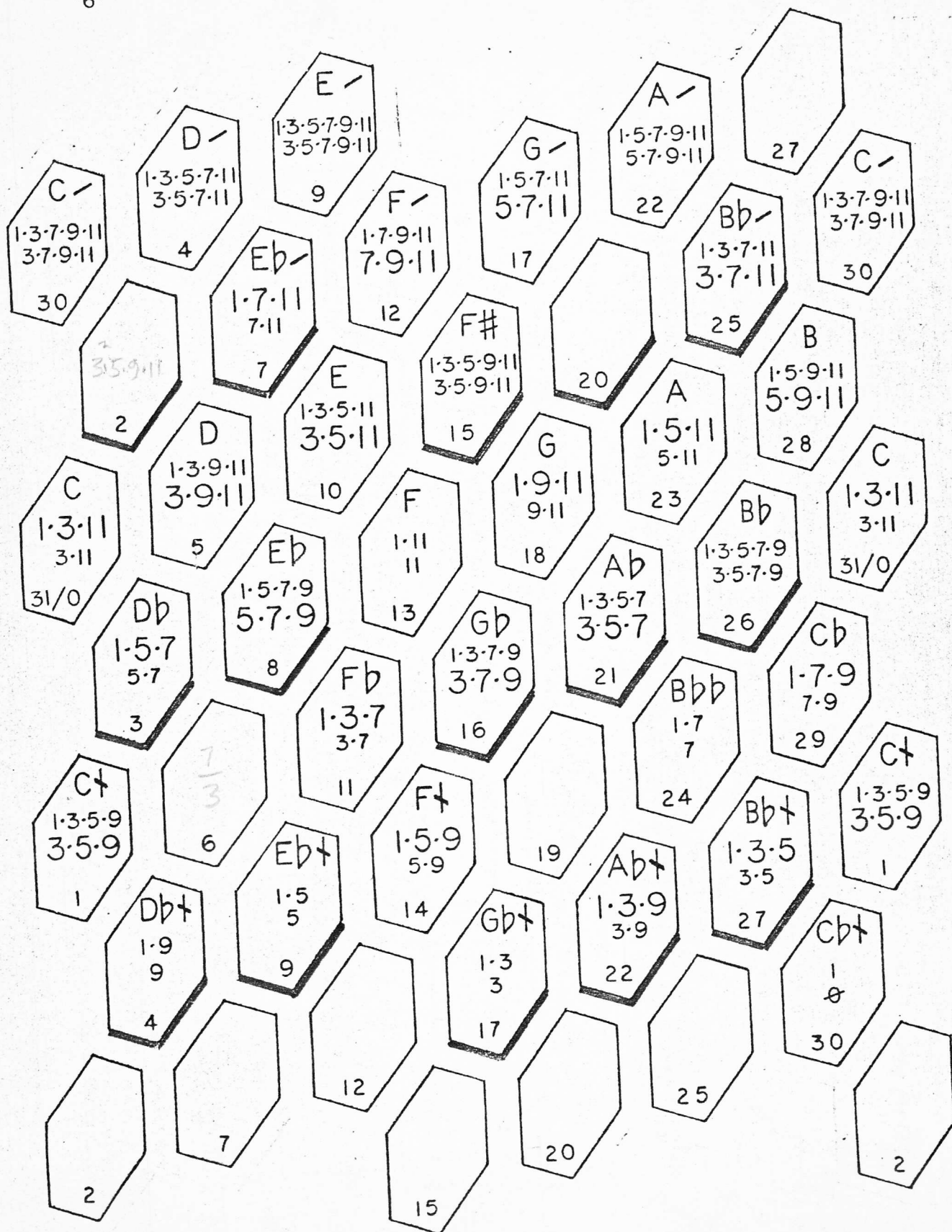
Design © 1974 by Ervin M. Wilson, all rights reserved.



Combination Sets ($\binom{0}{6}$) thru ($\binom{6}{6}$) 1.3.5.7.9.11
 Mapped over Modulus 31 and the Bosanquet Keyboard Scheme

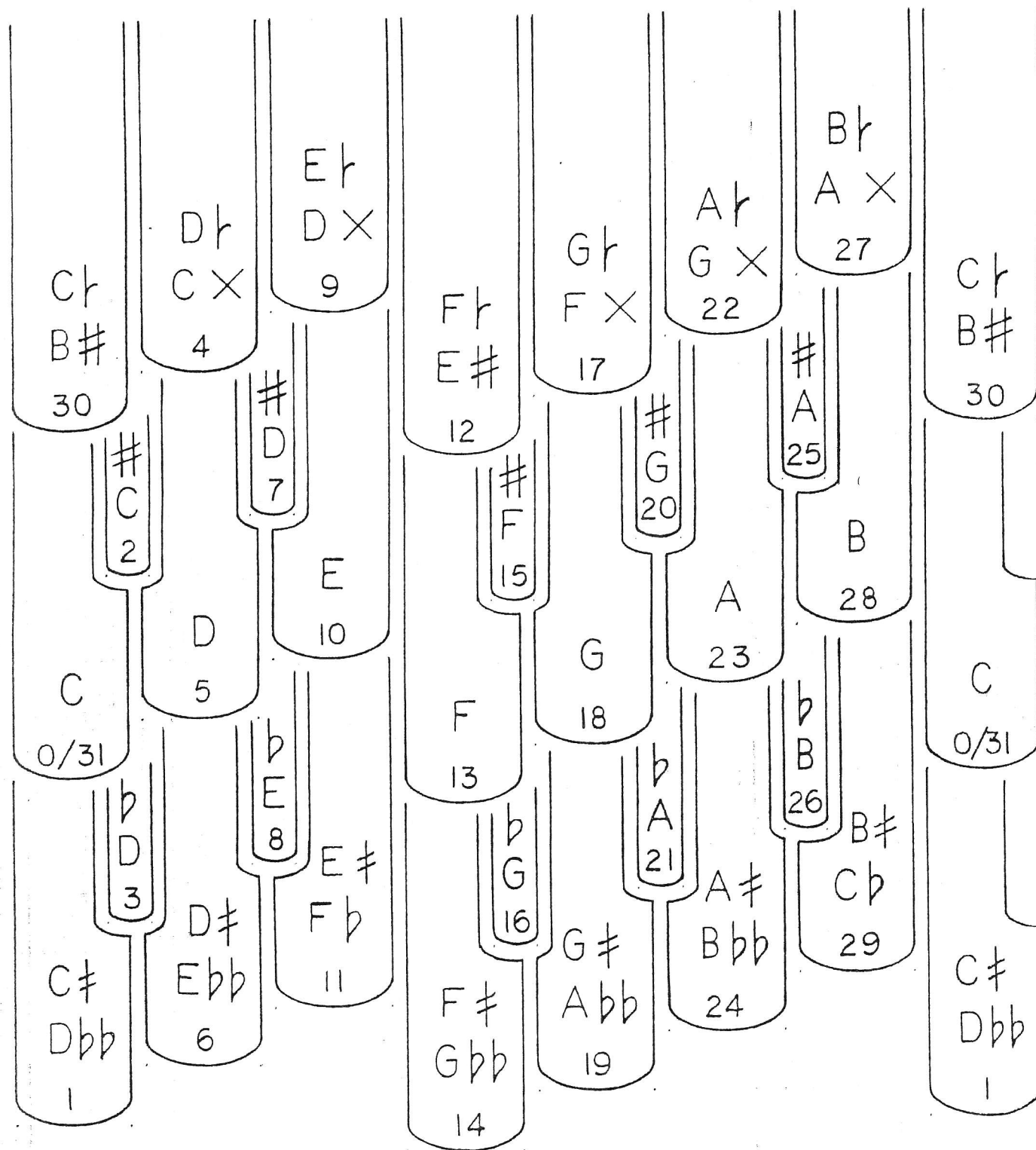
© 1975 by Ervin M. Wilson

($\binom{3}{6}$) 1.3.5.7.9.11 Eikosany shown in large numerals.

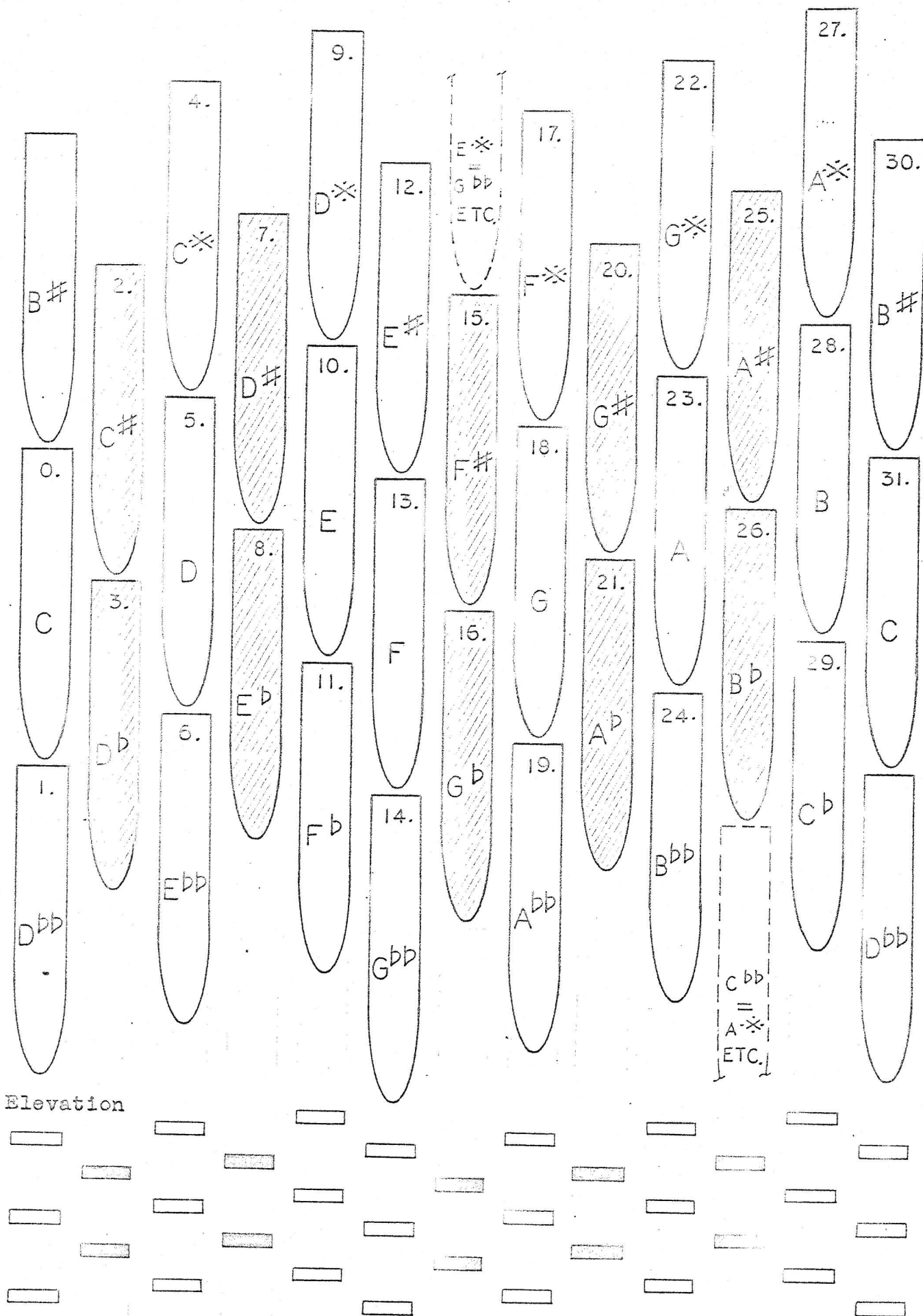


THE FABER-BOSANQUET KEYBOARD
(Showing the 31-tone Scale)

Design © 1974 by Ervin M. Wilson

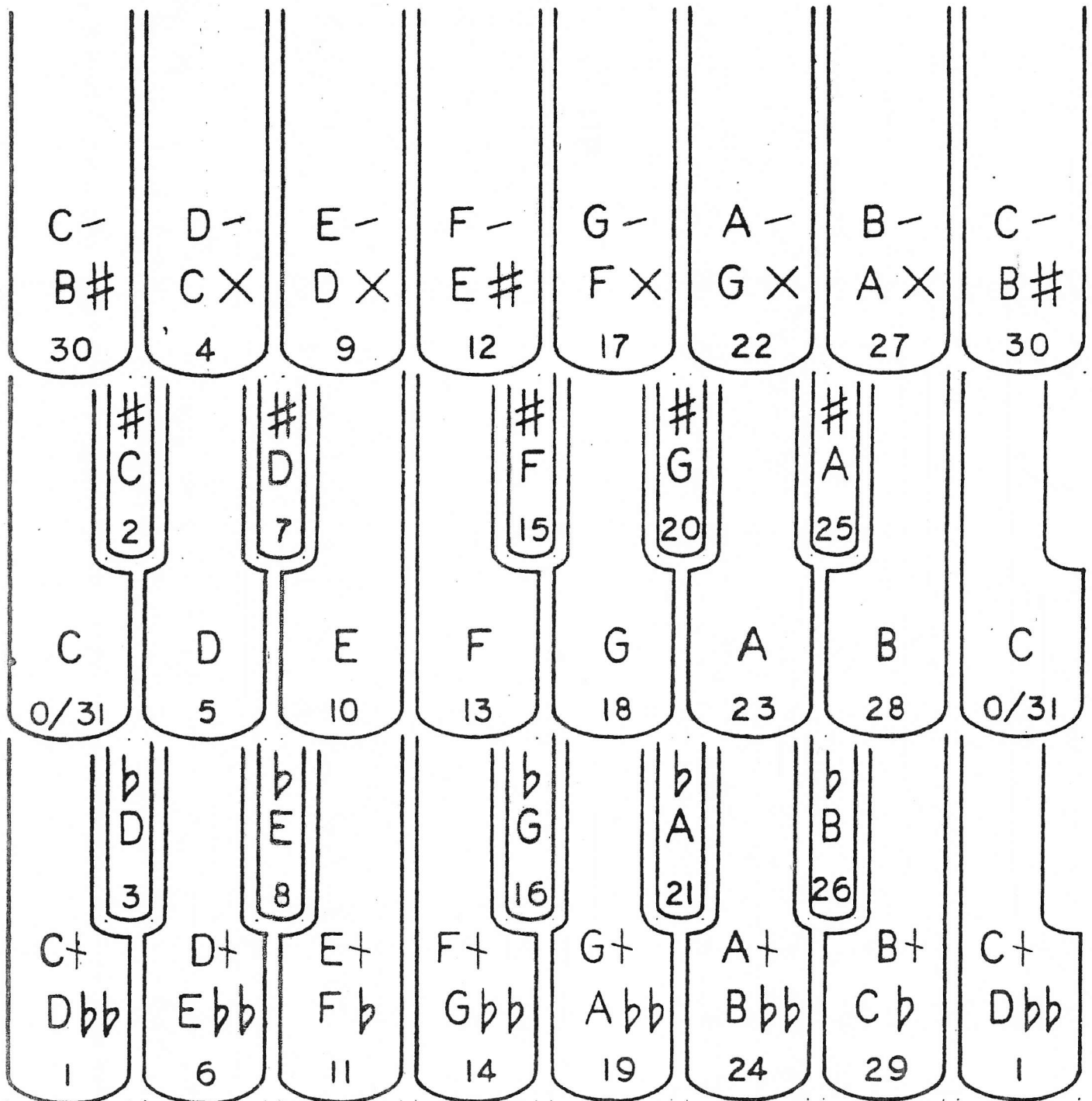


THE 31-TONE SCALE APPLIED TO BOSANQUET'S GENERALIZED KEYBOARD



7-5-7-5-7 KEYBOARD WITH 31-TONE SCALE

© 1974 BY ERVIN M. WILSON



U.S. PATENT NO. 3012460

FIG VI

	4.	9.	12.	17.	22.	27.	30.
B #	C ·✕·	D ·✕·	E #	F ·✕·	G ·✕·	A ·✕·	B #
c r	D r	E r	F r	G r	A r	B r	c r
	2.	7.		15.	20.	25.	
	C #	D #		F #	G #	A #	
	D r	E r		G r	A r	B r	
0.	5.	10.	13.	18.	23.	28.	31.
C	D	E	F	G	A	B	C
	3.	8.		16.	21.	26.	
	D b	E b		G b	A b	B b	
	c f	D f		F f	G f	A f	
1.	6.	11.	14.	19.	24.	29.	
D b b	E b b	F b	G b b	A b b	B b b	C b	D b b
c +	D +	E +	F +	G +	A +	B +	c +

Numerals merely indicate ascending panchromatic sequence.
 Standard notation is shown in large print.
 Exotic alteration is shown in type.

Example: 0. 1. 2. 3. 4. 5.
 C c + C # c f C ·✕· D
 C D b b D r D b D r D

+ = hard; f = acute; r = grave; r = soft

31-TONE PIPE ORGAN, Ref Fokker/Janko Keyboard

© 2006 by Ervin M. Wilson

	30.	4.	9.	14.	19.	24.	29.	3.	8.	
B	28.	2.	7.	12.	17.	22.	27.	1.	6.	11.
C	D	E								
0/31.	5.	10.	15.	20.	25.	30.	4.	9.		
29.	3.	8.	F	G	A	B				
			13.	18.	23.	28.	2.	7.	12.	
	1.	6.	11.	16.	21.	26.	C	D	E	
							0/31.	5.	10.	
30.	4.	9.	14.	19.	24.	29.	3.	8.	F	
									13.	

$$3 \times 19 = 57$$

$$57 \times 2 = 114$$

(ref Plamondon)

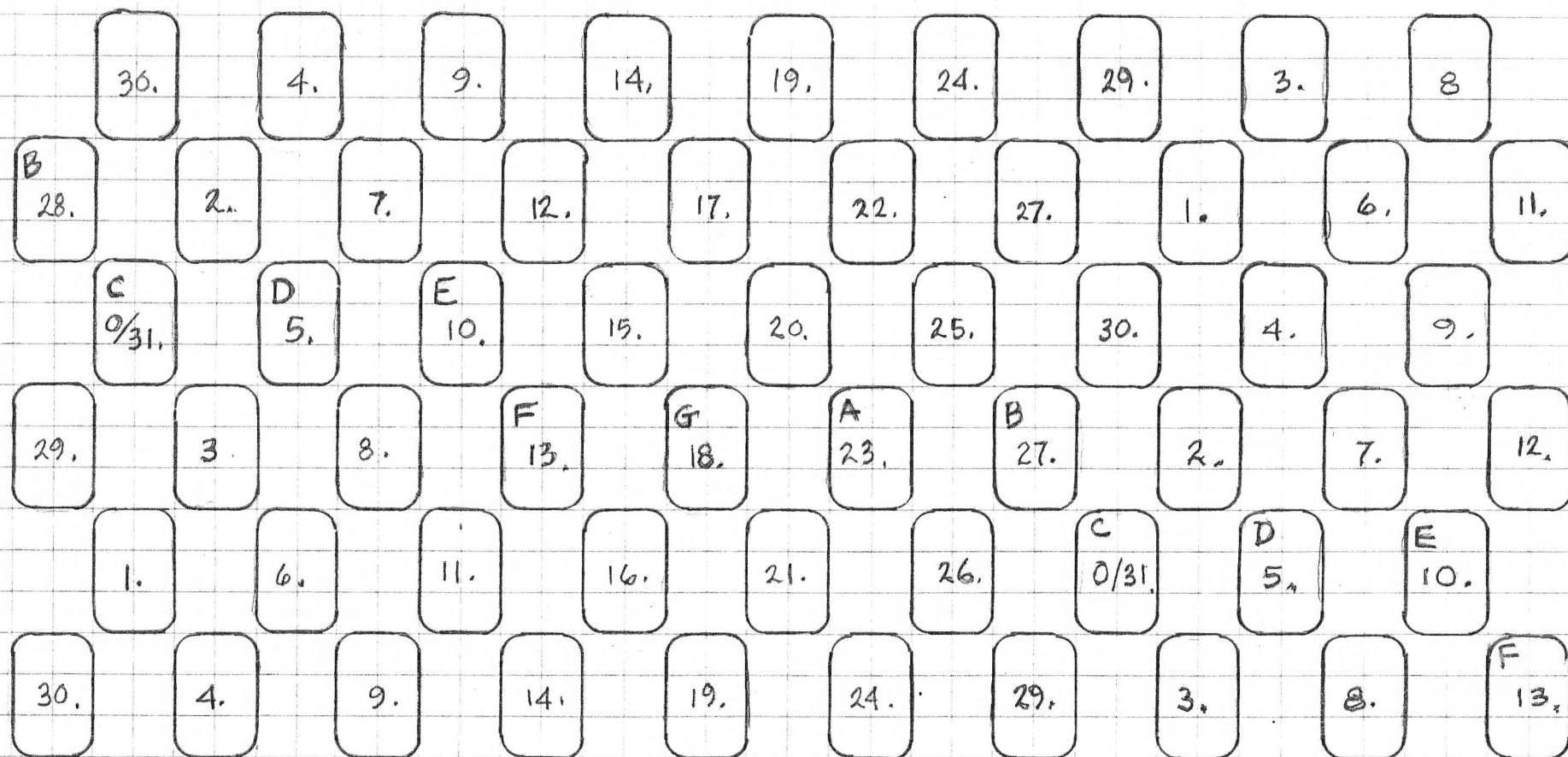
Janko

22Feb06.EW

31-TONE PIPE ORGAN, Ref Fokker/Janko Keyboard

© 2006 by Ervin M. Wilson

Digereedoo Tubes &
Australasian Gamelans



Alpine Pipes &
Harmonic Yodel

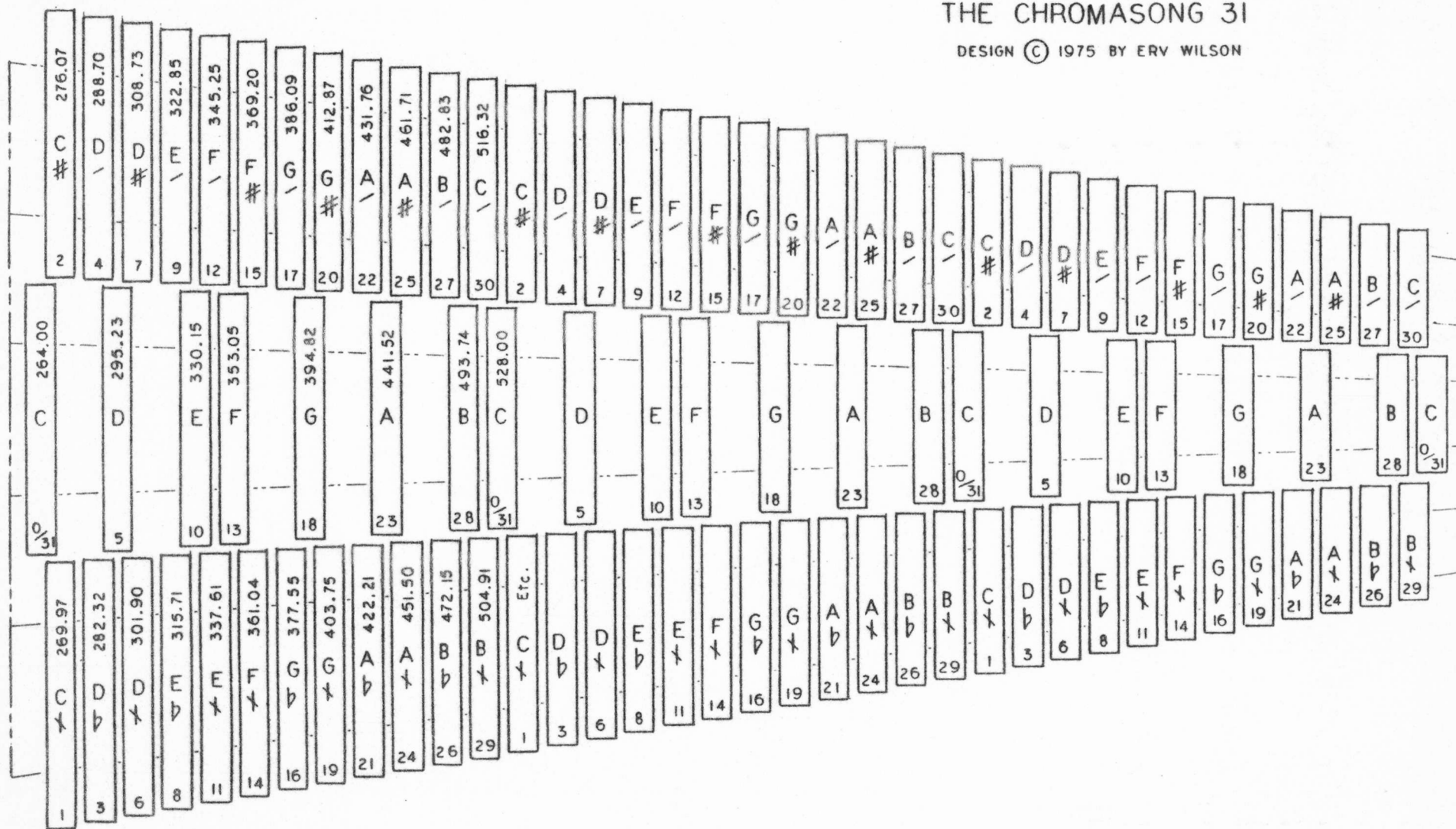
$$3 \times 19 = 57$$

$$57 \times 2 = 114$$

JANKO

THE CHROMASONG 31

DESIGN © 1975 BY ERV WILSON

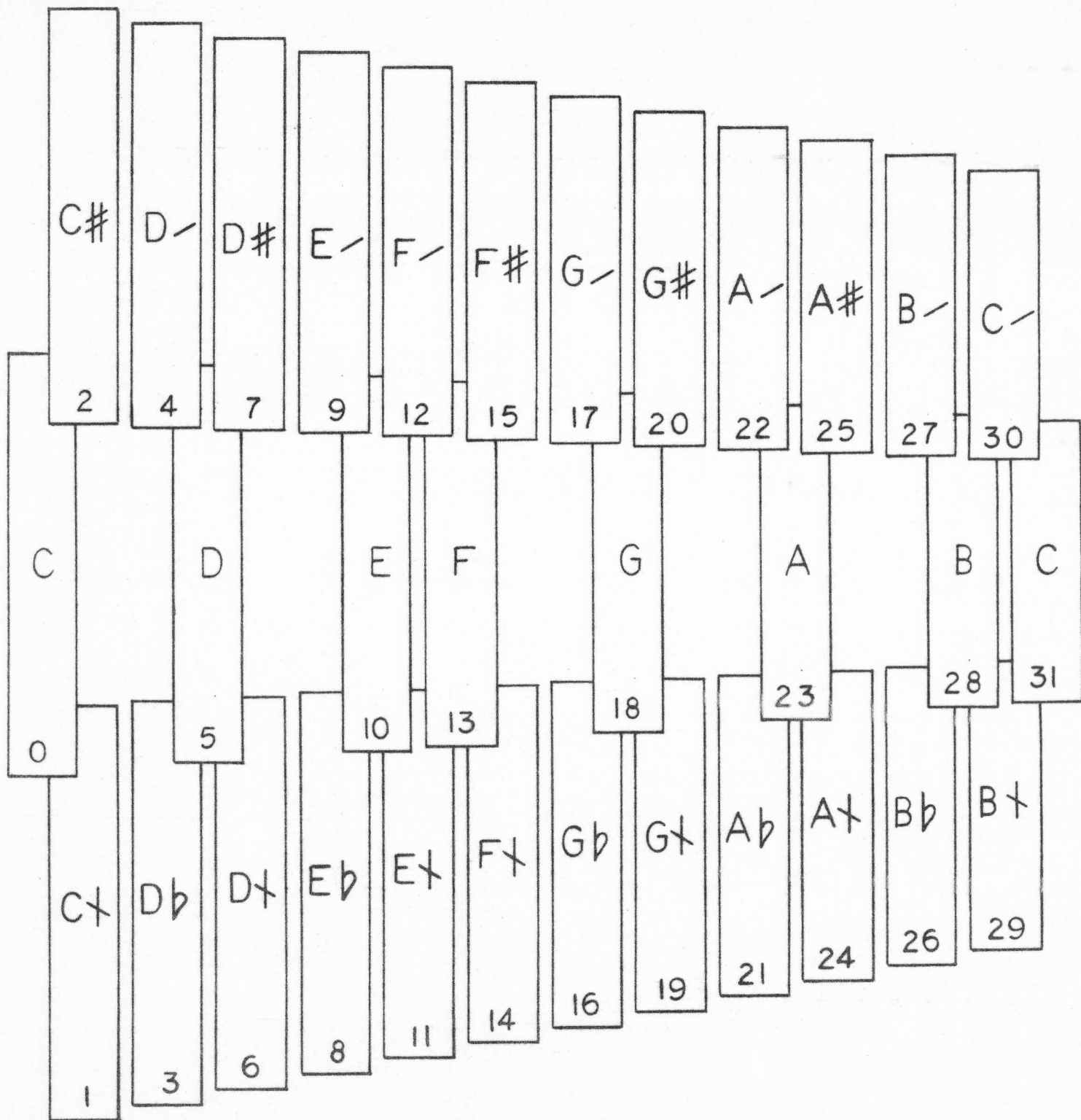


THE 31-TONE TUBULONG

DESIGN © 1975 BY ERV WILSON

28	B	30	\ C
29	+ B	0	C 264.00
		1	+ C 269.97
		2	# C 276.07
		3	∇ D 282.32
		4	\ D 288.70
		5	D 295.23
		6	+ D 301.90
		7	# D 308.73
		8	∇ E 315.71
		9	\ E 322.85
		10	E 330.15
		11	+ E 337.61
		12	\ F 345.25
		13	F 353.05
		14	+ F 361.04
		15	# F 369.20
		16	∇ G 377.55
		17	\ G 386.09
		18	G 394.82
		19	+ G 403.75
		20	# G 412.87
		21	∇ A 422.21
		22	\ A 431.76
		23	A 441.52
		24	+ A 451.50
		25	# A 461.71
		26	∇ B 472.15
		27	\ B 482.83
		28	B 493.74
		29	+ B 504.91
		30	\ C 516.32
		31	C 528.00

Pitches to base C264 are rounded-off from tables by John Chalmers

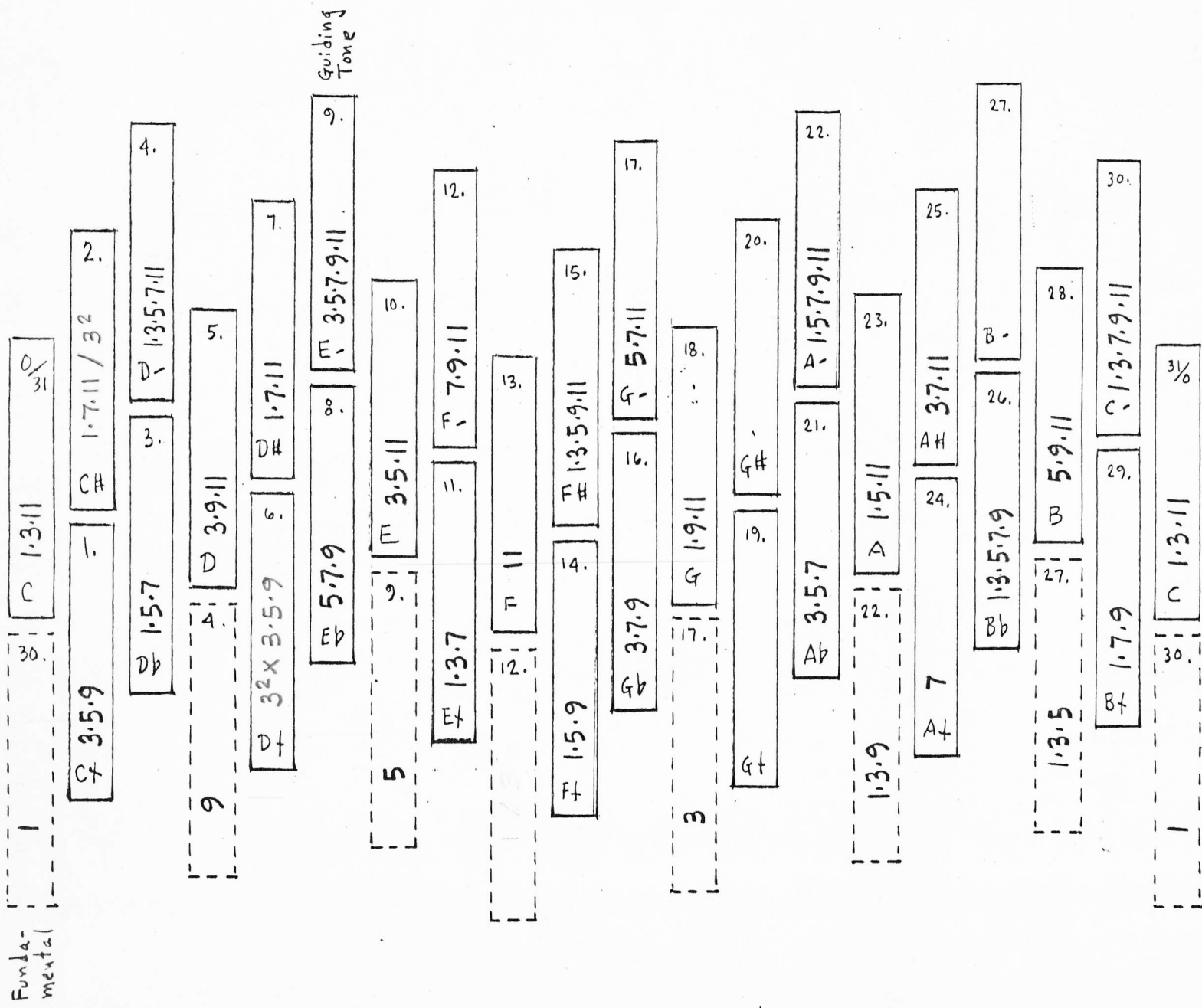


THE 31-TONE MARIMBA
DESIGN © 1975 BY ERV WILSON

© 1977 Erv Wilson

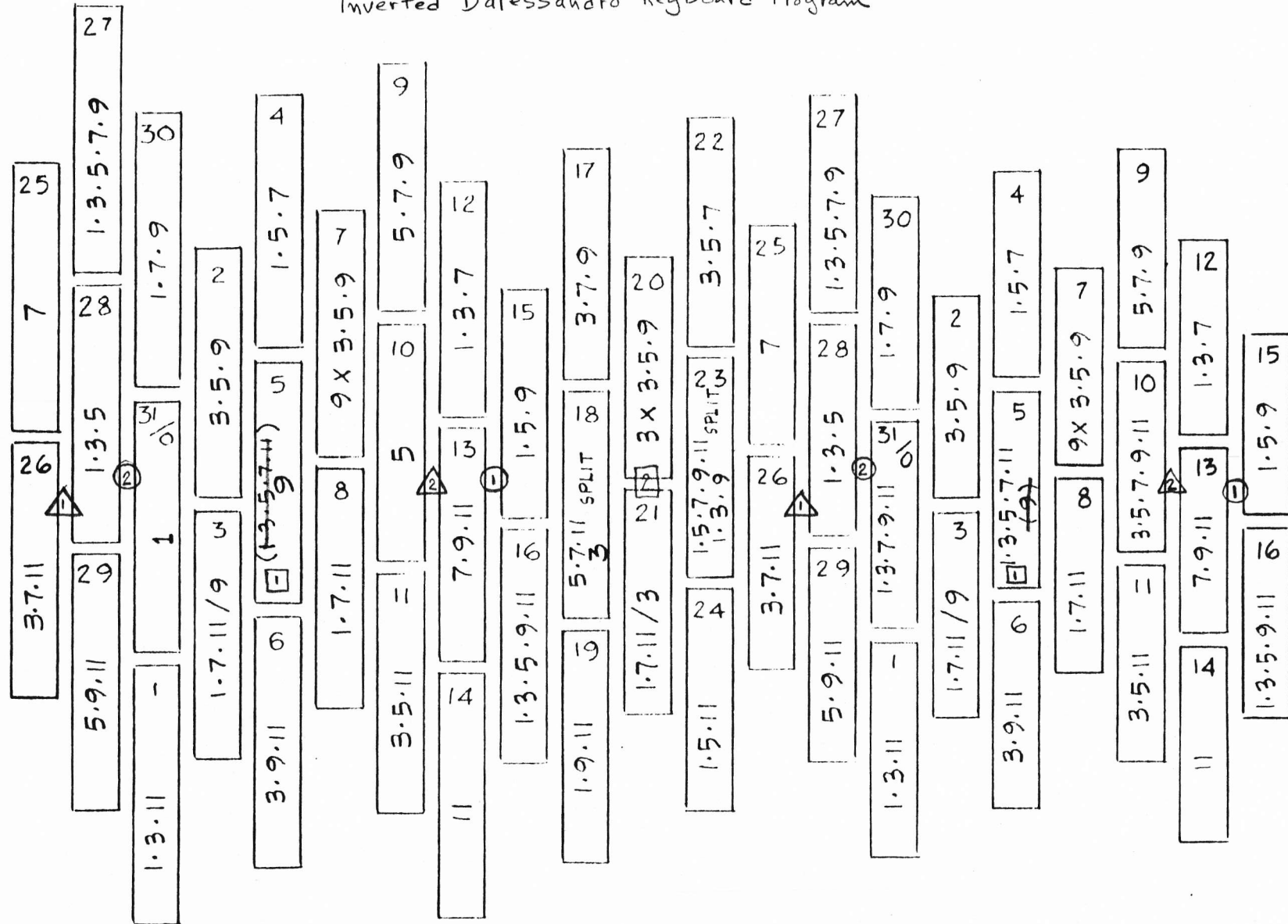
© 1984 by Eric Wilson

Generalized Pattern for Bass Marimba
Tubulms, showing modulus 31
Design ©. 1983 by. Eric Willson



Marimba design
 © 1986 by Erv Wilson
 Inverted D'alexandro Keyboard Program

861210
 Fig 3



□ & □ = Symetry Points of 1.3.5.7.9.11 Eikosany
 ① & ② = Symetry Points of 1.3.7.9.11.15 Eikosany
 △ & △ = Symetry Points of 1.3.7.9.11.15 ÷ 3 Eikosany

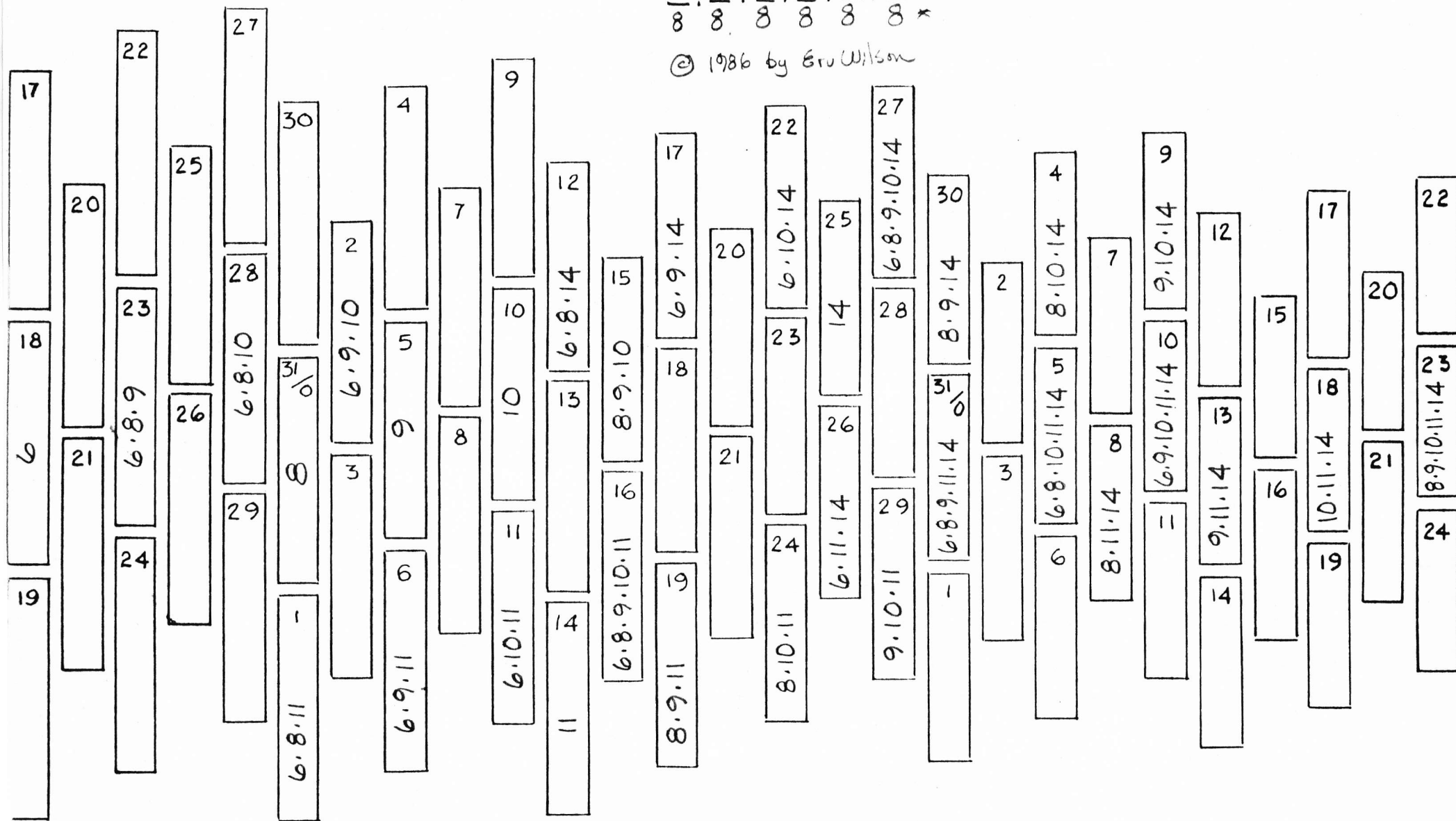
Literal Combination Product Array

861210

Fig 4

$$\frac{6}{8} \cdot \frac{8}{8} \cdot \frac{9}{8} \cdot \frac{10}{8} \cdot \frac{11}{8} \cdot \frac{14}{8} *$$

© 1986 by Gru Wilson

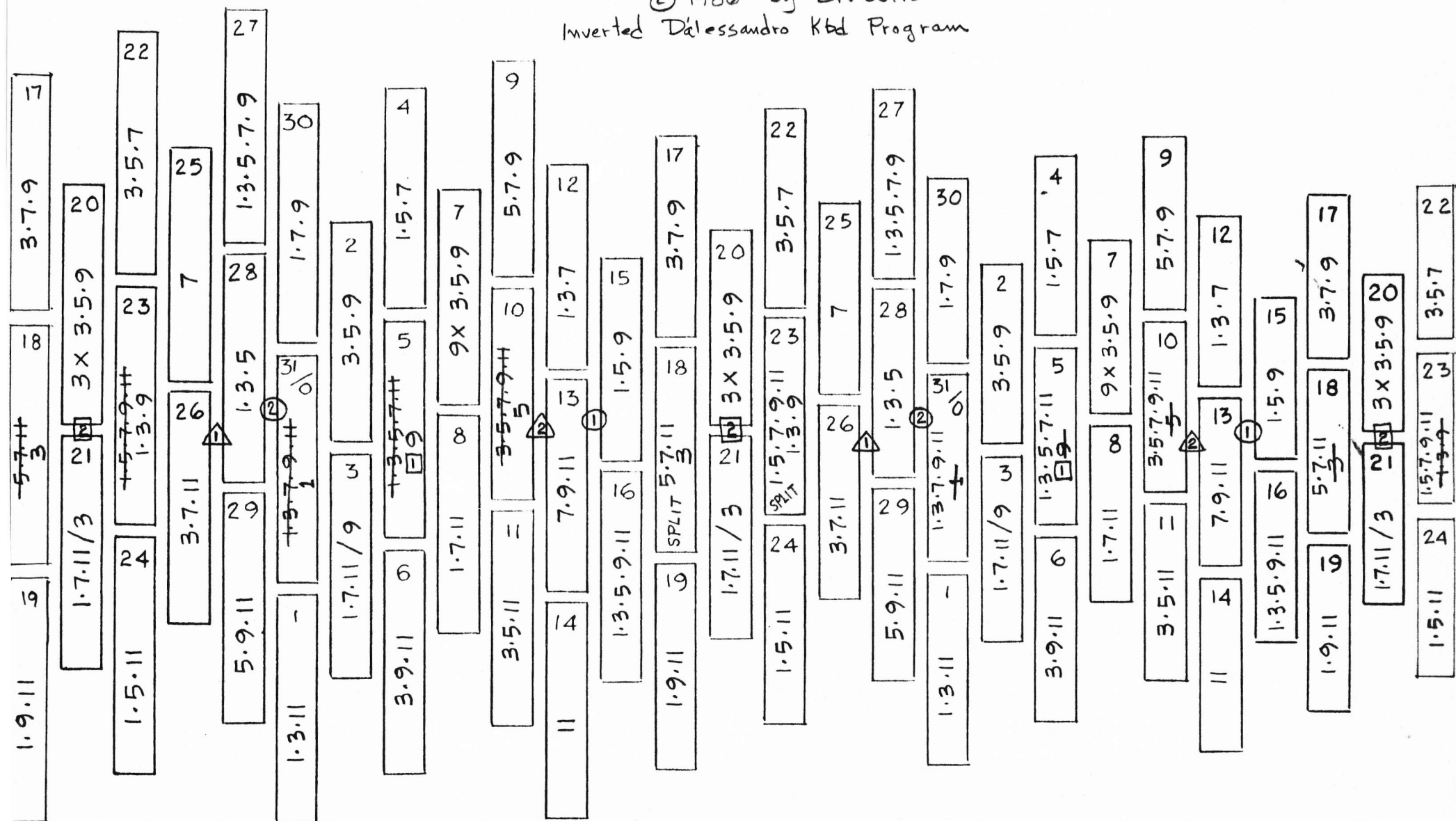


* g understood

861210

Fig 5

Marimba design

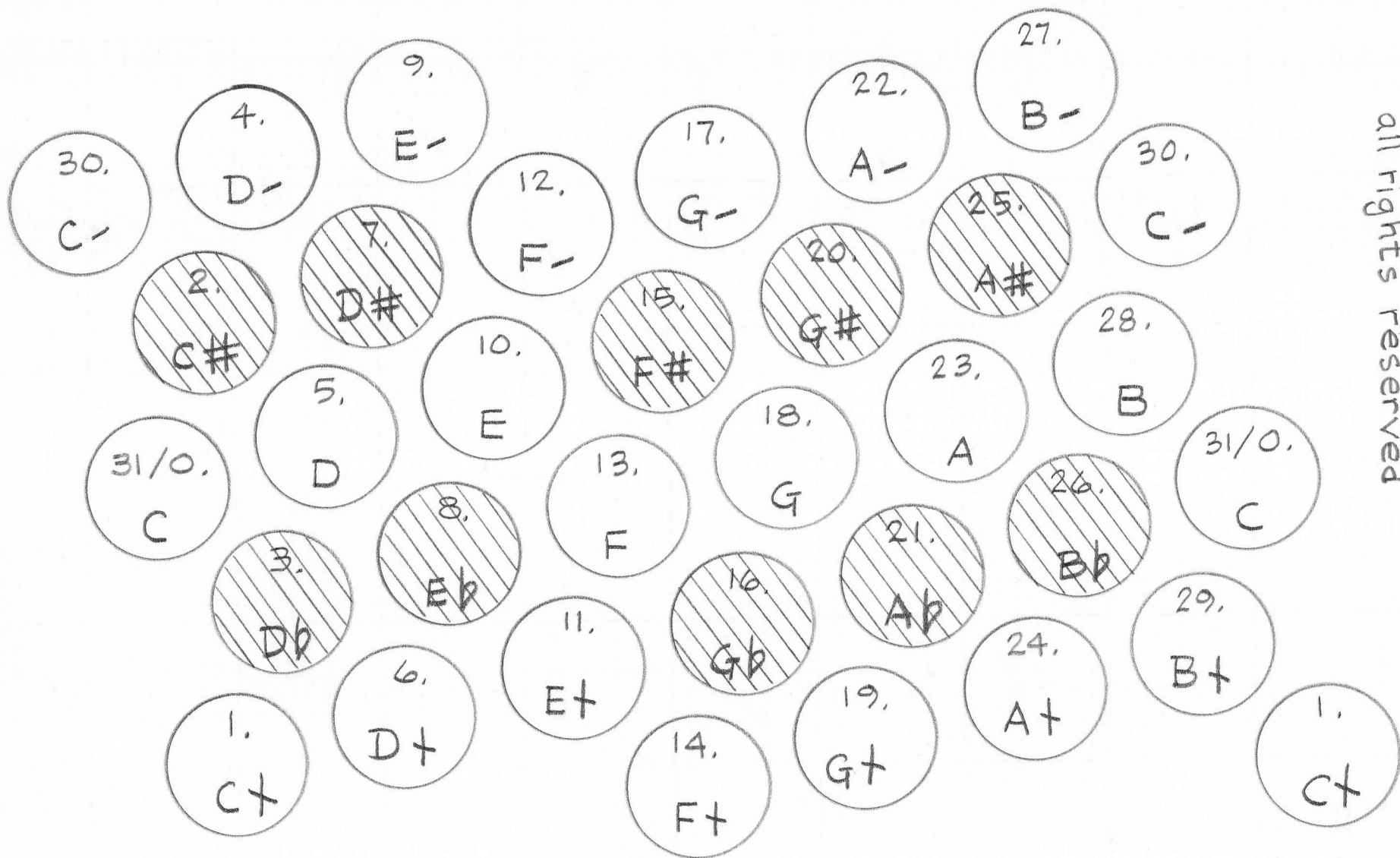
© 1986 by Erv Wilson
Inverted Dalessandro Kbd Program

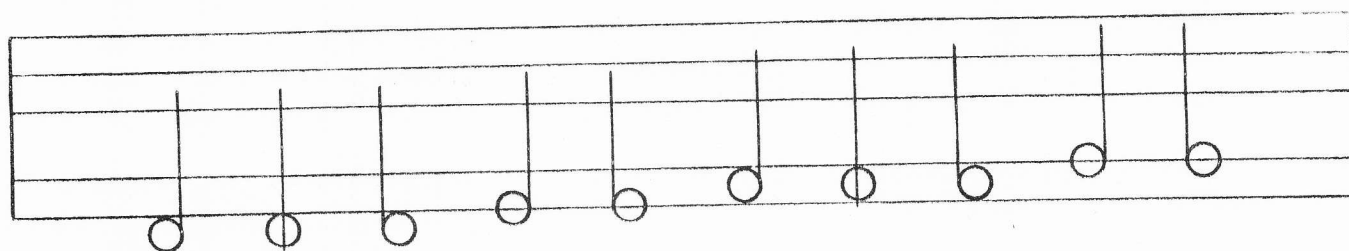
□ & ② = Symetry points of 1.3.5.7.9.11 Eikosany
 ○ & ② = Symetry Points of 1.3.7.9.11.15 Eikosany
 △ & ② = Symetry Points of 1.3.7.9.11.15 ÷ 3 Eikosany

Microtales-31

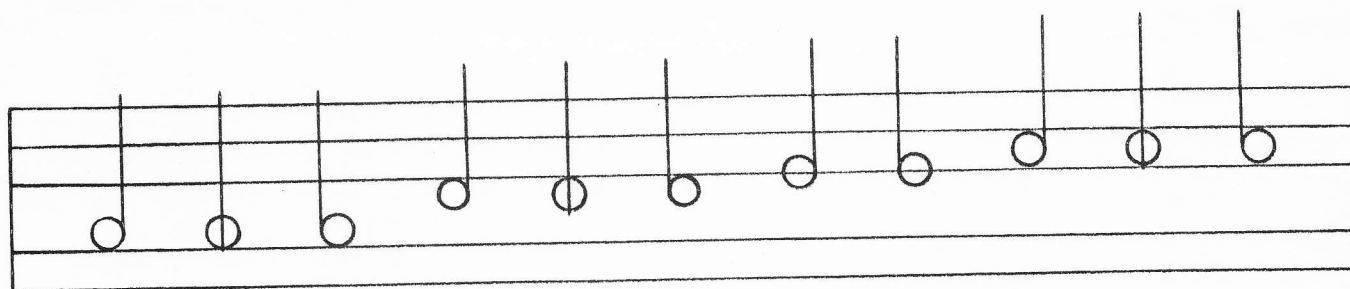
Design © 1978 by Erv Wilson

all rights reserved

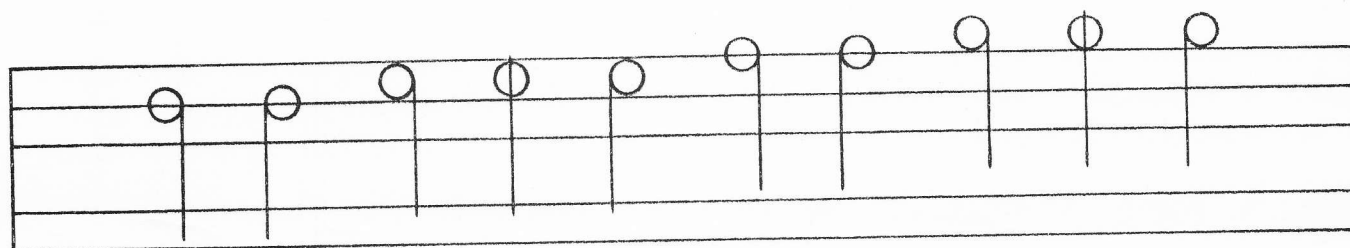




30 31-0 1 2 3 4 5 6 7 8
RU RA RI FO FE MU MA MI TO TE



9 10 11 12 13 14 15 16 17 18 19
ZU ZA ZI GU GA GI HO HE VU VA VI



20 21 22 23 24 25 26 27 28 29
KO KE DU DA DI SO SE PU PA PI

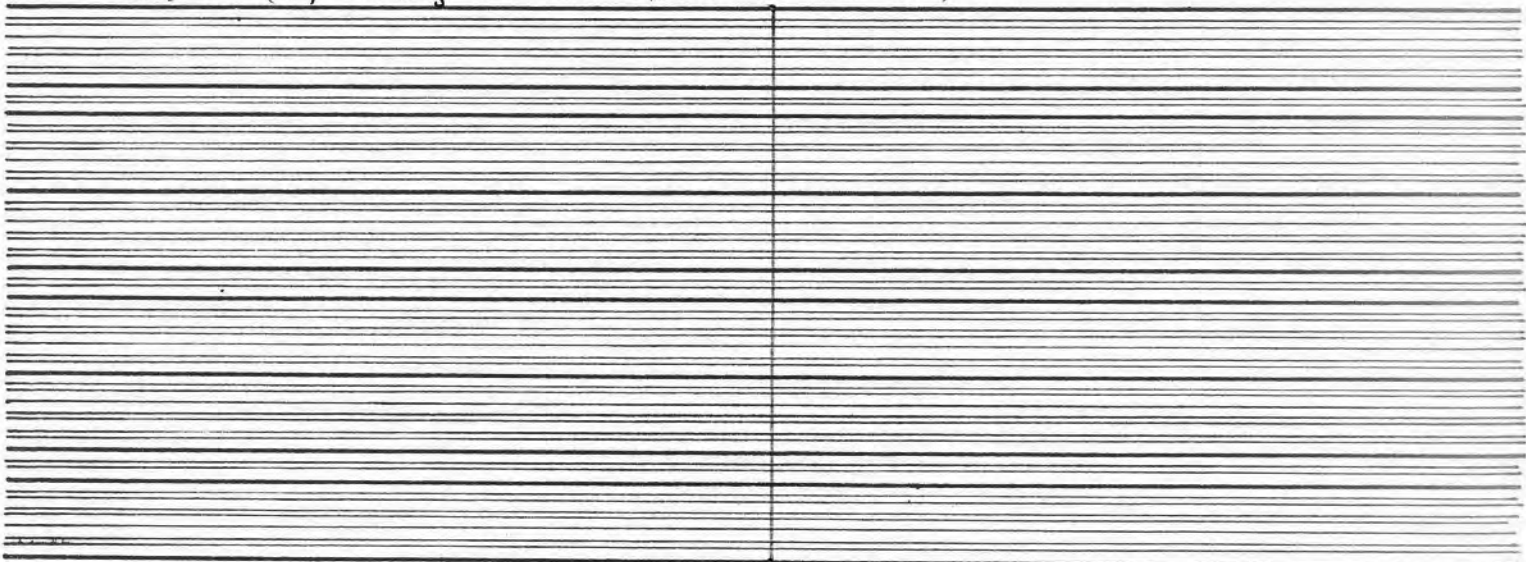
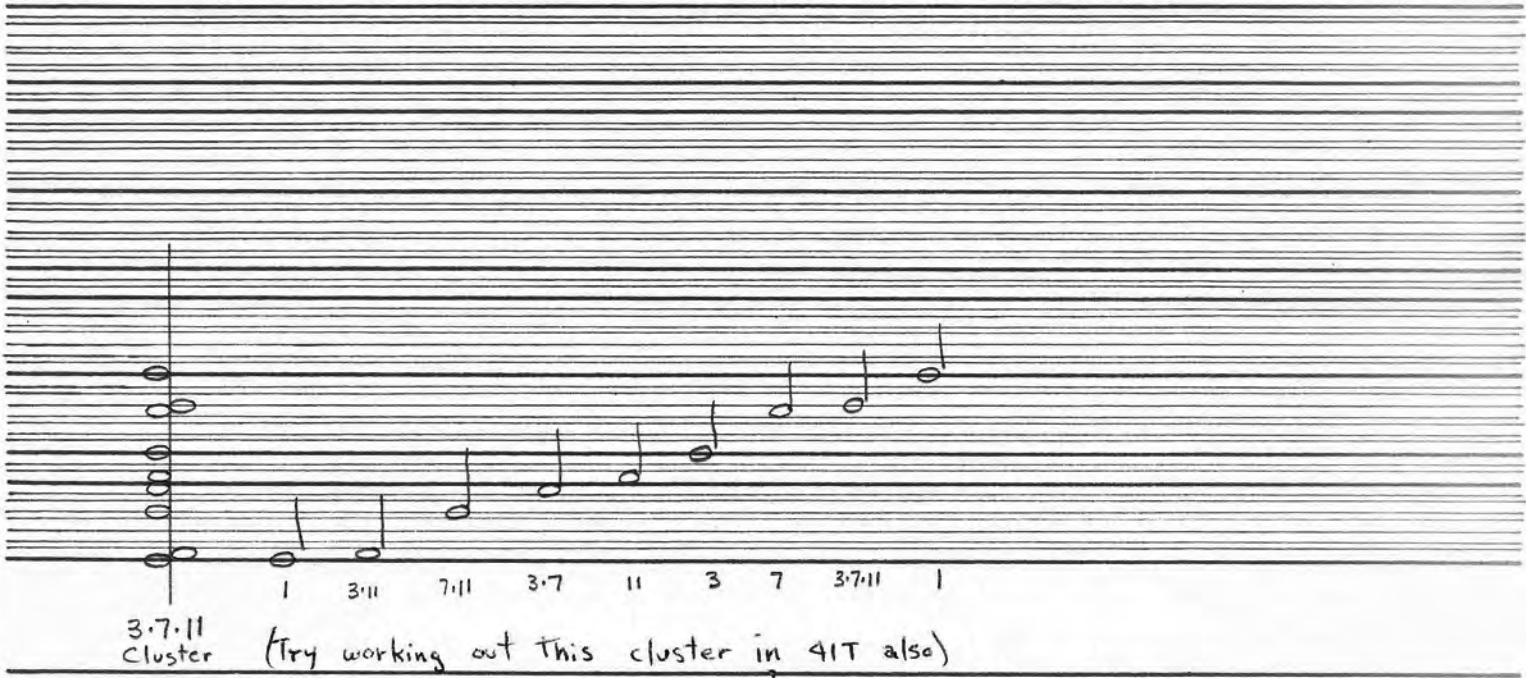
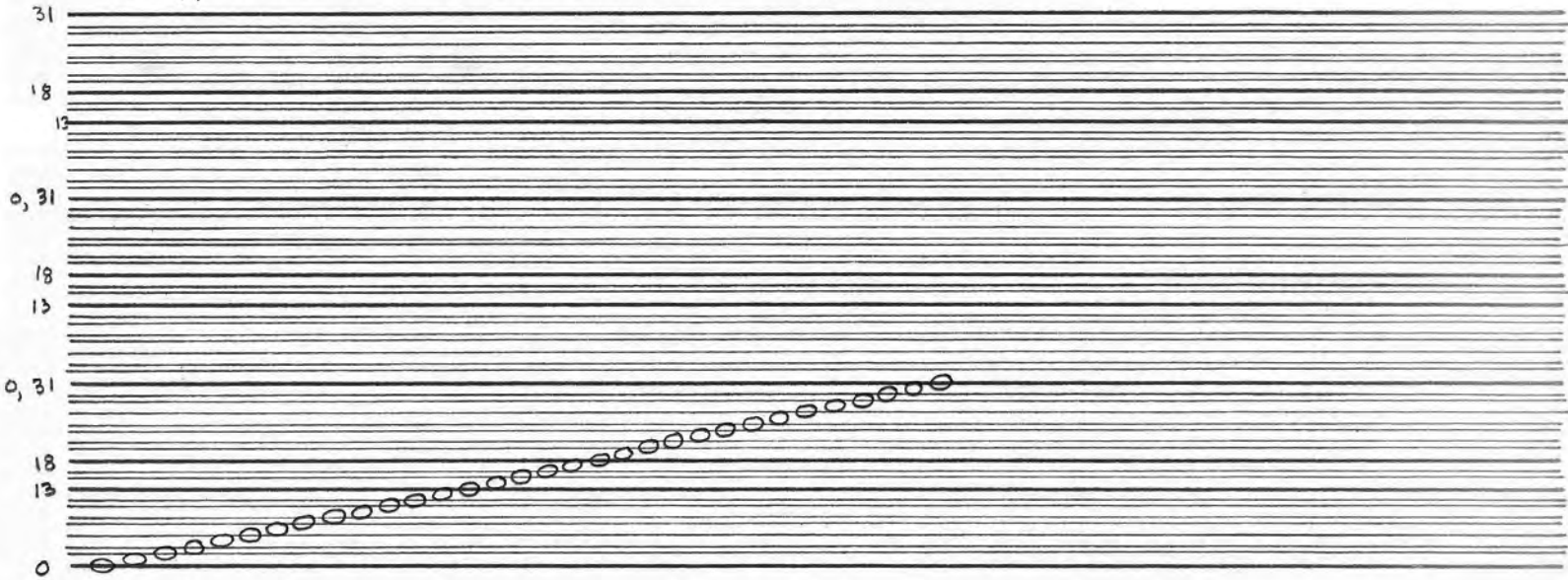
DIAGRAMATIC NOTATION FOR THE 31-TONE SCALE
TYPICAL FOR EACH OCTAVE OF STAFF

LETTER WILSON TO CHALMERS 13 SEP 1963

Sheet of blank notation paper

E. Wilson

Letter to John Chalmers from Err Wilson 16 Nov 64
 31-Tone graph-notation, 3 octaves per staff (The range of my xylophone or guitar)
 Staff based on symmetrical moment 19 of 31



[REDACTED]

[REDACTED]

[REDACTED]