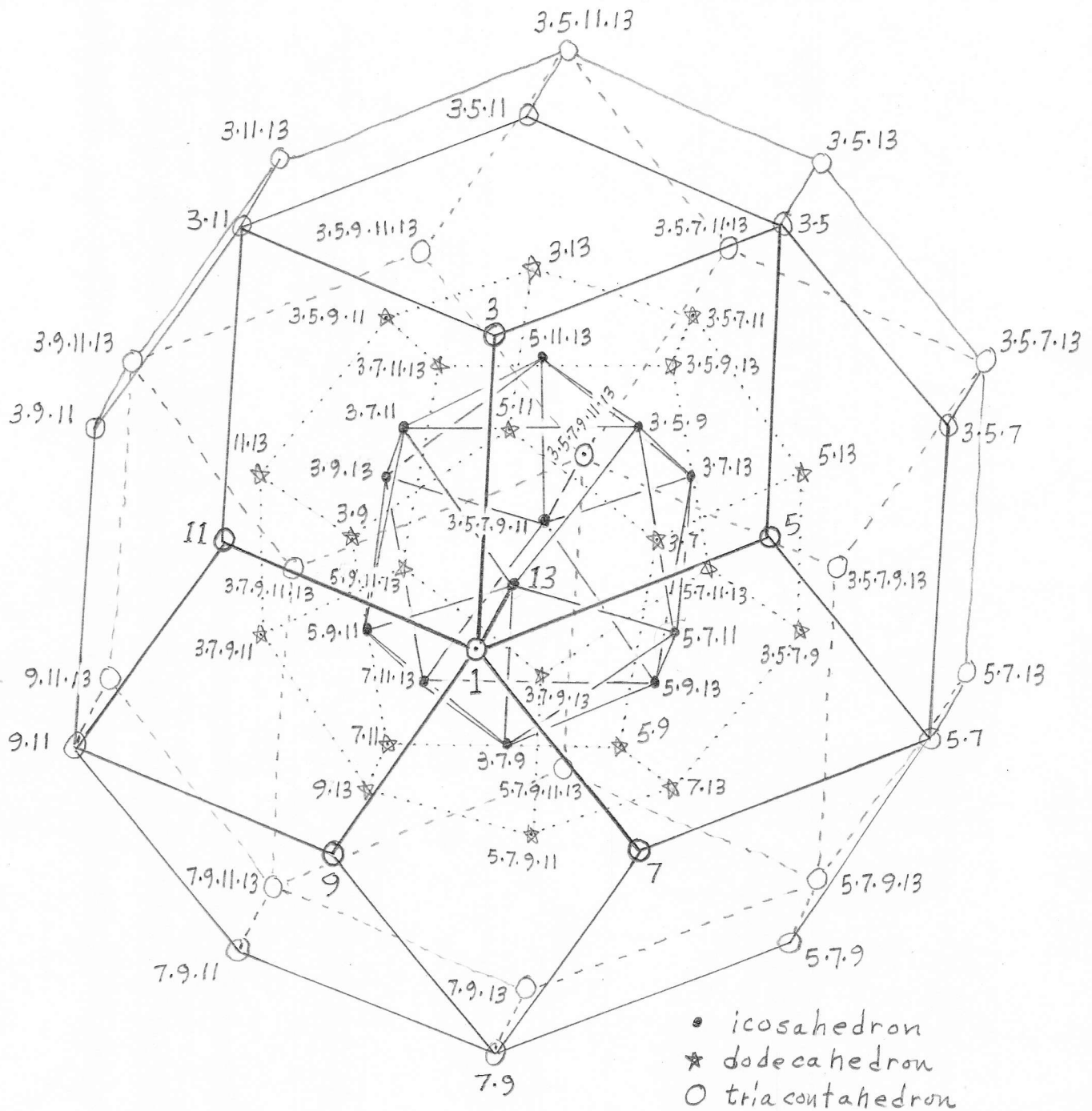


# Euler Genus (3.5.7.9.11.13) lattice within the triakontahedron

©1999 by Erv Wilson, work in progress -



## References:

Tentamen novae theoriae musicae, Leonhard Euler, 1739

D'alexandro like a hurricane, Erv Wilson 1989, fig 206, Xenharmonikon XII

# Euler genus (3,5,7,9,11,13) sorted

	genus	log2	increment	genus	log2	increment
fundamental	0. 1	.0000	2079/2048	33. 7.13	.5077	
	1. 3.7.9.11	.0216	2080/2079	→		1485/1456
	2. 5.13	.0223	2079/2048	34. 3.5.9.11	.5362	91/90
guidetone	3. 3.5.7.9.11.13	.0440	4096/4095	35. 3.7.11.13	.5521	144/143
	4. 3.11	.0443	65/64	36. 3.7.9	.5622	65/64
	5. 3.5.11.13	.0667	144/143	37. 3.5.7.9.13	.5846	4096/4095
	6. 3.5.9	.0768	91/90	38. 3	.5849	385/384
	7. 3.7.13	.0923	40/39	39. 5.7.11	.5887	78/77
	8. 5.7	.1295	1287/1280	40. 3.5.13	.6073	66/65
	9. 7.9.11.13	.1371	64/63	41. 9.11	.6293	65/64
	10. 11.13	.1598	144/143	42. 5.9.11.13	.6517	56/55
	11. 9	.1699	385/384	43. 7.9.13	.6777	64/63
	12. 3.5.7.11	.173	78/77	44. 13	.7004	105/104
	13. 5.9.13	.192	66/65	45. 3.5.7	.7142	1287/1280
	14. 3.9.11	.214	65/64	46. 3.7.9.11.13	.7221	64/63
	15. 3.5.9.11.13	.236	56/55	47. 3.11.13	.7448	144/143
	16. 3.7.9.13	.262	352/351	48. 3.9	.7548	385/384
	17. 7.11	.266	78/77	49. 5.7.9.11	.7586	78/77
	18. 3.13	.285	385/384	50. 3.5.9.13	.7772	352/351
	19. 5.7.11.13	.289	144/143	51. 5.11	.7813	56/55
	20. 5.7.9	.2992	64/63	52. 7	.8073	65/64
	21. 5	.321	1287/1280	53. 5.7.13	.8297	66/65
	22. 9.11.13	.329	105/104	54. 3.7.11	.8517	78/77
	23. 3.5.7.9.11	.343	64/63	55. 9.13	.8703	385/384
	24. 3.5.11	.366	56/55	56. 3.5.7.11.13	.8741	144/143
	25. 3.7	.392	65/64	57. 3.5.7.9	.8841	64/63
	26. 3.5.7.13	.414	66/65	58. 3.5	.9068	1287/1280
	27. 7.9.11	.436	78/77	59. 3.9.11.13	.9147	40/39
	28. 3.9.13	.455	385/384	60. 5.9.11	.9512	91/90
	29. 5.7.9.11.13	.4590	4096/4095	61. 7.11.13	.9672	144/143
	30. 11	.4594	65/64	62. 7.9	.9772	65/64
	31. 5.11.13	.4817	144/143	63. 5.7.9.13	.9996	4096/4095
	32. 5.9	.4918	91/90	64/0. 1	1.0000	(2079/2048)
	(33. 7.13	.5077)		(1. 3.7.9.11	.0216)	

Ø null	1.Ø	256.0000000000	5.7.9	1.5.7.9	315.0000000000
3	1.3	384.0000000000	5.7.11	1.5.7.11	385.0000000000
5	1.5	320.0000000000	5.7.13	1.5.7.13	455.0000000000
7	1.7	448.0000000000	5.9.11	1.5.9.11	495.0000000000
9	1.9	288.0000000000	5.9.13	1.5.9.13	292.5000000000
11	1.11	352.0000000000	5.11.13	1.5.11.13	357.5000000000
13	1.13	416.0000000000	7.9.11	1.7.9.11	346.5000000000
3.5	1.3.5	480.0000000000	7.9.13	1.7.9.13	409.5000000000
3.7	1.3.7	336.0000000000	7.11.13	1.7.11.13	500.5000000000
3.9	1.3.9	432.0000000000	9.11.13	1.9.11.13	321.7500000000
3.11	1.3.11	264.0000000000	3.5.7.9	1.3.5.7.9	472.5000000000
3.13	1.3.13	312.0000000000	3.5.7.11	1.3.5.7.11	288.7500000000
5.7	1.5.7	280.0000000000	3.5.7.13	1.3.5.7.13	341.2500000000
5.9	1.5.9	360.0000000000	3.5.9.11	1.3.5.9.11	371.2500000000
5.11	1.5.11	440.0000000000	3.5.9.13	1.3.5.9.13	438.7500000000
5.13	1.5.13	260.0000000000	3.5.11.13	1.3.5.11.13	268.1250000000
7.9	1.7.9	504.0000000000	3.7.9.11	1.3.7.9.11	259.8750000000
7.11	1.7.11	308.0000000000	3.7.9.13	1.3.7.9.13	307.1250000000
7.13	1.7.13	364.0000000000	3.7.11.13	1.3.7.11.13	375.3750000000
9.11	1.9.11	396.0000000000	3.9.11.13	1.3.9.11.13	482.6250000000
9.13	1.9.13	468.0000000000	5.7.9.11	1.5.7.9.11	433.1250000000
11.13	1.11.13	286.0000000000	5.7.9.13	1.5.7.9.13	511.8750000000
3.5.7	1.3.5.7	420.0000000000	5.7.11.13	1.5.7.11.13	312.8125000000
3.5.9	1.3.5.9	270.0000000000	5.9.11.13	1.5.9.11.13	402.1875000000
3.5.11	1.3.5.11	330.0000000000	7.9.11.13	1.7.9.11.13	281.5312500000
3.5.13	1.3.5.13	390.0000000000	3.5.7.9.11	1.3.5.7.9.11	324.8437500000
3.7.9	1.3.7.9	378.0000000000	3.5.7.9.13	1.3.5.7.9.13	383.9062500000
3.7.11	1.3.7.11	462.0000000000	3.5.7.11.13	1.3.5.7.11.13	469.2187500000
3.7.13	1.3.7.13	273.0000000000	3.5.9.11.13	1.3.5.9.11.13	301.6406250000
3.9.11	1.3.9.11	297.0000000000	3.7.9.11.13	1.3.7.9.11.13	422.2968750000
3.9.13	1.3.9.13	351.0000000000	5.7.9.11.13	1.5.7.9.11.13	351.9140625000
3.11.13	1.3.11.13	429.0000000000	3.5.7.9.11.13	1.3.5.7.9.11.13	263.935546875

Partitioned Genera Cross-Set,

21MAR99.EW

Genus (3.5.7) x (9.11.13)

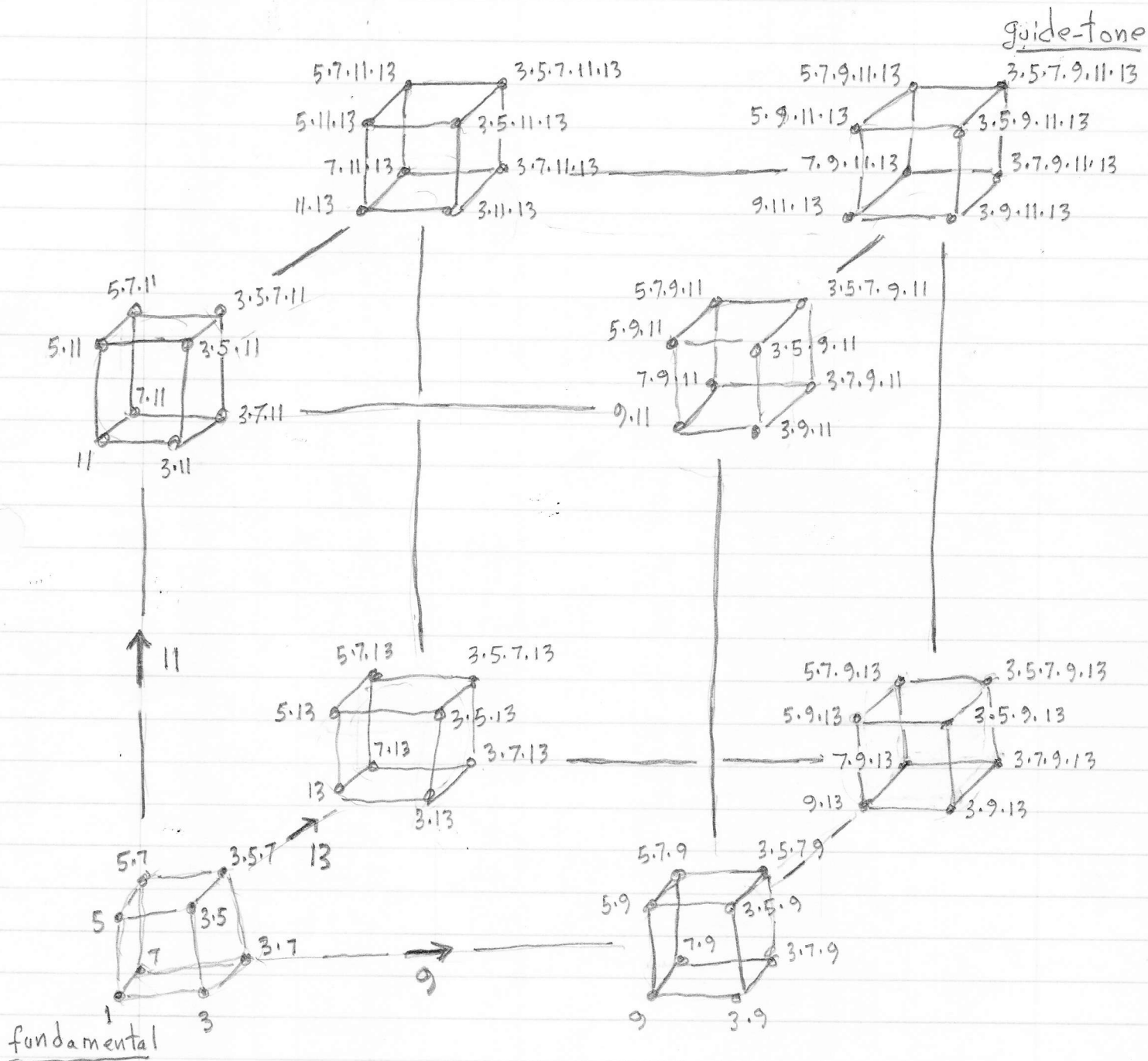
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	(null)	3	5	7	3.5	3.7	5.7	3.5.7
(null) $\emptyset$	$\emptyset$	3	5	7	3.5	3.7	5.7	3.5.7
9	9	3.9	5.9	7.9	3.5.9	3.7.9	5.7.9	3.5.7.9
11	11	3.11	5.11	7.11	3.5.11	3.7.11	5.7.11	3.5.7.11
13	13	3.13	5.13	7.13	3.5.13	3.7.13	5.7.13	3.5.7.13
9.11	9.11	3.9.11	5.9.11	7.9.11	3.5.9.11	3.7.9.11	5.7.9.11	3.5.7.9.11
9.13	9.13	3.9.13	5.9.13	7.9.13	3.5.9.13	3.7.9.13	5.7.9.13	3.5.7.9.13
11.13	11.13	3.11.13	5.11.13	7.11.13	3.5.11.13	3.7.11.13	5.7.11.13	3.5.7.11.13
9.11.13	9.11.13	3.9.11.13	5.9.11.13	7.9.11.13	3.5.9.11.13	3.7.9.11.13	5.7.9.11.13	3.5.7.9.11.13

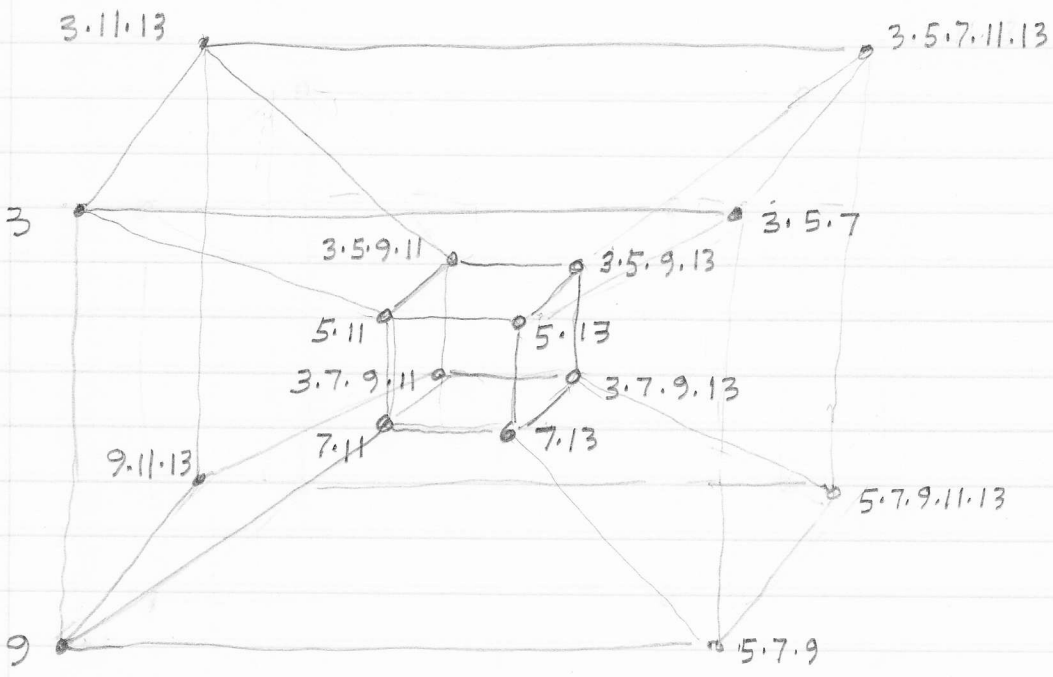
# Euler Genus (3.5.7.9.11.13) as the cube-of-cubes

20MAR99.EW

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Ref; Tentamen novae theoriae musicae, Leonhard Euler, 1739  
 New Music with 31 Notes, Adriaan D. Fokker, 1966 German; (1975 English, Gerdine)  
 D'alexandro like a hurricane, Erv Wilson 1989, Xenharmonikon VII



Nested Euler Genera (3), (3.5), (3.5.7), (3.5.7.9), (3.5.7.9.11), (3.5.7.9.11.13)

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Combination		Pitch	Increment	Chiasmus	Combination		Pitch	Increment	Chiasmus*
Fund.	Ø	Ø (256)			13 cont.	13			
Guide-Tone	3	3	5/3	b	3.13	39	5/3	b	
	5	5	3	a	5.13	65	3	a	
GT	3.5	15	7/15	c	3.5.13	195	7/15	c	
	7	7	3	a	7.13	91	3	a	
	3.7	21	5/3	b	3.7.13	273	5/3	b	
	5.7	35	3	a	5.7.13	455	3	a	
GT	3.5.7	105	9/105	d	3.5.7.13	1,365	9/105	d	
	9	9	3	a	9.13	117	3	a	
	3.9	27	5/3	b	3.9.13	351	5/3	b	
	5.9	45	3	a	5.9.13	585	3	a	
	3.5.9	135	7/15	c	3.5.9.13	1,755	7/15	c	
	7.9	63	3	a	7.9.13	819	3	a	
	3.7.9	189	5/3	b	3.7.9.13	2,457	5/3	b	
	5.7.9	315	3	a	5.7.9.13	4,095	3	a	
GT	3.5.7.9	945	11/945	e	3.5.7.9.13	12,285	11/945	e	
	11	11	3	a	11.13	143	3	a	
	3.11	33	5/3	b	3.11.13	429	5/3	b	
	5.11	55	3	a	5.11.13	715	3	a	
	3.5.11	165	7/15	c	3.5.11.13	2,145	7/15	c	
	7.11	77	3	a	7.11.13	1,001	3	a	
	3.7.11	231	5/3	b	3.7.11.13	3,003	5/3	b	
	5.7.11	385	3	a	5.7.11.13	5,005	3	a	
	3.5.7.11	1,155	9/105	d	3.5.7.11.13	15,015	9/105	d	
	9.11	99	3	a	9.11.13	1,287	3	a	
	3.9.11	297	5/3	b	3.9.11.13	3,861	5/3	b	
	5.9.11	495	3	a	5.9.11.13	6,435	3	a	
	3.5.9.11	1,485	7/15	c	3.5.9.11.13	19,305	7/15	c	
	7.9.11	693	3	a	7.9.11.13	9,009	3	a	
	3.7.9.11	2,079	5/3	b	3.7.9.11.13	27,027	5/3	b	
	5.7.9.11	3,465	3	a	5.7.9.11.13	45,045	3	a	
GT	3.5.7.9.11	10,395	13/10,395	f	GT 3.5.7.9.11.13	135,135	Ø/135,135	g	

ref: \* Chiasmus in Antiquity, John W. Welch 1981, ISBN 0-934893-33-0

# The Combinations of (3·5·7·9·11·13·etc), Binary-Sort ⑦

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#	combination						Pitch	Khiasmos	#	combination						Pitch	Khiasmos	
	∅	3	5	7	9	11				13	∅	3	5	7	9			11
1.							∅	a	33.							13	13	a
2.		3					3	b	34		3					13	39	b
3.			5				5	a	35.			5				13	65	a
4.		3	5				15	c	36.		3	5				13	195	c
5.				7			7	a	37.				7			13	91	a
6.		3		7			21	b	38.		3		7			13	273	b
7.			5	7			35	a	39.			5	7			13	455	a
8.		3	5	7			105	d	40.		3	5	7			13	1,365	d
9.					9		9	a	41.					9		13	117	a
10.		3			9		27	b	42.		3			9		13	351	b
11.			5		9		45	a	43.			5		9		13	585	a
12.		3	5		9		135	c	44.		3	5		9		13	1,755	c
13.				7	9		63	a	45.				7	9		13	819	a
14.		3		7	9		189	b	46.		3		7	9		13	2,457	b
15.			5	7	9		315	a	47.			5	7	9		13	4,095	a
16.		3	5	7	9		945	e	48.		3	5	7	9		13	12,285	e
17.					11		11	a	49.					11	13		143	a
18.		3			11		33	b	50.		3			11	13		429	b
19.			5		11		55	a	51.			5		11	13		715	a
20.		3	5		11		165	c	52.		3	5		11	13		2,145	c
21.				7	11		77	a	53.				7	11	13		1,001	a
22.		3		7	11		231	b	54.		3		7	11	13		3,003	b
23.			5	7	11		385	a	55.			5	7	11	13		5,005	a
24.		3	5	7	11		1,155	d	56.		3	5	7	11	13		15,015	d
25.					9	11	99	a	57.					9	11	13	1,287	a
26.		3			9	11	297	b	58.		3			9	11	13	3,861	b
27.			5		9	11	495	a	59.			5		9	11	13	6,435	a
28.		3	5		9	11	1,485	c	60.		3	5		9	11	13	19,305	c
29.				7	9	11	1,693	a	61.				7	9	11	13	9,009	a
30.		3		7	9	11	2,079	b	62.		3		7	9	11	13	27,027	b
31.			5	7	9	11	3,465	a	63.			5	7	9	11	13	45,045	a
32.		3	5	7	9	11	10,395	f	64.		3	5	7	9	11	13	135,135	f



# The Combinations of (3·5·7·9·11·13·15 etc.), Binary-Sort ⑧

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#	combinations							Pitch	Σ	#	combinations							Pitch	Σ
	∅	3	5	7	9	11	13				15	∅	3	5	7	9	11		
65.							15	15	a	97.						13 15	195	a	
66.		3					15	45	b	98.		3				13 15	585	b	
67.			5				15	75	a	99.			5			13 15	975	a	
68.		3	5				15	225	c	100.		3	5			13 15	2,925	c	
69.				7			15	105	a	101.				7		13 15	1,365	a	
70.		3		7			15	315	b	102.		3		7		13 15	4,095	b	
71.			5	7			15	525	a	103.			5	7		13 15	6,825	a	
72.		3	5	7			15	1,575	d	104.		3	5	7		13 15	20,475	d	
73.					9		15	135	a	105.					9	13 15	1,755	a	
74.		3			9		15	405	b	106.		3			9	13 15	5,265	b	
75.			5		9		15	675	a	107.			5		9	13 15	8,775	a	
76.		3	5		9		15	2,025	c	108.		3	5		9	13 15	26,325	c	
77.				7	9		15	945	a	109.				7	9	13 15	12,285	a	
78.		3		7	9		15	2,835	b	110.		3		7	9	13 15	36,855	b	
79.			5	7	9		15	4,725	a	111.			5	7	9	13 15	61,425	a	
80.		3	5	7	9		15	14,175	e	112.		3	5	7	9	13 15	184,275	e	
81.						11	15	165	a	113.					11	13 15	2,145	a	
82.		3				11	15	495	b	114.		3			11	13 15	6,435	b	
83.			5			11	15	825	a	115.			5		11	13 15	10,725	a	
84.		3	5			11	15	2,475	c	116.		3	5		11	13 15	32,175	c	
85.				7		11	15	1,155	a	117.				7	11	13 15	15,015	a	
86.		3		7		11	15	3,465	b	118.		3		7	11	13 15	45,045	b	
87.			5	7		11	15	5,775	a	119.			5	7	11	13 15	75,075	a	
88.		3	5	7		11	15	17,325	d	120.		3	5	7	11	13 15	225,225	d	
89.				9	11		15	1,485	a	121.					9	11	13 15	19,305	a
90.		3		9	11		15	4,455	b	122.		3		9	11	13 15	57,915	b	
91.			5	9	11		15	7,425	a	123.			5	9	11	13 15	96,525	a	
92.		3	5	9	11		15	22,275	c	124.		3	5	9	11	13 15	289,575	c	
93.				7	9	11	15	10,395	a	125.				7	9	11	13 15	135,135	a
94.		3		7	9	11	15	31,185	b	126.		3		7	9	11	13 15	405,405	b
95.			5	7	9	11	15	51,975	a	127.			5	7	9	11	13 15	675,675	a
96.		3	5	7	9	11	15	155,925	f	128.		3	5	7	9	11	13 15	2,027,025	f

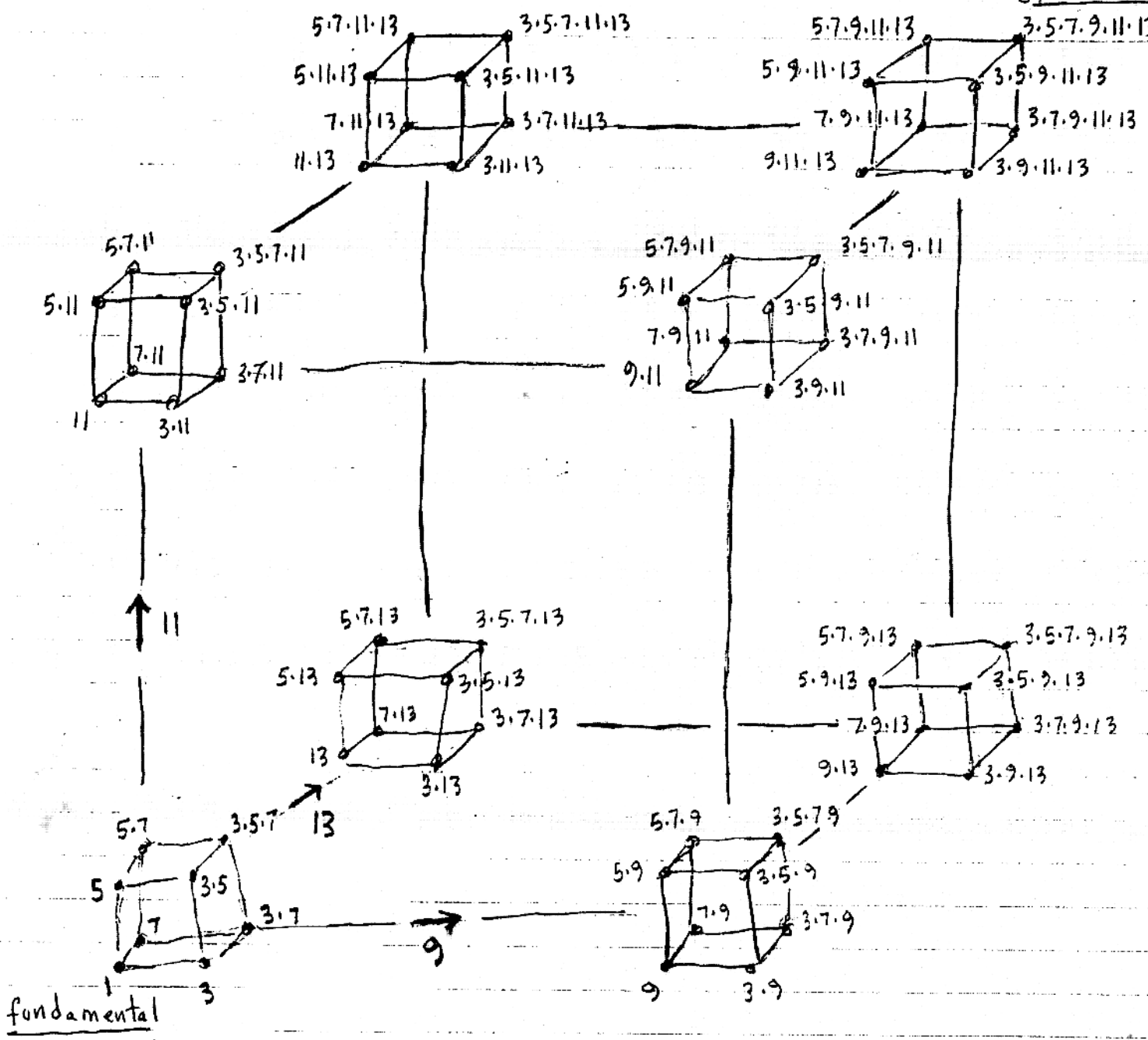
Euler Genus (3.5.7.9.11.13)  
as the cube-of-cubes

20MAR99.EW

④

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guide-tone



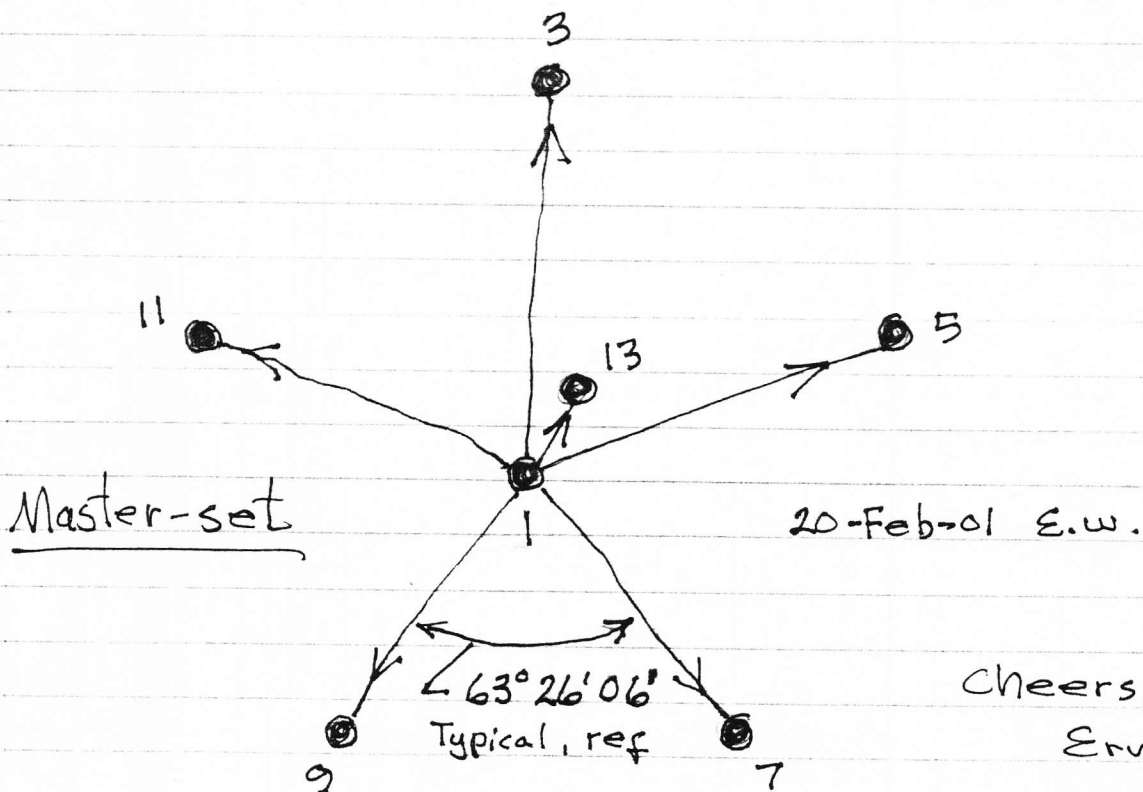
Ref; Tentamen novae theoriae musicae, Leonhard Euler, 1739  
 New Music with 31 Notes, Adriaan D. Fokker, 1966 German; (1975 English, Gerdine)  
 D'alexandro like a hurricane, Erv Wilson 1989, Xenharmonikon VII

P.S.

20 Feb 2001

David Keenan,

If you hold this double-photo of the tone-lattice for Genera (a.b.c.d.e.f) about 15 inches from your eyes; allow the eyes to cross effortlessly so that third image appears in the center; then rotate minutely and adjust the distance to suit — then the center image will slip into stereo. When you get the hang of it, its easy. I used the Zometool researcher kit to construct this model. Medium length struts color red, (all of equal length) are used to construct this figure, according to the master set from which is rigorously extrapolated (projected). Some struts are omitted because they obviously cannot pass through each other — (this is no problem with wire-frame.) In the finished lattice each ball will connect by red struts to 6 other balls. The master-set is mutable of course.



Cheers,  
Erin Wilson

Dr. Alan I. Goldman  
Iowa State University  
A501 Physics  
Ames, Iowa 50014

Jan 11, 2000 Tuesday

Dear Dr. Goldman,

The enclosed tone-lattice for the Euler genus (3.5.7.9.11.13), altho intended primarily for musical elucidation, embodies visual patterns that are the 6-dimensional (or 7-point) equivalent of Roger Penrose's tile patterns. (See A Quasicrystal Construction Kit, Ivars Peterson, Science News, Vol. 155 Jan 23, 1999) There are 64 tones<sup>vertices</sup> (points) connected by 192 lines (edges) of equal length. The inner sphere intersects 12 points of an icosahedron; the second sphere intersects 20 points of a dodecahedron; and the outer sphere intersects the 32 points of the triakontahedron. (The icosahedron and the dodecahedron come along free for the ride, and do not represent the essential features of the Euler lattice; and are shown as connected by lines to assist the untrained eye only.)

Each point of the essential Euler lattice is connected by lines of ~~of~~ equal length to six other points. Since tones/points are zero-small and lines/intervals are zero-thin they may pass thru each other without presenting any musical contradiction. Think not-solids, wire-frame.

Most of the 192 lines are not shown in this flat sheet diagram - but can be easily inferred ~~by~~ (assume "1" is at the "empty" set) by connecting those combinations ~~of elements~~ which share all but one harmonic element.

example 3.9.11, 3.9.11.13 are a connected pair. This lattice, is ~~mappe~~ ~~constructed~~ projected <sup>by combinations</sup> from an harmonic heptad of ~~of~~ ~~elements~~, ~~map~~ ~~as the master set~~, mapped to

the geometric master-set as seen at points 1, 3, 5, 7, 9, 11. This set of ~~of~~ combinations is congruent with its own reciprocal; it is Khiasmatic (XIAZMOE); what

goes upward from  $\mathbb{I}$  fundamental  $\mathbb{1}$ , comes downward from  $f$  ~~357~~ guiding-tone  $\mathbb{I}3.5.7.9.11$ . Euler knew that very well indeed.

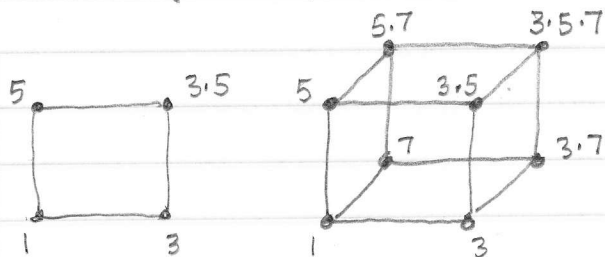
The vertices are tones; the edges are intervals; there are ~~32~~ ~~There are~~ 32 tones<sup>o</sup> in the master-set Triakontahedron are connected to the 12 tones<sup>#</sup> of the dodekahedron and to the 10 tones<sup>o</sup> of the icosahedron; the ~~64~~ total<sup>f</sup> 64 tones are connected by the 6 intervals of the master-set (6<sub>as</sub>)

~~There~~ The 7 tones of the master-set are connected by the 6 intervals <sup>(~~as~~ and are</sup> shown by associated <sup>with</sup> elements 1, 3, 5, 7, 9, 11, 13), which are projected by ~~at~~ the 64 combinations-out-of-6 (~~64 in all~~). The ~~the~~ dodekahedron and icosahedron are <sup>intriguing but</sup> secondary artifacts of this projection. This is a ~~Xiasmos~~  $\{XIAZMOZ\}$ ; and ~~is~~ can be repeated periodically to fill all Heptadic Tone-space.

The vertices are taken as musical tones, and the edges as intervals. The 7 tones of the master-set <sup>(1, 3, 5, 7, 9, 11, 13)</sup> are connected by 6 intervals, ~~and the~~ are shown ~~as~~ which are projected by the 64 combinations ~~of~~ out-of-6 (1+6+15+20+15+6+1). ~~The~~ ~~do~~ to generate a formal 64-tone, ~~XIASMOS~~, <sup>self-reversing</sup> ~~it will~~ fill a ~~tonal~~ ~~lattice~~ lattice, which is self-reversing (Xiasmatic) and can be repeated periodically to fill all Heptadic Tone-space.

There are 64 points in the full set, and 192 (32x6) lines (of equal length). Each point connects to 6 other points, and there are 32 lines in each of the 6 directions.

Other examples of simpler Euler genera mapped to regular geometry



Euler

The master-set of 7 elements may be repositioned on its companion geometric nucleus in  $7! = 5,040, (1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7)$  ways.

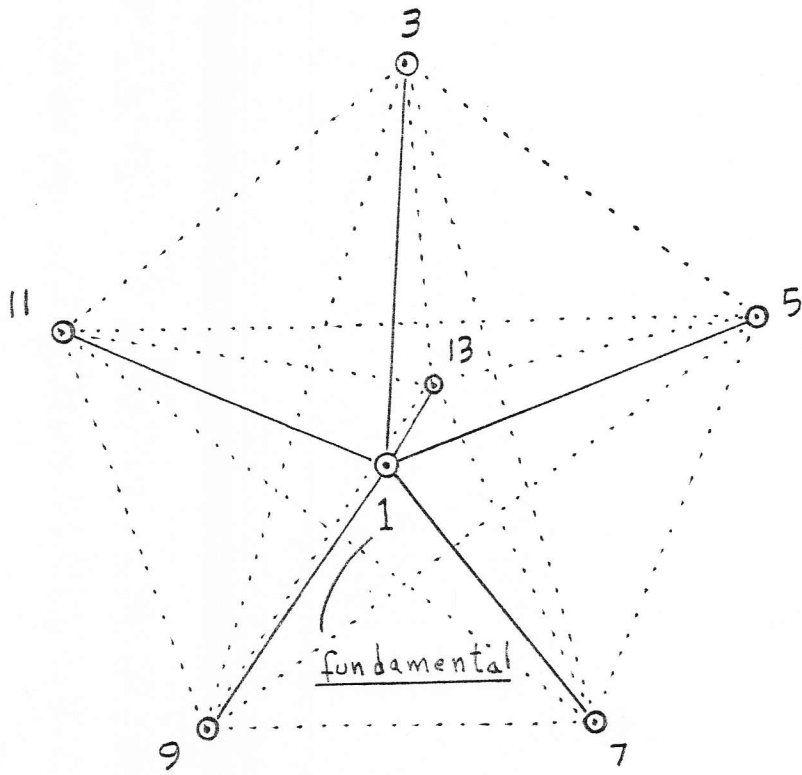
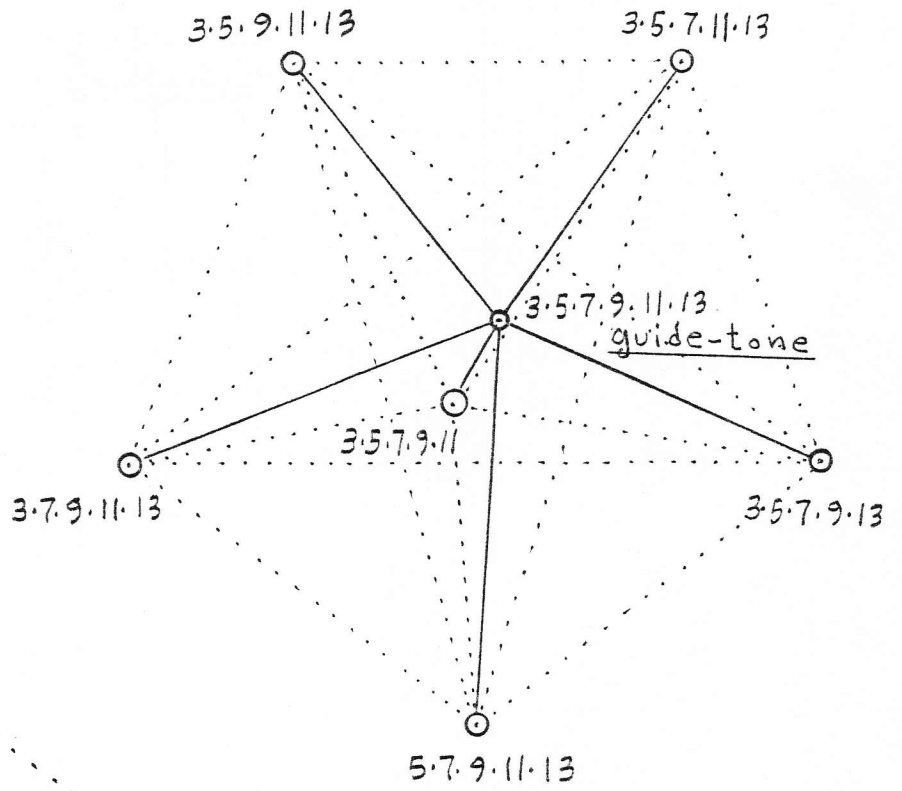
3.7.9.11

5.9.11.13

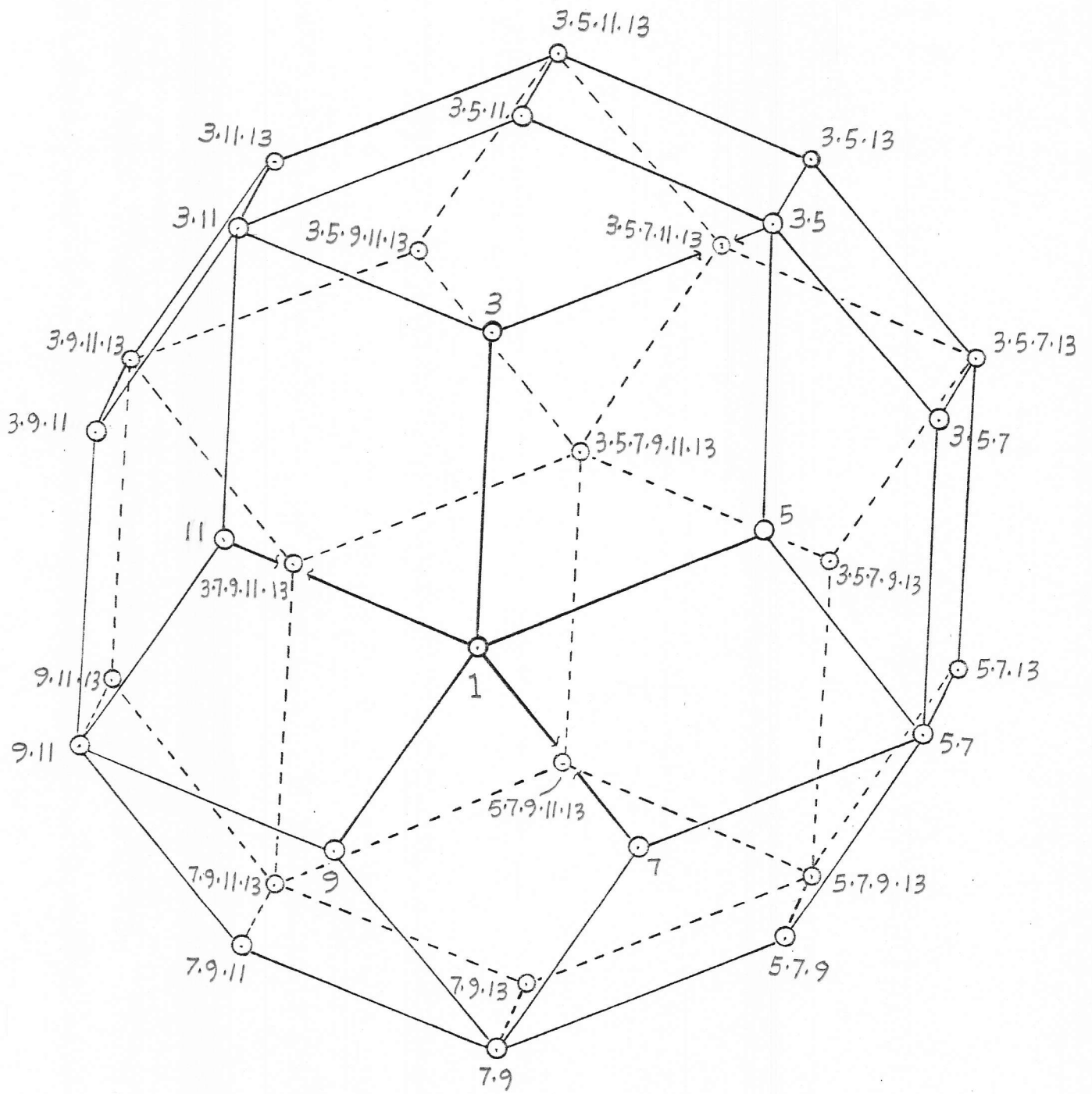
3.7.11.13

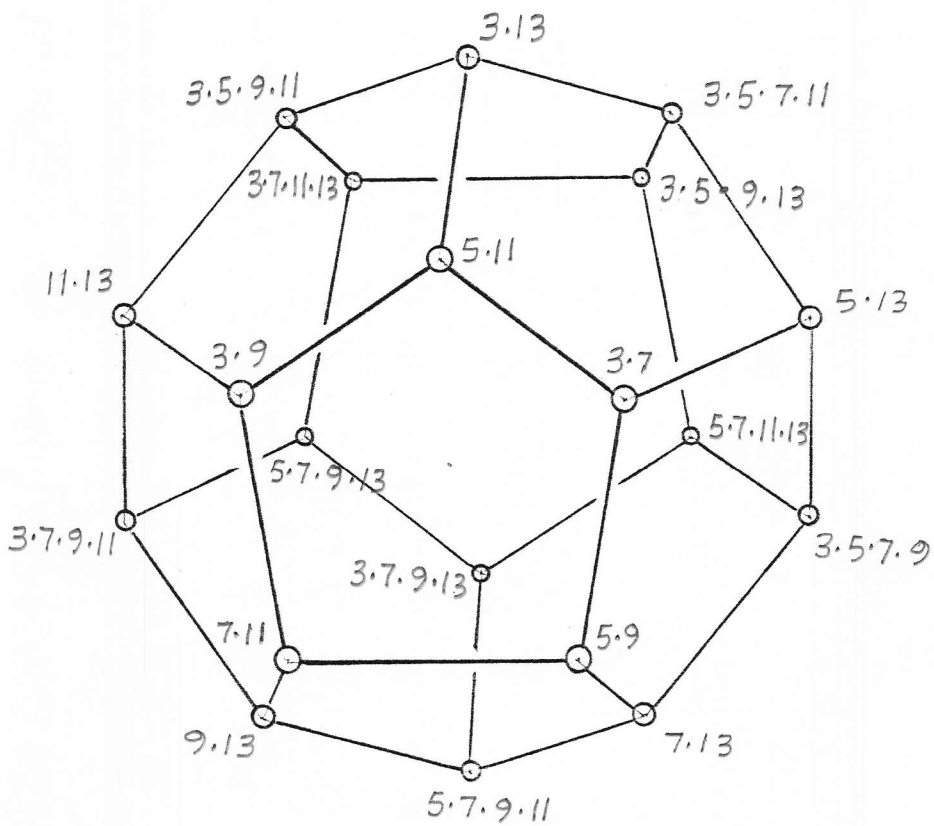
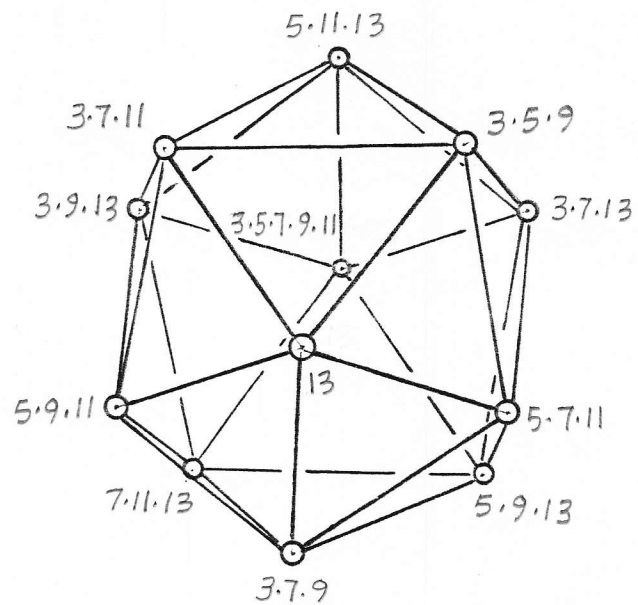
3.5.9.11

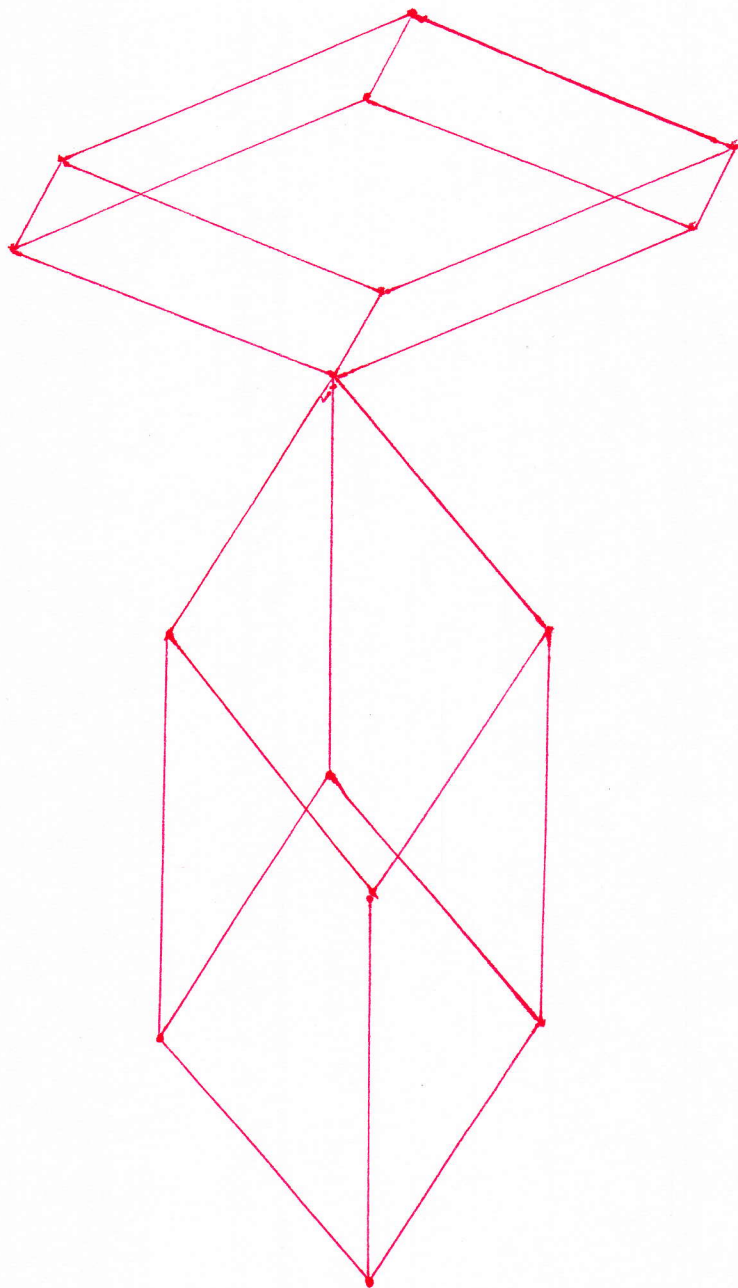
11.13





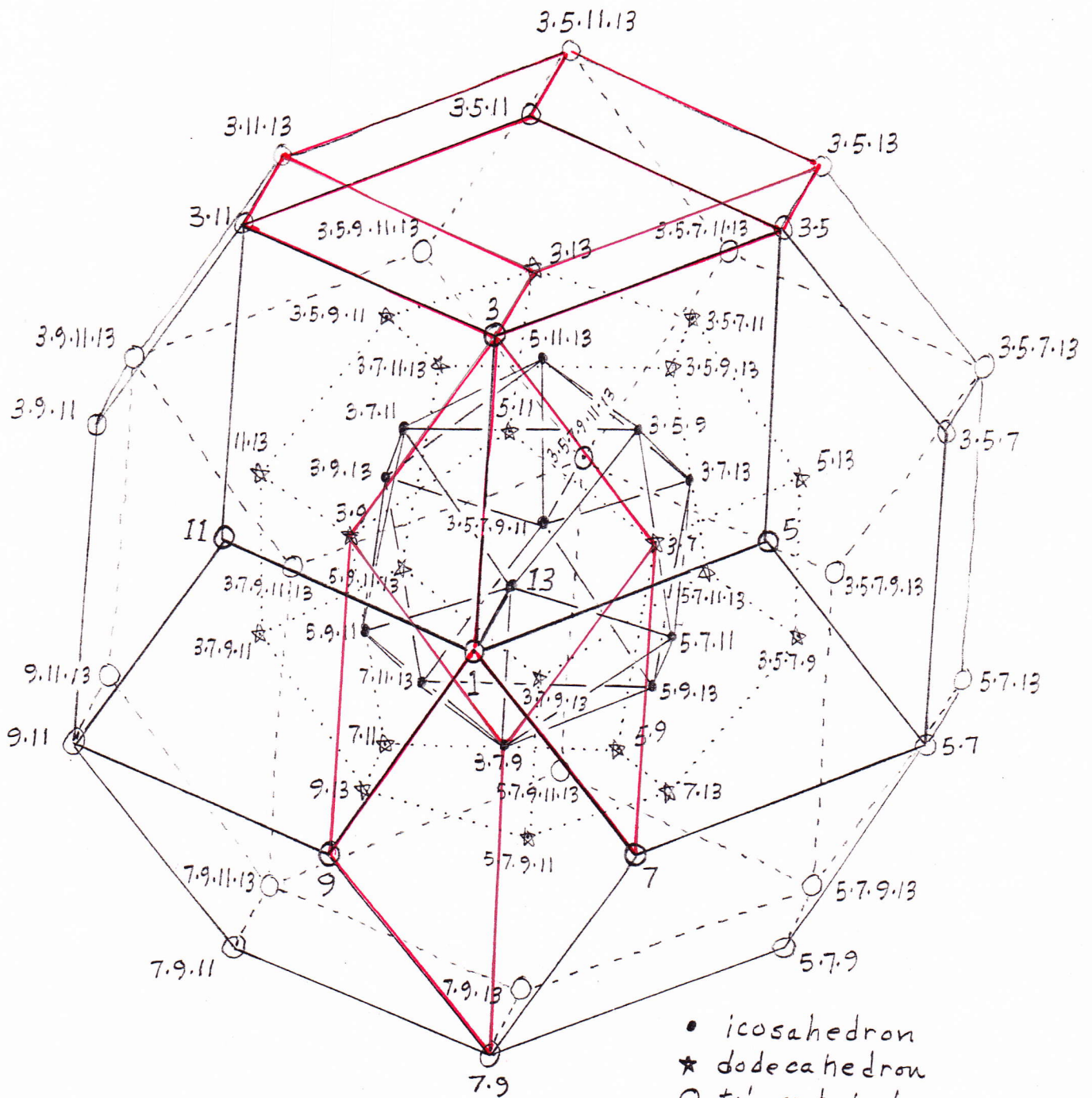






# Euler Genus (3.5.7.9.11.13) lattice within the triakontahedron

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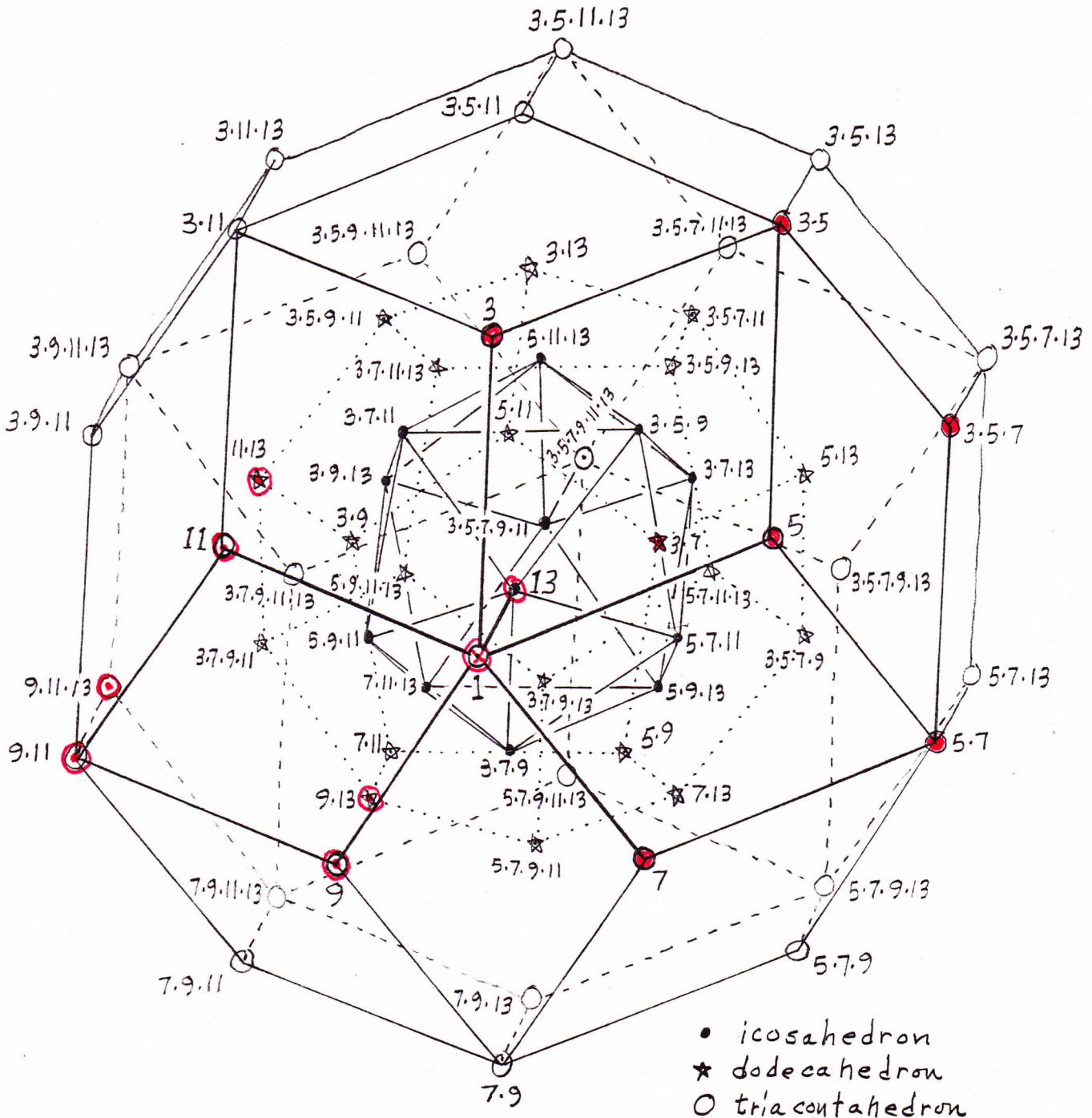
## References:

Tentamen novae theoriae musicae, Leonhard Euler, 1739

D'alessandro like a hurricane, Erv Wilson 1989, fig 206, Xenharmonikon XII

# Euler Genus (3.5.7.9.11.13) lattice within the triakontahedron

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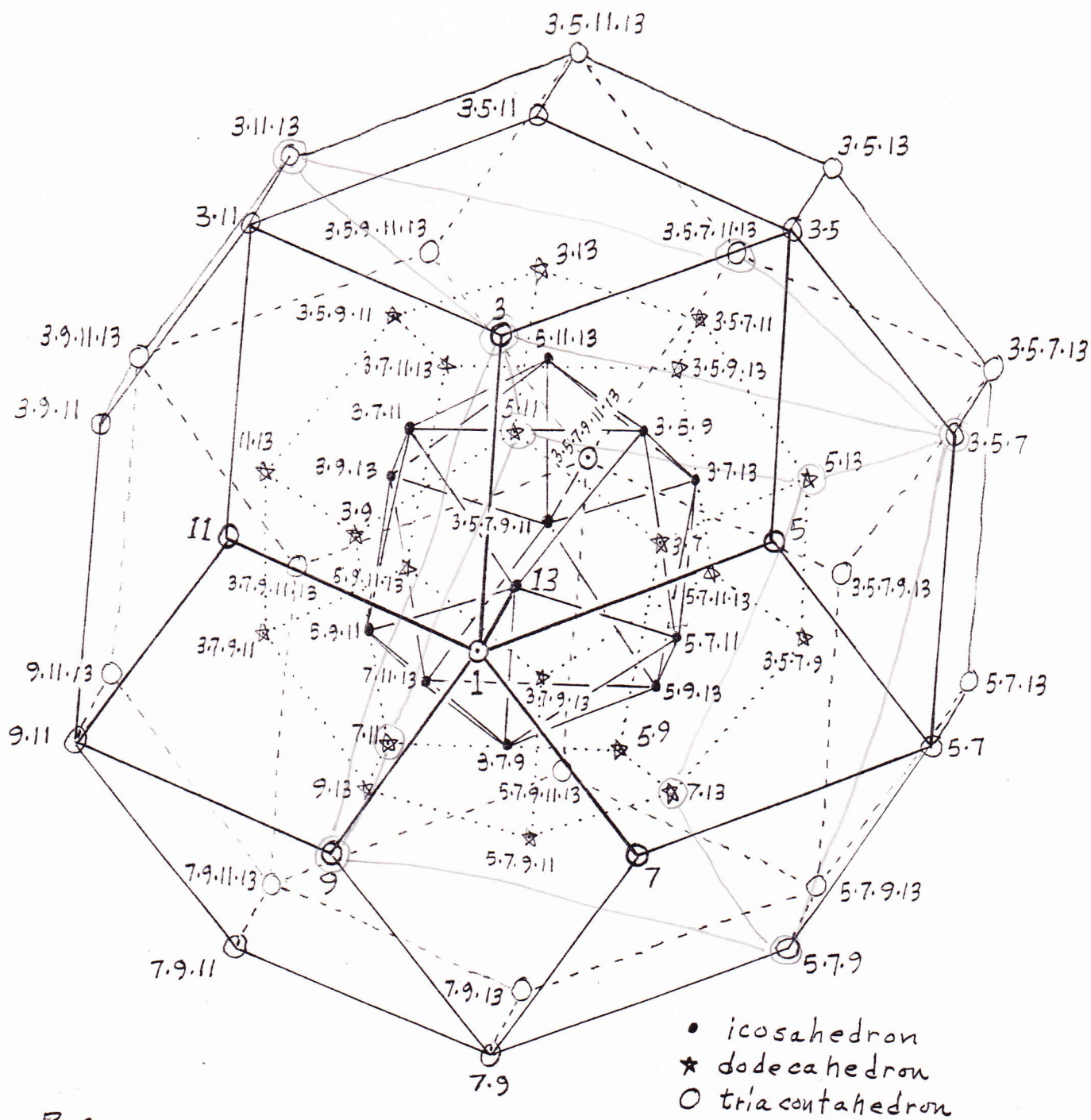
## References;

Tentamen novae theoriae musicae, Leonhard Euler, 1739

D'alexandro like a hurricane, Erv Wilson 1989, fig 206, Xenharmonikon XII

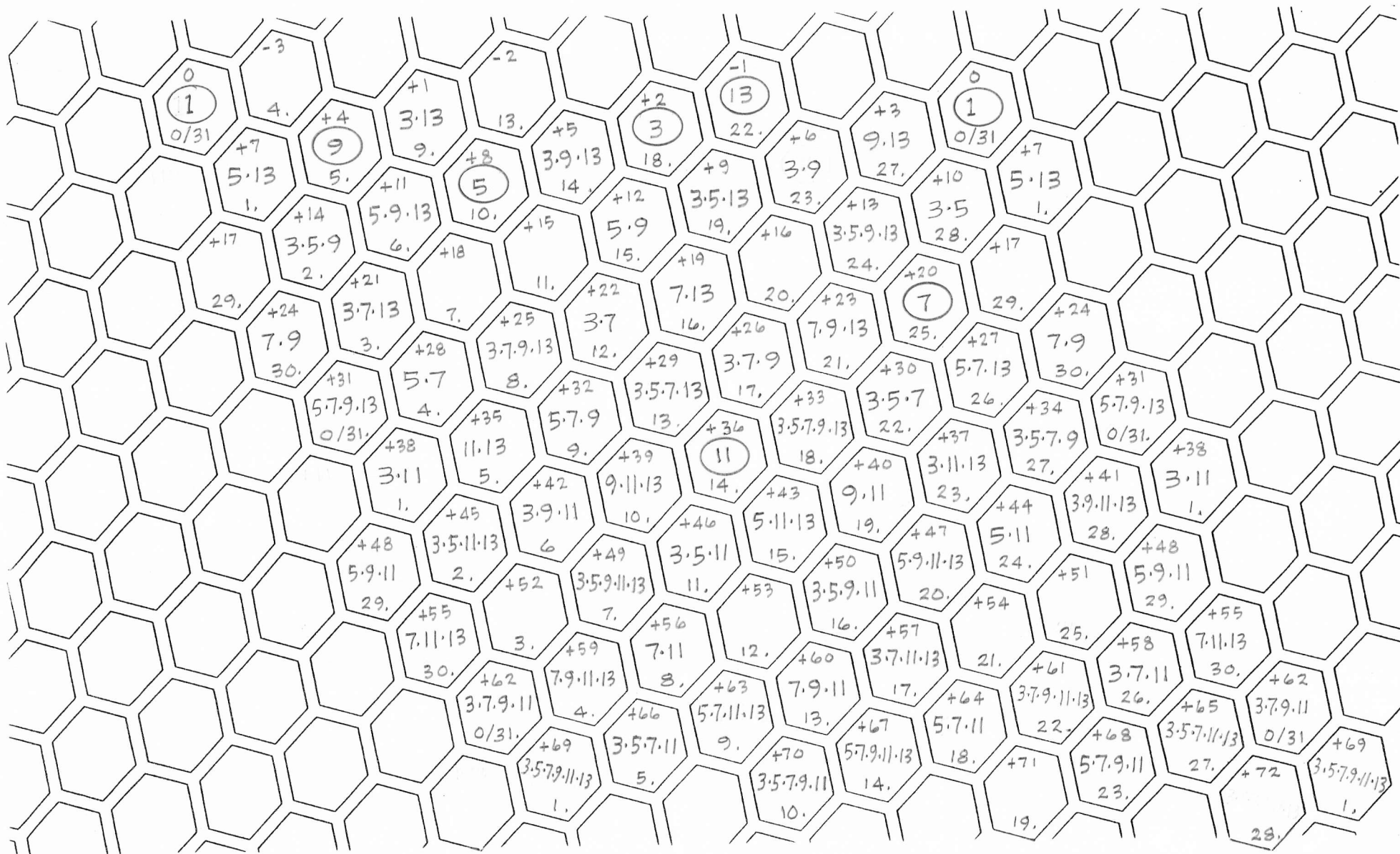
# Euler Genus (3.5.7.9.11.13) lattice within the triakontahedron

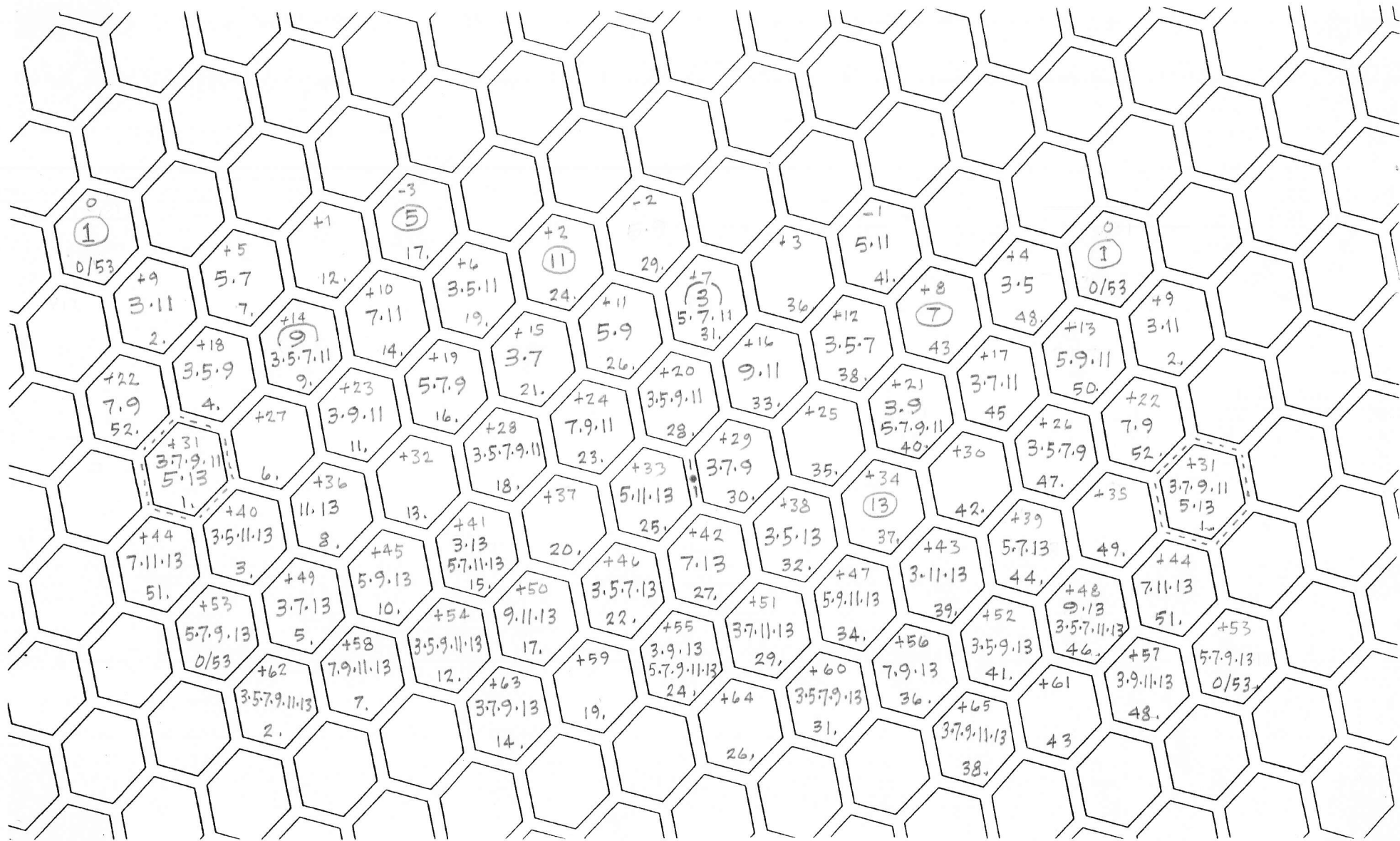
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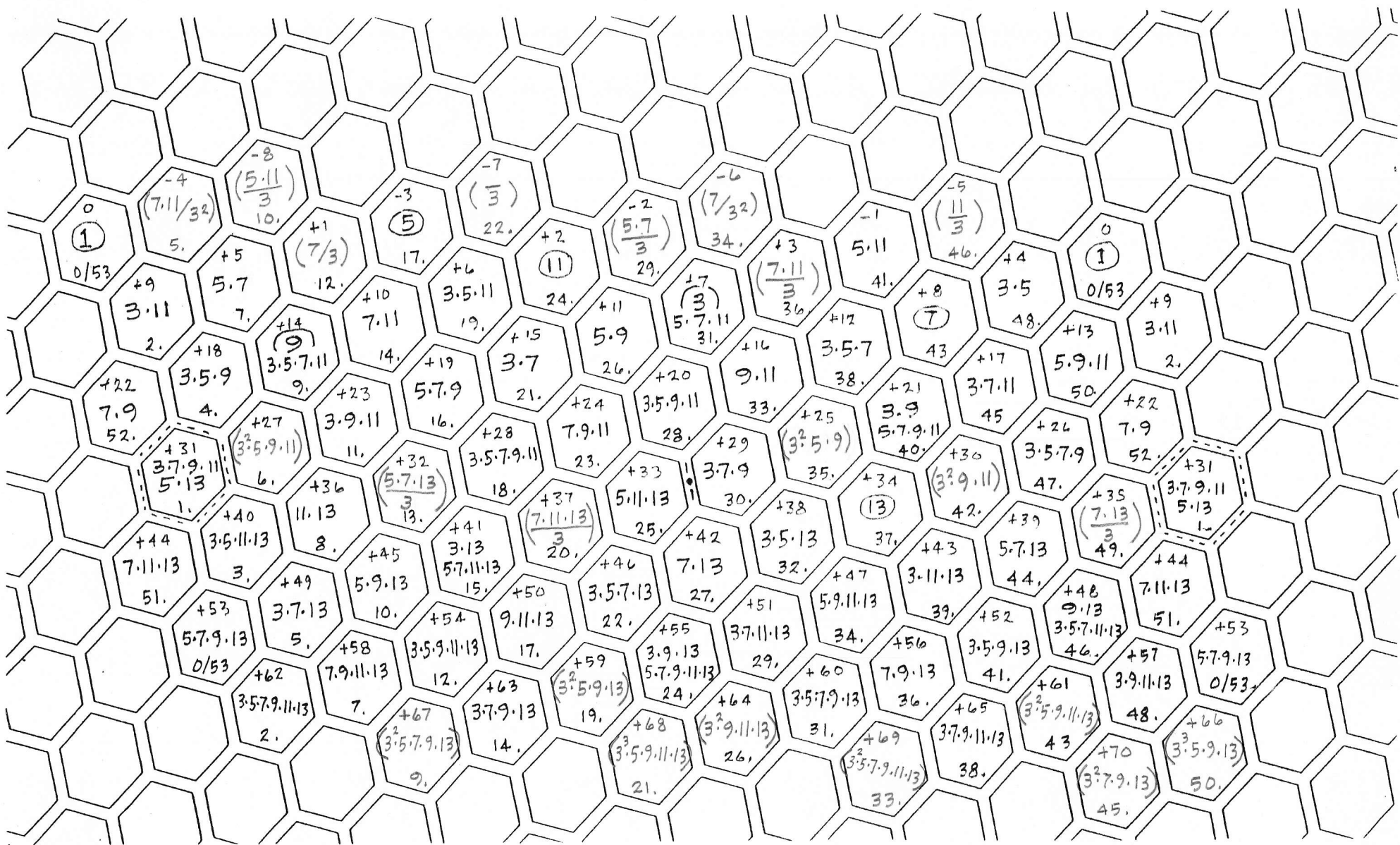
## References;

- Tentamen novae theoriae musicae, Leonhard Euler, 1739
- D'alessandro like a hurricane, Erv Wilson 1989, fig 206, Xenharmonikon XII









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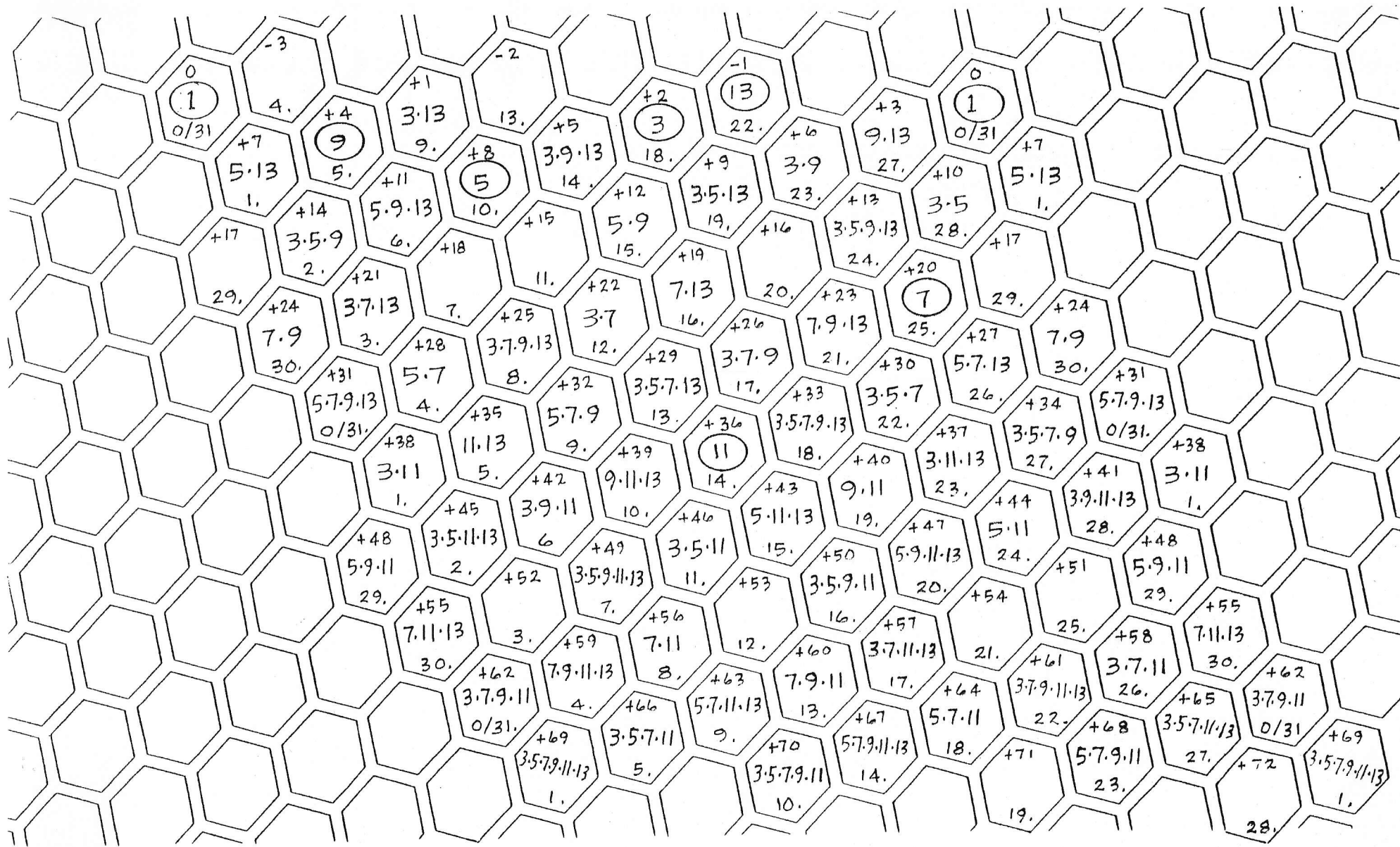
ADRIAAN, with melodic fillers in parentheses  
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13 JUL 99. EW

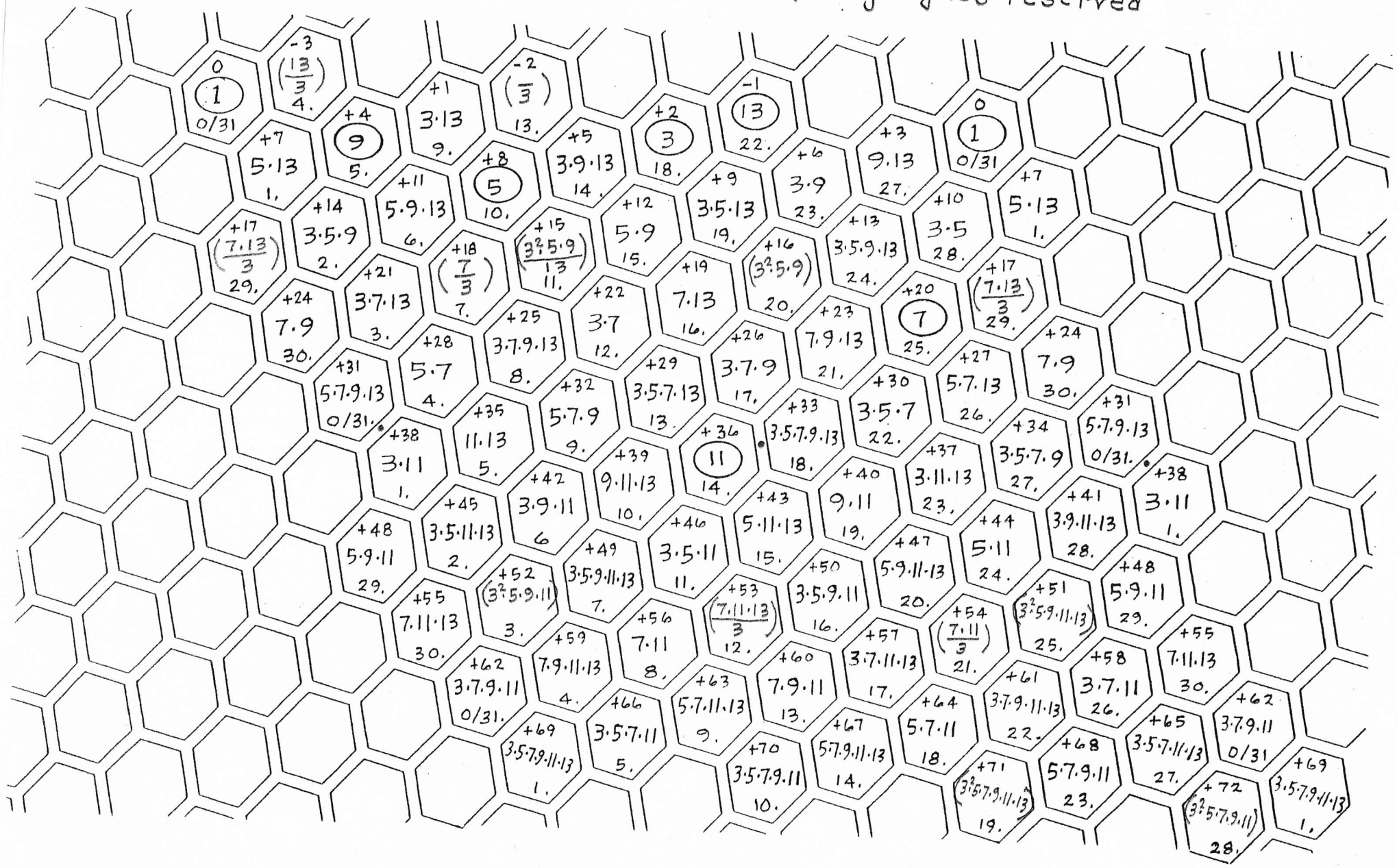
"LEONARD" Keyboard mapping for Euler Genus (3.5.7.9.11.13)

sh 2

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