

Definition:  $\frac{0}{1} \frac{1}{1} \frac{1}{0}$  is the most extensive Diophantine Triplet  $\frac{a}{b} \frac{c}{d} \frac{e}{f}$ ;  $be-af=1$   
 $bc-ad=1$   
 $de-cf=1$

On The Application of Diophantine Equations  
 To Musical Instrument Keyboard Format

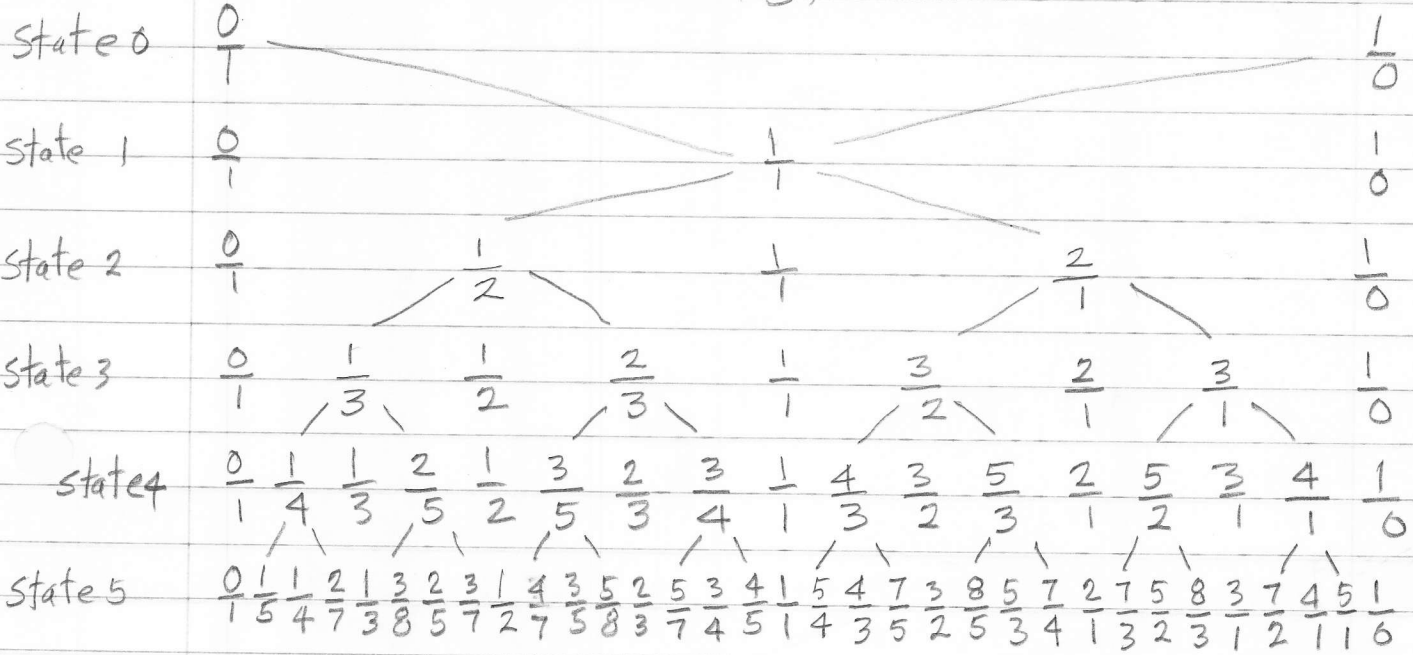
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 (work in progress)

"Keyboard noun --- arrangement of the keys as of an organ, piano, etc."  
 Funk & Wagnalls New College Standard Dictionary 1947

Diophantus of Alexandria was a 3rd century mathematician

His equation,  $b \cdot c - a \cdot d = 1$ , is applicable to

Musical Instrument Keyboard format. Where  $\frac{a}{b} \frac{c}{d}$ , the most comprehensive form is  $\frac{0}{1} \frac{1}{0}$ . Charles Sanders Peirce, 19th century logician embodies this Diophantine Couplet in his series (Peirce Series), which I call the Scale Tree. This is how it progresses; add the top numbers (a+c) and the bottom numbers (b+d) to get the intermediate fraction,  $\frac{0}{1} \frac{1}{1} \frac{1}{0}$ , the Diophantine Triplet  $\frac{a}{b} \frac{c}{d} \frac{e}{f}$ . Continue procedure to get  $\frac{0}{1} \frac{1}{2} \frac{1}{1} \frac{2}{1} \frac{1}{0}$ , and then  $\frac{0}{1} \frac{1}{3} \frac{1}{2} \frac{2}{3} \frac{1}{1} \frac{3}{2} \frac{2}{1} \frac{3}{1} \frac{1}{0}$  and so on, endlessly, Thus;



NOT SCRAP! .580482

about?  
28 JAN 00. EW

Method  
Peirce Triplet

where

then

and

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$1.000 \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$

$$\frac{c}{d} = \frac{a+e}{b+f}$$

(Diophantine equation)

$$b \cdot e - a \cdot f = 1$$

$$\text{Hence } b \cdot c - a \cdot d = 1$$

$$\text{and } d \cdot e - c \cdot f = 1$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

.500

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

.666

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 3 \end{pmatrix}$$

.600

$$\begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 5 \end{pmatrix}$$

.5714

$$\begin{pmatrix} 4 & 7 & 3 \\ 7 & 12 & 5 \end{pmatrix}$$

.5833

$$\begin{pmatrix} 4 & 11 & 7 \\ 7 & 19 & 12 \end{pmatrix}$$

.578947

$$\begin{pmatrix} 11 & 18 & 7 \\ 19 & 31 & 12 \end{pmatrix}$$

.580645

$$\begin{pmatrix} 11 & 29 & 18 \\ 19 & 50 & 31 \end{pmatrix}$$

.586000

$$\begin{pmatrix} 29 & 47 & 18 \\ 50 & 81 & 31 \end{pmatrix}$$

.580247

$$\begin{pmatrix} 47 & 65 & 18 \\ 81 & 112 & 31 \end{pmatrix}$$

achtung!  
application to  
Keyboard Sites

(Root)	Gen	8ve
○	○	○
$0x, 0y$	$1x, 3y$	$2x, 5y$
○	$ax, ey$	$bx, fy$
○	○	○
$0, 0$	$4x, 3y$	$7x, 5y$
○	○	○
$0, 0$	$4x, 7y$	$7x, 12$

Gen. info  
8ve. info

- Notes 1.  $x, y$  coordinates cannot be casually slapped on the scale-tree.
2. The complementary generators must be run to get all the keyboards.
3. The Peirce triplet is ubiquitous in the Scale-Tree, the Lambdoma & the Triangular's (imbue) substance  
The nuclear stuff

# Six Items revisited

①

"Six Items applied to Uath Keyboard" 23 Jan 00. EW  
These are urgent after-thoughts on Fred Kohler's software as of 23 Jan 2000. I expect to be able to do these sorts of things.

"1st Item: Every Keyboard is a Boomsliter/Creel Work Station; each note is ear-tunable."

Getting each note tuneable by ear is square one.  
(Alternate)

"2nd Item: Alter-Octaves are expressible in logarithms to the base of the respective alter-octave."

The Octave ( $\frac{2}{1}$ ) is expressible in logarithms to the base 2. In the same manner the Alternate-Octave ( $\frac{3}{1}$ ) is expressible in logarithms to the base 3. And likewise the Alternate-Octave ( $\frac{8}{3}$ ) is expressible in logarithms to the base ( $\frac{8}{3}$ ), and so forth.

## Six Items applied to Uath Keyboard 23JAN00.EW

1st Item: Every keyboard is a Boomsliter & Creel Work Station; each note is ear-tuneable.

2nd Item: Alter-Octaves are expressible in logarithms to the base of the respective alter-octave.

3rd Item: Notes assigned to the keyboard are optionally played in reciprocal; the same fingering will play the melody upside-down.

4th Item: Octaves are assignable to any key which is a co-prime move on the  $x-y$  grid from the Root.

5th Item: The generator of a linear series is mutable live, and can be hand-controlled or set into continuum. Example; With  $C_1^1$  as root, and  $C_1^2$  as the Octave, and  $G_2^3$  as the generator of a chain-of-Fifths — the generator may be changed in size to give Pythagorean tuning or  $\frac{1}{4}$ -comma meantone, or any linear tuning in between and beyond.

6th Item: The  $x, y$  grid is selectable on the keyboard, and may be rotated thru all its planes.



## Six Items applied to Uath Keyboard 23JAN00.EW

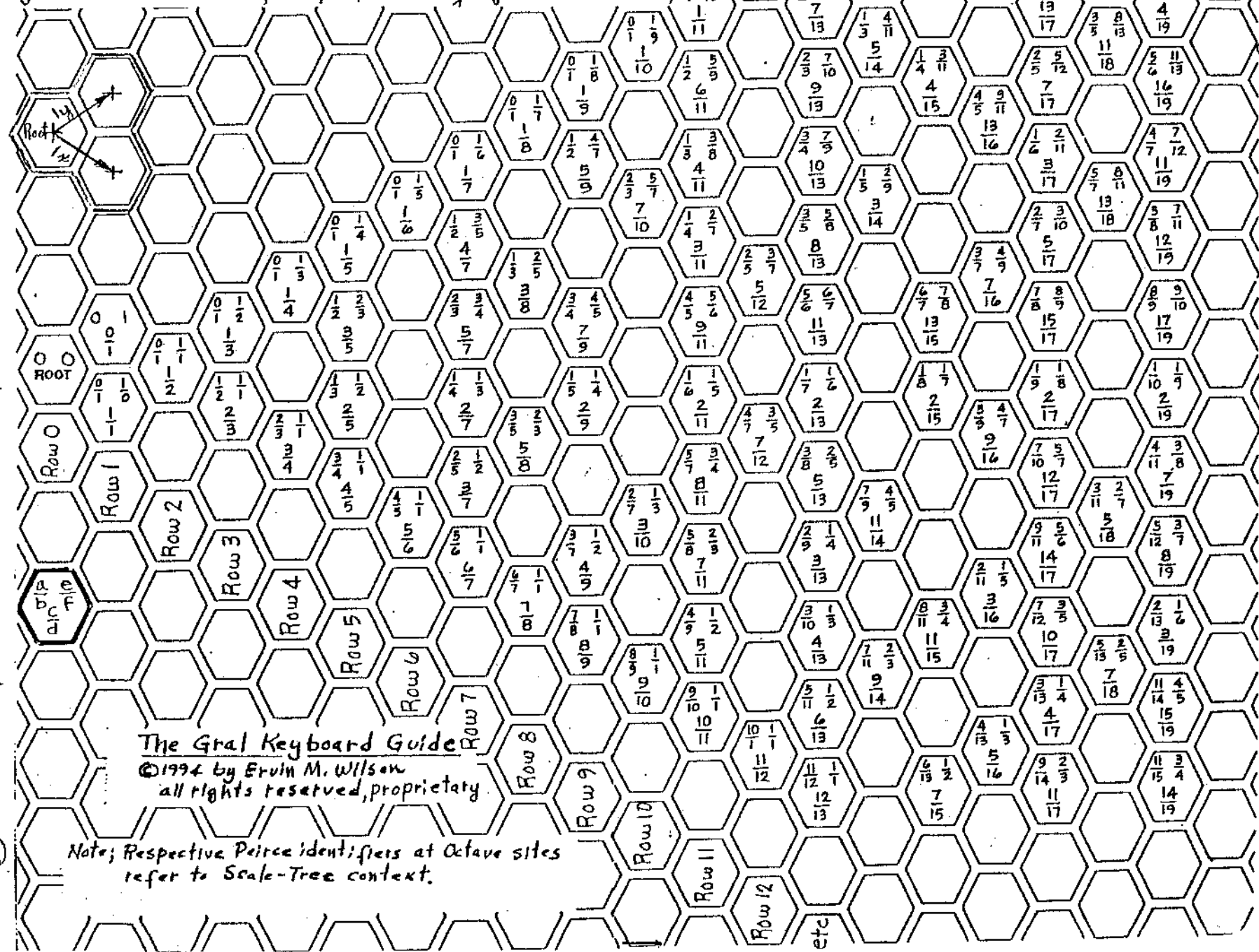
- 1st Item: Every keyboard is a Boomsliter & Creel Work Station; each note is ear-tuneable.
- 2nd Item: Alter-Octaves are expressible in logarithms to the base of the respective alter-octave.
- 3rd Item: Notes assigned to the keyboard are optionally played in reciprocal; the same fingering will play the melody upside-down.
- 4th Item: Octaves are assignable to any key which is a co-prime move on the  $x-y$  grid from the Root.
- 5th Item: The generator of a linear series is mutable live, and can be hand-controlled or set into continuum.  
Example; With  $C_1^2$  as root, and  $C_1^2$  as the Octave, and  $G_{\frac{3}{2}}$  as the generator of a chain-of-Fifths — the generator may be changed in size to give Pythagorean tuning or  $\frac{1}{4}$ -comma meantone, or any linear tuning in between and beyond.
- 6th Item: The  $x, y$  grid is selectable on the keyboard, and may be rotated thru all its planes.

# DIOPHANTINE TRIPLETS and $x, y$ Coordinates, Applied to the GRAAL KEYBOARD, Co-Prime Format.

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To Craig Grady from Erv Wilson 13 April 2001

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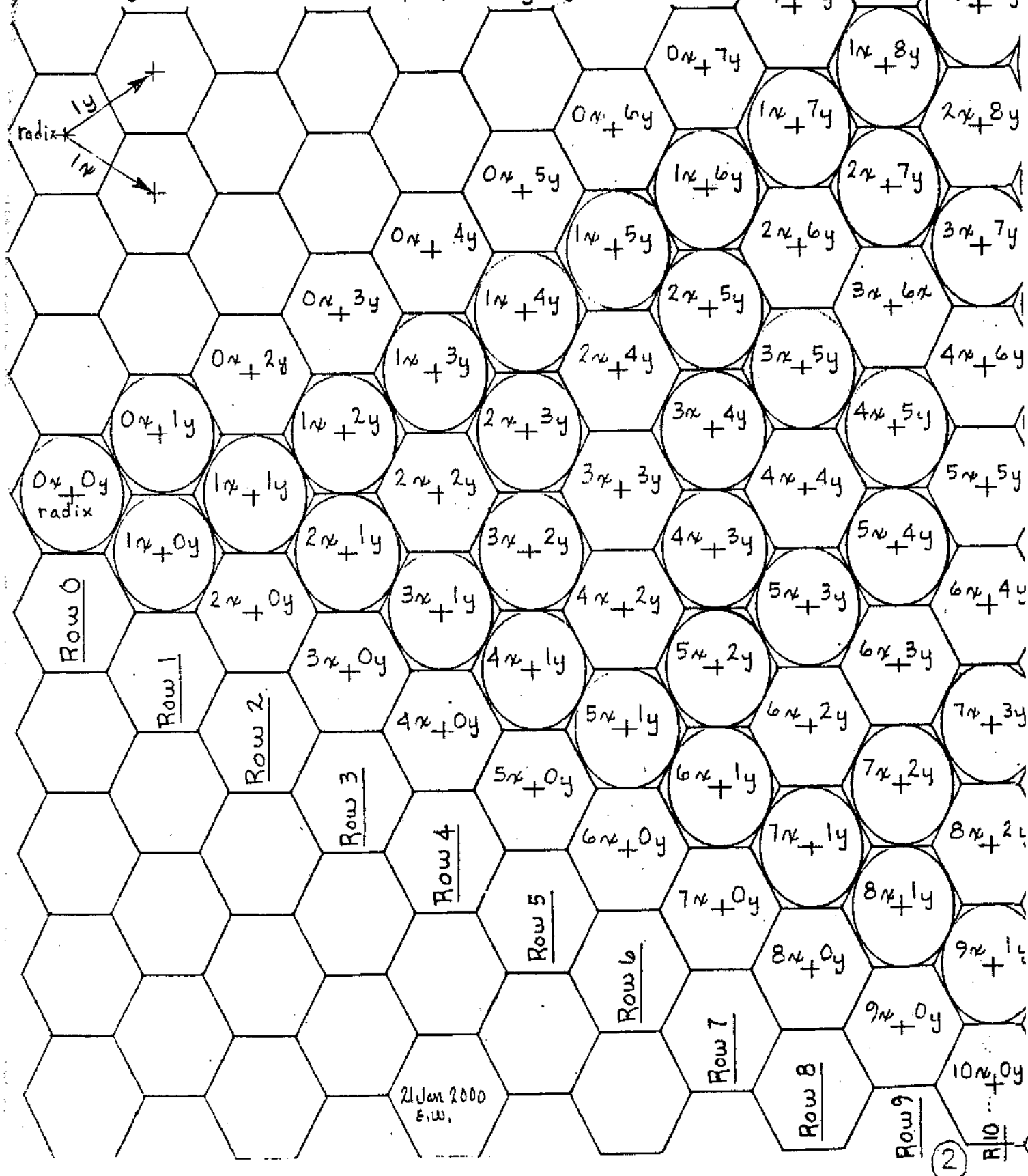
Note; Respective Peirce identifiers at Octave sites refer to Scale-Tree context.

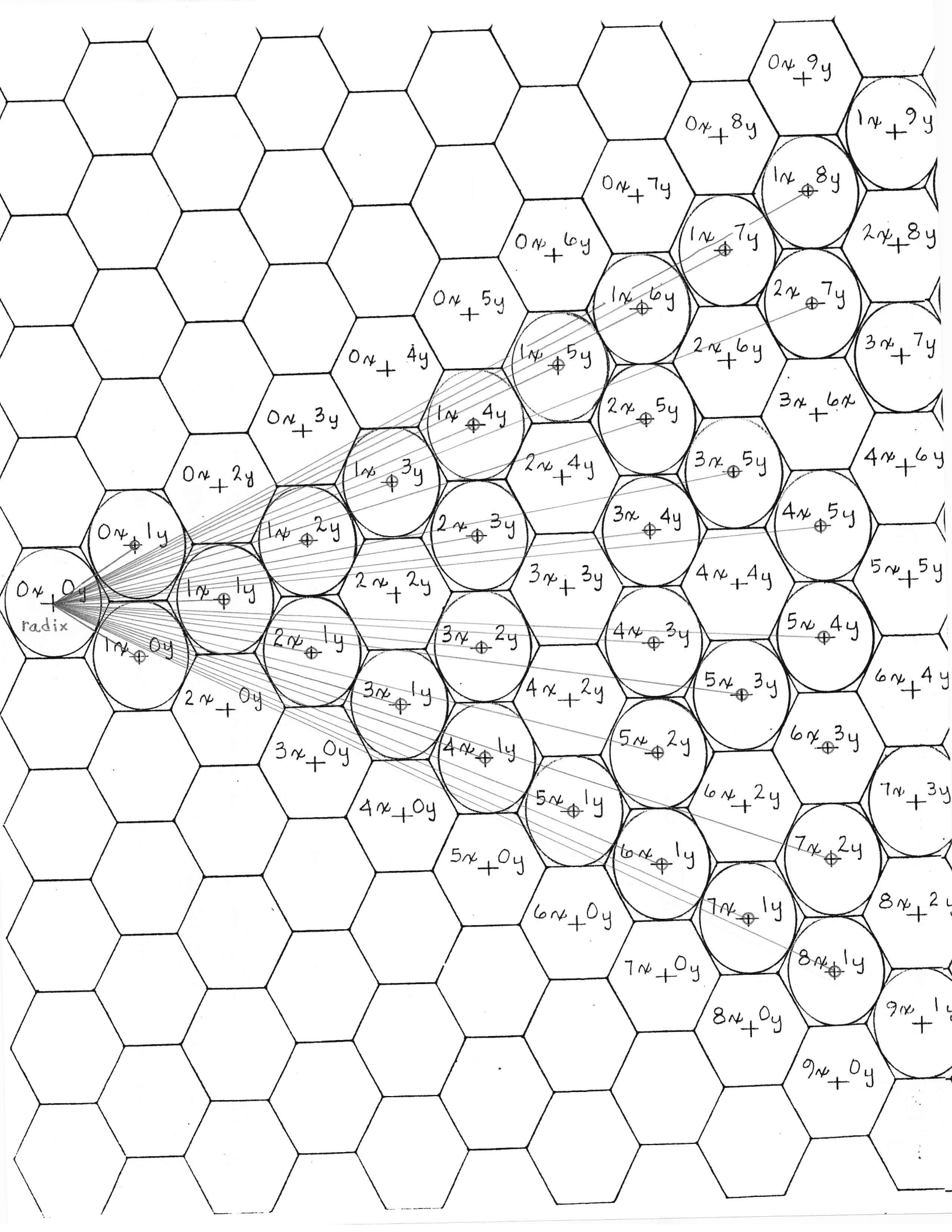
Sheet 1  
of 23 sheets

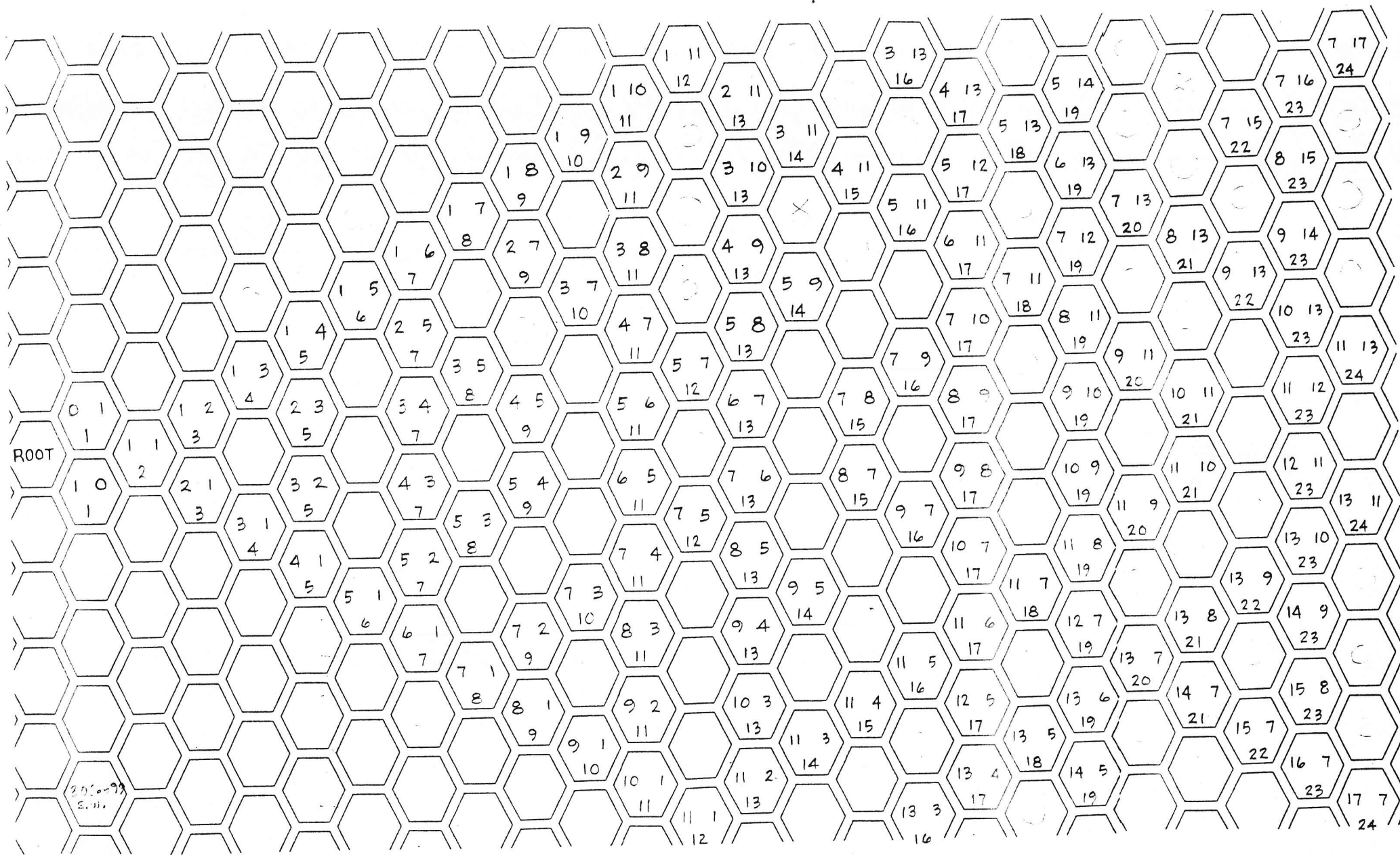
①

# CO-PRIME TRIANGULARIS, ellipsed, sum-cap 10.

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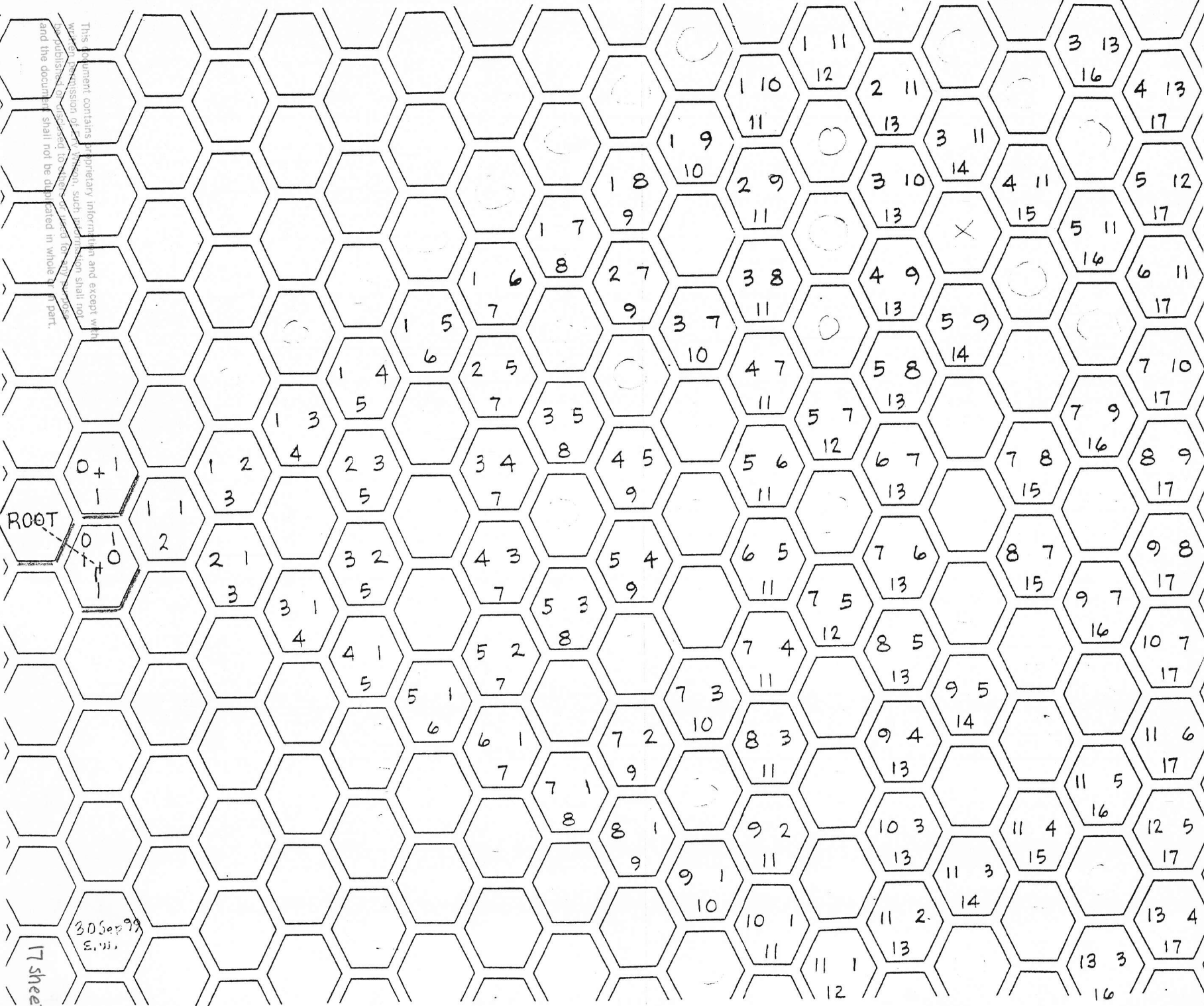






# Keyboards on the Incipient Keyboard Guide to Peirce State 6 on the Scale-Tree

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ROOT

30 Sep 99  
E.W.

17 sheets



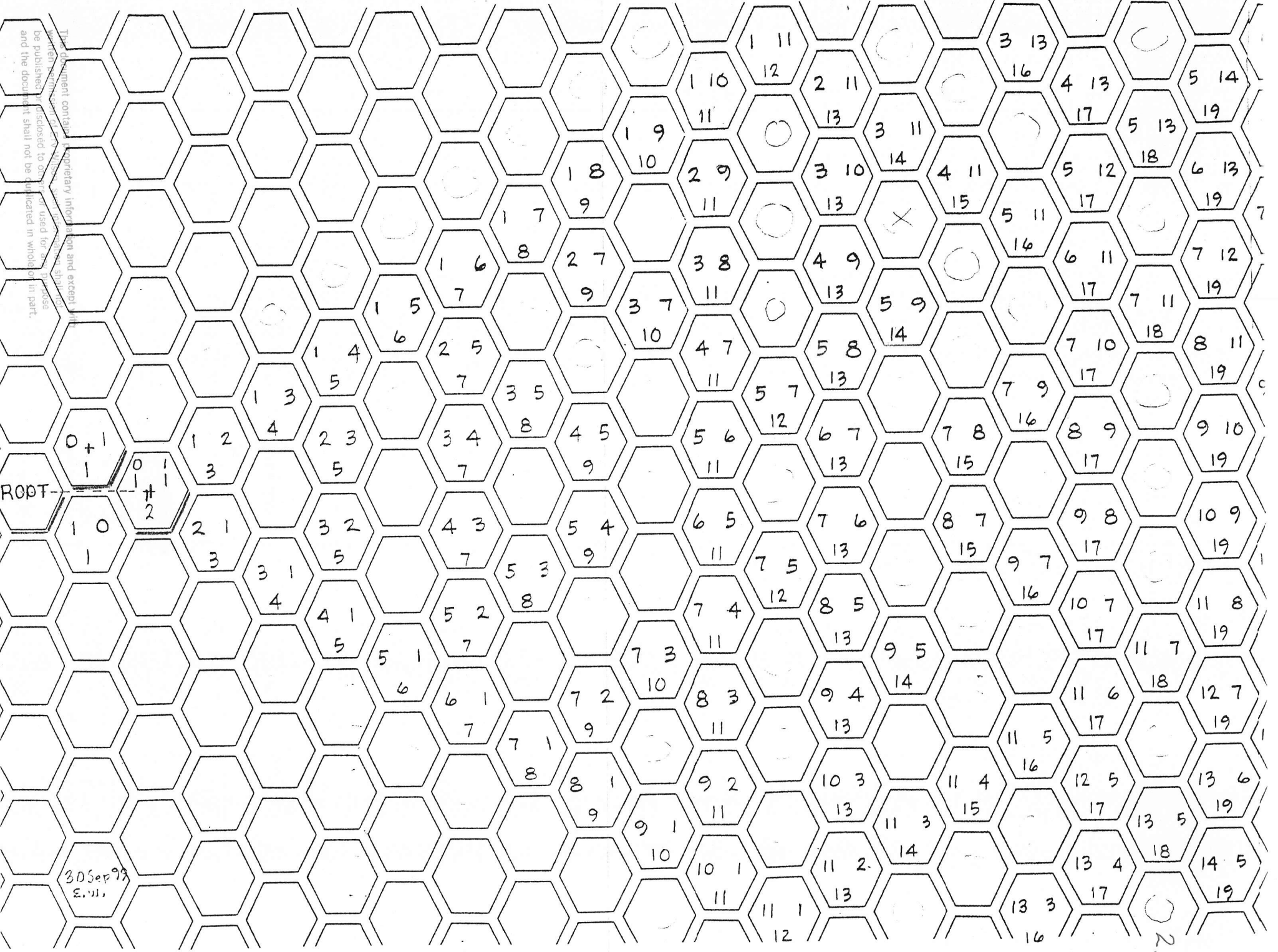
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ROPT

0+1  
1

0 1  
1 2

30 Sep 98  
E. W.



7

c

1

1

2

Yassarian Keyboard Guide

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0  
Root

$1+0 = 1$

$0+1 = 1$

$1+1 = 2$

$2+1 = 3$

$1+2 = 3$

$3+1 = 4$

$4+1 = 5$

$3+2 = 5$

$2+3 = 5$

$5+1 = 6$

$6+1 = 7$

$5+2 = 7$

$4+3 = 7$

$3+4 = 7$

$7+1 = 8$

$8+1 = 9$

$7+2 = 9$

$5+3 = 8$

$5+4 = 9$

$4+5 = 9$

$9+1 = 10$

$8+2 = 10$

$7+3 = 10$

$6+5 = 11$

$5+6 = 11$

$10+1 = 11$

$9+2 = 11$

$8+3 = 11$

$7+4 = 11$

$6+5 = 11$

$5+6 = 11$

$11+1 = 12$

$10+2 = 12$

$9+3 = 12$

$8+4 = 12$

$7+5 = 12$

$6+7 = 13$

$12+1 = 13$

$11+2 = 13$

$10+3 = 13$

$9+4 = 13$

$8+5 = 13$

$7+6 = 13$

$6+7 = 13$

$11+3 = 14$

$10+4 = 14$

$9+5 = 14$

$8+6 = 14$

$7+8 = 15$

$6+9 = 15$

$13+2 = 15$

$11+4 = 15$

$10+5 = 15$

$9+6 = 15$

$8+7 = 15$

$7+8 = 15$

$6+9 = 15$

$13+3 = 16$

$12+4 = 16$

$11+5 = 16$

$10+6 = 16$

$9+7 = 16$

$8+9 = 17$

$7+10 = 17$

$14+3 = 17$

$13+4 = 17$

$12+5 = 17$

$11+6 = 17$

$10+7 = 17$

$9+8 = 17$

$8+9 = 17$

$13+5 = 18$

$12+6 = 18$

$11+7 = 18$

$10+8 = 18$

$9+10 = 19$

$8+11 = 19$

$7+11 = 18$

$6+13 = 19$

$15+4 = 19$

$14+5 = 19$

$13+6 = 19$

$12+7 = 19$

$11+8 = 19$

$10+9 = 19$

$9+10 = 19$

$8+11 = 19$

$7+12 = 19$

$6+13 = 19$

$5+14 = 19$

$4+15 = 19$

(2x)  
7  
ret.

Hassan  
Roots

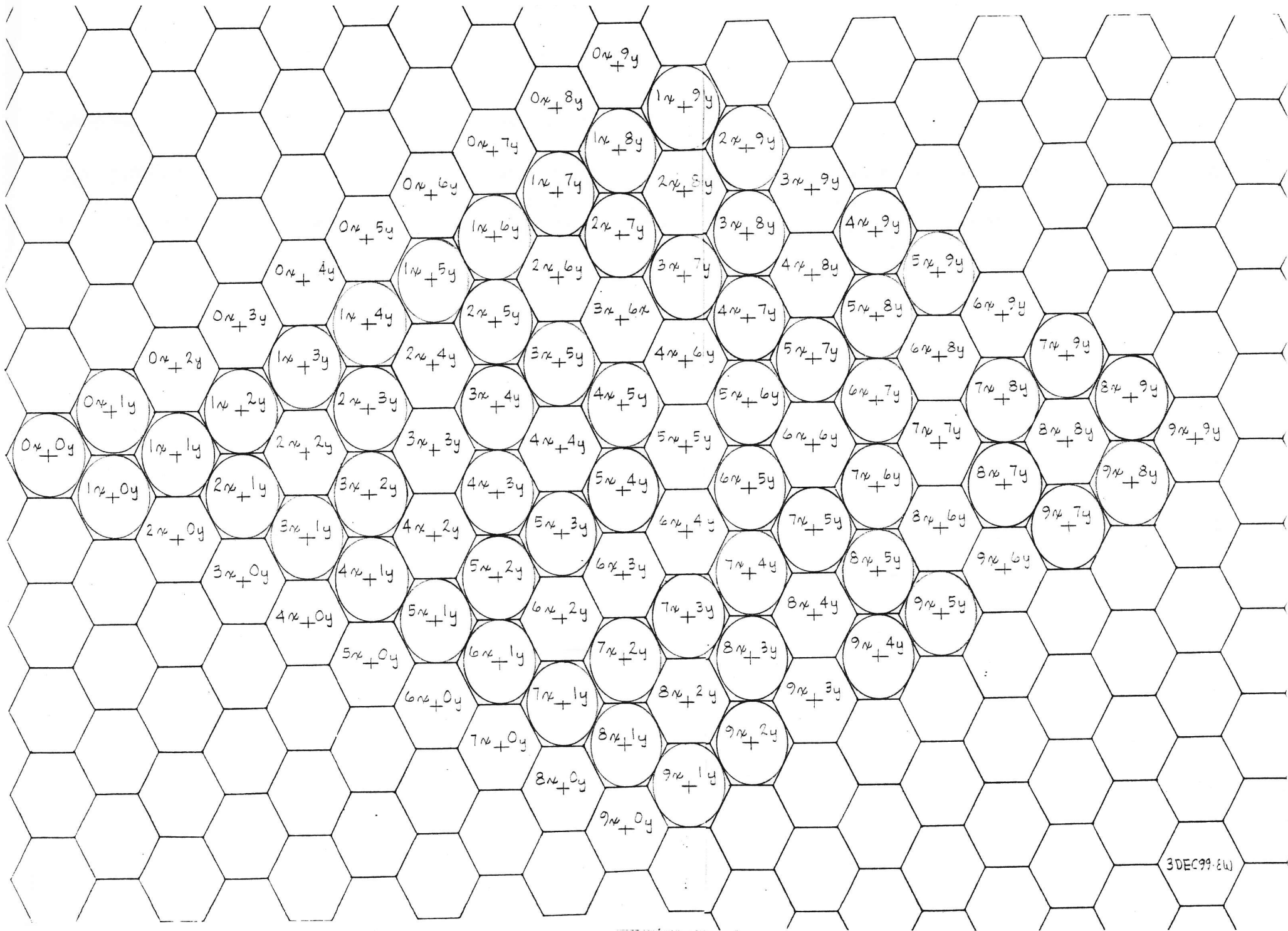
Basque

Hassan

Basque

Basque

Basque



$$2/\sqrt[3]{10/3} = 1.33886590017\dots$$

1/3 Comma Meantone, Fourth

$$\text{Log}_2 .421011468617\dots$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root Generator Octave
$0x, 0y$	$a_x, e_y$	$b_x, f_y$		
0	1	1	1.000000	$0x, 0y$ $0x, 1y$ $1x, 0y$
			←	
0	1/2	1	.500000	$0x, 0y$ $0x, 1y$ $1x, 1y$
			←	
0	1/3	2	.333333	$0x, 0y$ $0x, 1y$ $1x, 2y$
			→	
1	2/5	2	.400000	$0x, 0y$ $1x, 1y$ $3x, 2y$
			→	
2	3/7	2	.428571	$0x, 0y$ $2x, 1y$ $5x, 2y$
			←	
2	5/12	3/7	.416667	$0x, 0y$ $2x, 3y$ $5x, 7y$ ✓
			→	
5	8/19	3/7	.421053	$0x, 0y$ $5x, 3y$ $12x, 7y$
			←	
5	13/31	8/19	.419355	$0x, 0y$ $5x, 8y$ $12x, 19y$ ✓
			→	
13	21/50	8/19	.420000	$0x, 0y$ $13x, 8y$ $31x, 19y$
			→	
21	29/69	8/19	.420290	$0x, 0y$ $21x, 8y$ $50x, 19y$
			→	
29	37/88	8/19	.420455	$0x, 0y$ $29x, 8y$ $69x, 19y$
			→	
37	45/107	8/19	.420561	$0x, 0y$ $37x, 8y$ $88x, 19y$
			→	
45	53/126	8/19	.420635	$0x, 0y$ $45x, 8y$ $107x, 19y$
			→	
53	61/145	8/19	.420690	$0x, 0y$ $53x, 8y$ $126x, 19y$
			→	
61	69/164	8/19	.420732	$0x, 0y$ $61x, 8y$ $145x, 19y$
			→	
69	77/183	8/19	.420765	$0x, 0y$ $69x, 8y$ $164x, 19y$
			→ etc	

66 places

15APROO.EW

$$2/\sqrt[3]{10/3} = 1.33886590017\dots$$

1/3 Comma Meantone, Fourth

$$\text{Log}_2 .421011468617\dots$$

a	c	e	$\frac{c}{d}$	dec.	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$		$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000		$0n, 0y$	$0n, 1y$	$1n, 0y$
				←			
0	1	2	.500000		$0n, 0y$	$0n, 1y$	$1n, 1y$
				←			
0	1	3	.333333		$0n, 0y$	$0n, 1y$	$1n, 2y$
				→			
1	3	2	.400000		$0n, 0y$	$1n, 1y$	$3n, 2y$
				→			
2	5	2	.428571		$0n, 0y$	$2n, 1y$	$5n, 2y$
				←			
2	5	3	.416667		$0n, 0y$	$2n, 3y$	$5n, 7y$
				→			
5	12	3	.421053		$0n, 0y$	$5n, 3y$	$12n, 7y$
				←			
5	12	8	.419355		$0n, 0y$	$5n, 8y$	$12n, 19y$
				→			
13	31	8	.420000		$0n, 0y$	$13n, 8y$	$31n, 19y$
				→			
21	50	8	.420290		$0n, 0y$	$21n, 8y$	$50n, 19y$
				→			
29	69	8	.420455		$0n, 0y$	$29n, 8y$	$69n, 19y$
				→			
37	88	8	.420561		$0n, 0y$	$37n, 8y$	$88n, 19y$
				→			
45	107	8	.420635		$0n, 0y$	$45n, 8y$	$107n, 19y$
				→			
53	126	8	.420690		$0n, 0y$	$53n, 8y$	$126n, 19y$
				→			
61	145	8	.420732		$0n, 0y$	$61n, 8y$	$145n, 19y$
				→			
69	164	8	.420765		$0n, 0y$	$69n, 8y$	$164n, 19y$
				→			
66 places				→ etc			

$$\sqrt[3]{\frac{5}{2}} = 1.35720880830\dots$$

$$\log_2 .440642698299\dots$$

Neo-Pèlog, Fourth

a	c	e	c	Root	Generator	Octave
b	d	f	d dec.	$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
1	1	0				
			←			
0	1	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
1	2	1				
			←			
0	1	1	.333333	$0n, 0y$	$0n, 1y$	$1n, 2y$
1	3	2				
			→			
1	2	1	.400000	$0n, 0y$	$1n, 1y$	$3n, 2y$
3	5	2				
			→			
2	3	1	.428571	$0n, 0y$	$2n, 1y$	$5n, 2y$
5	7	2				
			→			
3	4	1	.444444	$0n, 0y$	$3n, 1y$	$7n, 2y$
7	9	2				
			←			
3	7	4	.437500	$0n, 0y$	$3n, 4y$	$7n, 9y$
7	16	9				
			→			
7	11	4	.440000	$0n, 0y$	$7n, 4y$	$16n, 9y$
16	25	9				
			→			
11	15	4	.441176	$0n, 0y$	$11n, 4y$	$25n, 9y$
25	34	9				
			←			
11	26	15	.440678	$0n, 0y$	$11n, 15y$	$25n, 34y$
25	59	34				
			←			
11	37	26	.440476	$0n, 0y$	$11n, 26y$	$25n, 59y$
25	84	59				
			→			
37	63	26	.440559	$0n, 0y$	$37n, 26y$	$84n, 59y$
84	143	59				
			→			
63	89	26	.440594	$0n, 0y$	$63n, 26y$	$143n, 59y$
143	202	59				
			→			
89	115	26	.440613	$0n, 0y$	$89n, 26y$	$202n, 59y$
202	261	59				
			→			
115	141	26	.440625	$0n, 0y$	$115n, 26y$	$261n, 59y$
261	320	59				
			→			
141	167	26	.440633	$0n, 0y$	$141n, 26y$	$320n, 59y$
320	379	59				
			→			
167	193	26	.440639	$0n, 0y$	$167n, 26y$	$379n, 59y$
379	438	59				
			→			
193	219	26	.440644	$0n, 0y$	$193n, 26y$	$438n, 59y$
438	497	59				
			←			

9x29

✓



$$\sqrt[3]{16/5} = 1.47361259945\dots$$

Neo-Pèlog, Fifth

$$\text{Log}_2 .559357301701\dots$$

a	c	e	$\frac{c}{d}$	dec.	Root Generator Octave
b	d	f	$\frac{c}{d}$		$0n, 0y$ $a_n, e_y$ $b_n, f_y$
0	1	1	1.000000		$0n, 0y$ $0n, 1y$ $1n, 0y$
1	1	0		←	
0	1/2	1	.500000		$0n, 0y$ $0n, 1y$ $1n, 1y$
1	2	1		→	
1	2/3	1	.666667		$0n, 0y$ $1n, 1y$ $2n, 1y$
1	3	2		←	
2	5	3	.600000		$0n, 0y$ $1n, 2y$ $2n, 3y$
1	4	3		←	
2	7	5	.571429		$0n, 0y$ $1n, 3y$ <u><math>2n, 5y</math></u> ✓
1	5	4		←	
2	9	7	.555556		$0n, 0y$ $1n, 4y$ $2n, 7y$
5	9	4		→	
9	16	7	.562500		$0n, 0y$ $5n, 4y$ $9n, 7y$
5	14	9		←	
9	25	16	.560000		$0n, 0y$ $5n, 9y$ $9n, 16y$
5	19	14		←	
9	34	25	.558824		$0n, 0y$ $5n, 14y$ $9n, 25y$
19	33	14		→	
34	59	25	.559322		$0n, 0y$ $19n, 14y$ $34n, 25y$
33	47	14		→	
59	84	25	.559524		$0n, 0y$ $33n, 14y$ $59n, 25y$
33	80	47		←	
59	143	84	.559441		$0n, 0y$ $33n, 47y$ $59n, 84y$
33	113	80		←	
59	202	143	.559406		$0n, 0y$ $33n, 80y$ $59n, 143y$ ✓
33	146	113		←	
59	261	202	.559387		$0n, 0y$ $33n, 113y$ $59n, 202y$
33	179	146		←	
59	320	261	.559375		$0n, 0y$ $33n, 146y$ $59n, 261y$
33	212	179		←	
59	379	320	.559367		$0n, 0y$ $33n, 179y$ $59n, 320y$
33	245	212		←	
59	438	379	.559361		$0n, 0y$ $33n, 212y$ $59n, 379y$
33	278	245		←	
59	497	438	.559356		$0n, 0y$ $33n, 245y$ $59n, 438y$
				→	

$$\sqrt[6]{5/2} = 1.16499305075\dots$$

Neo-Pèlog, subminorThird

$$\text{Log}_2 .220321349146\dots$$

a	c	e	c	Root	Generator	Octave
b	d	f	d	$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
1	1	0				
			←			
0	1	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
1	2	1				
			←			
0	1	2	.333333	$0n, 0y$	$0n, 1y$	$1n, 2y$
1	3	2				
			←			
0	1	3	.250000	$0n, 0y$	$0n, 1y$	$1n, 3y$
1	4	3				
			←			
0	1	4	.200000	$0n, 0y$	$0n, 1y$	$1n, 4y$
1	5	4				
			→			
1	2	1	.222222	$0n, 0y$	$1n, 1y$	$5n, 4y$
5	9	4				
			←			
1	3	2	.214286	$0n, 0y$	$1n, 2y$	$5n, 9y$
5	14	9				
			→			
3	5	2	.217391	$0n, 0y$	$3n, 2y$	$14n, 9y$
14	23	9				
			→			
5	7	2	.218750	$0n, 0y$	$5n, 2y$	$23n, 9y$
23	32	9				
			→			
7	9	2	.219512	$0n, 0y$	$7n, 2y$	$32n, 9y$
32	41	9				
			→			
9	11	2	.220000	$0n, 0y$	$9n, 2y$	$41n, 9y$
41	50	9				
			→			
11	13	2	.220339	$0n, 0y$	$11n, 2y$	$50n, 9y$
50	59	9				
			←			
11	24	13	.220183	$0n, 0y$	$11n, 13y$	$50n, 59y$
50	109	59				
			→			
24	37	13	.220238	$0n, 0y$	$24n, 13y$	$109n, 59y$
109	168	59				
			→			
37	50	13	.220264	$0n, 0y$	$37n, 13y$	$168n, 59y$
168	227	59				
			→			
50	63	13	.220280	$0n, 0y$	$50n, 13y$	$227n, 59y$
227	286	59				
			→			
15 places		etc				

$$2/\sqrt[6]{5/2} = 1.71674843787\dots \quad \text{Neo-Pèlog, supramajor Sixth}$$

$$\text{Log}_2 .779678650856\dots$$

a	c	e	$\frac{c}{d}$	dec.	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$		$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	0	1.000000		$0n, 0y$	$0n, 1y$	$1n, 0y$
				←			
0	1	1	.500000		$0n, 0y$	$0n, 1y$	$1n, 1y$
				→			
1	2	1	.666667		$0n, 0y$	$1n, 1y$	$2n, 1y$
				→			
2	3	1	.750000		$0n, 0y$	$2n, 1y$	$3n, 1y$
				→			
3	4	1	.800000		$0n, 0y$	$3n, 1y$	$4n, 1y$
				←			
3	7	4	.777778		$0n, 0y$	$3n, 4y$	$4n, 5y$
				→			
7	11	4	.785714		$0n, 0y$	$7n, 4y$	$9n, 5y$
				←			
7	18	11	.782609		$0n, 0y$	$7n, 11y$	$9n, 14y$
				←			
7	25	18	.781250		$0n, 0y$	$7n, 18y$	$9n, 23y$
				←			✓ new good
7	32	25	.780488		$0n, 0y$	$7n, 25y$	$9n, 32y$
				←			
7	39	32	.780000		$0n, 0y$	$7n, 32y$	$9n, 41y$
				←			
7	46	39	.779661		$0n, 0y$	$7n, 39y$	$9n, 50y$
				→			
46	85	39	.779817		$0n, 0y$	$46n, 39y$	$59n, 50y$
				←			
46	131	85	.779762		$0n, 0y$	$46n, 85y$	$59n, 109y$
				←			
46	177	131	.779736		$0n, 0y$	$46n, 131y$	$59n, 168y$
				←			
46	223	177	.779720		$0n, 0y$	$46n, 177y$	$59n, 227y$
				←			

$$\sqrt[4]{5} = 1.49534878122\dots$$

1/4-comma Meantone, Fifth

$$\rightarrow \text{Log}_2 .580482023721\dots$$

5FEB00·EW

	a	c	e		Root	Generator	Octave
	$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	$0x, 0y$	$a x, e y$	$b x, f y$
	0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
	1	1	0				
	⋮	⋮	⋮	←			
	0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
	1	2	1				
	1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
	2	3	1				
	1	3	2	.600000	$0x, 0y$	$1x, 2y$	$2x, 3y$
Infra Bosanquet	2	5	3				
	1	4	3	.571429	$0x, 0y$	$1x, 3y$	$2x, 5y$
Uath	2	7	5				↙
	4	7	3	.583333	$0x, 0y$	$4x, 3y$	$7x, 5y$
Bosanquet	7	12	5				
	4	11	7	.578947	$0x, 0y$	$4x, 7y$	$7x, 12y$
	7	19	12				↙
	11	18	7	.580645	$0x, 0y$	$11x, 7y$	$19x, 12y$
	19	31	12				
	11	29	18	.580000	$0x, 0y$	$11x, 18y$	$19x, 31y$
	19	50	31				
	29	47	18	.580247	$0x, 0y$	$29x, 18y$	$50x, 31y$
	50	81	31				
	47	65	18	.580357	$0x, 0y$	$47x, 18y$	$81x, 31y$
	81	112	31				
→	65	83	18	.580420	$0x, 0y$	$65x, 18y$	$112x, 31y$
	112	143	31				
	83	101	18	.580460	$0x, 0y$	$83x, 18y$	$143x, 31y$
	143	174	31				
	101	119	18	.580488	$0x, 0y$	$101x, 18y$	$174x, 31y$
	174	205	31				
	101	220	119	.580475	$0x, 0y$	$101x, 119y$	$174x, 205y$
	174	379	205				
	220	339	119	.580479	$0x, 0y$	$220x, 119y$	$379x, 205y$
	379	584	205				

checked

$$2/\sqrt[4]{5} = 1.33748060995\dots$$

1/4-comma Meantone, Fourth

$$\rightarrow \text{Log}_2 .419517976279\dots$$

	a	c	e	$\frac{c}{d}$	Root	Generator	Octave
	b	d	f	$\frac{c}{d}$	$0x, 0y$	$a_x, e_y$	$b_x, f_y$
gen.	0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
Oct.	1	1	0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
				←			
	0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
	1	2	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
				←			
	0	1	1	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
	1	3	2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
				→			
	1	2	1	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
	3	5	2	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
				→			
	2	3	1	.428571	$0x, 0y$	$2x, 1y$	$5x, 2y$
	5	7	2	.428571	$0x, 0y$	$2x, 1y$	$5x, 2y$
				←			
	2	5	3	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
	5	12	7	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
				→			
	5	8	3	.421053	$0x, 0y$	$5x, 3y$	$12x, 7y$
	12	19	7	.421053	$0x, 0y$	$5x, 3y$	$12x, 7y$
				←			
	5	13	8	.419355	$0x, 0y$	$5x, 8y$	$12x, 19y$
	12	31	19	.419355	$0x, 0y$	$5x, 8y$	$12x, 19y$
				→			
	13	21	8	.420000	$0x, 0y$	$13x, 8y$	$31x, 19y$
	31	50	19	.420000	$0x, 0y$	$13x, 8y$	$31x, 19y$
				←			
	13	34	21	.419753	$0x, 0y$	$13x, 21y$	$31x, 50y$
	31	81	50	.419753	$0x, 0y$	$13x, 21y$	$31x, 50y$
				←			
	13	47	34	.419643	$0x, 0y$	$13x, 34y$	$31x, 81y$
	31	112	81	.419643	$0x, 0y$	$13x, 34y$	$31x, 81y$
				←			
	13	60	47	.419580	$0x, 0y$	$13x, 47y$	$31x, 112y$
	31	143	112	.419580	$0x, 0y$	$13x, 47y$	$31x, 112y$
				←			
	13	73	60	.419540	$0x, 0y$	$13x, 60y$	$31x, 143y$
	31	174	143	.419540	$0x, 0y$	$13x, 60y$	$31x, 143y$
				←			
	13	86	73	.419512	$0x, 0y$	$13x, 73y$	$31x, 174y$
	31	205	174	.419512	$0x, 0y$	$13x, 73y$	$31x, 174y$
				→			
	86	159	73	.419525	$0x, 0y$	$86x, 73y$	$205x, 174y$
	205	379	174	.419525	$0x, 0y$	$86x, 73y$	$205x, 174y$
				←			
	86	245	159	.419521	$0x, 0y$	$86x, 159y$	$205x, 379y$
	205	584	379	.419521	$0x, 0y$	$86x, 159y$	$205x, 379y$
				←			

$$\sqrt[5]{\frac{15}{2}} = 1.49627786974\dots$$

1/5-Comma Meantone, Fifth

$$\rightarrow \log_2 .581378119122\dots$$

a	c	e	$\frac{c}{d}$ dec.	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$		$0x, 0y$	$a_x, e_y$	$b_x, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0	←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1	→			
1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
2	3	1	←			
1	3	2	.600000	$0x, 0y$	$1x, 2y$	$2x, 3y$
2	5	3	←			
1	4	3	.571429	$0x, 0y$	$1x, 3y$	$2x, 5y$
2	7	5	→			
4	7	3	.583333	$0x, 0y$	$4x, 3y$	$7x, 5y$
7	12	5	←			
4	11	7	.578947	$0x, 0y$	$4x, 7y$	$7x, 12y$
7	19	12	→			
11	18	7	.580645	$0x, 0y$	$11x, 7y$	$19x, 12y$
19	31	12	→			
18	25	7	.581395	$0x, 0y$	$18x, 7y$	$31x, 12y$
31	43	12	←			
18	43	25	.581081	$0x, 0y$	$18x, 25y$	$31x, 43y$
31	74	43	→			
43	68	25	.581197	$0x, 0y$	$43x, 25y$	$74x, 43y$
74	117	43	→			
68	93	25	.581250	$0x, 0y$	$68x, 25y$	$117x, 43y$
117	160	43	→			
93	118	25	.581281	$0x, 0y$	$93x, 25y$	$160x, 43y$
160	203	43	→			
118	143	25	.581301	$0x, 0y$	$118x, 25y$	$203x, 43y$
203	246	43	→			
143	168	25	.581315	$0x, 0y$	$143x, 25y$	$246x, 43y$
246	289	43	→			
168	193	25	.581325	$0x, 0y$	$168x, 25y$	$289x, 43y$
289	332	43	→			

41x6!

17x17



$$\sqrt[3]{10/3} = 1.49380158218\dots$$

$\frac{1}{3}$ -Comma Meantone, Fifth

$$\rightarrow \text{Log}_2 .578988531383\dots$$

a	c	e	$\frac{c}{d}$	dec.	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$		$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000		$0n, 0y$	$0n, 1y$	$1n, 0y$
				←			
0	1	1	.500000		$0n, 0y$	$0n, 1y$	$1n, 1y$
				→			
1	2	1	.666667		$0n, 0y$	$1n, 1y$	$2n, 1y$
				←			
1	2	3	.600000		$0n, 0y$	$1n, 2y$	$2n, 3y$
				←			
1	2	3	.571429		$0n, 0y$	$1n, 3y$	<u><math>2n, 5y</math></u>
				→			
4	7	3	.583333		$0n, 0y$	$4n, 3y$	$7n, 5y$
				←			
4	7	12	.578947		$0n, 0y$	$4n, 7y$	$7n, 12y$
				→			
11	18	7	.580645		$0n, 0y$	$11n, 7y$	$19n, 12y$
				←			
11	29	18	.580000		$0n, 0y$	$11n, 18y$	$19n, 31y$
				←			
11	40	29	.579710		$0n, 0y$	$11n, 29y$	$19n, 50y$
				←			
11	51	40	.579545		$0n, 0y$	$11n, 40y$	$19n, 69y$
				←			
11	62	51	.579439		$0n, 0y$	$11n, 51y$	$19n, 88y$
				←			
11	73	62	.579365		$0n, 0y$	$11n, 62y$	$19n, 107y$
				←			
5 × 29	11	84	73	.579310	$0n, 0y$	$11n, 73y$	$19n, 126y$
				←			
4 × 41	11	95	84	.579268	$0n, 0y$	$11n, 84y$	$19n, 145y$
				←			
	11	106	95	.579235	$0n, 0y$	$11n, 95y$	$19n, 164y$

See  $744/1285$  66 places ← etc

$$2/\sqrt[5]{15/2} = 1.33665012392\dots$$

1/5-comma Meantone, Fourth

$$\rightarrow \text{Log}_2 .418621880078\dots$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root $0x, 0y$	Generator $a x, e y$	Octave $b x, f y$
0/1	1/1	1/0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0/1	1/2	1/1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0/1	1/3	1/2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			→			
1/3	2/5	1/2	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
			→			
2/5	3/7	1/2	.428571	$0x, 0y$	$2x, 1y$	$5x, 2y$
			←			
2/5	5/12	3/7	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
			→			
5/12	8/19	3/7	.421053	$0x, 0y$	$5x, 3y$	$12x, 7y$
			←			
5/12	13/31	8/19	.419355	$0x, 0y$	$5x, 8y$	$12x, 19y$
			←			
5/12	18/43	13/31	.418605	$0x, 0y$	$5x, 13y$	$12x, 31y$
			→			
18/43	31/74	13/31	.418919	$0x, 0y$	$18x, 13y$	$43x, 31y$
			←			
18/43	49/117	31/74	.418803	$0x, 0y$	$18x, 31y$	$43x, 74y$
			←			
18/43	67/160	49/117	.418750	$0x, 0y$	$18x, 49y$	$43x, 117y$
			←			
18/43	85/203	67/160	.418719	$0x, 0y$	$18x, 67y$	$43x, 160y$
			←			
18/43	103/246	85/203	.418699	$0x, 0y$	$18x, 85y$	$43x, 203y$
			←			
18/43	121/289	103/246	.418685	$0x, 0y$	$18x, 103y$	$43x, 246y$
			←			
18/43	139/332	121/289		$0x, 0y$	$18x, 121y$	$43x, 289y$
			←			

32x5

172

Checked

$$3/2 = 1.5$$

# Pythagorean Fifth

$$\rightarrow \text{Log}_2 .584962500721\dots$$

gen. Oct.	a b	c d	e f	$\frac{c}{d}$ dec.	Root $0x, 0y$	Generator $a, n, e, y$	Octave $b, n, f, y$	
	0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$	
				←				
	0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$	
				→				
	1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$	
				←				
	1	3	2	.600000	$0x, 0y$	$1x, 2y$	$2x, 3y$	
				←				
	1	4	3	.571429	$0x, 0y$	$1x, 3y$	<u><math>2x, 5y</math></u>	✓
				→				
	4	7	3	.583333	$0x, 0y$	$4x, 3y$	$7x, 5y$	
				→				
	7	10	3	.588235	$0x, 0y$	$7x, 3y$	$12x, 5y$	
				←				
	7	17	10	.586207	$0x, 0y$	$7x, 10y$	$12x, 17y$	✓
				←				
	7	24	17	.585366	$0x, 0y$	$7x, 17y$	$12x, 29y$	✓ remarkable
				←				
	7	31	24	.584906	$0x, 0y$	$7x, 24y$	$12x, 41y$	✓
				→				
	31	55	24	.585106	$0x, 0y$	$31x, 24y$	$53x, 41y$	
				←				
	31	86	55	.585034	$0x, 0y$	$31x, 55y$	$53x, 94y$	
				←				
	31	117	86	.585000	$0x, 0y$	$31x, 86y$	$53x, 147y$	
				←				
	31	148	117	.584980	$0x, 0y$	$31x, 117y$	$53x, 200y$	
				←				
	31	179	148	.584967	$0x, 0y$	$31x, 148y$	$53x, 253y$	
				←				
	31	210	179	.584958	$0x, 0y$	$31x, 179y$	$53x, 306y$	
				→				
	210	389	179	.5849624	$0x, 0y$	$210x, 179y$	$359x, 306y$	

$4/3 = 1.333333333333 \dots$   
 $\rightarrow \text{Log}_2 .415037499279 \dots$

### Pythagorean Fourth

gen. Oct.	$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root $0x, 0y$	Generator $a_n, e_y$	Octave $b_n, f_y$
		0	1	1	1.000000	$0n, 0y$	$0n, 1y$
	1	1	0	←			
	0	1	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
	1	2	1	←			
	0	1	2	.333333	$0n, 0y$	$0n, 1y$	$1n, 2y$
	1	3	2	→			
	1	2	1	.400000	$0n, 0y$	$1n, 1y$	$3n, 2y$
	2	3	1	→			
	2	3	1	.428571	$0n, 0y$	$2n, 1y$	$5n, 2y$
	5	7	2	←			
	2	5	3	.416667	$0n, 0y$	$2n, 3y$	$5n, 7y$
	5	12	7	←			
	2	7	5	.411765	$0n, 0y$	<u><math>2n, 5y</math></u>	$5n, 12y$
	5	17	12	→			
	7	12	5	.413793	$0n, 0y$	$7n, 5y$	$17n, 12y$
	17	29	12	→			
	12	17	5	.414634	$0n, 0y$	$12n, 5y$	$29n, 12y$
	29	41	12	→			
	17	22	5	.415094	$0n, 0y$	$17n, 5y$	$41n, 12y$
	41	53	12	←			
	17	39	22	.414894	$0n, 0y$	$17n, 22y$	$41n, 53y$
	41	94	53	→			
	39	61	22	.414966	$0n, 0y$	$39n, 22y$	$94n, 53y$
	94	147	53	→			
	61	83	22	.415000	$0n, 0y$	$61n, 22y$	$147n, 53y$
	147	200	53	→			
	83	105	22	.415020	$0n, 0y$	$83n, 22y$	$200n, 53y$
	200	253	53	→			
	105	127	22	.415033	$0n, 0y$	$105n, 22y$	$253n, 53y$
	253	306	53	→			
	127	149	22	.415042	$0n, 0y$	$127n, 22y$	$306n, 53y$
	306	359	53	→			
	127	276	149	←			
	306	665	359	.4150376	$0n, 0y$	$127n, 149y$	$306n, 359y$

✓

✓

$8\sqrt{10} = 1.33352143216\dots$  Helmholtz  $\frac{1}{8}$ -skhisma, Fourth  
 $\log_2 .415241011858\dots$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root	Generator	Octave
				$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0/1	1/1	0/0	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
			←			
0/1	1/2	1/1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
			←			
0/1	1/3	2/2	.333333	$0n, 0y$	$0n, 1y$	$1n, 2y$
			→			
1/3	2/5	1/2	.400000	$0n, 0y$	$1n, 1y$	$3n, 2y$
			→			
2/5	3/7	1/2	.428571	$0n, 0y$	$2n, 1y$	$5n, 2y$
			←			
2/5	5/12	3/7	.416667	$0n, 0y$	$2n, 3y$	$5n, 7y$
			←			
2/5	7/17	5/12	.411765	$0n, 0y$	<u><math>2n, 5y</math></u>	$5n, 12y$
			→			
7/17	12/29	5/12	.413793	$0n, 0y$	$7n, 5y$	$17n, 12y$
			→			
12/29	17/41	5/12	.414634	$0n, 0y$	$12n, 5y$	$29n, 12y$
			→			
17/41	22/53	5/12	.415094	$0n, 0y$	$17n, 5y$	$41n, 12y$
			→			
22/53	27/65	5/12	.415385	$0n, 0y$	$22n, 5y$	$53n, 12y$
			←			
22/53	49/118	27/65	.415254	$0n, 0y$	$22n, 27y$	$53n, 65y$
			←			
22/53	71/171	49/118	.415205	$0n, 0y$	$22n, 49y$	$53n, 118y$
			→			
71/171	120/289	49/118	.415225	$0n, 0y$	$71n, 49y$	$171n, 118y$
			→			
120/289	169/407	49/118	.415233	$0n, 0y$	$120n, 49y$	$289n, 118y$
			→			
169/407	218/525	49/118	.415238	$0n, 0y$	$169n, 49y$	$407n, 118y$
			→			
218/525	267/643	49/118	.4152410575	$0n, 0y$	$218n, 49y$	$525n, 118y$
			←			

$$\sqrt[8]{128/5} = 1.49978841866 \dots \text{ Helmholtz } \frac{1}{8} \text{skhisma, Fifth}$$

$$\text{Log}_2 .584758988142 \dots$$

a	c	e	$\frac{c}{d}$	dec.	Root Generator Octave
b	d	f			$0x, 0y$ $a_n, e_y$ $b_n, f_y$
0	1	1	1.000000		$0x, 0y$ $0x, 1y$ $1x, 0y$
			←		
0	1/2	1	.500000		$0x, 0y$ $0x, 1y$ $1x, 1y$
			→		
1	2/3	1	.666667		$0x, 0y$ $1x, 1y$ $2x, 1y$
			←		
1	3/5	2/3	.600000		$0x, 0y$ $1x, 2y$ $2x, 3y$
			←		
1	4/7	3/5	.571429		$0x, 0y$ $1x, 3y$ <u><math>2x, 5y</math></u> ✓
			→		
4	7/12	3/5	.583333		$0x, 0y$ $4x, 3y$ $7x, 5y$
			→		
7	10/17	3/5	.588235		$0x, 0y$ $7x, 3y$ $12x, 5y$
			←		
7	17/29	10/17	.586207		$0x, 0y$ $7x, 10y$ $12x, 17y$
			←		
7	24/41	17/29	.585366		$0x, 0y$ $7x, 17y$ $12x, 29y$ ✓ remarkable
			←		
7	31/53	24/41	.584906		$0x, 0y$ $7x, 24y$ $12x, 41y$
			←		
7	38/65	31/53	.584615		$0x, 0y$ $7x, 31y$ $12x, 53y$
			→		
38	69/118	31/53	.584746		$0x, 0y$ $38x, 31y$ $65x, 53y$
			→		
69	100/171	31/53	.584795		$0x, 0y$ $69x, 31y$ $118x, 53y$
			←		
69	169/289	100/171	.584775		$0x, 0y$ $69x, 100y$ $118x, 171y$
			←		
69	238/407	169/289	.584767		$0x, 0y$ $69x, 169y$ $118x, 289y$
			←		
69	307/525	238/407	.584762		$0x, 0y$ $69x, 238y$ $118x, 407y$
			←		
69	376/643	307/525	.584758942		$0x, 0y$ $69x, 307y$ $118x, 525y$

and then 52 places → 1 pl.



$$\sqrt[8]{5} = 1.22284454499\dots$$

Neutral-Third,  $\sqrt{\quad}$  Variation  
on  $1/4$ -comma meantone

$$\text{Log}_2 .290241011856\dots$$

a	c	e	$\frac{c}{d}$	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	$0x, 0y$	$a_x, e_y$	$b_x, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0		$0x, 0y$		
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1				
			←			
0	1	1	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
1	3	2				
			←			
0	1	1	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
1	4	3				
			→			
1	2	1	.285714	$0x, 0y$	$1x, 1y$	$4x, 3y$
4	7	3				
			→			
2	3	1	.300000	$0x, 0y$	$2x, 1y$	$7x, 3y$
7	10	3				
			←			
2	5	3	.294118	$0x, 0y$	$2x, 3y$	$7x, 10y$
7	17	10				
			←			
2	7	5	.291667	$0x, 0y$	<u><math>2x, 5y</math></u>	$7x, 17y$
7	24	17				
			←			
2	9	7	.290323	$0x, 0y$	$2x, 7y$	$7x, 24y$
7	31	24				
			←			
2	11	9	.289474	$0x, 0y$	$2x, 9y$	$7x, 31y$
7	38	31				
			→			
11	20	9	.289855	$0x, 0y$	$11x, 9y$	$38x, 31y$
38	69	31				
			→			
20	29	9	.290000	$0x, 0y$	$20x, 9y$	$69x, 31y$
69	100	31				
			→			
29	38	9	.290076	$0x, 0y$	$29x, 9y$	$100x, 31y$
100	131	31				
			→			
38	47	9	.290123	$0x, 0y$	$38x, 9y$	$131x, 31y$
131	162	31				
			→			
47	56	9	.290155	$0x, 0y$	$47x, 9y$	$162x, 31y$
162	193	31				
			→			
56	65	9	.290179	$0x, 0y$	$56x, 9y$	$193x, 31y$
193	224	31				
			→			

$$2/\sqrt[8]{5} = 1.63553086792\dots$$

Neutral Sixth,  $\sqrt{}$  variation  
on 1/4-comma Meantone

$$\text{Log}_2 .709758988144\dots$$

a	c	e	$\frac{c}{e}$	Root	Generator	Octave
b	d	f	$\frac{d}{f}$	$0x, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0		$0x, 0y$		
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1				
			→			
1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
2	3	1				
			→			
2	3	1	.750000	$0x, 0y$	$2x, 1y$	$3x, 1y$
3	4	1				
			←			
2	5	3	.714286	$0x, 0y$	$2x, 3y$	$3x, 4y$
3	7	4				
			←			
2	7	5	.700000	$0x, 0y$	<u><math>2x, 5y</math></u>	$3x, 7y$
3	10	7				
			→			
7	12	5	.705882	$0x, 0y$	$7x, 5y$	$10x, 7y$
10	17	7				
			→			
12	17	5	.708333	$0x, 0y$	$12x, 5y$	$17x, 7y$
17	24	7				
			→			
17	22	5	.709677	$0x, 0y$	$17x, 5y$	$24x, 7y$
24	31	7				
			→			
22	27	5	.710526	$0x, 0y$	$22x, 5y$	$31x, 7y$
31	38	7				
			←			
22	49	27	.710145	$0x, 0y$	$22x, 27y$	$31x, 38y$
31	69	38				
			←			
22	71	49	.710000	$0x, 0y$	$22x, 49y$	$31x, 69y$
31	100	69				
			←			
22	93	71	.709924	$0x, 0y$	$22x, 71y$	$31x, 100y$
31	131	100				
			←			
22	115	93	.709877	$0x, 0y$	$22x, 93y$	$31x, 131y$
31	162	131				
			←			
22	137	115	.709845	$0x, 0y$	$22x, 115y$	$31x, 162y$
31	193	162				
			←			
22	159	137	.709821	$0x, 0y$	$22x, 137y$	$31x, 193y$
31	224	193				
			←			

$$2 \sqrt{\frac{192}{5}} = 1.49981192623 \dots$$

$$\text{Log}_2 = .584781600652 \dots$$

1/9 skhisma  
Fifth

a	c	e	c	Root Generator Octave
b	d	f	d dec.	$0n, 0y$ $a_n, e_y$ $b_n, f_y$
0	1	1	1.000000	$0n, 0y$ $0n, 1y$ $1n, 0y$
			←	
0	1	1	.500000	$0n, 0y$ $0n, 1y$ $1n, 1y$
			→	
1	2	1	.666667	$0n, 0y$ $1n, 1y$ $2n, 1y$
			←	
1	3	2	.600000	$0n, 0y$ $1n, 2y$ $2n, 3y$
			←	
1	4	3	.571429	$0n, 0y$ $1n, 3y$ $2n, 5y$
			→	
4	7	3	.583333	$0n, 0y$ $4n, 3y$ $7n, 5y$
			→	
7	10	3	.588235	$0n, 0y$ $7n, 3y$ $12n, 5y$
			←	
7	17	10	.586207	$0n, 0y$ $7n, 10y$ $12n, 17y$
			←	
7	24	17	.585366	$0n, 0y$ $7n, 17y$ $12n, 29y$
			←	
7	31	24	.584906	$0n, 0y$ $7n, 24y$ $12n, 41y$
			←	
7	38	31	.584615	$0n, 0y$ $7n, 31y$ $12n, 53y$
			→	
38	69	31	.584746	$0n, 0y$ $38n, 31y$ $65n, 53y$
			→	
69	100	31	.584795	$0n, 0y$ $69n, 31y$ $118n, 53y$
			←	
69	169	100	.584775	$0n, 0y$ $69n, 100y$ $118n, 171y$
			→	
169	269	100	.584783	$0n, 0y$ $169n, 100y$ $289n, 171y$
			←	
169	438	269	.584780	$0n, 0y$ $169n, 269y$ $289n, 460y$
			→	

28MAY00.EW

$$\sqrt{\frac{3}{2}} = 1.22474487139\dots$$

Neutral Third,  $\sqrt{\phantom{x}}$  variation on Pythagorean

$$\rightarrow \text{Log}_2 .292481250359\dots$$

a	c	e	$\frac{c}{d}$	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	$0x, 0y$	$a'x, e'y$	$b'x, f'y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0				
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1				
			←			
0	1	1	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
1	3	2				
			←			
0	1	1	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
1	4	3				
			→			
1	2	1	.285714	$0x, 0y$	$1x, 1y$	$4x, 3y$
4	7	3				
			→			
2	3	1	.300000	$0x, 0y$	$2x, 1y$	$7x, 3y$
7	10	3				
			←			
2	5	3	.294118	$0x, 0y$	$2x, 3y$	$7x, 10y$
7	17	10				
			←			
2	7	5	.291667	$0x, 0y$	$2x, 5y$	$7x, 17y$
7	24	17				
			→			
7	12	5	.292683	$0x, 0y$	$7x, 5y$	$24x, 17y$
24	41	17				
			←			
7	19	12	.292308	$0x, 0y$	$7x, 12y$	$24x, 41y$
24	65	41				
			→			
19	31	12	.292453	$0x, 0y$	$19x, 12y$	$65x, 41y$
65	106	41				
			→			
31	43	12	.292517	$0x, 0y$	$31x, 12y$	$106x, 41y$
106	147	41				
			←			
31	74	43	.292490	$0x, 0y$	$31x, 43y$	$106x, 147y$
106	253	147				
			←			
31	105	74	.292479	$0x, 0y$	$31x, 74y$	$106x, 253y$
106	359	253				
			→			
105	179	74	.292484	$0x, 0y$	$105x, 74y$	$359x, 253y$
359	612	253				
			←			
105	284	179	.292482	$0x, 0y$	$105x, 179y$	$359x, 612y$
359	971	612				
			←			

$$2/\sqrt{\frac{3}{2}} = 1.63299316186\dots$$

Neutral Sixth,  $\sqrt{\text{variation}}$   
on Pythagorean

$$\rightarrow \text{Log}_2 \underline{.707518749641\dots}$$

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	Root	Generator	Octave
				$0x, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
←						
0	1	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
→						
1	2	1	.666667	$0n, 0y$	$1n, 1y$	$2n, 1y$
→						
2	3	1	.750000	$0n, 0y$	$2n, 1y$	$3n, 1y$
←						
2	5	3	.714286	$0n, 0y$	$2n, 3y$	$3n, 4y$
←						
2	7	5	.700000	$0n, 0y$	<u><math>2n, 5y</math></u>	$3n, 7y$
→						
7	12	5	.705882	$0n, 0y$	$7n, 5y$	$10n, 7y$
→						
12	17	5	.708333	$0n, 0y$	$12n, 5y$	$17n, 7y$
←						
12	29	17	.707317	$0n, 0y$	$12n, 17y$	$17n, 24y$
→						
29	46	17	.707692	$0n, 0y$	$29n, 17y$	$41n, 24y$
←						
29	75	46	.707547	$0n, 0y$	$29n, 46y$	$41n, 65y$
←						
29	104	75	.707483	$0n, 0y$	$29n, 75y$	$41n, 106y$
→						
104	179	75	.707510	$0n, 0y$	$104n, 75y$	$147n, 106y$
→						
179	254	75	.707521	$0n, 0y$	$179n, 75y$	$253n, 106y$
←						
179	433	254	.707516	$0n, 0y$	$179n, 254y$	$253n, 359y$
→						
433	687	254	.707518	$0n, 0y$	$433n, 254y$	$612n, 359y$
→						

$$\sqrt[6]{3} = 1.20093695518\dots$$

$$\rightarrow \text{Log}_2 .264160416792\dots$$

Tanaka/Hanson  
Kleismatic Temperament  
Thirds

a	c	e	$\frac{c}{d}$ dec.	Root	Generator	Octave
b	d	f		$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
1	1	0	←			
0	1	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
1	2	1	←			
0	1	1	.333333	$0n, 0y$	$0n, 1y$	$1n, 2y$
1	3	2	←			
0	1	1	.250000	$0n, 0y$	$0n, 1y$	$1n, 3y$
1	4	3	→			
1	2	1	.285714	$0n, 0y$	$1n, 1y$	$4n, 3y$
4	7	3	←			
1	3	2	.272727	$0n, 0y$	$1n, 2y$	$4n, 7y$
4	11	7	←			
1	4	3	.266667	$0n, 0y$	$1n, 3y$	$4n, 11y$
4	15	11	←			
1	5	4	.263158	$0n, 0y$	$1n, 4y$	$4n, 15y$
4	19	15	→			
5	9	4	.264706	$0n, 0y$	$5n, 4y$	$19n, 15y$
19	34	15	←			
5	14	9	.264151	$0n, 0y$	$5n, 9y$	$19n, 34y$
19	53	34	→			
14	23	9	.264368	$0n, 0y$	$14n, 9y$	$53n, 34y$
53	87	34	←			
14	37	23	.264286	$0n, 0y$	$14n, 23y$	$53n, 87y$
53	140	87	←			
14	51	37	.264249	$0n, 0y$	$14n, 37y$	$53n, 140y$
53	193	140	←			
14	65	51	.264228	$0n, 0y$	$14n, 51y$	$53n, 193y$
53	246	193	←			
14	79	65	.264214	$0n, 0y$	$14n, 65y$	$53n, 246y$
53	299	246	←			
14	93	79	.264205	$0n, 0y$	$14n, 79y$	$53n, 299y$
53	352	299	←			
36 Places	etc.		←			



$$2/\sqrt[6]{3} = 1.66536635531\dots$$

$$\rightarrow \text{Log}_2 .735839583208\dots$$

Tanaka/Hanson  
Kleismatic Temperament  
Sixths

a	c	e	$\frac{c}{d}$	dec.	Root	Generator	Octave
b	d	f	d		$0x, 0y$	$a_x, e_y$	$b_x, f_y$
0	1	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
←							
0	1	1	2	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
→							
1	2	1	3	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
→							
2	3	1	4	.750000	$0x, 0y$	$2x, 1y$	$3x, 1y$
←							
2	5	3	7	.714286	$0x, 0y$	$2x, 3y$	$3x, 4y$
→							
5	8	3	11	.727273	$0x, 0y$	$5x, 3y$	$7x, 4y$
→							
8	11	3	15	.733333	$0x, 0y$	$8x, 3y$	$11x, 4y$
→							
11	14	3	19	.736842	$0x, 0y$	$11x, 3y$	$15x, 4y$
←							
11	25	14	34	.735294	$0x, 0y$	$11x, 14y$	$15x, 19y$
→							
25	39	14	53	.735849	$0x, 0y$	$25x, 14y$	$34x, 19y$
←							
25	64	39	87	.735632	$0x, 0y$	$25x, 39y$	$34x, 53y$
→							
64	103	39	140	.735714	$0x, 0y$	$64x, 39y$	$87x, 53y$
→							
103	142	39	193	.735751	$0x, 0y$	$103x, 39y$	$140x, 53y$
→							
142	181	39	246	.735772	$0x, 0y$	$142x, 39y$	$193x, 53y$
→							
181	220	39	299	.735786	$0x, 0y$	$181x, 39y$	$246x, 53y$
→							
220	259	39	352	.735795	$0x, 0y$	$220x, 39y$	$299x, 53y$
→							

$$\sqrt[3]{3} = 1.16993081276\dots$$

Subminor-Third

$$\rightarrow \text{Log}_2 .226423214390\dots$$

a	c	e	c	Root	Generator	Octave
b	d	f	d	$0x, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0	1	1	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			←			
0	1	1	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
			←			
0	1	1	.200000	$0x, 0y$	$0x, 1y$	$1x, 4y$
			→			
1	2	1	.222222	$0x, 0y$	$1x, 1y$	$5x, 4y$
			→			
2	3	1	.230769	$0x, 0y$	$2x, 1y$	$9x, 4y$
			←			
2	5	3	.227273	$0x, 0y$	$2x, 3y$	$9x, 13y$
			←			
2	7	5	.225806	$0x, 0y$	<u><math>2x, 5y</math></u>	$9x, 22y$
			→			
7	12	5	.226415	$0x, 0y$	$7x, 5y$	$31x, 22y$
			→			
12	17	5	.226667	$0x, 0y$	$12x, 5y$	$53x, 22y$
			←			
12	29	17	.226563	$0x, 0y$	$12x, 17y$	$53x, 75y$
			←			
12	41	29	.226519	$0x, 0y$	$12x, 29y$	$53x, 128y$
			←			
12	53	41	.226496	$0x, 0y$	$12x, 41y$	$53x, 181y$
			←			
12	65	53	.226481	$0x, 0y$	$12x, 53y$	$53x, 234y$
			←			
12	77	65	.226471	$0x, 0y$	$12x, 65y$	$53x, 287y$
			←			

$$2/\sqrt[3]{3} = 1.70950279981\dots$$

Supramajor - Sixth

$$\rightarrow \text{Log}_2 .773576785610\dots$$

a	c	e	$\frac{c}{d}$	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	$0x, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
1	1	0	←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
1	2	1	→			
1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
2	3	1	→			
2	3	1	.750000	$0x, 0y$	$2x, 1y$	$3x, 1y$
3	4	1	→			
3	4	1	.800000	$0x, 0y$	$3x, 1y$	$4x, 1y$
3	7	4	←			
3	7	4	.777778	$0x, 0y$	$3x, 4y$	$4x, 5y$
4	9	5	←			
3	10	7	.769231	$0x, 0y$	$3x, 7y$	$4x, 9y$
4	13	9	→			
10	17	7	.772727	$0x, 0y$	$10x, 7y$	$13x, 9y$
13	22	9	→			
17	24	7	.774194	$0x, 0y$	$17x, 7y$	$22x, 9y$
22	31	9	←			
17	41	24	.773585	$0x, 0y$	$17x, 24y$	$22x, 31y$
22	53	31	←			
17	58	41	.773333	$0x, 0y$	$17x, 41y$	$22x, 53y$
22	75	53	→			
58	99	41	.773438	$0x, 0y$	$58x, 41y$	$75x, 53y$
75	128	53	→			
99	140	41	.773481	$0x, 0y$	$99x, 41y$	$128x, 53y$
128	181	53	→			
140	181	41	.773504	$0x, 0y$	$140x, 41y$	$181x, 53y$
181	234	53	→			
181	222	41	.773519	$0x, 0y$	$181x, 41y$	$234x, 53y$
234	287	53	→			
222	263	41	.773529	$0x, 0y$	$222x, 41y$	$287x, 53y$
287	340	53	→			

41x7

$$\sqrt[2]{40} = 1.50663019029\dots$$

$$\text{Log}_2 = .591325343871\dots$$

$$\sqrt[2]{\frac{20,480}{19,683}}$$

Fifth

a	c	e	c	Root Generator Octave
b	d	f	d dec.	$0n, 0y$ $a_n, e_y$ $b_n, f_y$
0	1	1	1.000000	$0n, 0y$ $0n, 1y$ $1n, 0y$
			←	
0	1	1	.500000	$0n, 0y$ $0n, 1y$ $1n, 1y$
			→	
1	2	1	.666667	$0n, 0y$ $1n, 1y$ $2n, 1y$
			←	
1	3	2	.600000	$0n, 0y$ $1n, 2y$ $2n, 3y$
			←	
1	4	3	.571429	$0n, 0y$ $1n, 3y$ $2n, 5y$
			→	
4	7	3	.583333	$0n, 0y$ $4n, 3y$ $7n, 5y$
			→	
7	10	3	.588235	$0n, 0y$ $7n, 3y$ $12n, 5y$
			→	
10	13	3	.590909	$0n, 0y$ $10n, 3y$ $17n, 5y$
			→	
13	16	3	.592593	$0n, 0y$ $13n, 3y$ $22n, 5y$
			←	
13	29	16	.591837	$0n, 0y$ $13n, 16y$ $22n, 27y$
			←	
13	42	29	.591549	$0n, 0y$ $13n, 29y$ $22n, 49y$ ✓
			←	
13	55	42	.591398	$0n, 0y$ $13n, 42y$ $22n, 71y$
			←	
13	68	55	.591304	$0n, 0y$ $13n, 55y$ $22n, 93y$
			→	
68	123	55	.591346	$0n, 0y$ $68n, 55y$ $115n, 93y$
			←	
68	191	123	.591331	$0n, 0y$ $68n, 123y$ $115n, 208y$
			←	
68	259	191	.591324	$0n, 0y$ $68n, 191y$ $115n, 323y$
			→	

$$\sqrt[9]{64/5} = 1.32746576624\dots$$

$$\text{Log}_2 = .408674656129\dots$$

$$\sqrt[9]{\frac{20,480}{19,683}}$$

Fourth

a	c	e	c	Root Generator	Octave	
b	d	f	d dec.	$0x, 0y$	$a_x, e_y$	$b_x, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1/2	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0	1/3	1	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			→			
1	2/5	1	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
			→			
2	3/7	1	.428571	$0x, 0y$	$2x, 1y$	$5x, 2y$
			←			
2	5/12	3	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
			←			
2	7/17	5	.411765	$0x, 0y$	$2x, 5y$	$5x, 12y$
			←			
2	9/22	7	.409091	$0x, 0y$	$2x, 7y$	$5x, 17y$
			←			
2	11/27	9	.407407	$0x, 0y$	$2x, 9y$	$5x, 22y$
			→			
11	20/49	9	.408163	$0x, 0y$	$11x, 9y$	$27x, 22y$
			→			
20	29/71	9	.408451	$0x, 0y$	$20x, 9y$	$49x, 22y$
			→			
29	38/93	9	.408602	$0x, 0y$	$29x, 9y$	$71x, 22y$
			→			
38	47/115	9	.408696	$0x, 0y$	$38x, 9y$	$93x, 22y$
			←			
38	85/208	47	.408654	$0x, 0y$	$38x, 47y$	$93x, 115y$
			→			
85	132/323	47	.408669	$0x, 0y$	$85x, 47y$	$208x, 115y$
			→			
132	179/438	47	.408676	$0x, 0y$	$132x, 47y$	$323x, 115y$
			←			

$$115 + (22 \times 25) = 665$$

$$\sqrt[2]{\frac{40}{3}} = 1.33350053098\dots$$

$$\text{Log}_2 = \underline{.415218399348\dots}$$

1/9 Skhisma  
Fourth

a	c	e	c	Root Generator	Octave	
b	d	f	d dec.	$0x, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0	1	2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			→			
1	2	2	.400000	$0x, 0y$	$1x, 1y$	$3x, 2y$
			→			
2	3	2	.428471	$0x, 0y$	$2x, 1y$	$5x, 2y$
			←			
2	5	3	.416667	$0x, 0y$	$2x, 3y$	$5x, 7y$
			←			
2	7	5	.411765	$0x, 0y$	$2x, 5y$	$5x, 12y$
			→			
7	12	5	.413793	$0x, 0y$	$7x, 5y$	$17x, 12y$
			→			
12	17	5	.414634	$0x, 0y$	$12x, 5y$	$29x, 12y$
			→			
17	22	5	.415094	$0x, 0y$	$17x, 5y$	$41x, 12y$
			→			
22	27	5	.415385	$0x, 0y$	$22x, 5y$	$53x, 12y$
			←			
22	49	27	.415254	$0x, 0y$	$22x, 27y$	$53x, 65y$
			←			
22	71	49	.415205	$0x, 0y$	$22x, 49y$	$53x, 118y$
			→			
71	120	49	.415225	$0x, 0y$	$71x, 49y$	$171x, 118y$
			←			
71	191	120	.415217	$0x, 0y$	$71x, 120y$	$171x, 289y$
			→			
191	311	120	.415220	$0x, 0y$	$191x, 120y$	$460x, 289y$
			←			



$$\sqrt[9]{\frac{192}{5}} = 1.49981192623\dots$$

$$\log_2 = .584781600652\dots$$

1/9 skhisma  
Fifth

$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$ dec.	Root $0n, 0y$	Generator $a_n, e_y$	Octave $b_n, f_y$
0/1	1/1	1/0	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
			←			
0/1	1/2	1/1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
			→			
1/2	2/3	1/1	.666667	$0n, 0y$	$1n, 1y$	$2n, 1y$
			←			
1/2	3/5	2/3	.600000	$0n, 0y$	$1n, 2y$	$2n, 3y$
			←			
1/2	4/7	3/5	.571429	$0n, 0y$	$1n, 3y$	$2n, 5y$
			→			
4/7	7/12	3/5	.583333	$0n, 0y$	$4n, 3y$	$7n, 5y$
			→			
7/12	10/17	3/5	.588235	$0n, 0y$	$7n, 3y$	$12n, 5y$
			←			
7/12	17/29	10/17	.586207	$0n, 0y$	$7n, 10y$	$12n, 17y$
			←			
7/12	24/41	17/29	.585366	$0n, 0y$	$7n, 17y$	$12n, 29y$
			←			
7/12	31/53	24/41	.584906	$0n, 0y$	$7n, 24y$	$12n, 41y$
			←			
7/12	38/65	31/53	.584615	$0n, 0y$	$7n, 31y$	$12n, 53y$
			→			
38/65	69/118	31/53	.584746	$0n, 0y$	$38n, 31y$	$65n, 53y$
			→			
69/118	100/171	31/53	.584795	$0n, 0y$	$69n, 31y$	$118n, 53y$
			←			
69/118	169/289	100/171	.584775	$0n, 0y$	$69n, 100y$	$118n, 171y$
			→			
169/289	269/460	100/171	.584783	$0n, 0y$	$169n, 100y$	$289n, 171y$
			←			
169/289	438/749	269/460	.584780	$0n, 0y$	$169n, 269y$	$289n, 460y$
			→			

(31)

28MAY00.EW

$$2/\sqrt[5]{5} = \begin{matrix} \nearrow 1.63553086792\dots \\ \nearrow .709758988144\dots \\ \text{Log}_2 \end{matrix}$$

Neutral-Sixth variation  
on  $\frac{1}{4}$ -Comma meantone

a	c	e	$\frac{c}{d}$	Root	Generator	Octave
$\frac{a}{b}$	$\frac{c}{d}$	$\frac{e}{f}$	$\frac{c}{d}$	$0x, 0y$	$a/x, e/y$	$b/x, f/y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
←						
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
→						
1	2	1	.666667	$0x, 0y$	$1x, 1y$	$2x, 1y$
→						
2	3	1	.750000	$0x, 0y$	$2x, 1y$	$3x, 1y$
←						
2	5	3	.714286	$0x, 0y$	$2x, 3y$	$3x, 4y$ ✓
←						
2	7	5	.700000	$0x, 0y$	<u><math>2x, 5y</math></u>	$3x, 7y$ ✓
→						
7	12	5	.705882	$0x, 0y$	$7x, 5y$	$10x, 7y$
→						
12	17	5	.708333	$0x, 0y$	$12x, 5y$	$17x, 7y$
→						
17	22	5	.709677	$0x, 0y$	$17x, 5y$	$24x, 7y$
→						
22	27	5	.710526	$0x, 0y$	$22x, 5y$	$31x, 7y$
←						
22	49	27	.710145	$0x, 0y$	$22x, 27y$	$31x, 38y$
←						
22	71	49	.710000	$0x, 0y$	$22x, 49y$	$31x, 69y$
←						
22	93	71	.709924	$0x, 0y$	$22x, 71y$	$31x, 100y$
←						
22	115	93	.709877	$0x, 0y$	$22x, 93y$	$31x, 131y$
←						
22	137	115	.709845	$0x, 0y$	$22x, 115y$	$31x, 162y$
←						
22	159	137	.709821	$0x, 0y$	$22x, 137y$	$31x, 193y$
←						

checked  
8FE300.EW

Log<sub>2</sub>

.773576785610  $\frac{2}{\sqrt{3}} = 1.70950279981...$

$\frac{3}{1} \div (\frac{7}{6})^7$  Temperament

a	c	e	$\frac{c}{d}$	Root	Generator	Octave
b	d	f	$\frac{c}{d}$	$0n, 0y$	$a_n, e_y$	$b_n, f_y$
0	1	1	1.000000	$0n, 0y$	$0n, 1y$	$1n, 0y$
1	1	0	←			
0	1/2	1	.500000	$0n, 0y$	$0n, 1y$	$1n, 1y$
1	2	1	→			
1	2/3	1	.666667	$0n, 0y$	$1n, 1y$	$2n, 1y$
2	3	1	→			
2	3/4	1	.750000	$0n, 0y$	$2n, 1y$	$3n, 1y$
3	4	1	→			
3	4/5	1	.800000	$0n, 0y$	$3n, 1y$	$4n, 1y$
4	5	1	←			
3	7/9	4/5	.777778	$0n, 0y$	$3n, 4y$	$4n, 5y$
4	9	5	←			
3	10/13	7/9	.769231	$0n, 0y$	$3n, 7y$	$4n, 9y$
4	13	9	→			✓
10	17/22	7/9	.772727	$0n, 0y$	$10n, 7y$	$13n, 9y$
13	22	9	→			
17	24/31	7/9	.774194	$0n, 0y$	$17n, 7y$	$22n, 9y$
22	31	9	←			
17	41/53	24/31	.773585	$0n, 0y$	$17n, 24y$	$22n, 31y$
22	53	31	←			
17	58/75	41/53	.773333	$0n, 0y$	$17n, 41y$	$22n, 53y$
22	75	53	→			✓
53	99/128	41/53	.773438	$0n, 0y$	$58n, 41y$	$75n, 53y$
75	128	53	→			
99	140/181	41/53	.773481	$0n, 0y$	$99n, 41y$	$128n, 53y$
128	181	53	→			
140	181/234	41/53	.773504	$0n, 0y$	$140n, 41y$	$181n, 53y$
181	234	53	→			
181	222/287	41/53	.773519	$0n, 0y$	$181n, 41y$	$234n, 53y$
234	287	53	→			
222	263/340	41/53	.773529	$0n, 0y$	$222n, 41y$	$287n, 53y$
287	340	53	→			

41x7

INC  
checked ✓  
8FEB00·EW

$\log_2 .226423214390... \quad \sqrt[7]{3} = 1.16993081276...$

$\frac{3}{1} \div (\frac{7}{6})^7$  Temperament

a	c	e	$\frac{c}{d}$	Root	Generator	Octave	
b	d	f	$\frac{c}{d}$	$0x, 0y$	$a, e, y$	$b, f, y$	$\frac{3}{1}$
0	1	0	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$	<del><math>\frac{3}{1}</math></del>
			←				$\frac{823,543}{279,936}$
0	1	1	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$	$\frac{839,808}{823,543}$
			←				
0	1	2	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$	
			←				
0	1	3	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$	
			←				
0	1	4	.200000	$0x, 0y$	$0x, 1y$	$1x, 4y$	
			→				
1	2	4	.222222	$0x, 0y$	$1x, 1y$	$5x, 4y$	
			→				
2	3	4	.230769	$0x, 0y$	$2x, 1y$	$9x, 4y$	
			←				
2	5	13	.227273	$0x, 0y$	$2x, 3y$	$9x, 13y$	✓
			←				
2	7	22	.225806	$0x, 0y$	<u><math>2x, 5y</math></u>	$9x, 22y$	✓
			→				
7	12	22	.226415	$0x, 0y$	$7x, 5y$	$31x, 22y$	
			→				
12	17	22	.226667	$0x, 0y$	$12x, 5y$	$53x, 22y$	
			←				
12	29	75	.226563	$0x, 0y$	$12x, 17y$	$53x, 75y$	
			←				
12	41	128	.226519	$0x, 0y$	$12x, 29y$	$53x, 128y$	✓
			←				
12	53	181	.226496	$0x, 0y$	$12x, 41y$	$53x, 181y$	
			←				
12	65	234	.226481	$0x, 0y$	$12x, 53y$	$53x, 234y$	
			←				
12	77	287	.226471	$0x, 0y$	$12x, 65y$	$53x, 287y$	
			←				

inc

checked ✓

$$2^{\left(\frac{7}{72}\right)} = 1.06971184581\dots$$

$$\rightarrow \text{Log}_2 .0972222222\dots$$

a	c	e	$\frac{c}{d}$ dec.	Root	Generator	Octave
b	d	f		$0x, 0y$	$a_x, e_y$	$b_x, f_y$
0	1	1	1.000000	$0x, 0y$	$0x, 1y$	$1x, 0y$
			←			
0	1	2	.500000	$0x, 0y$	$0x, 1y$	$1x, 1y$
			←			
0	1	3	.333333	$0x, 0y$	$0x, 1y$	$1x, 2y$
			←			
0	1	4	.250000	$0x, 0y$	$0x, 1y$	$1x, 3y$
			←			
0	1	5	.200000	$0x, 0y$	$0x, 1y$	$1x, 4y$
			←			
0	1	6	.166667	$0x, 0y$	$0x, 1y$	$1x, 5y$
			←			
0	1	7	.142857	$0x, 0y$	$0x, 1y$	$1x, 6y$
			←			
0	1	8	.125000	$0x, 0y$	$0x, 1y$	$1x, 7y$
			←			
0	1	9	.111111	$0x, 0y$	$0x, 1y$	$1x, 8y$
			←			
0	1	10	.100000	$0x, 0y$	$0x, 1y$	$1x, 9y$
			←			
0	1	11	.090909	$0x, 0y$	$0x, 1y$	$1x, 10y$
			→			
1	2	1	.095238	$0x, 0y$	$1x, 1y$	$11x, 10y$
			→			
2	3	1	.096774	$0x, 0y$	$2x, 1y$	$21x, 10y$
			→			
3	4	1	.097561	$0x, 0y$	$3x, 1y$	$31x, 10y$
			←			
3	7	4	<u>.097222</u>	$0x, 0y$	$3x, 4y$	$31x, 41y$

Ref; A New Look at the Partch Monophonic Fabric, George Secor 1975, XH3

23 JUN 01 EW

Zig-Zag Series (1←, 1→), Limit Kornerup's Golden Fifth  $\frac{c}{d}$

$\log_2 1.580178728295$

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Sheet 1 of 2  
27 JUL 2000 E.W.

a	c	e	$\frac{c}{d}$ dec
b	d	f	
1	5	2	.600000
2	5	3	←
2	4	3	.571429
4	7	5	→
7	12	5	.583333
			←
4	11	7	.578947
7	19	12	→
11	18	7	.580645
19	31	12	←
11	29	18	.580000
19	50	31	→
29	47	18	.580247
50	81	31	←
29	76	47	.580153
50	131	81	→
76	123	47	.580189
131	212	81	←
76	199	123	.580175
131	343	212	→
199	322	123	.580180
343	555	212	←
199	521	322	.5801782
343	898	555	→
521	843	322	.5801789
898	1453	555	←
521	1,364	843	.5801786
898	2,351	1453	→
1,364	2,207	843	.5801788
2,351	3804	1453	←
1,364	3571	2207	.5801787
2,351	6155	3804	

Ref: Acoustic Methods of Work, Thorvald Kornerup, 1934  
 A Theory of Evolving Tonality, Joseph Yasser, 1932  
 Diophantine Triplets and  $N, Y$  Coordinates, Ervin M. Wilson, 2000



Zig-Zag Series (1→, 1←), Limit Kornerup's Golden Fourth  $\frac{c}{d}$

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Sheet 2.

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a	c	e	$\frac{c}{d}$ dec
1	2	1	.400000
3	5	2	→
2	3	1	.428571
5	7	2	←
2	5	3	.416667
5	12	7	→
5	8	3	.421053
12	19	7	←
5	13	8	.419355
12	31	19	→
13	21	8	.420000
31	50	19	←
13	34	21	.419753
31	81	50	→
34	55	21	.419847
81	131	50	←
34	89	55	.419811
81	212	131	→
89	144	55	.419825
212	343	131	←
89	233	144	.419820
212	555	343	→
233	377	144	.4198218
555	898	343	←
233	610	377	.4198211
555	1453	898	→
610	987	377	.4198214
1453	2351	898	←
610	1597	987	.41982124
1453	3804	2351	→
1597	2584	987	.41982128
3804	6155	2351	