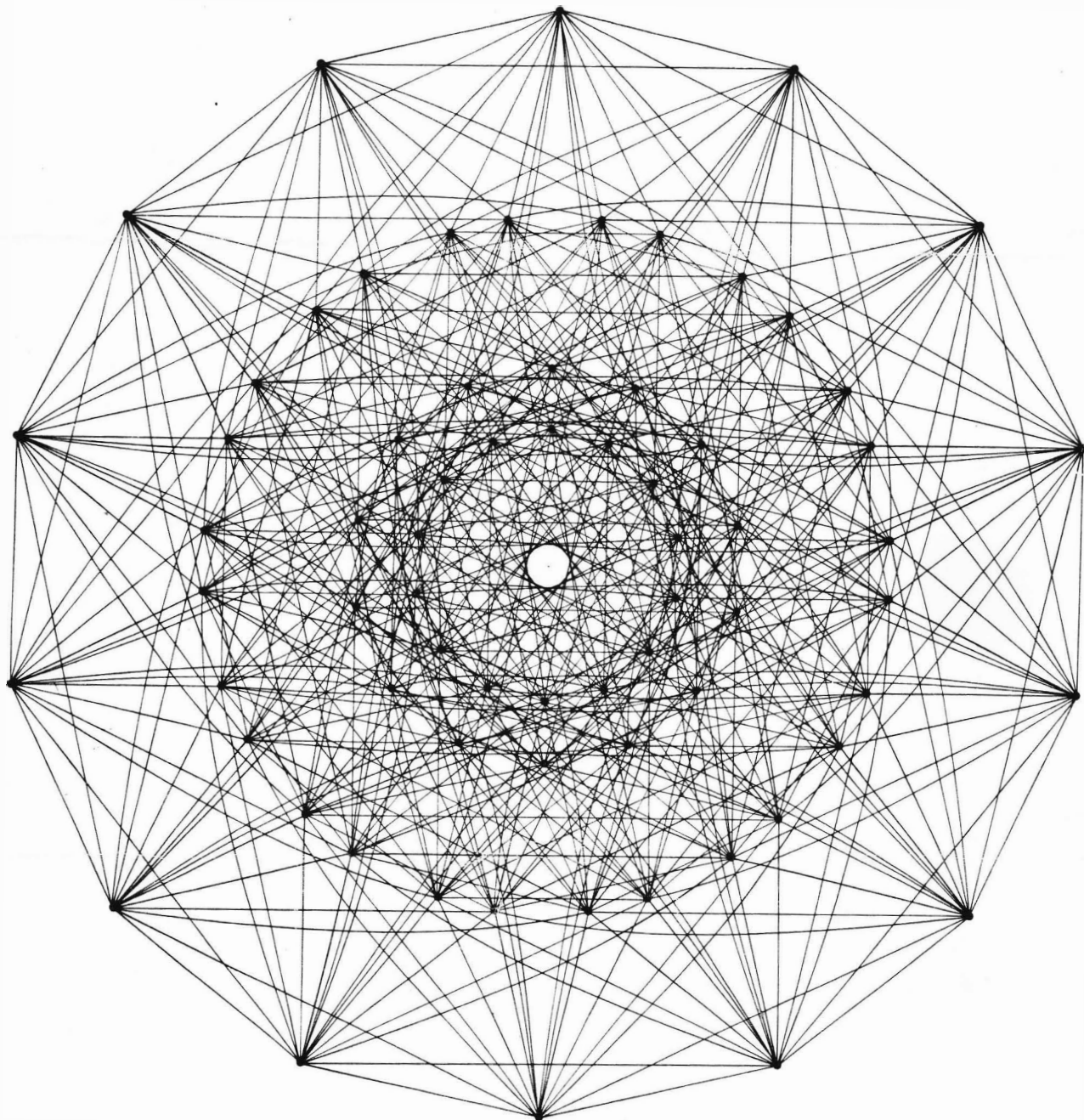


# Hebdomekontang Notes

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(Please do not take)



# HEBDOMEKONTANY PITCHES, ©1989 by Ern Wilson

SH 1

(1 ÷ 15)

1 2 3 4 5 6 7 8 9 10

✓ -	36	+C	3.5.9.15	• 270	P	86	- 2 3 - - - 8 9 -
✓ -	37	"δ	{ 3.7.13.15	273		87%	- - - 4 5 - - 8 9 -
✓ -	38	"δ	{ 5.7.9.13	"			- - - - - - - -
✓ -	39	δ	3.7.9.11	277.2		2	1 - - - 5 6 - 8 9 -
✓ -	40	δ	1.5.7.15	• 280	P	3	1 - - - - - 7 8 9 -
✓ -	41	"D	{ 1.11.13.15	286		6	1 2 - - - 6 7 8 9 -
✓ -	42	"D	{ 3.5.11.13	"			- - - - - - - -
✓ -	43	+δ	1.3.5.9	• 288	P	7	1 - 3 4 - 6 7 8 9 -
✓ -	44	-D <sub>1</sub>	1.3.7.13	291.2		8	1 2 3 4 5 6 7 8 9 -
✓ -	45	-D <sub>2</sub>	5.9.13.15	292.5		9	1 - 3 - - - - - 10
✓ -	46	D <sub>2</sub>	3.9.11.15	• 297	P	11	1 - 3 4 5 - - - 10
✓ -	47	≡ E	7.9.11.13	300.3		12	1 - 3 4 - 6 - - - 10
✓ -	48	-E <sub>1</sub>	{ 1.7.11.15	• 308	P	15	- 2 - - 5 - 7 - - 10
✓ -	49	-E <sub>1</sub>	{ 3.5.7.11	"			- - - - - - - -
✓ -	50	-E <sub>2</sub>	{ 1.3.13.15	312		17	1 - 3 - - 6 7 - - 10
✓ -	51	-E <sub>2</sub>	{ 1.5.9.13	"			- - - - - - - -
✓ -	52	E <sub>1</sub>	5.7.9.15	• 315	P	18	1 2 - - 5 6 7 - - 10
✓ -	53	E <sub>2</sub>	1.3.9.11	316.8		19	1 2 - 4 5 6 7 - - 10
✓ -	54	"E	9.11.13.15	321.75		21	1 - - - 5 - - 8 - 10
✓ -	55	-E	3.7.9.13	327.6		23	1 - - 4 - 6 - 8 - 10
✓ -	56	*E	3.5.11.15	• 330	P	24	1 2 - - 5 6 - 8 - 10
✓ -	57	≡ F	5.7.11.13	333.666		25	- 2 - - - - 7 8 - 10
✓ -	58	+E	{ 1.3.7.15	• 336	P	26	- - 3 4 - - 7 8 - 10
✓ -	59	+E	{ 1.5.7.9	"			- - - - - - - -
✓ -	60	"E	1.9.11.13	343.2		29	1 2 3 - - 6 7 8 - 10
✓ -	61	-F <sub>1</sub>	7.9.11.15	• 346.5		30	- - 3 - 5 6 7 8 - 10
✓ -	62	-F <sub>2</sub>	3.9.13.15	351		32	- - 3 - - - - 9 10
✓ -	63	F	1.3.5.11	• 352	P	32	- - - 4 - - - 9 10
✓ -	64	≡ Y	5.11.13.15	357.5		34	- - 3 4 5 - - - 9 10
✓ -	65	+F	1.5.9.15	• 360	P	35	- - 3 - - 6 - - 9 10
✓ -	66	"Y	{ 1.7.13.15	364		36	- 2 - - 5 6 - - 9 10
✓ -	67	"Y	{ 3.5.7.13	"			- - - - - - - -
✓ -	68	-Y <sub>1</sub>	1.7.9.11	369.6		39	1 - 3 - - - 7 - 9 10
✓ -	69	-Y <sub>2</sub>	1.3.9.13	374.4		40	- - 3 - 5 - 7 - 9 10
✓ -	70	Y	3.7.9.15	• 378		41	- - - - - 6 7 - 9 10

+D 1.5.7  
 "E 3.7.9.13.15  
 +E 1.5.15

# HEBDOMEKONTAN PITCHES (1 ÷ 15)

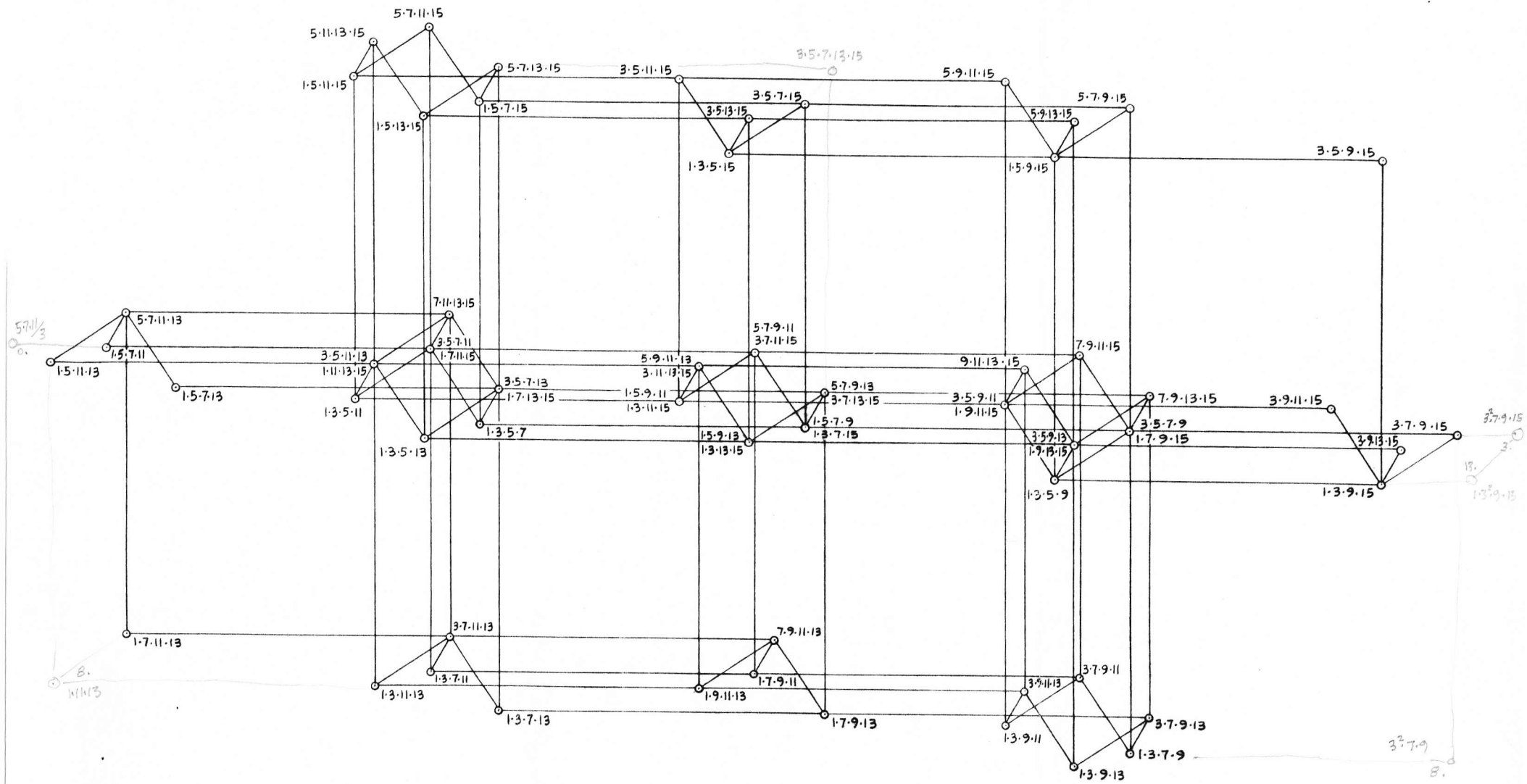
SH 2

				1	2	3	4	5	6	7	8	9	10
✓	1	≡ G	1.5.11.13	381.333	-	2	-	4	-	6	7	-	9 10
✓	2	∩ G <sub>1</sub>	5.7.11.15	• 385 P (384=3)	1	-	3	-	5	6	7	-	9 10
✓	3	∩ G <sub>2</sub>	3.5.13.15	390	-	-	3	-	-	-	-	8	9 10
✓	4	G	{ 1.9.11.15	• 396 P	1	-	3	-	5	-	-	8	9 10
✓	5		{ 3.5.9.11	"	<hr/>								
✓	6	≡ α	3.7.11.13	400.4	-	2	-	-	-	6	-	8	9 10
✓	7	+ G	1.3.7.9	403.2	-	2	-	4	-	6	-	8	9 10
✓	8	≡ α	7.9.13.15	409.5	1	2	-	4	5	6	-	8	9 10
✓	9	∩ α <sub>1</sub>	1.5.7.11	• 410.666 P	-	2	3	4	5	6	-	8	9 10
✓	10	∩ α <sub>2</sub>	1.3.5.13	13 416	-	-	3	4	-	-	7	8	9 10
✓	11	α	3.5.7.15	• 420 P	-	2	3	-	5	-	7	8	9 10
✓	12	≡ A	{ 3.11.13.15	429	-	-	3	4	-	6	7	8	9 10
✓	13		{ 5.9.11.13	"	<hr/>								
✓	14	+ α	1.3.9.15	• 432	-	-	3	-	5	6	7	8	9 10
✓	15	∩ A	1.7.9.13	436.8	1	2	3	4	5	6	7	8	9 10
✓	16	A	1.5.11.15	• 440 P	1	-	3	4	-	-	-	-	-
✓	17	+ A	1.3.5.7	7 • 448 P	1	-	-	-	5	6	-	-	-
✓	18	≡ β <sub>1</sub>	5.7.13.15	455	-	-	-	-	5	7	-	-	-
✓	19	≡ β <sub>2</sub>	1.3.11.13	457.6	1	2	-	4	5	-	7	-	-
✓	20	∩ β <sub>1</sub>	{ 3.7.11.15	• 462 P	-	2	3	4	-	6	7	-	-
✓	21		{ 5.7.9.11	"	<hr/>								
✓	22	∩ β <sub>2</sub>	{ 1.9.13.15	468	1	2	3	-	-	-	-	8	-
✓	23		{ 3.5.9.13	"	<hr/>								
✓	24	+ β	1.3.5.15	15 • 480 P	1	2	3	-	5	6	-	8	-
✓	25	∩ B	1.5.7.13	485.333	1	2	-	4	-	-	7	8	-
✓	26	B <sub>1</sub>	1.3.7.11	492.8	1	2	3	-	-	6	7	8	-
✓	27	B <sub>2</sub>	5.9.11.15	• 495 P	1	2	3	4	-	6	7	8	-
✓	28	≡ C	7.11.13.15	500.5	1	2	-	-	-	-	-	9	-
✓	29	+ B	{ 1.7.9.15	• 504 P	1	2	3	4	-	-	-	-	9
✓	30		{ 3.5.7.9	"	<hr/>								
✓	31	≡ C	3.9.11.13	514.8	1	-	3	-	5	6	-	-	9
✓	32	∩ C	1.5.13.15	520	-	2	3	-	-	-	7	-	9
✓	33	C	{ 1.3.11.15	C • 528 P	-	-	-	-	-	6	7	-	9
✓	34		{ 1.5.9.11	"	<hr/>								
✓	35	≡ δ	1.7.11.13	533.666	-	2	-	-	5	6	7	-	9

≡ β 3.7.9.11.13  
 ∩ β 1.5.11  
 ≡ B 3.9.11.13.15

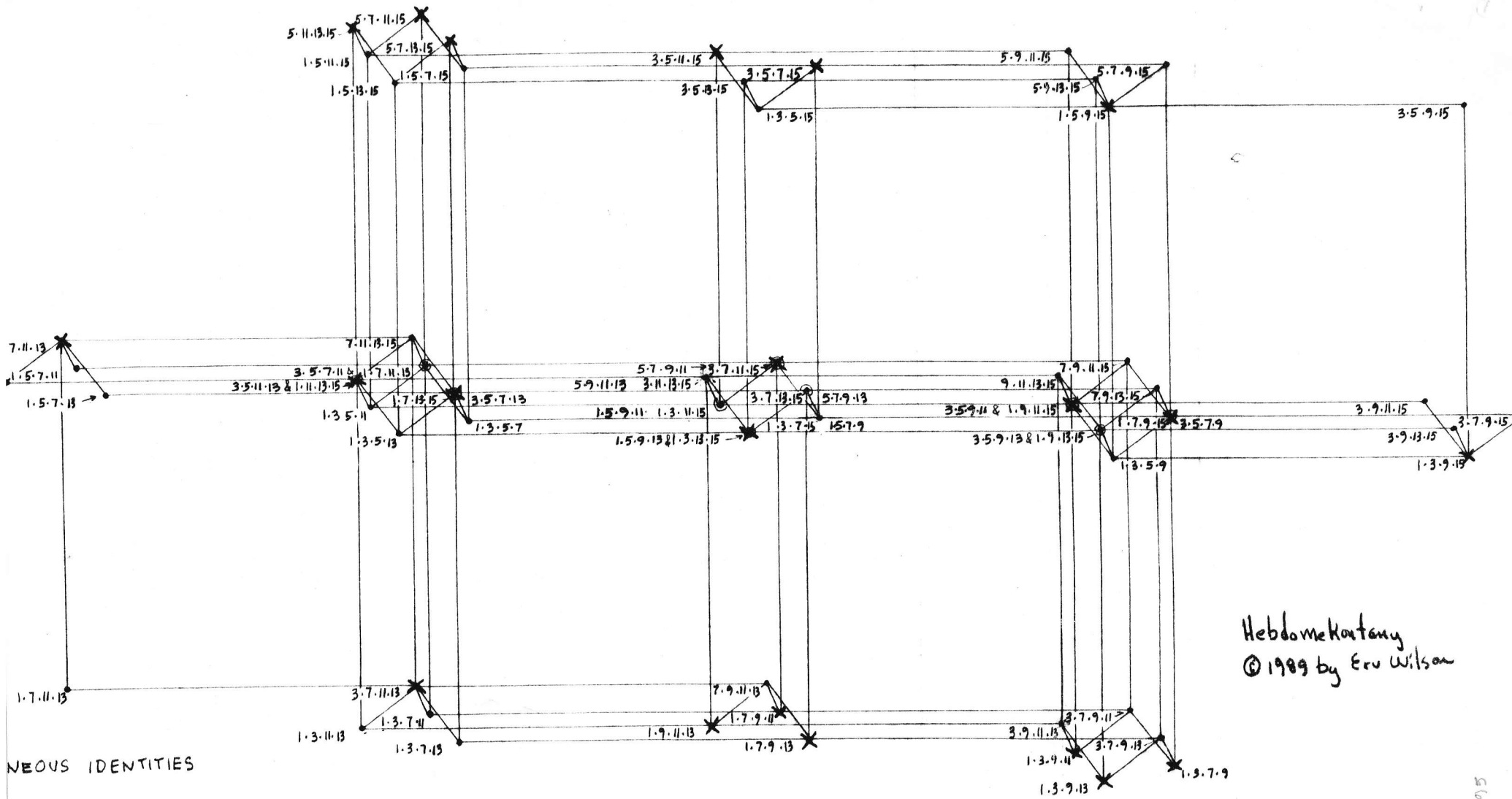
385 fused in 87  
 384

352 fused in 87  
 351



1-3-5-7-9-11-13-15  
 Hebdomekontany  
 (drawn circa 1967-68)

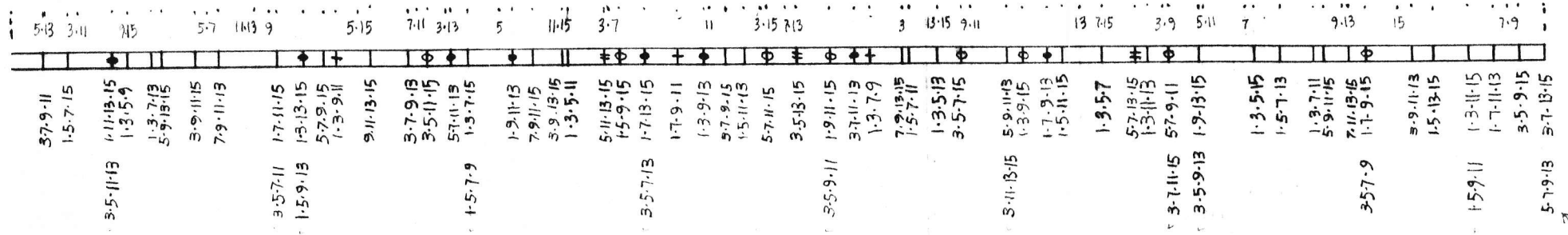




Hebdomerontony  
 © 1989 by Eric Wilson

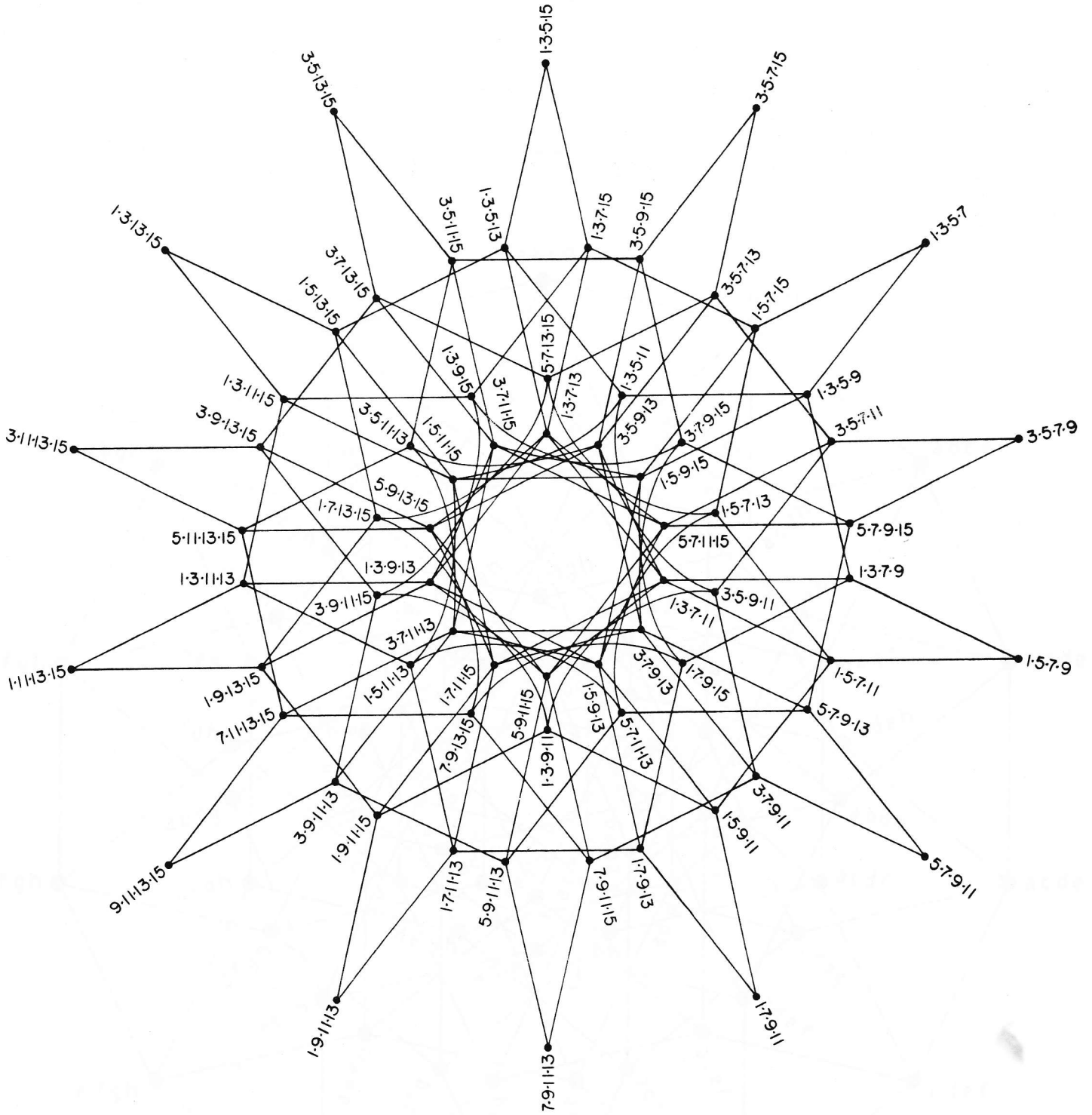
NEOUS IDENTITIES

1 x



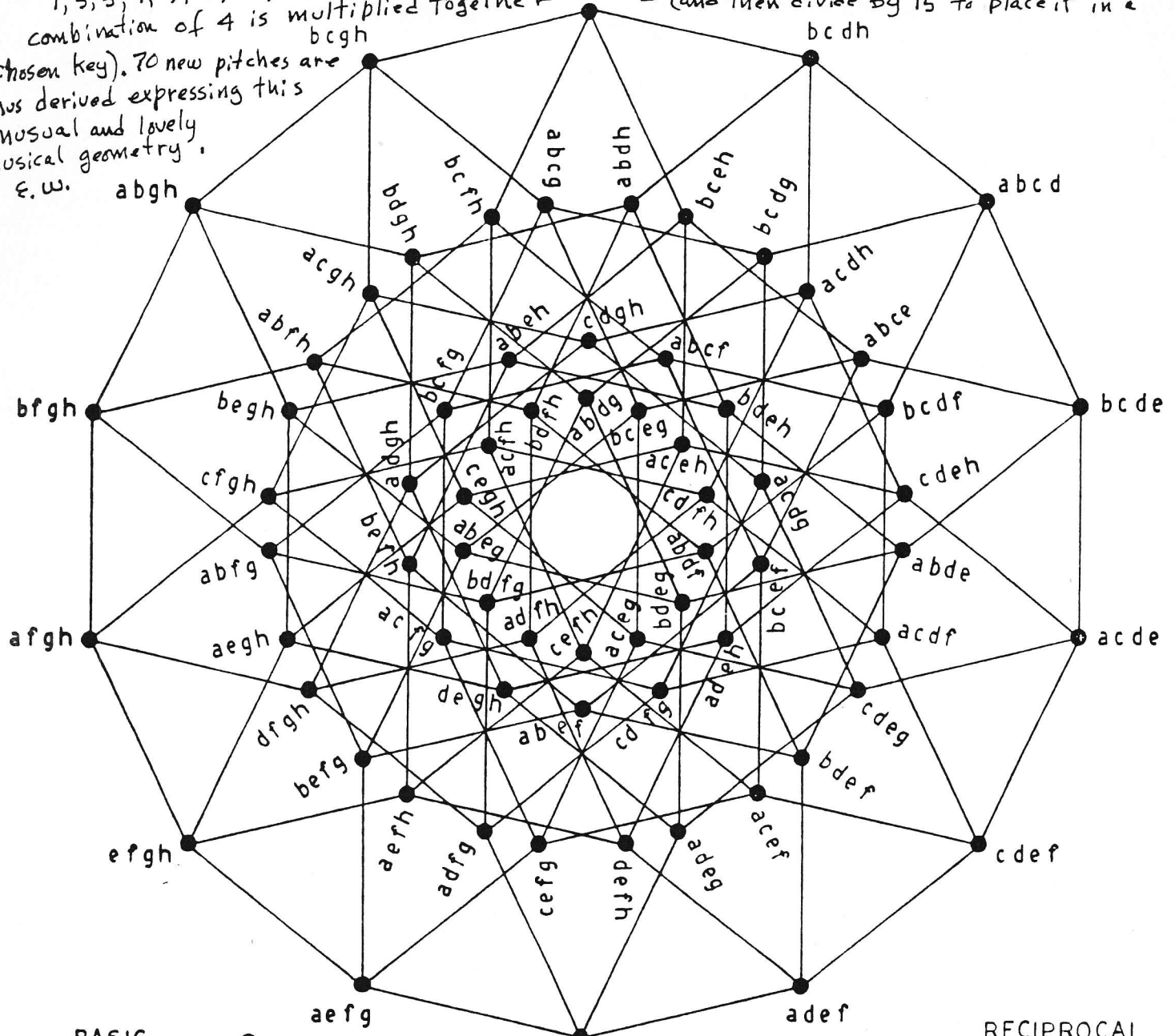
4096/4095

↑

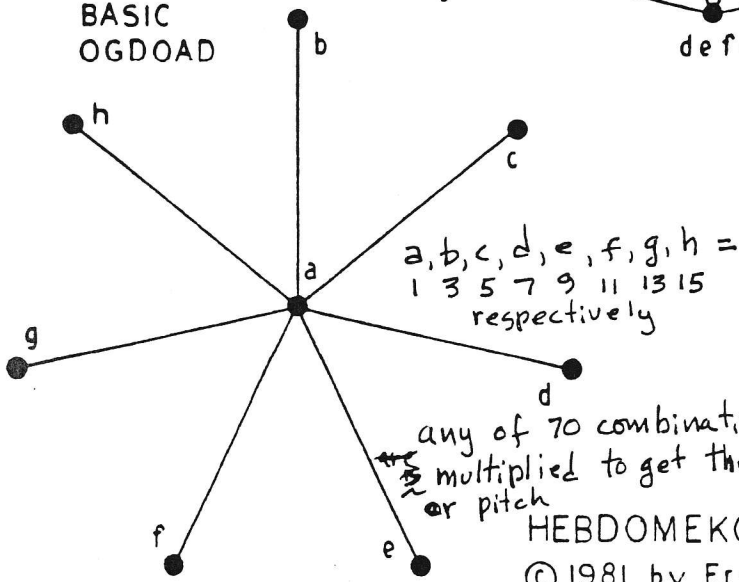


1-3-5-7-9-11-13-15 HEBDOMEKONTANY

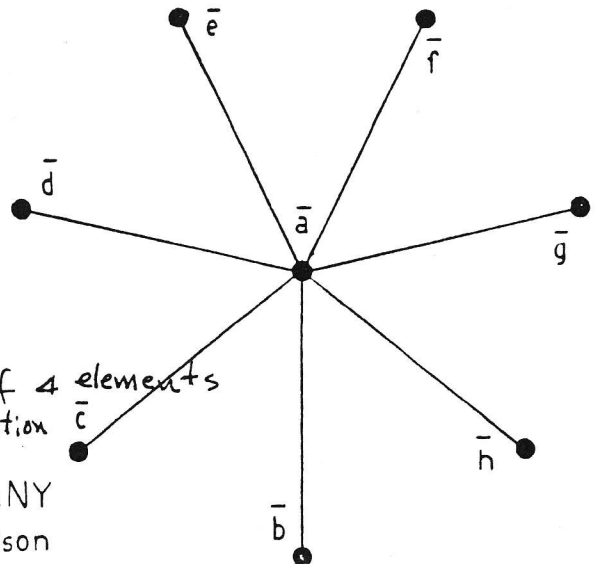
This is a geometric expression of one of the "hyper-spherical" patterns occurring in the set of 70 combinations of 4-out-of-8. 1, 3, 5, 7, 9, 11, 13, 15 refer to members of the harmonic series. Each combination of 4 is multiplied together -  $abch$  - (and then divide by 15 to place it in a chosen key). 70 new pitches are thus derived expressing this unusual and lovely musical geometry. E.W.



BASIC  
OGDOAD



RECIPROCAL



HEBDOMEKONTANY

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# Partitioned Cross-Sets of the Hebdomekontany

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Item 1. In the 4)8 Hebdomekontany the 1)2 dyany subset has 28 varieties, each of which occurs 20 times, in a 3)6 Eikosony partitioned cross-set.

Example; the  $\binom{1}{2} | 3$  dyany  $\times$  the  $\binom{3}{6} | 5 7 9 11 13 15$  Eikosony,  $\left\{ \binom{1}{2} | 3 \right\} \times \left\{ \binom{3}{6} | 5 7 9 11 13 15 \right\} =$



x	1	3
5.7.9	1.5.7.9	3.5.7.9
5.7.11	1.5.7.11	3.5.7.11
5.7.13	1.5.7.13	3.5.7.13
5.7.15	1.5.7.15	3.5.7.15
5.9.11	1.5.9.11	3.5.9.11
5.9.13	1.5.9.13	3.5.9.13
5.9.15	1.5.9.15	3.5.9.15
5.11.13	1.5.11.13	3.5.11.13
5.11.15	1.5.11.15	3.5.11.15
5.13.15	1.5.13.15	3.5.13.15
7.9.11	1.7.9.11	3.7.9.11
7.9.13	1.7.9.13	3.7.9.13
7.9.15	1.7.9.15	3.7.9.15
7.11.13	1.7.11.13	3.7.11.13
7.11.15	1.7.11.15	3.7.11.15
7.13.15	1.7.13.15	3.7.13.15
9.11.13	1.9.11.13	3.9.11.13
9.11.15	1.9.11.15	3.9.11.15
9.13.15	1.9.13.15	3.9.13.15
11.13.15	1.11.13.15	3.11.13.15

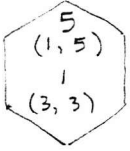
Item 2. The  $\{(4) | 3 5 7 9 11 13 15\}$  Hebdomekany may be partitioned into the following 13 <sup>illustrative</sup> types of cross-sets;   
 variations or permutations of Partition

$\{(0) \emptyset\}_{(1y)} \times \{(4) a b c d e f g h\}_{(70y)}$	1
$\{(0) a\}_{(1y)} \times \{(4) b c d e f g h\}_{(35y)}$	8
$\{(1) a\}_{(1y)} \times \{(3) b c d e f g h\}_{(35y)}$	8
$\{(0) a b\}_{(1y)} \times \{(4) c d e f g h\}_{(15y)}$	28
$\{(1) a b\}_{(2y)} \times \{(3) c d e f g h\}_{(20y)}$	28
$\{(2) a b\}_{(1y)} \times \{(2) c d e f g h\}_{(15y)}$	28
$\{(0) a b c\}_{(1y)} \times \{(4) d e f g h\}_{(15y)}$	56
$\{(1) a b c\}_{(3y)} \times \{(3) d e f g h\}_{(10ny)}$	56
$\{(2) a b c\}_{3ny} \times \{(2) d e f g h\}_{10ny}$	56
$\{(3) a b c\}_{1ny} \times \{(1) d e f g h\}_{5ny}$	56
$\{(0) a b c d\}_{1ny} \times \{(4) e f g h\}_{1ny}$	70
$\{(1) a b c d\}_{4ny} \times \{(3) e f g h\}_{4ny}$	70
$\{(2) a b c d\}_{6ny} \times \{(2) e f g h\}_{6ny}$	$\frac{70}{2} = (35)$

12 remaining cross-sets (total 25) are in effect a re-ordering of the first 12 cross-sets above,



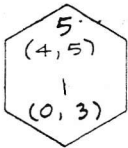
Item 3; In the 4-out-of-8, (4,8) hebdomekontany matrix the (1,5) Pentany subset has 56 variations, each of which occurs once, in a (3,3) Monany, partitioned <sup>complementary</sup> cross-set. Example; the  $\binom{1}{5} | 3 5 7 9$  Pentany partition/cross with  $\binom{3}{3} || 13 15$  Monany can be expressed,



$$\left\{ \binom{1}{5} | 3 5 7 9 \right\} \times \left\{ \binom{3}{3} || 13 15 \right\} =$$

x	1	3	5	7	9
11.13.15	1.11.13.15	3.11.13.15	5.11.13.15	7.11.13.15	9.11.13.15

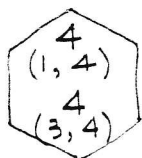
Item 4; In the (4,8) Hebdomekontany matrix the (4,5) Pentany subset has 56 variations, each of which occurs once, in a cross-set with its partitioned complementary (0,3) monany. Note; because the (4,5) Pentany conceals a subharmonic hexad, it is helpful to show that <sup>addition</sup> relationship, thus - Example;  $\left\{ \binom{4}{5} | 3 5 7 9 \right\} \times \left\{ \binom{0}{3} || 13 15 \right\}$ ,



added step	1	3	5	7	9	(subharmonic Pentad)
		x 1.3.5.7.9 =				
x	3.5.7.9	1.5.7.9	1.3.7.9	1.3.5.9	1.3.5.7	(4/5) set
(empty) ∅	3.5.7.9	1.5.7.9	1.3.7.9	1.3.5.9	1.3.5.7	

The last step of this operation, (multiplication of of the (4,5) Pentany by the (0,3) Monany [an empty set, indicated ∅]) is obviously academic.

Item 5; In the 4-out-of-8, (4,8) Hebdomekontany matrix the (1,4) Tetrany forms a partitioned cross-set with the (3,4) Tetrany. There are 70 combinations by which this partitioning may occur. Note; the (3,4) Tetrany carries a concealed sub-harmonic tetrad which is identified thus; Example



	( T	3	5	7 )	x 1.3.5.7 =	
	x	3.5.7	1.5.7	1.3.7	1.3.5	← (3,4) Tetrany
(1,4) Tetrany ↓	9	3.5.7.9	1.5.7.9	1.3.7.9	1.3.5.9	
	11	3.5.7.11	1.5.7.11	1.3.7.11	1.3.5.11	
	13	3.5.7.13	1.5.7.13	1.3.7.13	1.3.5.13	
	15	3.5.7.15	1.5.7.15	1.3.7.15	1.3.5.15	

Now, then — By adding a single tone, 1.3.5.7, to the 4 harmonic tetrads and to the 4 subharmonic tetrads, a set of 4 plus 4 Pentads will occur, each of which shares the 1.3.5.7 in common. A 17-tone quasi-diamond, providing 8 ways to harmonize the member 1.3.5.7. If the entire set above is divided by 1.3.5.7 this simple relationship emerges;

	x	T	3	5	7	
	9	9/1	9/3	9/5	9/7	9/9
	11	11/1	11/3	11/5	11/7	11/11
x 1.3.5.7 =	13	13/1	13/3	13/5	13/7	13/13
	15	15/1	15/3	15/5	15/7	15/15
add →	1, 3, 5, 7	1/1	3/3	5/5	7/7	

One sees (by  $\frac{1.3.5.7}{1.3.5.7} = 1$ ) the 8 harmonizing roles that 1.3.5.7 can play; 4 harmonic, partitioned w. 4 subharmonic senses.

Item 5 continued;

Another way to show this is to simply add 1.3.5.7 to each of the two tetrads, thus

	(1)	(3)	(5)	(7)	$\overline{9} \ \overline{11} \ \overline{13} \ \overline{15}$ <small>sequence</small>
	3.5.7	1.5.7	1.3.7	1.3.5	<u>add</u> ↓
9	3.5.7.9	1.5.7.9	1.3.7.9	1.3.5.9	1.3.5.7
11	3.5.7.11	1.5.7.11	1.3.7.11	1.3.5.11	1.3.5.7
13	3.5.7.13	1.5.7.13	1.3.7.13	1.3.5.13	1.3.5.7
15	3.5.7.15	1.5.7.15	1.3.7.15	1.3.5.15	1.3.5.7
<small>sequences</small> 1.3.5.7	1.3.5.7	1.3.5.7	1.3.5.7	1.3.5.7	1.3.5.7

add ↗

The horizontal lines may be seen as partitioned cross-sets  $\{(4,5) a b c d e\} \times \{(0,3) f g h\}$ .

The vertical lines may be seen as partitioned cross-sets  $\{(1,5) a b c d e\} \times \{(3,3) f g h\}$ .

But that doesn't really tie it all together as well as I'd wish. —

What we have in this "quasi-diamond" is that set of (4,5) Pentanies <sup>horizontal</sup> which share 1.3.5.7. This simultaneously gives us the material for that set of (1,5) Pentanies <sup>vertical</sup> which, also, share the 1.3.5.7. These are the 17 points that would be shared by the ogdoadic cross-set (1 3 5 7 9 11 13 15) × (1 3 5 7 9 11 13 15) with its 1/4 at key 1.3.5.7. [This kind of cross-set is usually called a "Diamond" after Partch's usage.] These 17 points (in their 70 partitioned permutations) are the 70 bonding-sites between centered and centerless modules in ogdoadic tone-space. They are the basis for modulating from Hebdomekontanies to 8adic Diamonds & visa versa thruout the infinite realms of open 8adic tone-space.

Item 6;



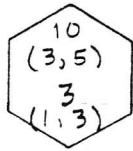
within the (4,8) hebdomekontany the (2,5) Dekany forms a partitioned cross-set with the (2,3) Triany. There are 56 ways the 8 elements of the master set can be partitioned into 2 sets of 3 and 5 elements.

Example;

$$\{(2,5) \mid 3 \ 5 \ 7 \ 9\} \times \{(2,3) \mid 11 \ 13 \ 15\} =$$

	(15)	(13)	(11)	
x	11.13	11.15	13.15	→ (2,3) triany
(2,5) Dekany ↓	1.3	1.3.11.13	1.3.11.15	1.3.13.15
	1.5	1.5.11.13	1.5.11.15	1.5.13.15
	1.7	1.7.11.13	1.7.11.15	1.7.13.15
	1.9	1.9.11.13	1.9.11.15	1.9.13.15
	3.5	3.5.11.13	3.5.11.15	3.5.13.15
	3.7	3.7.11.13	3.7.11.15	3.7.13.15
	3.9	3.9.11.13	3.9.11.15	3.9.13.15
	5.7	5.7.11.13	5.7.11.15	5.7.13.15
	5.9	5.9.11.13	5.9.11.15	5.9.13.15
	7.9	7.9.11.13	7.9.11.15	7.9.13.15

Item 7;



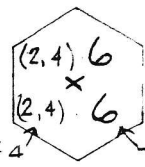
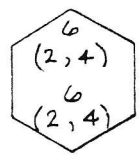
In the (4,8) Hebdomekontany [70ny] matrix the (3,5) Dekany [10ny] forms a partitioned cross-set with the (1,3) Triany [3ny]. There are 56 ways a set of 8 elements may be partitioned into 2 sets, of 5 and 3 elements each. (Hence 56 cross-sets)

Example;  $\{(3,5) | 3\ 5\ 7\ 9\} \times \{(1,3) | 11\ 13\ 15\}$

x	11	13	15
1.3.5	1.3.5.11	1.3.5.13	1.3.5.15
1.3.7	1.3.7.11	1.3.7.13	1.3.7.15
1.3.9	1.3.9.11	1.3.9.13	1.3.9.15
1.5.7	1.5.7.11	1.5.7.13	1.5.7.15
1.5.9	1.5.9.11	1.5.9.13	1.5.9.15
1.7.9	1.7.9.11	1.7.9.13	1.7.9.15
3.5.7	3.5.7.11	3.5.7.13	3.5.7.15
3.5.9	3.5.9.11	3.5.9.13	3.5.9.15
3.7.9	3.7.9.11	3.7.9.13	3.7.9.15
5.7.9	5.7.9.11	5.7.9.13	5.7.9.15



Item 8:



(4,8) 70

In the (4,8) Hebdomekontany matrix the (2,4) Hexany forms a partitioned cross-set with the (2,4) Hexany. The master-set of 8 elements may be partitioned into 2 sets, of 4 and 4 elements each, in 35 ways only ( $70 \div 2 = 35$ ), because of the mirroring nature of the entire theoretical group of 70 ways.

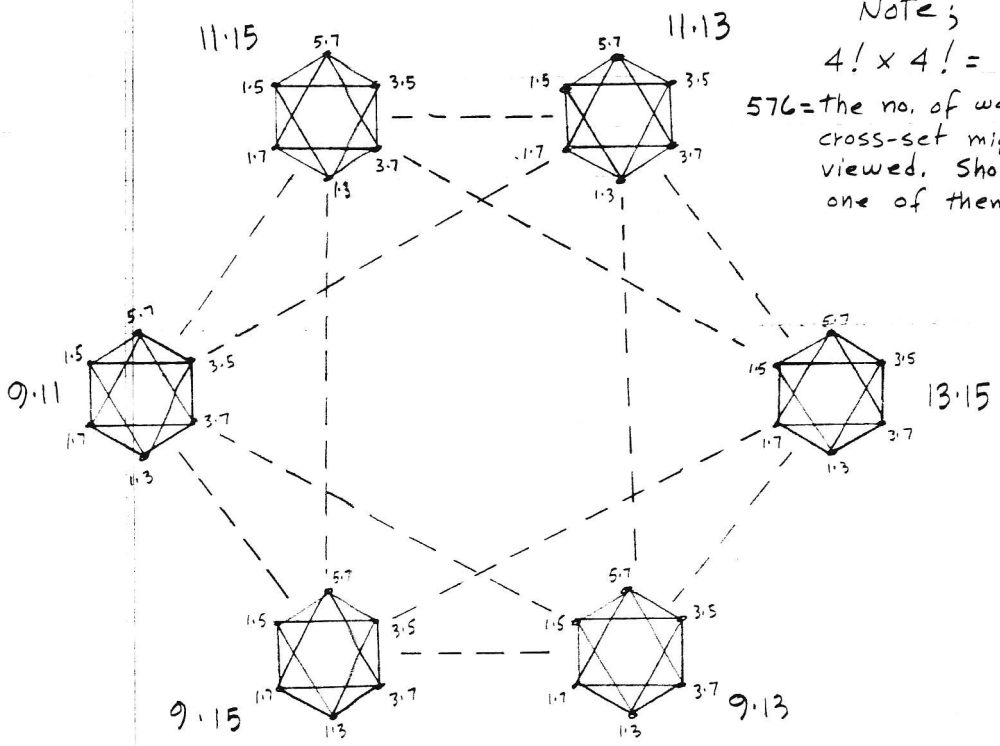
Example;  $\{(2,4) 1357\} \times \{(2,4) 9111315\}$

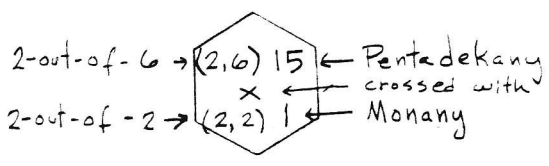
x	1.3	1.5	1.7	3.5	3.7	5.7
9.11	1.3.9.11	1.5.9.11	1.7.9.11	3.5.9.11	3.7.9.11	5.7.9.11
9.13	1.3.9.13	1.5.9.13	1.7.9.13	3.5.9.13	3.7.9.13	5.7.9.13
9.15	1.3.9.15	1.5.9.15	1.7.9.15	3.5.9.15	3.7.9.15	5.7.9.15
11.13	1.3.11.13	1.5.11.13	1.7.11.13	3.5.11.13	3.7.11.13	5.7.11.13
11.15	1.3.11.15	1.5.11.15	1.7.11.15	3.5.11.15	3.7.11.15	5.7.11.15
13.15	1.3.13.15	1.5.13.15	1.7.13.15	3.5.13.15	3.7.13.15	5.7.13.15

Note;

$4! \times 4! = 576$

576 = the no. of ways this cross-set might be viewed. Shown, is but one of them.





Item 9;

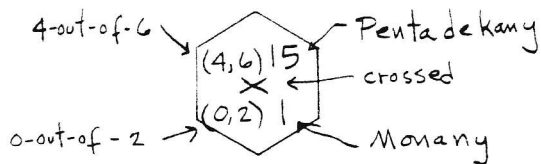
Contained in the hebdomekontany matrix, the (2,6) Pentadekany forms a partitioned cross-set with the (2,2) Monany. The ogdoad (8ad) partitions into 2 sets, of 6 and 2 elements, in 28 different ways. (Hence 28 different forms of this cross-set.)

$$\{(2,6) 1 2 5 7 9 11\} \times \{(2,2) 13 15\}$$

Example;

	X	13.15
1.3		1.3.13.15
1.5		1.5.13.15
1.7		1.7.13.15
1.9		1.9.13.15
1.11		1.11.13.15
3.5		3.5.13.15
3.7		3.7.13.15
3.9		3.9.13.15
3.11		3.11.13.15
5.7		5.7.13.15
5.9		5.9.13.15
5.11		5.11.13.15
7.9		7.9.13.15
7.11		7.11.13.15
9.11		9.11.13.15

Item 10;



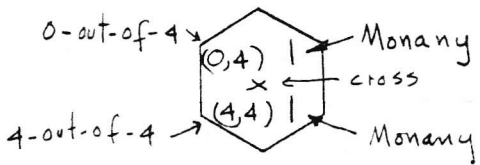
In the (4,8) Hebdomekontany [70ny] the (4,6) Pentadekany forms a partition cross-set with the (0,2) Monany. There are 28 expressions of this cross-set, based on the 28 ways 6 and 2 elements may be partitioned out of 8 elements.

Example;  $\{(4,6) 1 3 5 7 9 11\} \times \{(0,2) 13 15\}$

$\times$	$\emptyset (= \text{empty set})$
1.3.5.7	1.3.5.7
1.3.5.9	1.3.5.9
1.3.5.11	1.3.5.11
1.3.7.9	1.3.7.9
1.3.7.11	1.3.7.11
1.3.9.11	1.3.9.11
1.5.7.9	1.5.7.9
1.5.7.11	1.5.7.11
1.5.9.11	1.5.9.11
1.7.9.11	1.7.9.11
3.5.7.9	3.5.7.9
3.5.7.11	3.5.7.11
3.5.9.11	3.5.9.11
3.7.9.11	3.7.9.11
5.7.9.11	5.7.9.11

Note; Caution - the "empty set" ( $\emptyset$ ) is not equivalent to zero (0). This notwithstanding, the cross is academic, and done to maintain a useful format.

Item 11;



In the (4,8) Hebdomekontany [70my] the (0,4) Monany forms a partitioned cross-set with the (4,4) Monany.

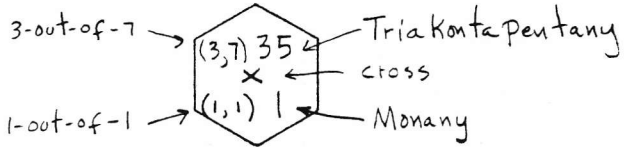
There are 70 expressions of this cross-set, according to the number of ways 4 and 4 elements may be partitioned from 8 elements.

Example;  $\{(0,4) | 3\ 5\ 7\} \times \{(4,4) | 9\ 11\ 13\ 15\}$

$$\begin{array}{r} \times \quad 9 \cdot 11 \cdot 13 \cdot 15 \\ \hline \emptyset \quad 9 \cdot 11 \cdot 13 \cdot 15 \end{array}$$

Note; obviously, going thru all 70 expressions of this cross-set would be a futile exercise. Dont do it.

Item 12;



In the (4,8) Hebdomekontany the (3,7) Triakontapentany forms a partitioned cross-set with the (1,1) Monany.

There are 8 expressions of this cross-set, according to the number of ways 7 and 1 elements may be partitioned from 8 elements.

Example;  $\{(3,7) | 3\ 5\ 7\ 9\ 11\ 13\} \times \{(1,1) | 15\}$

$$\begin{array}{r} \times \quad 15 \\ \hline 1 \cdot 3 \cdot 5 \quad 1 \cdot 3 \cdot 5 \cdot 15 \\ 1 \cdot 3 \cdot 7 \quad 1 \cdot 3 \cdot 7 \cdot 15 \\ 1 \cdot 3 \cdot 9 \quad 1 \cdot 3 \cdot 9 \cdot 15 \\ \text{etc} \quad \text{etc} \end{array}$$

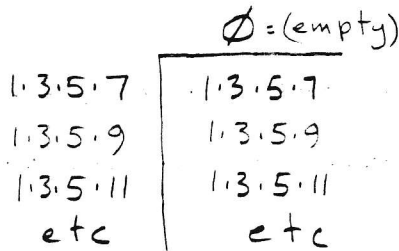
Item 13;



In the  $(4,8) 70_{ny}$  the  $(4,7) 35_{ny}$  forms a partitioned cross-set with the  $(0,1) 1_{ny}$ .

There are 8 expressions of this cross-set, as there are 8 ways in which a set of 8 elements may be partitioned into 2 sets, of 7 and 1 elements.

Example;  $\{(4,7) | 3 5 7 9 11 13\} \times \{(0,1) | 15\}$



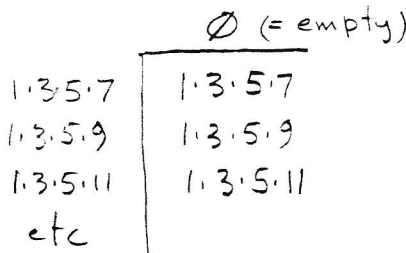
Item 14;



And finally, in the  $(4,8) 70_{ny}$ , the  $(4,8) 70_{ny}$ , itself, forms a partitioned cross-set with the  $(0,0) 1_{ny}$ .

There is only 1 expression of this cross-set. This corresponds to the 1 way a set of 8 elements may be partitioned into 2 sets, of 8 and 1 elements.

Example:  $\{(4,8) | 3 5 7 9 11 13 15\} \times \{(0,0) | \emptyset (\text{empty})\}$





# Part II

## Partitioned Cross-sets of the Hexany

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Pascal's Triangle;

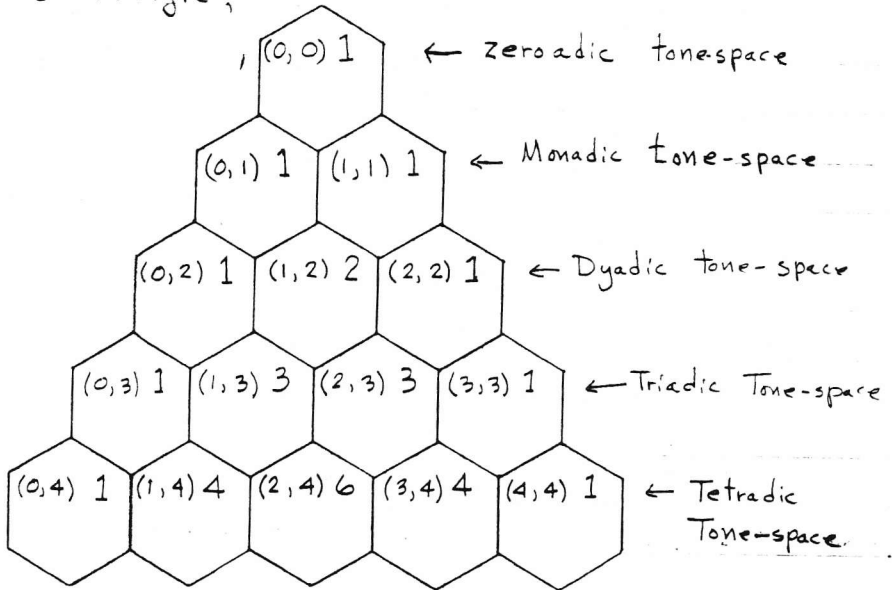
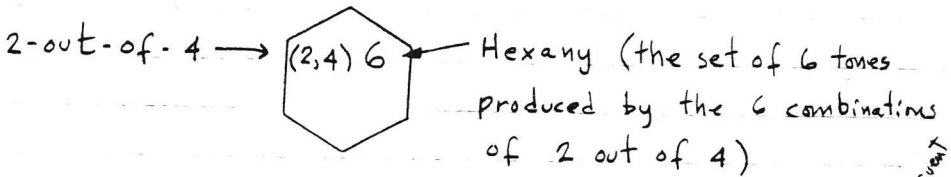


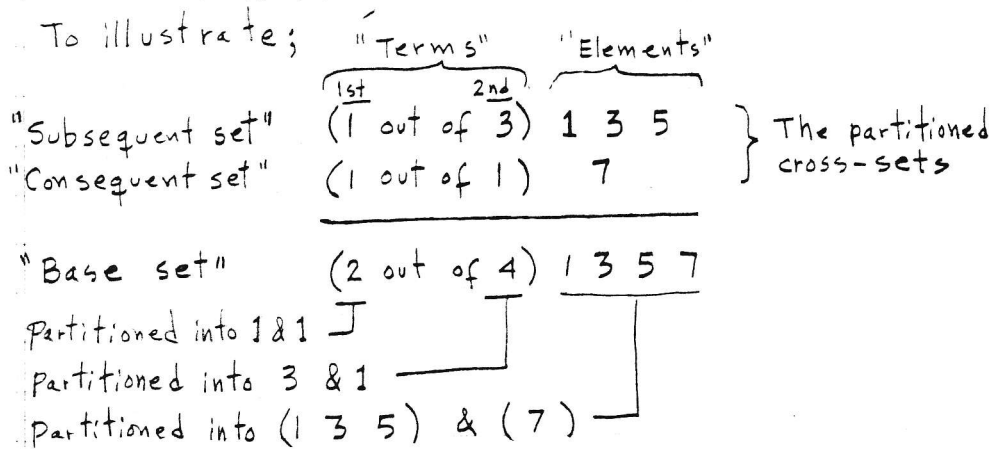
Fig 1.

Pascal's Triangle (Fig 1) may be used to codify the combination-product sets.



- A. Each <sup>Base</sup> combination-product set contains <sup>all</sup> the <sup>subsequent</sup> combination-product sets obliquely above it in the trianguler heirarchy.
- B. Further, each "subsequent" combination-product set occurs a predictable number of times in the "base" combination-product set. The pattern of this multiple occurence is (interestingly enough) also a combination-product set. A cross-set is produced. The second set is labeled "consequent" set.

C. The partitioned nature of the cross-set is seen in the numerical terms of combination as well as in the elements of the set.



D. Finally, the partitioned cross-set, itself, has a predictable number of "expressions" (permutations) based on the no. of ways the elements of the "base set" can be partitioned into the appropriate no. of elements in the partitioned sets. For example; when the "base" set has the 4 elements (1 3 5 7), and is being partitioned into sets of 3 and 1 elements, the "expressions" or permutations are (135)(7), (1 3 7)(5), (1 5 7)(3), and (3 5 7)(1); 4 Permutations.

E. When the various "consequent" sets of a "base" set are assigned to their proper niches in the Triangle they produce, among them, the 180° rotation of the Triangle (see Fig 2). The apex has now become the nadir, which is juxtaposed upon the "base" set, (see Fig 3).

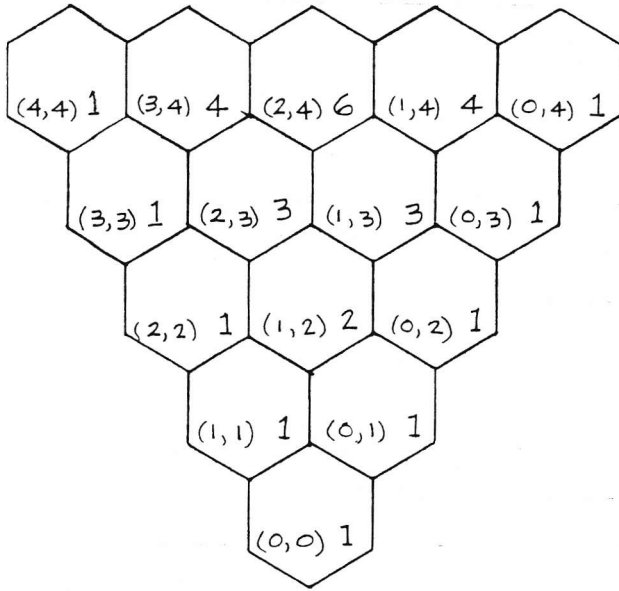


Figure 2  
Pascal's Triangle rotated 180°

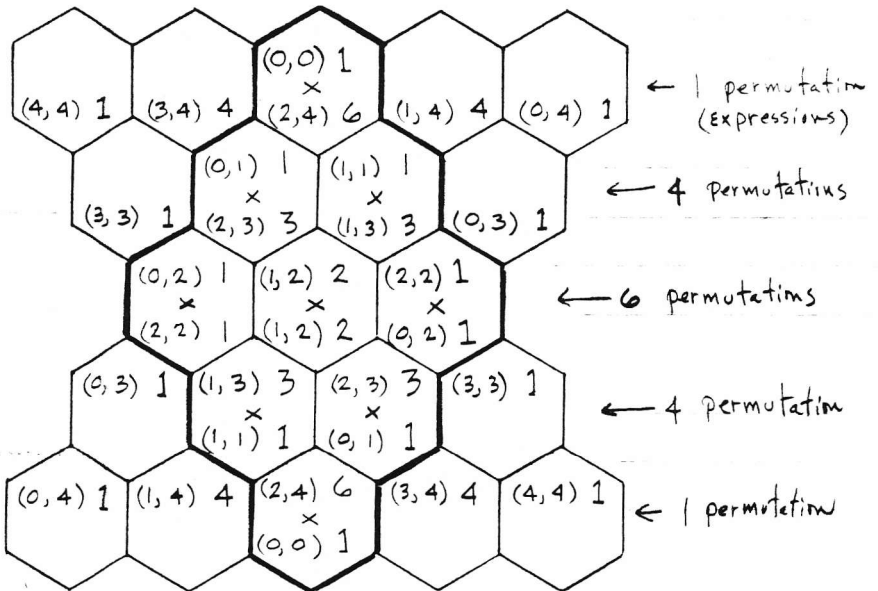


Figure 3

180° rotation of Pascal's Triangle rising from the (2,4) hexany.  
Note: this is the key for all the partitioned cross-sets of the 2-out-of-4, (2,4) Hexany, within the outlined rhombus the cross-sets are cross-indexed, as well.

## Notes on Terminology

1.	<u>No. of Tones</u>	<u>Name of Combination-product Set</u> — abbrev.	<u>Name of Primary Module</u>
	0		Zeroad
	1	Monany 1ny	Monad
	2	Dyany 2ny	Dyad
	3	Triany 3ny	Triad
	4	Tetrazy 4ny	Tetrad
	5	Pentany 5ny	Pentad
	6	Hexany 6ny	Hexad
	7	Heptany 7ny	Heptad
	8	OKtany 8ny	Ogdoad
	9	Enneany 9ny	Ennead
	10	Dekany 10ny	Dekad
	15	Pentadekany 15ny	
	20	Eikosany 20ny	
	21	Eikosi monany 21ny	
	28	Eikosi oktany 28ny	
	35	Tria kontapentany 35ny	
	70	Hebdomekontany 70ny	

- The suffix "-ny" or "-any" is reserved for combination-product sets.
- The "Primary Module" is a master set; that set from which the combination-product set is derived.
- Combination-product Set; a set of combinations taken from the elements of a master set, & where the elements of each combination are multiplied. Example; the 2-out-of-4 combination-product set of the master set, 1 3 5 7, is 1.3, 1.5, 1.7, 3.5, 3.7, 5.7.

# Notes on Terminology (continued)

5.  $\binom{3}{6}$  or  $(3,6) = 3$  out of 6

6.  $(4,8)$  hebdomekontang = a combination-product set of 70 members, derived by taking <sup>combinations of</sup> 4-out-of-8 elements (usually the harmonics 1 3 5 7 9 11 13 15, but they could be any appropriate set of 8) and multiplying together the 4 elements of each combination, thus; 1.3.5.7, 1.3.5.9, 1.3.5.11, etc., to 9.11.13.15.



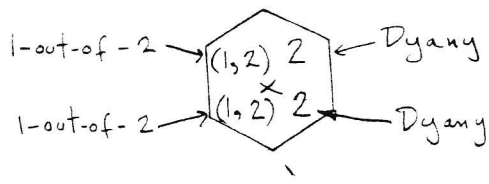
# Partitioned Cross-sets of the Hexany (cont.)

Referring to Figure 3 the outlined rhombus shows the following Partitioned Cross-sets:

	<u>Permutations</u>
$\{(0,0) \emptyset\} \times \{(2,4) a b c d\}$	1
$\{(0,1) a\} \times \{(2,3) b c d\}$	4
$\{(1,1) a\} \times \{(1,3) b c d\}$	4
$\{(0,2) a b\} \times \{(2,2) c d\}$	6
* $\{(1,2) a b\} \times \{(1,2) c d\}$	6
$\{(2,2) a b\} \times \{(0,2) c d\}$	6
* $\{(1,3) a b c\} \times \{(1,1) d\}$	4
* $\{(2,3) a b c\} \times \{(0,1) d\}$	4
$\{(2,4) a b c d\} \times \{(0,0) \emptyset\}$	1

(in the body of permutations)  
 In effect the 1st half of the series merely cross-indexes the 2nd half. Further, the sets on the 4 corners of the rhombus are academic. The 3 cross-sets indicated (\*) will be discussed below.

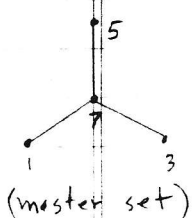
Item 15,



In the (2,4) Hexany matrix the (1,2) Dyany forms a partitioned cross-set with the (1,2) Dyany. This cross-set is a rare mirror of itself. Therefore, altho normally there would be 6 permutations of the ways 2 can be taken from 4, in this partitioned circumstance 3 of the cross-sets thus derived are merely a re-ordering of the remaining 3.

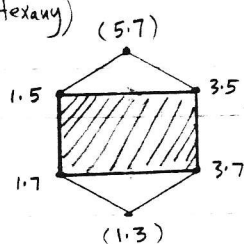
- Example: 1.  $\{(1,2) 13\} \times \{(1,2) 57\}$   
of re-ordered 2.  $\{(1,2) 15\} \times \{(1,2) 37\}$   
sets 3.  $\{(1,2) 17\} \times \{(1,2) 35\}$   
4.  $\{(1,2) 35\} \times \{(1,2) 17\}$   
5.  $\{(1,2) 37\} \times \{(1,2) 15\}$   
6.  $\{(1,2) 57\} \times \{(1,2) 13\}$
- 

Examples of cross-sets; (within the (2,4) 1357 Hexany)



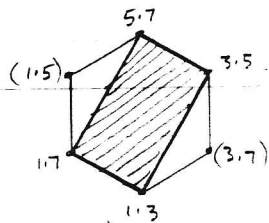
(1.)

X	1	3
5	1.5	3.5
7	1.7	3.7



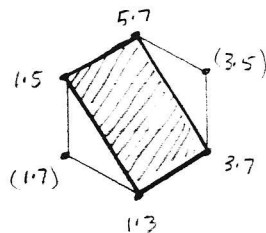
(2.)

X	1	5
3	1.3	5.3
7	1.7	5.7



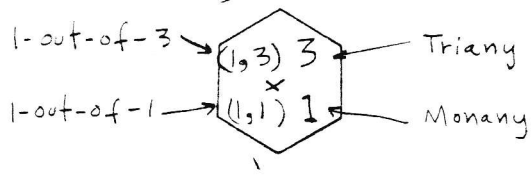
(3.)

X	1	7
3	1.3	7.3
5	1.5	7.5



Hexang (cont)

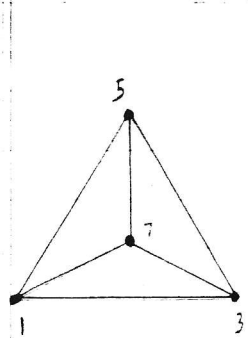
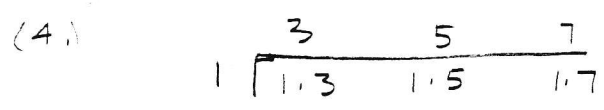
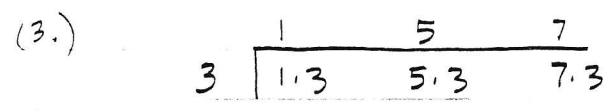
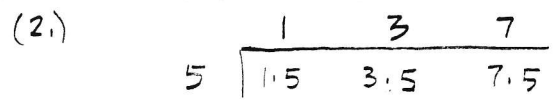
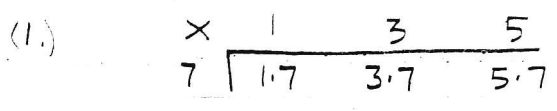
Item 16:



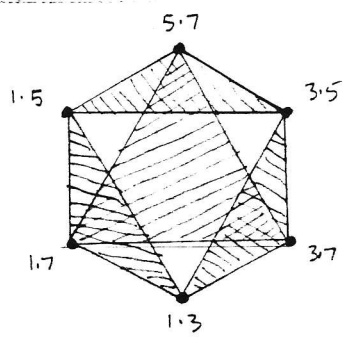
In the (2,4) Hexang matrix the (1,3) Triangy forms a partitioned cross-set with the (1,1) Monangy. There are 4 permutations of this cross-set, according to the combinations of 3 and/or 1 out of 4.

4 Permutations

- Example; (1)  $\{(1,3) | 3 | 5\} \times \{(1,1) | 7\}$   
 (2)  $\{(1,3) | 3 | 7\} \times \{(1,1) | 5\}$   
 (3)  $\{(1,3) | 5 | 7\} \times \{(1,1) | 3\}$   
 (4)  $\{(1,3) | 3 | 5 | 7\} \times \{(1,1) | 1\}$

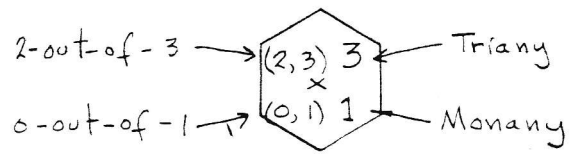


master set



The 4 (1,3) x (1,1) cross-sets are shown as shaded triangles

Item 17;



In the (2,4) Hexang matrix the (2,3) triang forms a partitioned cross-set with the (0,1) Monang. There are 4 permutations of this cross-set, resulting from the 4 combinations of 3 and/or 1 out of 4.

The 4 permutations; Example

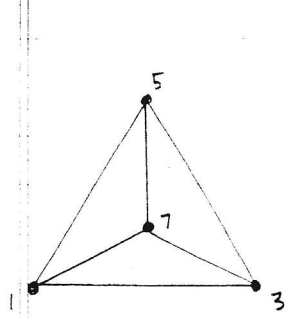
1.  $\{(2,3) 135\} \times \{(0,1) 7\}$
2.  $\{(2,3) 137\} \times \{(0,1) 5\}$
3.  $\{(2,3) 157\} \times \{(0,1) 3\}$
4.  $\{(2,3) 357\} \times \{(0,1) 1\}$

(1.) 
$$\begin{array}{c} \times \quad 1.3 \quad 1.5 \quad 3.5 \\ \hline \emptyset \quad 1.3 \quad 1.5 \quad 3.5 \end{array} \rightarrow (\times \overline{1.35} = \overline{5} \overline{3} \overline{1})$$

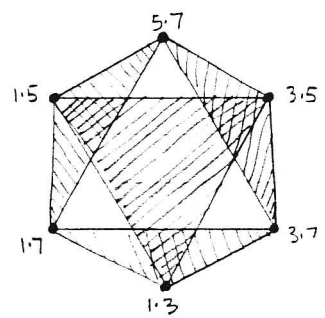
(2.) 
$$\begin{array}{c} \times \quad 1.3 \quad 1.7 \quad 3.7 \\ \hline \emptyset \quad 1.3 \quad 1.7 \quad 3.7 \end{array} \rightarrow (\times \overline{1.37} = \overline{7} \overline{3} \overline{1})$$

(3.) 
$$\begin{array}{c} \times \quad 1.5 \quad 1.7 \quad 5.7 \\ \hline \emptyset \quad 1.5 \quad 1.7 \quad 5.7 \end{array} \rightarrow (\times \overline{1.5.7} = \overline{7} \overline{5} \overline{1})$$

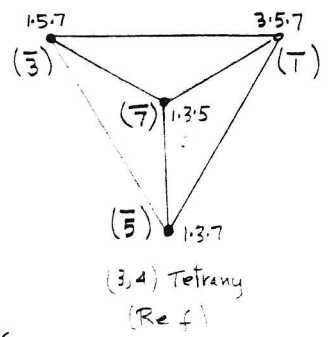
(4.) 
$$\begin{array}{c} \times \quad 3.5 \quad 3.7 \quad 5.7 \\ \hline \emptyset \quad 3.5 \quad 3.7 \quad 5.7 \end{array} \rightarrow (\times \overline{3.5.7} = \overline{7} \overline{5} \overline{3})$$



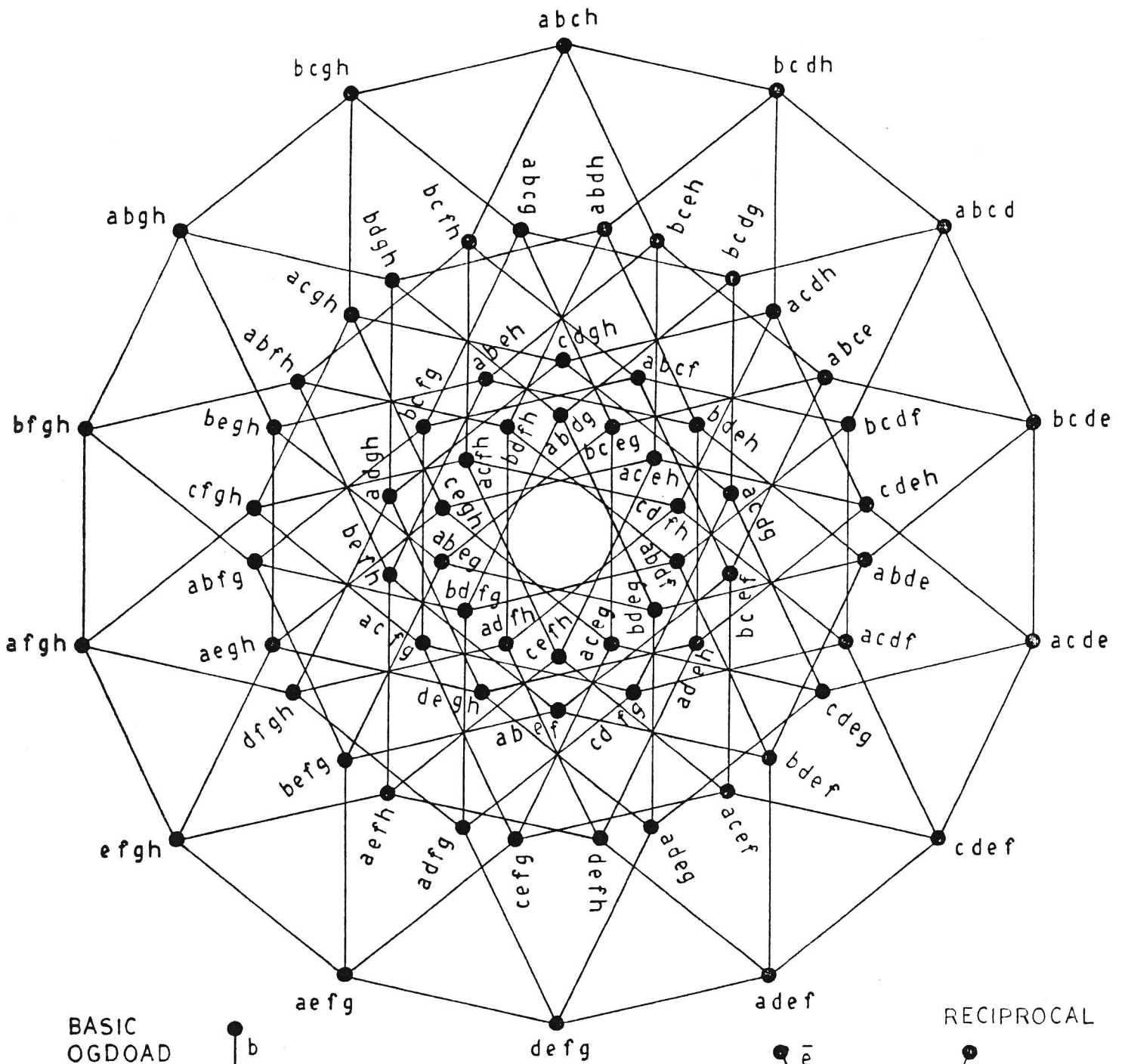
(1,4) Tetrang  
also Master Set  
(Ref)



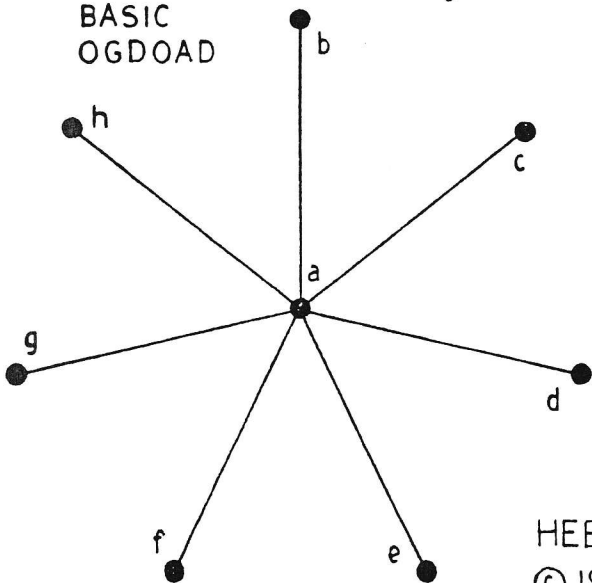
The 4 (2,3) x (0,1) cross-sets  
are shown as shaded Triangles



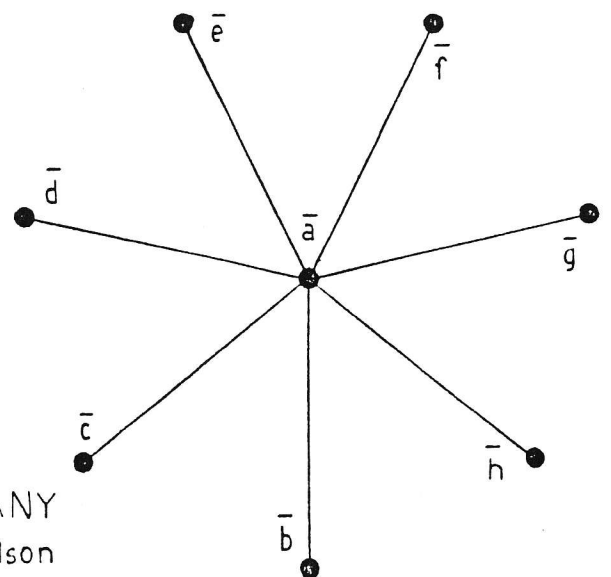
(3,4) Tetrang  
(Ref)



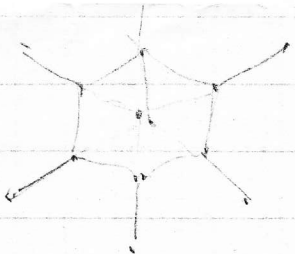
BASIC  
OGDOAD



RECIPROCAL



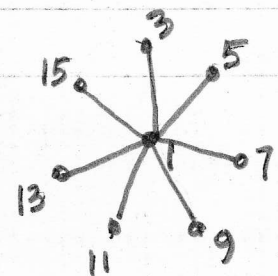
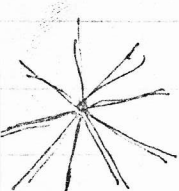
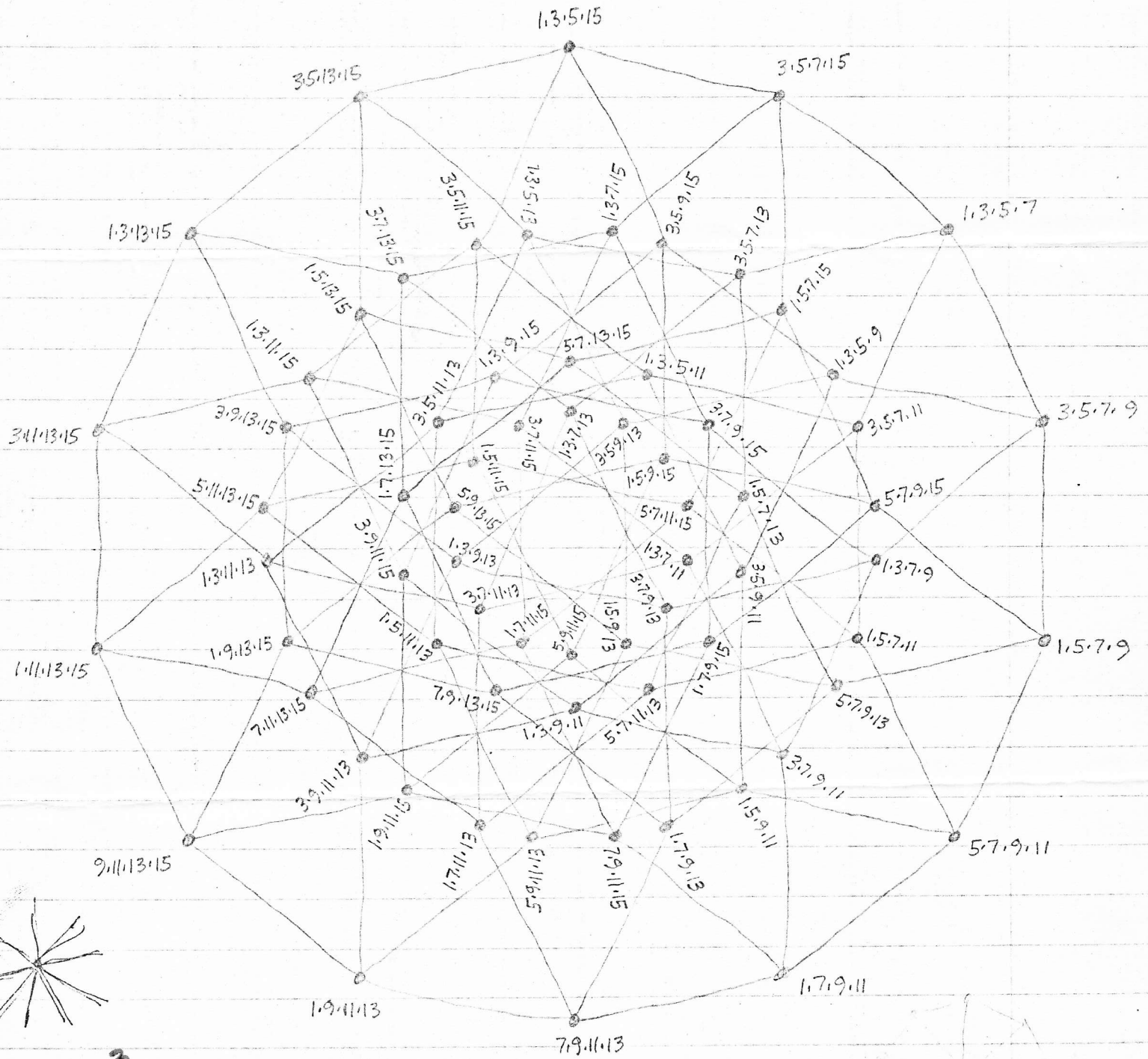
HEBDOMEKONTANY  
©1981 by Erv Wilson



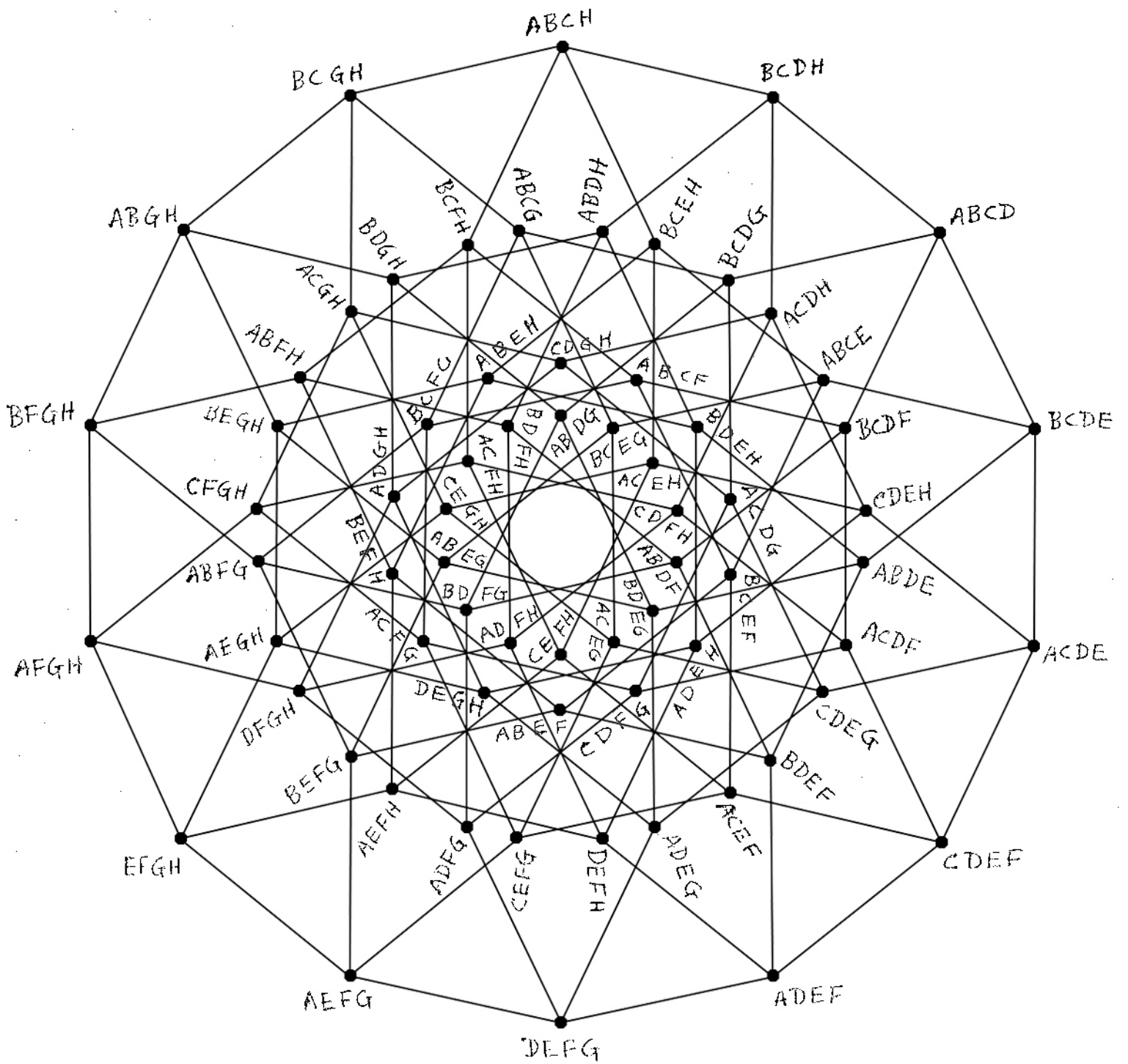
$$10 \overline{) 1440} \quad 20 \overline{) 1440}$$

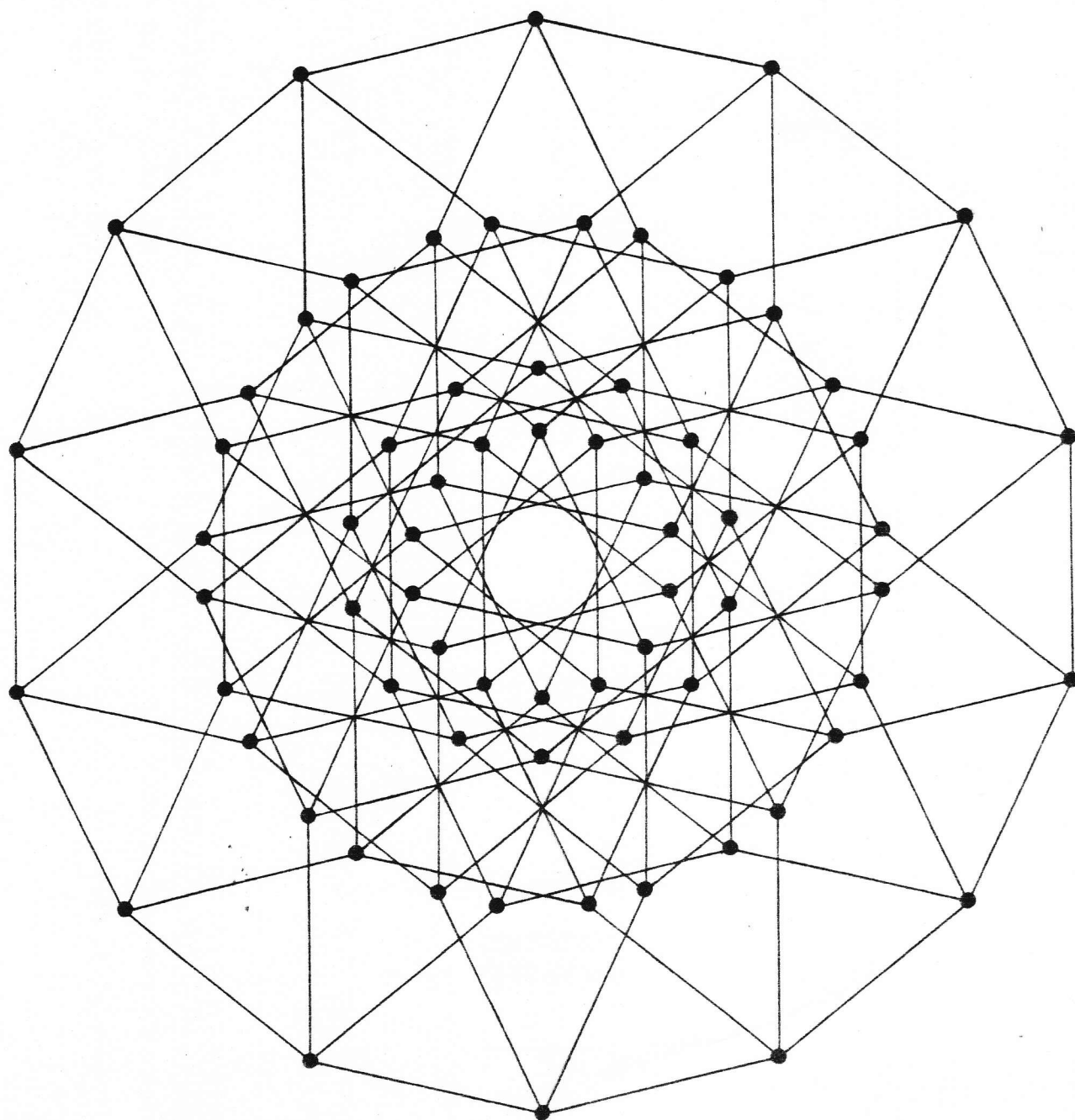
$$\underline{1440} \quad \underline{1440}$$

$$0 \quad 0$$

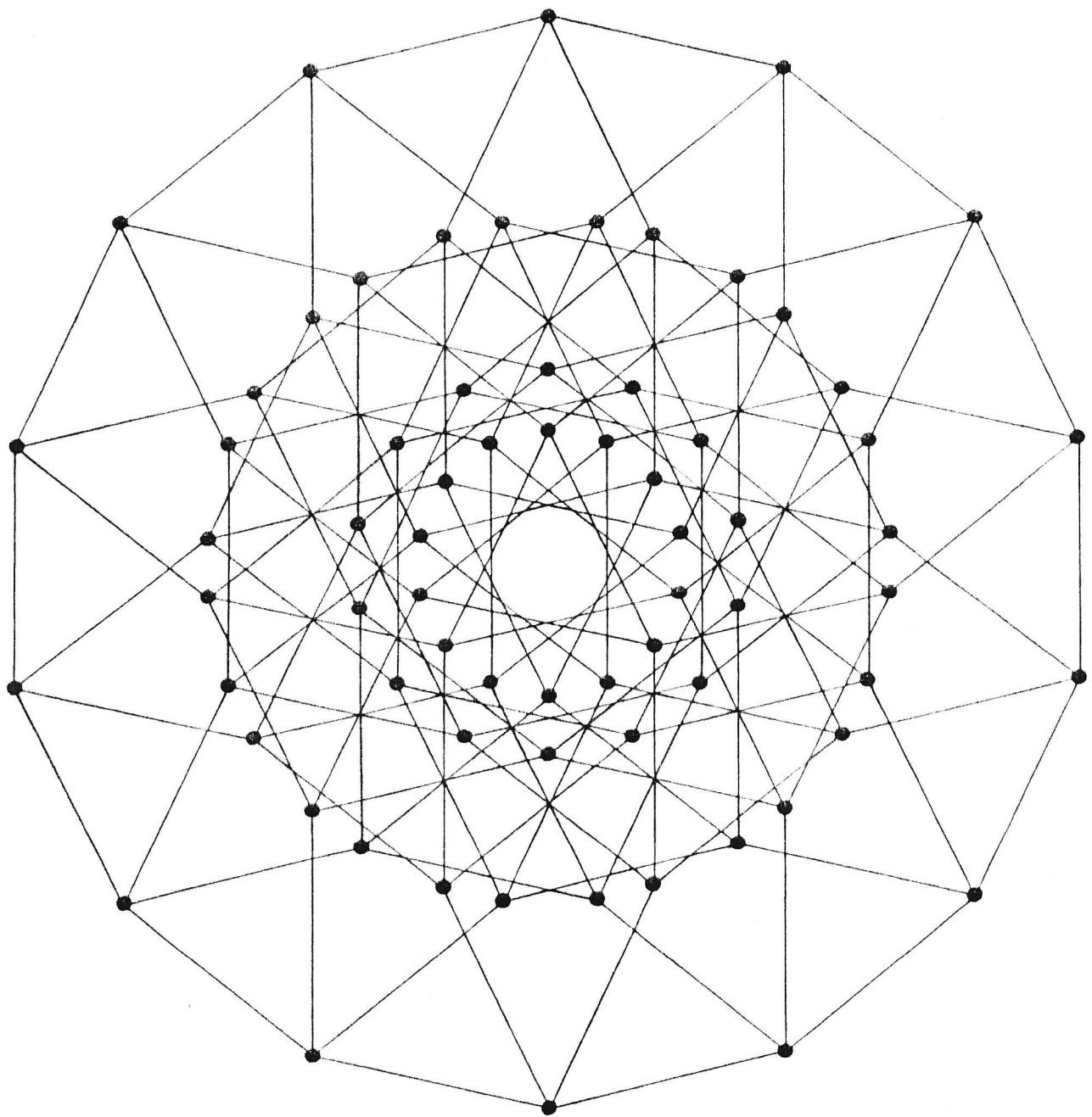


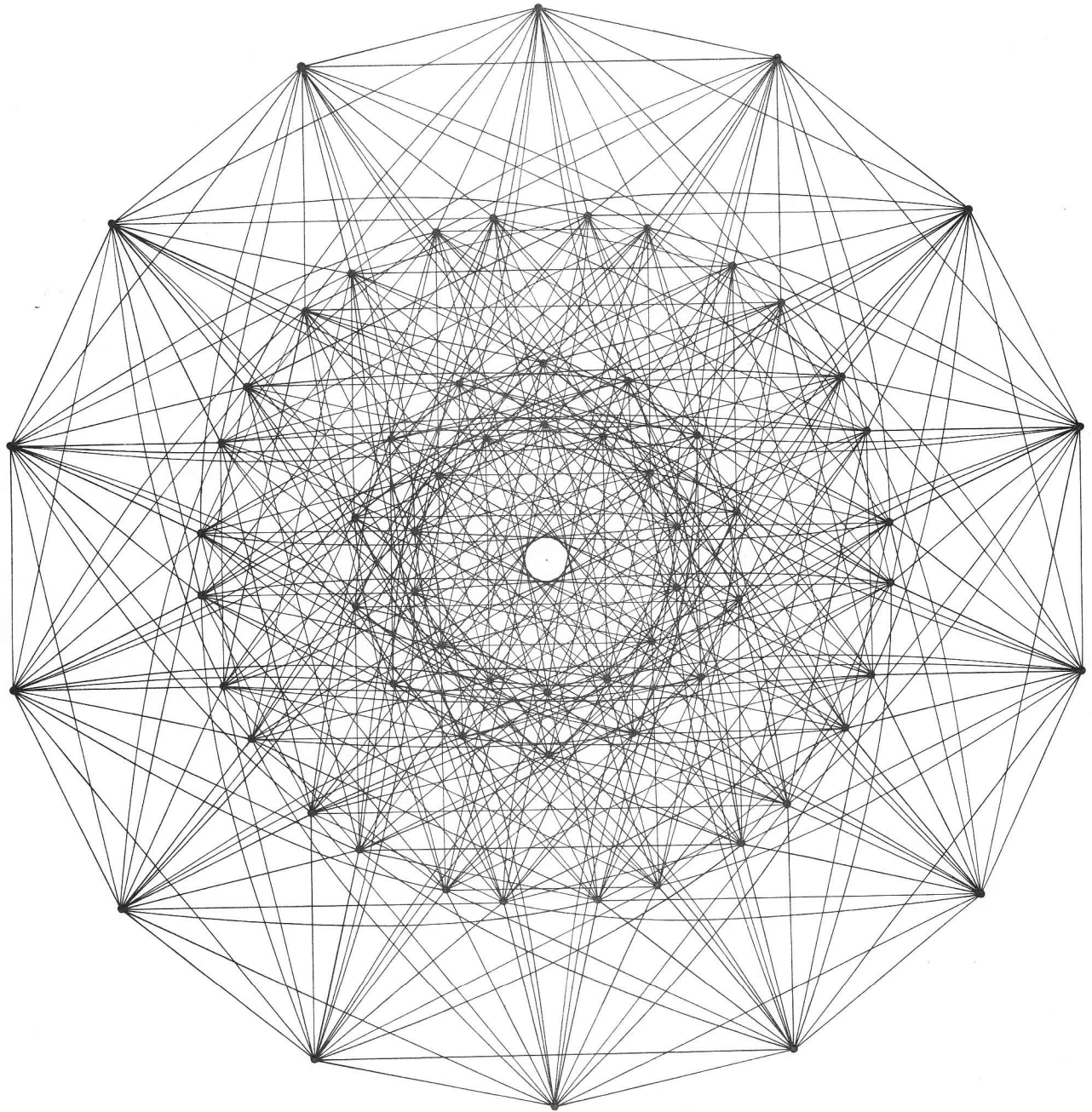
There will be 51 radial species of Dekateragrams.  $7 \times 51 = 357 + 3 = 360$   
 $(40320 \div 14 = 2880 \div 8 = 360)$   
 $[8!]$  3 concentric species.

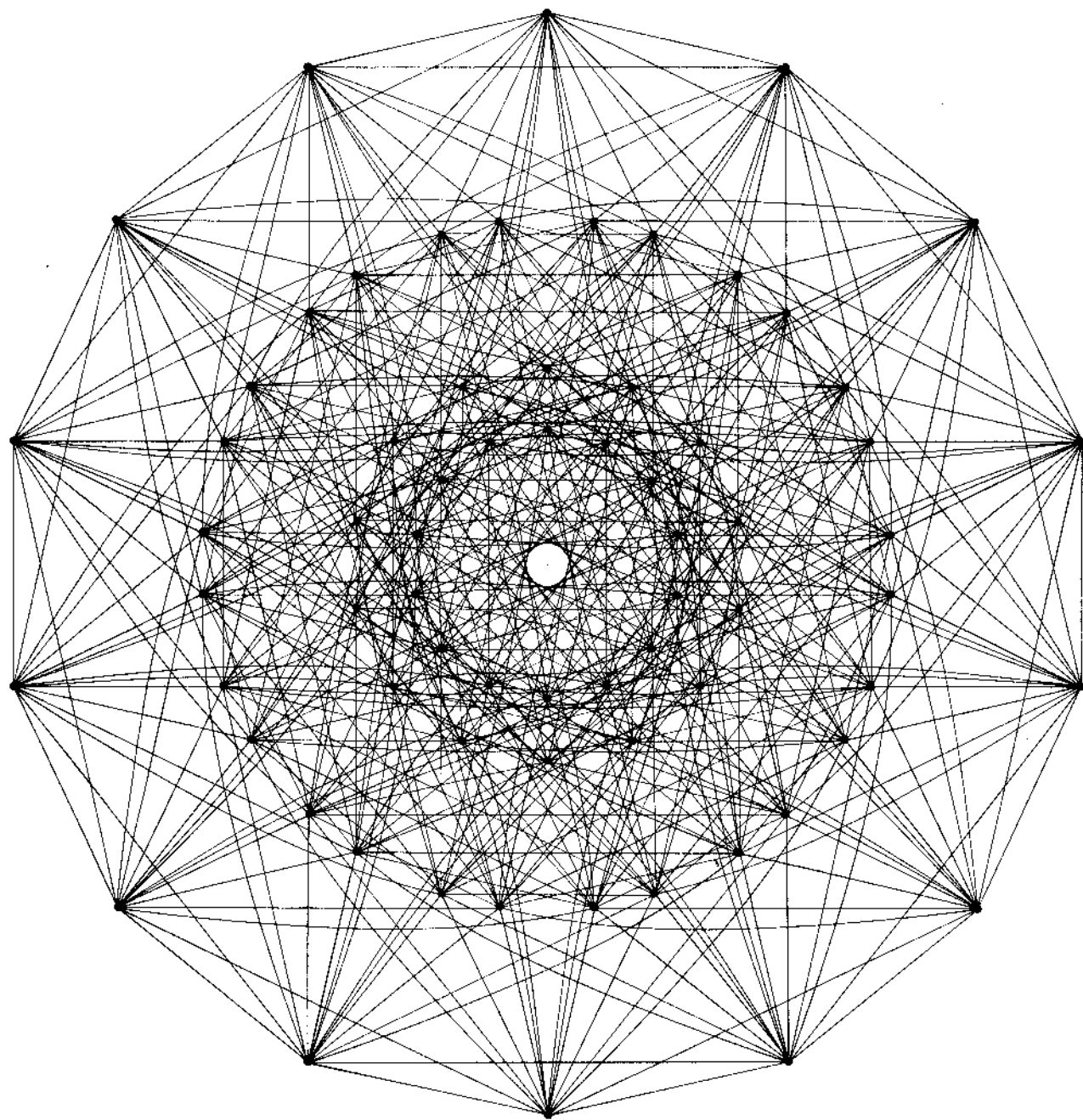


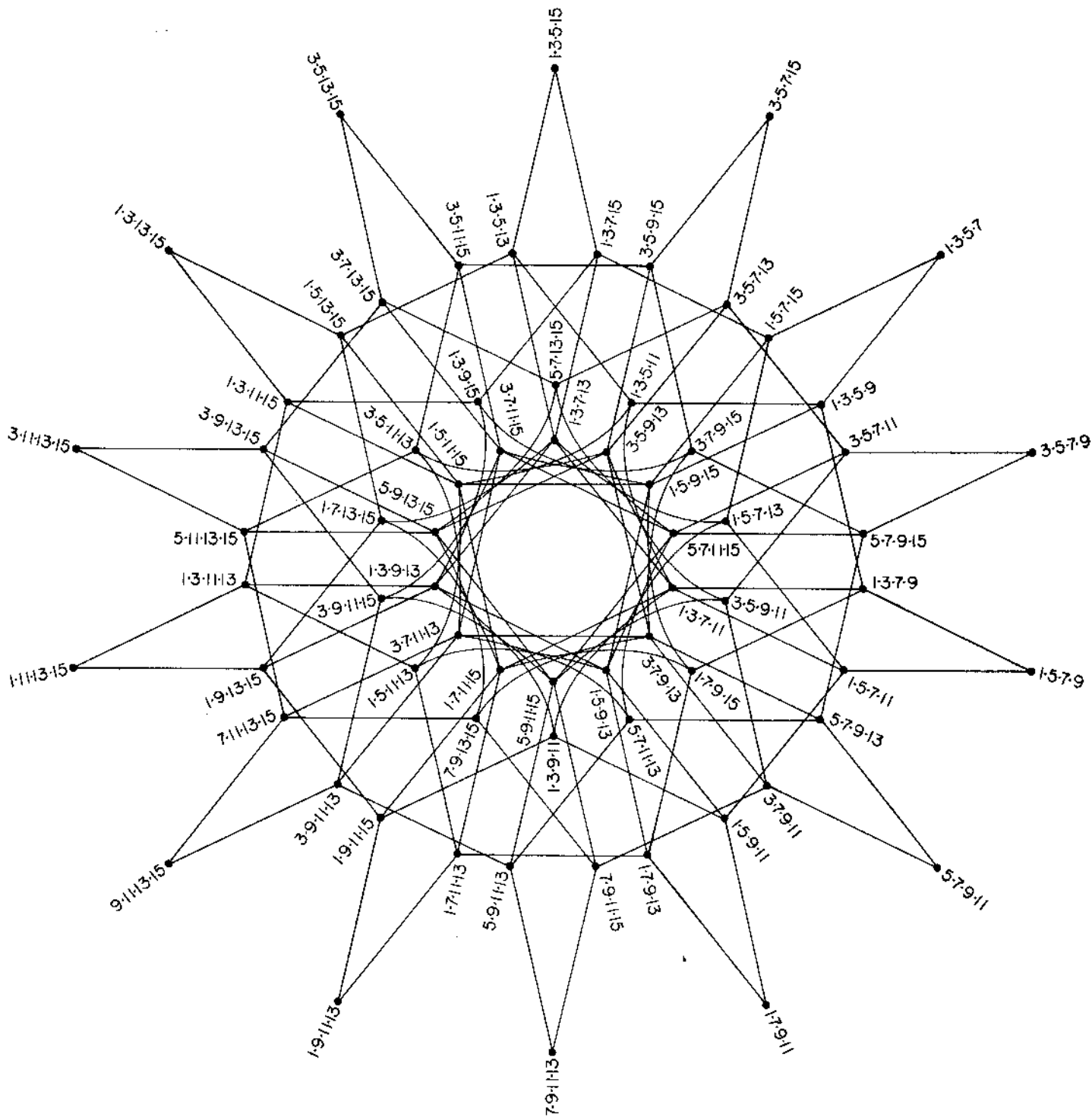




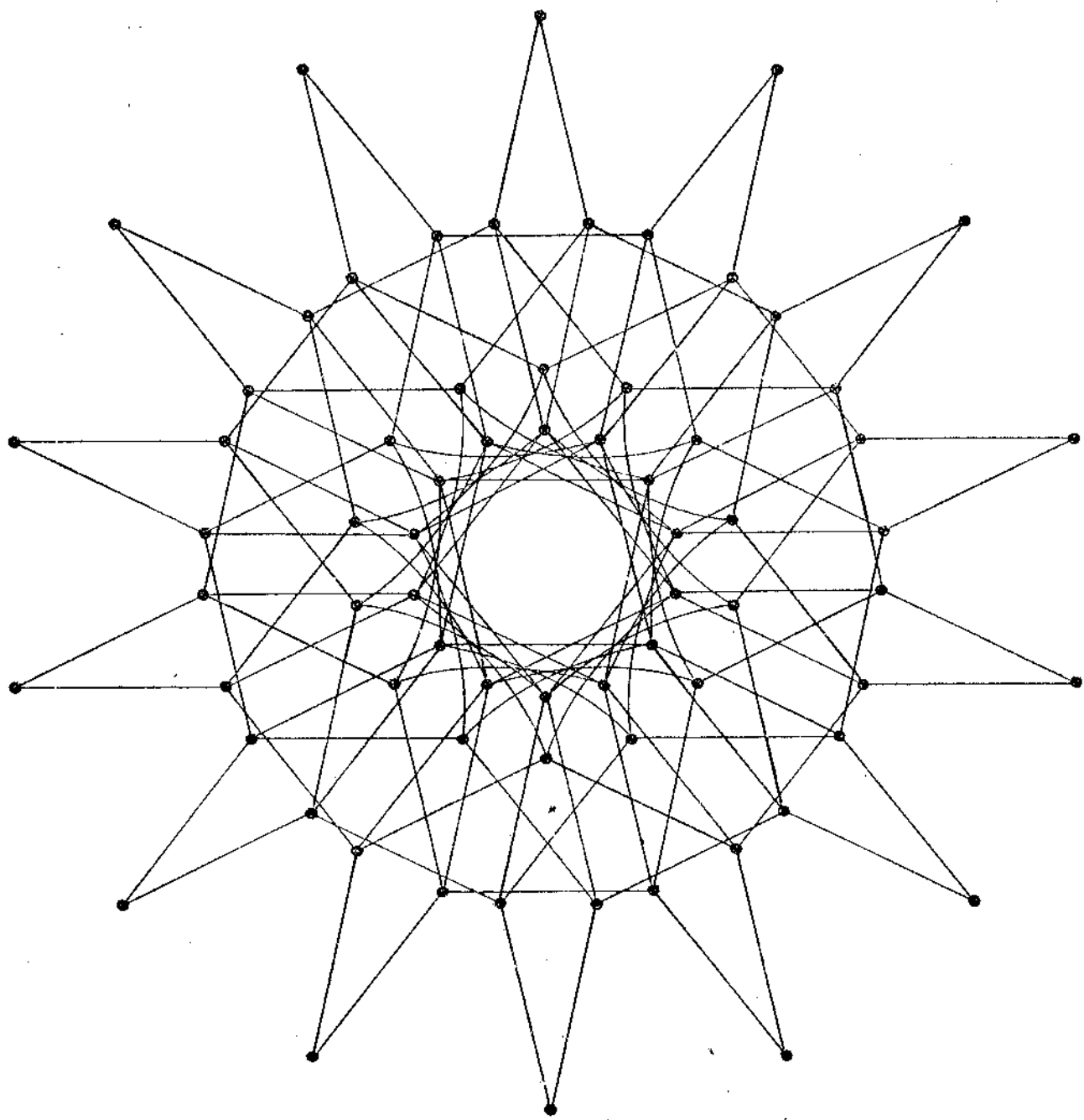








1-3-5-7-9-11-13-15 HEBDOMEKONTANY



Letter to John Chalmers

844 N. Ave 65  
Los Angeles, CA 90042  
April 27, 1992

Copy to Kraig Grady

Dear John Chalmers,

Harmonic 8ad →	<u>1</u>	<u>3</u>	<u>5</u>	<u>7</u>	<u>9</u>	<u>11</u>	<u>13</u>	<u>15</u>	
Convert to 72 →	0/72	42	23	58	12	33	50	65	(x1, modulus 72)
Transform to →	±0	+6	+5	+22	+12	(+51)	+14	+11	(x19, modulus 72)
chain position Template				use → -21					

using the above code the Hebdomekontang can be transferred to Hanson's generalized keyboard with rather nice results. The keyboard position is derived by summing the 4-out-of-8 combinations of the chain position (template). These 70 combinations all fall within a 72-tone chain of (unequal) minor thirds ( $19/72$ ). There are duplicate entries at positions +42 and +7, respectively, which require special handling. 16 "empties" are left, which can be filled (as shown in parentheses) to construct a formal 72-tone scale. This can be notated and fingered/malletted in an orderly and predictable (homogeneous) way. Article follows.

References; Sistema Natural base del Natural-Aproximado, Augusto Novaro, Mexico D.F. 1927  
(Library of Congress call number ML3805.N69 1927)

### Development of a 53-Tone Keyboard Layout

Larry Hanson 1989, Xenharmonikon XII (Frog Peak Music Box A36, Hanover NH 03755)

yours,

Ervin M. Wilson

Novaro's use of the number 72 to measure the harmonic ogdoad makes the placement of the Hebdomekontang on the Hanson generalized keyboard possible. This is a significant step. It's amazing what else he was doing (in 1927!).

			+52	+56	+60										
			(3·5·7·9·15)	(3²·7·9·13)											
		52,	56,	60,											
	+37	+41	+45	+49	+53	+57									
	55,	59,	63,	67,	71,	3,	Spare 5·9·13·15 *								
	+22	+26	+30	+34	+38	+42	+46	+50	+54	+58					
58,	62	66,	70,	2,	6,*	10,	14,	18,	22,						
	+11	+15	+19	+23	+27	+31	+35	+39	+43	+47	+51	+55	+59		
65,	69,	1,	5,	9,	13,	17,	21,	25,	29,	33,	37,	41,			
	±0	+4	+8	+12	+16	+20	+24	+28	+32	+36	+40	+44	+48	+52	
(5·7·11/3)	1·11·13·15 3·5·11·13	3·9·11·15	1·7·11·15 3·5·7·11	9·11·13·15	5·7·11·13	7·9·11·15	1·5·9·15	1·3·9·13	3·5·13·15	1·3·7·9	3·5·7·15	1·7·9·13	5·7·13·15		
0/72.	4,	8,	12,	16,	20,	24,	28,	32,	36,	40,	44,	48,	52,	56,	
	-11	-7	-3	+1	+5	+9	+13	+17	+21	+25	+29	+33	+37	+41	
(5·11·15/3)	(1·11·13)	1·3·9·11	3·5·11·15	1·9·11·13	5·11·13·15	1·7·9·11	5·7·11·15	3·7·11·13	1·3·5·13	1·3·9·15	1·3·5·7	1·9·13·15 3·5·9·13	1·5·7·13		
7,	11,	15,	19,	23,	27,	31,	35,	39,	43,	47,	51,	55,	59,	63,	
				-10	-6	-2	+2	+6	+10	+14	+18	+22	+26	+30	
			1·3·5·11	(5·7·11/3²)	1·5·11·13	1·9·11·15 3·5·9·11	1·5·7·11	3·11·13·15 5·9·11·13	(5·7·11·13/3)	3·7·11·15 5·7·9·11	1·3·5·15	7·11·13·15	1·5·13·15		
			26,	30,	34,	38,	42,	46,	50,	54,	58,	62,	66,	70,	
						-9	-5	-1	+3	+7	+11	+15	+19		
						(1·9·11)	1·5·11·15	1·3·11·13	(5·11·13·15/3)	1·3·7·11 5·9·11·15	3·9·11·13	1·7·11·13			
						45,	49,	53,	57,	61,	65,	69,	73,		
									Spare 1·3·7·11 †	-8 (5·11·13/3)	-4 1·3·11·15 1·5·9·11	±0 (5·7·11/3)	+4	+8	
									64,	68,	72/0	4,	8,		
											-11				
											7,				

(4)  
 (8) 1, 3, 5, 7, 9, 11, 13, 15 Hebdome Kontany,  
 19/72 scale, 3/11 Key board  
 © 1992 by Ervin M. Wilson  
 (Hanson Keyboard geometry used by permission)

Notation; (□) b \ ♯ / #  
72-scale -3 -6 -1 0 +1 +6

Tree Notes;  
□

a ailm b beth c coll d duir e eadha f fearn g gort

		+52	+56	+60		Normal Notes; d	A	B	C	D	E	F	G	
		52.	56. A#	60. b/		d								
	+37	+41	+45	+49	+53	+57	Spare							
	55.	59. b	63. B/	67. B#\#	71. C#	3. C/\#	5.9.13.15 *							
	+22	+26	+30	+34	+38	+42	+46	+50	+54	+58				
58.	62 B	66. c/	70. c#\#	2. C#	6. *d/	10. d#\#	14. D#	18. e/	22. e#\#					
	+11	+15	+19	+23	+27	+31	+35	+39	+43	+47	+51	+55	+59	
65. c	69. C/	1. C#\#	5. d	9. D/	13. D#\#	17. e	21. E/	25. E#\#	29. f#	33. F/\#	37. F*\*	41. g#\#		
	± 0	+4	+8	+12	+16	+20	+24	+28	+32	+36	+40	+44	+48	+52
(5.7.11/3) d/b 0/72.	1.11.13.15 3.5.11.13	4. d\	8. D	12. e/b	16. e\	20. E	24. f/	28. f#\#	32. F#	36. g/	40. g#\#	44. G#	48. a/	52. a#\#
	-11	-7	-3	+1	+5	+9	+13	+17	+21	+25	+29	+33	+37	+41
(5.11.15/3)	(1.11.13)	7. D\	11. eb	15. E/b	19. E\	23. f	27. F/	31. F#\#	35. g	39. G/	43. G#\#	47. a	51. A/	55. A#\#
			-10	-6	-2	+2	+6	+10	+14	+18	+22	+26	+30	
			1.3.5.11	(5.7.11/3 <sup>2</sup> )	1.5.11.13	1.9.11.15	1.5.7.11	3.11.13.15	(5.7.11.13/3)	3.7.11.15	1.3.5.15	7.11.13.15	1.5.13.15	
			26. F	30. g/b	34. g#\#	38. G	42. a/b	46. a\	50. A	54. b/b	58. b\	62. B	66. c/	
							-9	-5	-1	+3	+7	+11	+15	+19
							(1.9.11)	1.5.11.15	1.3.11.13	(5.11.13.15/3)	1.3.7.11	3.9.11.13	1.7.11.13	
							45. A/b	49. A\	53. bb	57. B/b	61. B\	65. c	69. C/	
									spare	-8	-4	± 0	+4	+8
									1.3.7.11	(5.11.13/3)	1.3.11.15	(5.7.11/3)		
									†	64. c\	68. C	72. 0	†.	8.
												-11		

(4)  
(8) 1,3,5,7,9,11,13,15 Hebdome Kontany,  
19/72 scale, 3/11 keyboard  
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7.



Notation; ( $\square$ )  $\flat$   $\backslash$   $\sharp$  /  $\#$   
 72-Scale -3 -6 -1 0 +1 +6

Tree Notes;

a ailm b beth c coll d duir e eadha f fearn g gort

Normal Notes;

A B C D E F G

		+52	+56	+60															
		52.	56. A#	60. b/															
	+37	+41	+45	+49	+53	+57													
	55.	59. b	1.7.9.15 3.5.7.9 63. B/	(3 <sup>2</sup> .9.13.15) 67. B#	3.7.13.15 5.7.9.13 71. C#	(3 <sup>2</sup> .7.9.15) 3. C#	Letter to	Kraig Grady	/	John Chalmers									
							April	27, 1992		from	Erin Wilson								
	+22	+26	+30	+34	+38	+42	+46	+50	+54	+58									
58.	62 B	66. C/	1.5.13.15 3.5.9.15 70. C#	1.5.7.15 2. C#	1.3.7.13 5.9.13.15 6. *d/	(1.3 <sup>2</sup> .7.9) 10. d#	5.7.9.15 14. D#	3.7.9.13 18. e/	(3.5.7.13.15) 22. e#										
	+11	+15	+19	+23	+27	+31	+35	+39	+43	+47	+51	+55	+59						
65. C	69. C/	1. C#	3.7.9.11 1.3.5.9 5. d	7.9.11.13 9. D/	1.3.13.15 1.5.9.13 13. D#	(1.3 <sup>2</sup> .9.15) 15. e	1.3.7.15 1.5.7.9 21. E/	3.9.13.15 25. E#	1.7.13.15 3.5.7.13 29. f#	3.7.9.15 33. F#	(3 <sup>2</sup> .9.13.15) 37. F#	7.9.13.15 41. g#							
(1) ± 0	+4	+8	(9) +12	+16	+20	+24	+28	+32	+36	+40	+44	+48	+52						
(5.7.11/3) d/b 0/12.	1.11.13.15 3.5.11.13 4. d\	3.9.11.15 8. D	1.7.11.15 3.5.7.11 12. e/b	9.11.13.15 16. e\	5.7.11.13 20. E	7.9.11.15 24. f/	1.5.9.15 28. f#	1.3.9.13 32. F#	3.5.13.15 36. g/	1.3.7.9 40. g#	3.5.7.15 44. G#	1.7.9.13 48. a/	5.7.13.15 52. a#	A#					
	-11	-7	-3	+1	(5) +5	+9	+13	+17	+21	+25	+29	+33	+37	+41					
(5.11.15/3) 7. D\	(1.11.13) 11. eb	1.3.9.11 15. E/b	3.5.11.15 19. E\	1.9.11.13 23. f	5.11.13.15 27. F/	1.7.9.11 31. F#	5.7.11.15 35. g	3.7.11.13 39. G/	1.3.5.13 43. G#	1.3.9.15 47. a	1.3.5.7 51. A/	1.9.13.15 3.5.9.13 55. A#	1.5.7.13 59. b						
Basic chord in red (X)			-10	-6	-2	12	(3) +6	+10	+14	+18	(7) +22	+26	+30						
			1.3.5.11 26. F	(5.7.11/3 <sup>2</sup> ) 30. g/b	1.5.11.13 34. g#	1.9.11.15 3.5.9.11 38. G	1.5.7.11 42. a/b	3.11.13.15 5.9.11.13 46. a\	(5.7.11.13/3) 50. A	3.7.11.15 5.7.9.11 54. b/p	1.3.5.15 58. b\	7.11.13.15 62. B	1.5.13.15 66. C/	70					
			(11) -21			-9	-5	-1	+3	+7	(15) +11	+15	+19						
			33.			(1.9.11) 45. A/b	1.5.11.15 49. A\	1.3.11.13 53. bb	(5.11.13.15/3) 57. B/b	1.3.7.11 5.9.11.15 61. B\	3.9.11.13 65. C	1.7.11.13 69. C/	1.						
									-8	-4	(1) ± 0	+4	+8						
									3.7.11.15 5.9.11.13 64. C\	(5.11.13/3) 68. C	1.3.11.15 1.5.9.11 72. 0	(5.7.11/3) d/b	4.	8.					

(4) 1, 3, 5, 7, 9, 11, 13, 15 Hebdome Kontany,  
 19/72 scale, 3/11 keyboard  
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except for the placement of the 2 "spares" all fingering/malletting in homogeneous.

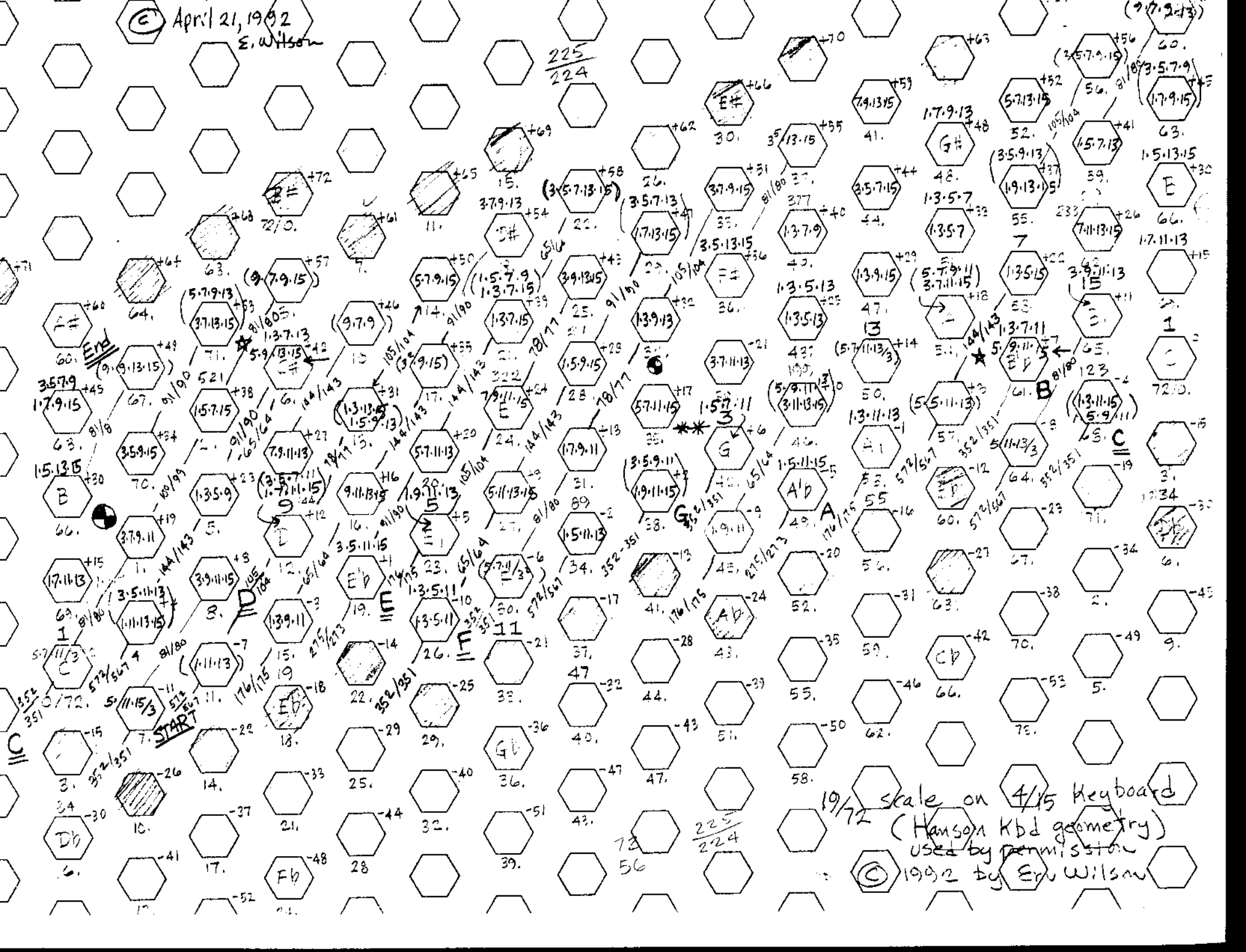
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E. Wilson

$\frac{225}{224}$

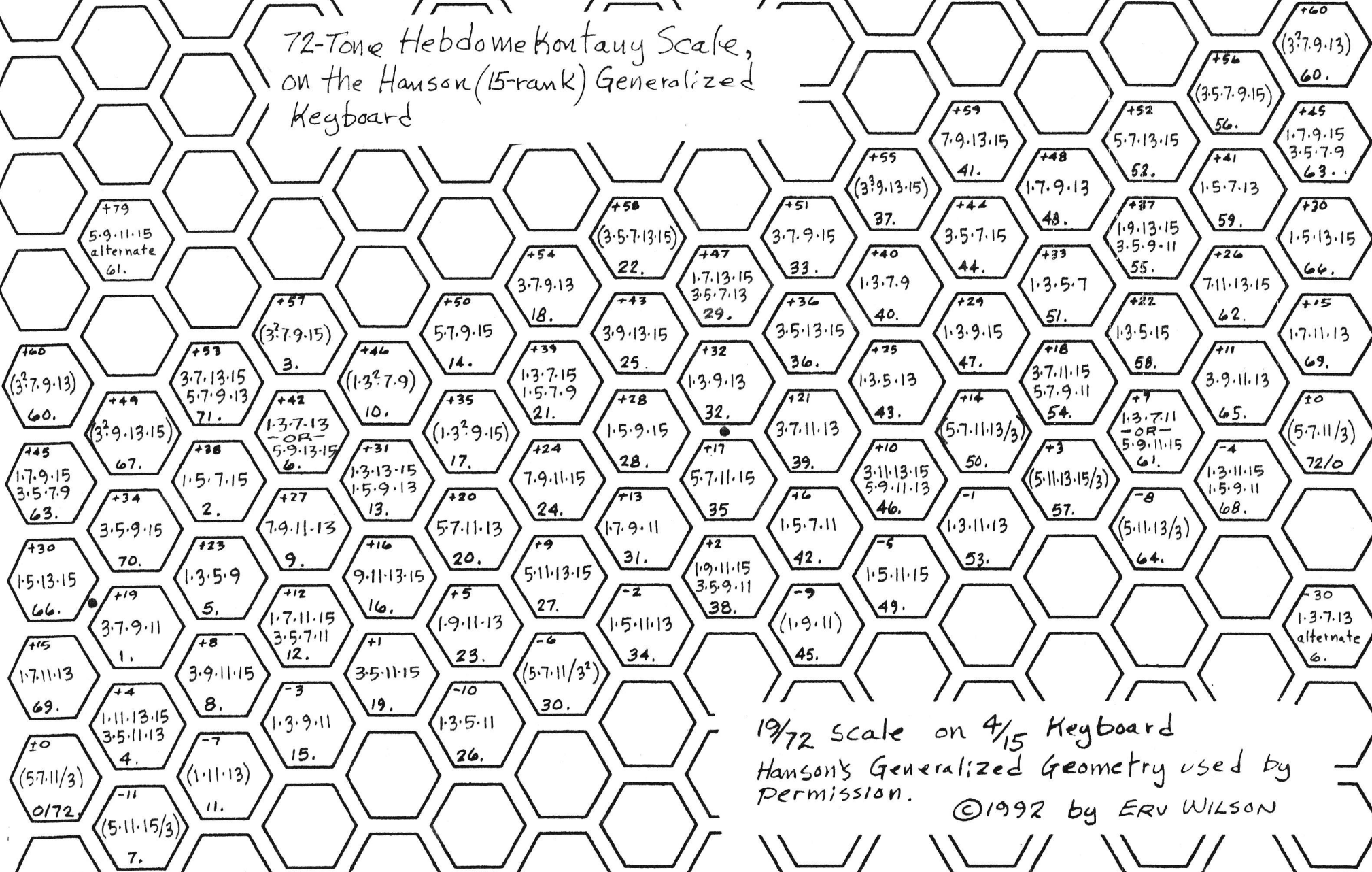
$\frac{225}{224}$

$\frac{225}{224}$

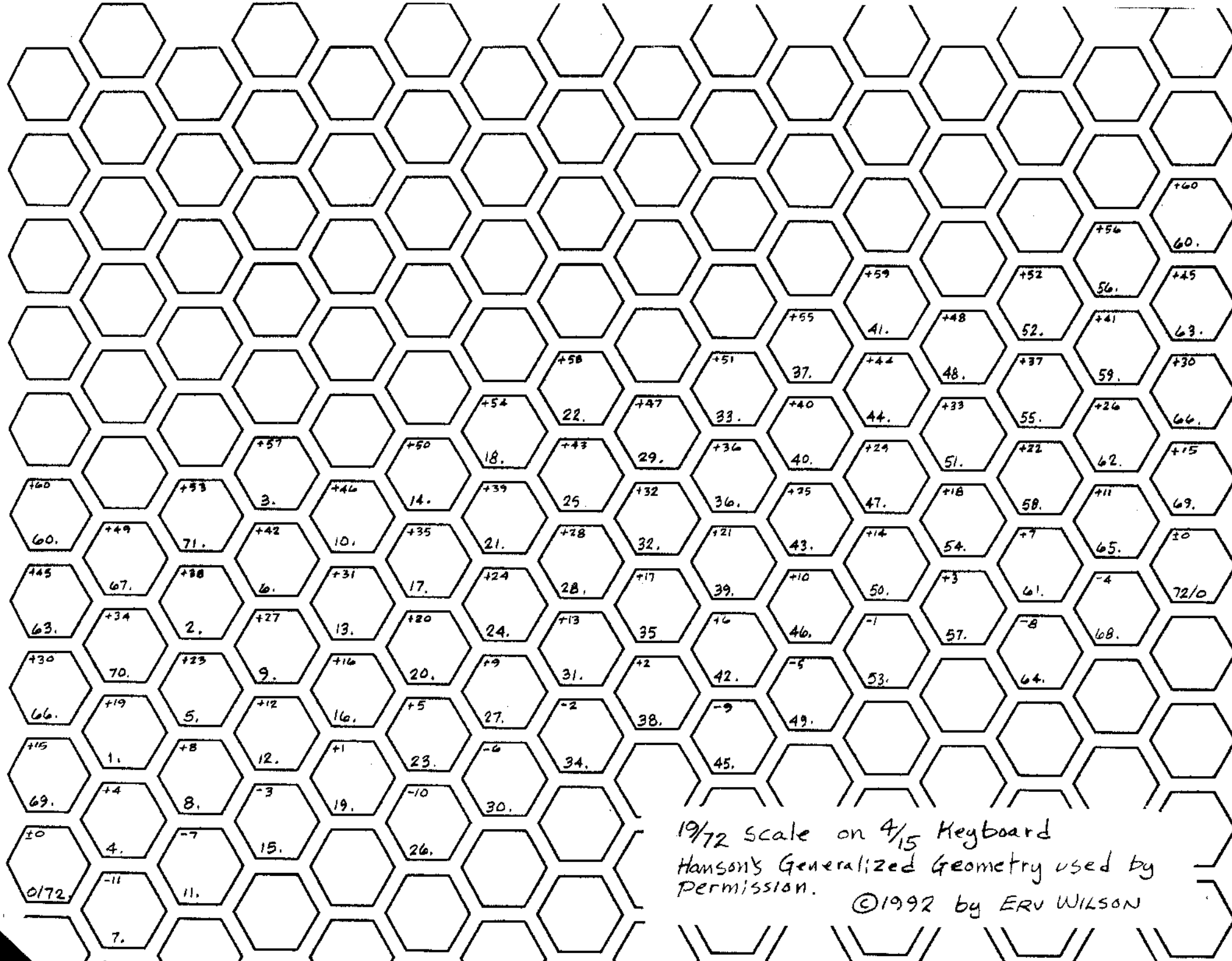
19/72 scale on 4/15 keyboard  
(Hanson Kbd geometry)  
used by permission  
© 1992 by Eric Wilson



72-Tone Hebdomekontauy Scale,  
on the Hanson (15-rank) Generalized  
Keyboard

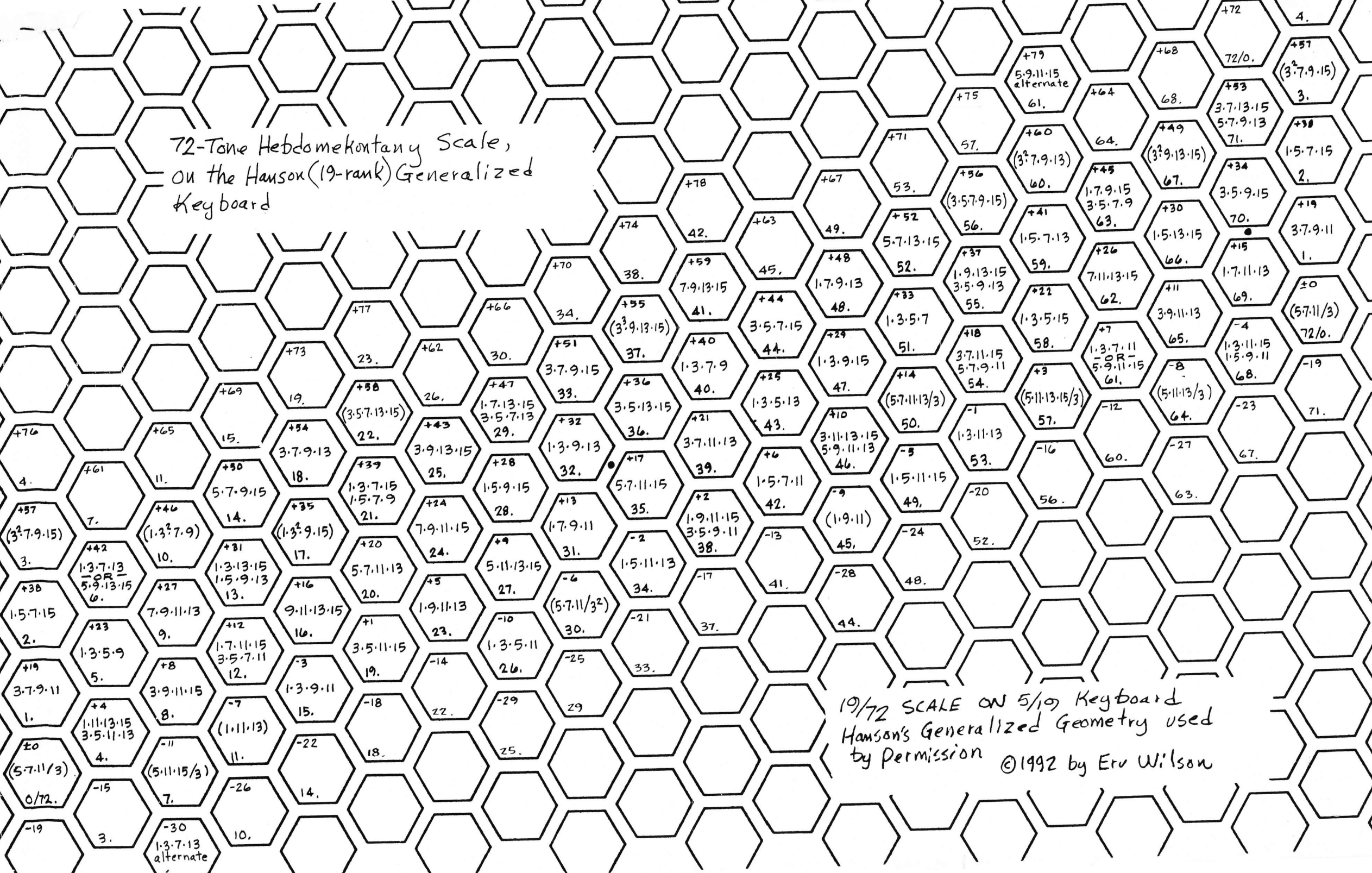


$19/72$  scale on  $4/15$  Keyboard  
Hanson's Generalized Geometry used by  
Permission. ©1992 by ERU WILSON



19/72 scale on 4/5 Keyboard  
 Hanson's Generalized Geometry used by  
 Permission. ©1992 by ERV WILSON

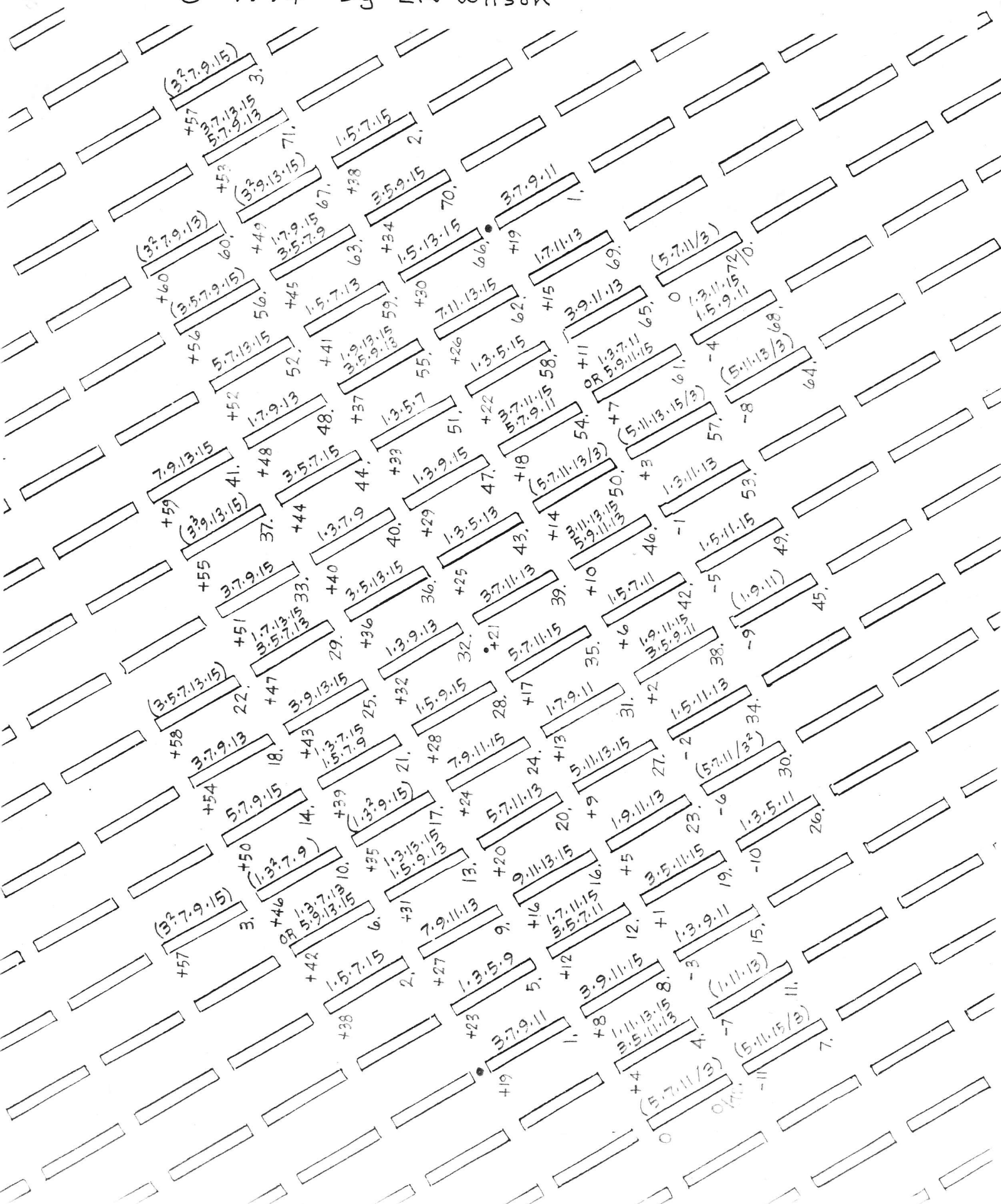
72-Tone Hebdomekontany Scale,  
 on the Hanson (19-rank) Generalized  
 Keyboard



19/72 SCALE ON 5/10 Keyboard  
 Hanson's Generalized Geometry used  
 by permission ©1992 by Eru Wilson



(4) 1.3.5.7.9.11.13.15 HEBDOMEKONTANY, in a 72-tone scale  
 Shown with Hanson Generalized Geometry, on Starr-switch grid  
 © 1994 by Erv Wilson



9.	1.7.11.13	5+	
10.	3.5.9.15	3+	
12.	(5.7.11/3)	0+	
1.	3.7.9.11	9+	
2.	1.5.7.15	3+	
4.	1.1.13.15, 3.5.11.13	4+	
5.	1.3.5.9	2+	
7.	(5.11.15/3)	=1	
8.	3.9.11.15	8+	
9.	7.9.11.13	2+	
11.	(1.11.13)	=1	
12.	1.7.11.15, 3.5.7.11	12+	
13.	1.3.13.15, 1.5.9.13	2+	
15.	1.3.9.11	5+	
16.	9.11.13.15	6+	
17.	(1.3.2.9.15)	3+	
19.	3.5.11.15	-+	
20.	5.7.11.13	2+	
21.	1.3.7.15, 1.5.7.9	3+	
23.	1.9.11.13	5+	
24.	7.9.11.15	2+	
26.	1.3.5.11	0+	
27.	5.11.13.15	9+	
28.	1.5.9.15	2+	
30.	(5.7.11/3 <sup>2</sup> )	6+	
31.	1.7.9.11	3+	
32.	1.3.9.13	3+	
34.	1.5.11.13	2+	
35.	5.7.11.15	7+	
36.	3.5.13.15	2+	
38.	1.9.11.15, 3.5.9.11	2+	
39.	3.7.11.13	2+	
40.	1.3.7.9	4+	
42.	1.5.7.11	6+	
43.	1.3.5.13	3+	
45.	(1.9.11)	9+	
46.	3.11.13.15, 5.9.11.13	0+	
47.	1.3.9.15	2+	
49.	1.5.11.15	5+	
50.	(5.7.11.13/3)	14+	
51.	1.3.5.7	3+	
53.	1.3.11.13	=1	
54.	3.7.11.15, 5.7.9.11	16+	
55.	1.9.13.15, 3.5.9.13	3+	
57.	(5.11.13.15/3)	3+	
58.	1.3.5.15	2+	
59.	1.5.7.13	4+	
61.	*1.3.7.11, 5.9.11.15	4+	
62.	7.11.13.15	2+	
64.	(5.11.13/3)	0+	
65.	3.9.11.13	11+	
66.	1.5.13.15	3+	
68.	1.3.11.15, 1.5.9.11	4+	
69.	1.7.11.13	15+	
70.	3.5.9.15	3+	
9.	3.7.13.15	5.7.9.13	53+
11.	(3.2.7.9.15)	57+	
13.	1.3.7.13	5.9.13.15*	42+
15.	(1.3.2.7.9)	46+	
17.	3.7.9.13	54+	
19.	(3.5.7.13.15)	58+	
21.	3.9.13.15	43+	
23.	7.9.13.15	59+	
25.	3.5.7.15	44+	
27.	1.7.9.13	48+	
29.	5.7.13.15	52+	
31.	3.7.9.13	56+	
33.	1.3.5.15	22+	
35.	1.5.7.13	41+	
37.	(3.2.7.9.13)	60↑	
39.	1.7.9.15, 3.5.7.9	45+	
41.	3.9.11.13	11+	
43.	1.5.13.15	3+	
45.	1.3.11.13	15+	
47.	3.5.9.15	34+	

(\* 2 pitches at one site requires special handling.)

72-Tone Tubulong,  
with Hebdomekantang  
on 19-34-19 Keyboard  
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59	1-7-11-13	5+
60	3-5-9-15	2+
61	(5-7-11/3)	0+
62	3-7-9-11	5+
63	C# 1-5-7-15	3+
64	1-11-13, 3-5-11-13	4+
65	1-3-5-9	3+
66	(5-11-15/3)	1+
67	1-3-7-13	5-9-13-15*
68	3-9-11-15	0+
69	7-9-11-13	2+
70	(1-11-13)	4+
71	1-7-11-15, 3-5-7-11	1+
72	1-3-13-15, 1-5-9-13	2+
73	1-3-9-11	5-7-9-15
74	9-11-13-15	6+
75	(1-3-9-15)	3+
76	3-5-11-15	3-7-9-13
77	E 5-7-11-13	2+
78	1-3-7-15, 1-5-7-9	3+
79	1-9-11-13	5+
80	7-9-11-15	2+
81	1-3-5-11	3-9-13-15
82	5-11-13-15	0+
83	1-5-9-15	2+
84	(5-7-11/4)	0+
85	1-7-9-11	5+
86	F# 1-3-9-13	3+
87	1-5-11-13	3-7-9-15
88	5-7-11-15	4+
89	3-5-13-15	2+
90	G 1-9-11-15, 3-5-9-11	3-9-13-15
91	3-7-11-13	2+
92	1-3-7-9	4+
93	1-5-7-11	6+
94	7-9-13-15	3+
95	(1-9-11)	3+
96	G# 3-5-7-15	4+
97	3-11-13-15, 5-9-11-13	0+
98	1-3-9-15	3+
99	1-5-11-15	5+
100	A (5-7-11-13/3)	4+
101	1-3-5-7	3+
102	1-3-11-13	5+
103	3-7-11-15, 5-7-9-11	0+
104	1-9-13-15, 3-5-9-13	2+
105	(5-11-13-15/3)	3+
106	A# (3-5-7-9-15)	5+
107	1-3-5-15	2+
108	1-5-7-13	4+
109	* 1-3-7-11, 5-9-11-15	4+
110	(3-7-9-13)	13-7-11-6+
111	B 7-11-13-15	2+
112	(5-11-13/3)	0+
113	1-7-9-15, 3-5-7-9	4+
114	3-9-11-13	2+
115	1-5-13-15	3+
116	C 1-3-11-15, 1-5-9-11	4+
117	3-9-13-15	4+
118	1-7-11-13	5+
119	3-5-9-15	2+
120	2-6-4	2+

(\* 2 pitches at one site requires special handling.)

→ 5-9-13-15 optional, but homogeneity is lost.

optional, but homogeneity is lost

To Kraig Grady  
April 27, 1992 from  
Erv Wilson

72-Tone Tubulog,  
with Hebdomekontany  
on 19-34-19 Keyboard  
© 1992 by Erv Wilson



02	1.7.11.13	5+
	3.5.9.15	3+
	(5.7.11/3)	0+
	3.7.9.11	5+
	1.5.7.15	3+
+	1.1.13.15, 3.5.11.13	4+
	(3.7.9.15)	5+
	1.3.5.9	5+
→	(5.11.15/3)	1+
	1.3.7.13	5.9.13.15
	3.9.11.15	0+
	7.9.11.13	2+
	(1.11.13)	4+
	1.7.11.15, 3.5.7.11	5+
	1.3.13.15, 1.5.9.13	5+
5	1.3.9.11	5.7.9.15
	3.11.13.15	6+
	(1.3.9.15)	5+
9	3.5.11.15	-+
	5.7.11.13	8+
	1.3.7.15, 1.5.7.9	3+
	7.9.11.15	8+
	(3.5.7.13.15)	5+
21	1.3.5.11	5.11.13.15
	5.11.13.15	0+
	1.5.9.15	3+
	1.7.13.15, 3.5.7.13	4+
	1.7.9.11	5+
	1.3.9.13	3+
	3.7.9.15	5+
	5.7.11.15	3+
	3.5.13.15	2+
38	1.9.11.15, 3.5.9.11	5.9.13.15
	3.7.11.13	5+
	1.3.7.9	8+
42	1.5.7.11	6+
	7.9.13.15	5+
43	1.3.5.13	5+
45	(1.9.11)	5.7.15
	3.11.13.15, 5.9.11.13	8+
	1.3.9.15	3+
49	1.5.1.15	1.7.9.13
	(5.7.11.13/3)	4+
	1.3.5.7	3+
53	1.3.11.13	5.7.13.15
	3.7.11.15, 5.7.9.11	8+
	1.9.13.15, 3.5.9.13	3+
57	(5.11.13.15/3)	5+
	1.3.5.15	2+
	1.5.7.13	4+
61	* 1.3.7.11, 5.9.11.15	(3.7.9.13)
	7.11.13.15	6+
64	(5.11.13/3)	1.7.9.15, 3.5.7.9
	3.9.11.13	1+
	7.5.13.15	8+
68	1.3.11.15, 1.5.9.11	3.9.13.15
	1.7.11.13	5+
	3.5.9.15	3+

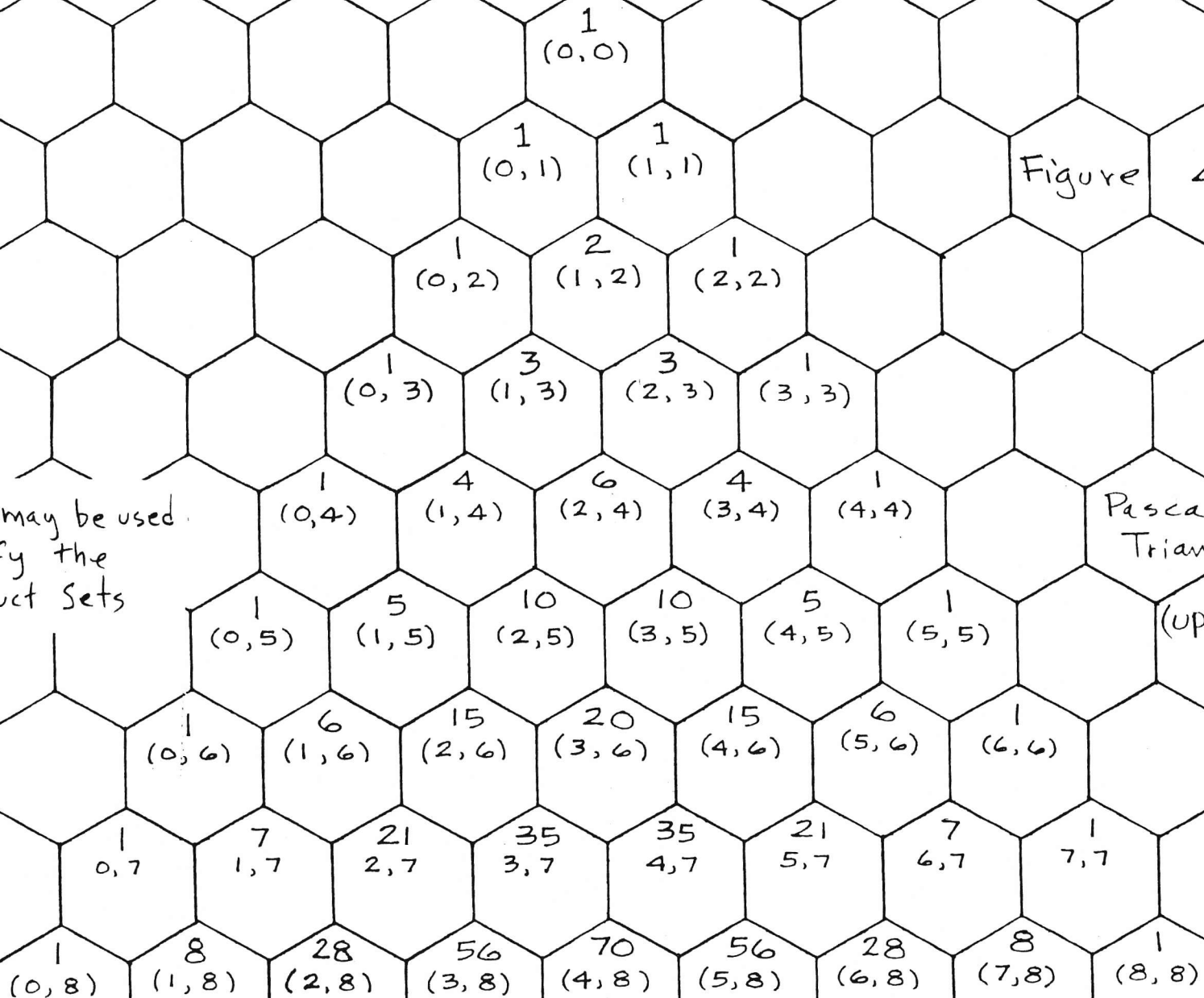
(\* 2 pitches at one site requires special handling.)

72-Tone Tubulung,  
with Hebdomekontany  
on 19-34-19 Keyboard  
©1992 by Erv Wilson

Figure 4

Pascal's Triangle may be used  
to codify the  
Combination-Product Sets

Pascal's  
Triangle  
(upright)



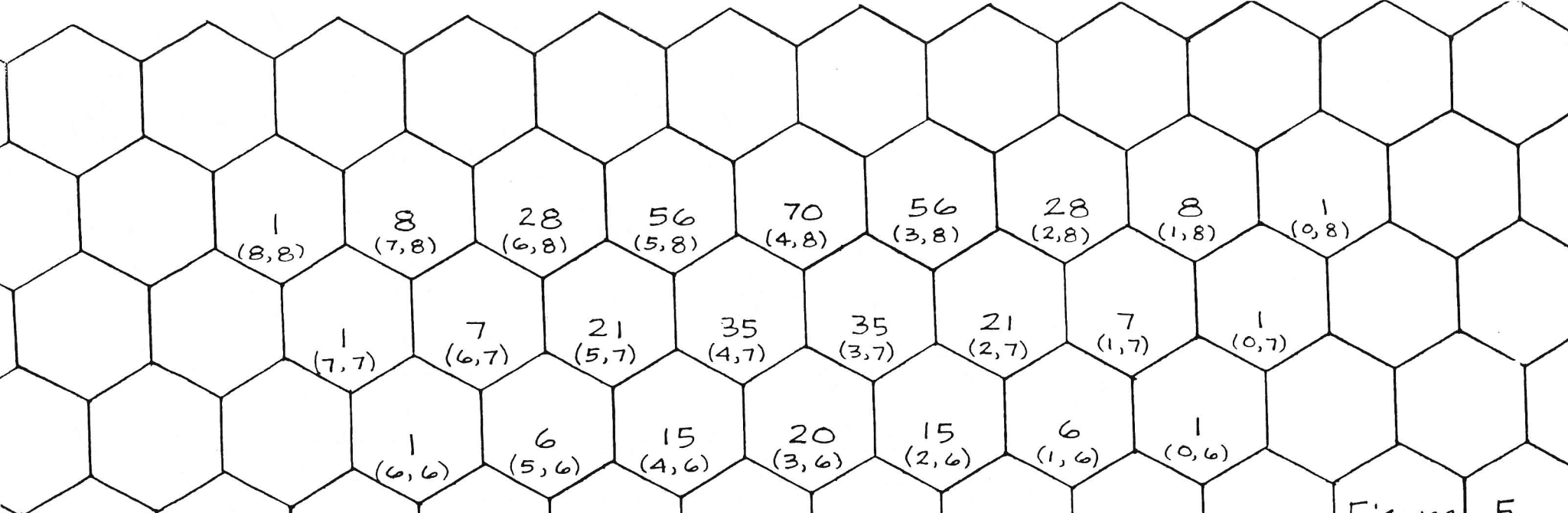
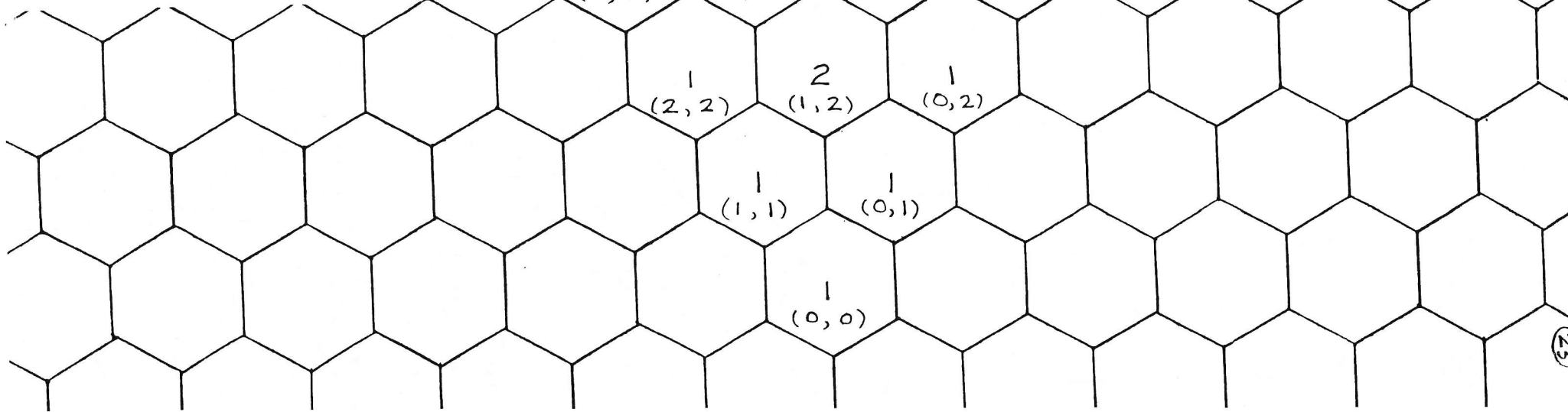


Figure 5

If the nadir of this triangle is juxtaposed over any particular set of the upright triangle, the partitioned cross-sets of that set may be identified.

see Fig. 5, 6, & 7

(Rotated 180°)  
Pascal's Triangle



(0 0)  
70  
(4 8)

← 1 variation

(0 1)  
35  
(4 7)

(1 1)  
35  
(3 7)

← 2 variations

(0 2)  
15  
(4 6)

2  
(1 2)  
20  
(3 6)

1  
(2 2)  
15  
(2 6)

← 28 variations

(0 3)  
5  
(4 5)

3  
(1 3)  
10  
(3 5)

3  
(2 3)  
10  
(2 5)

1  
(3 3)  
5  
(1 5)

← 56 variations

(0 4)  
4  
(4 4)

4  
(1 4)  
4  
(3 4)

6  
(2 4)  
6  
(2 4)

4  
(3 4)  
4  
(1 4)

1  
(4 4)  
1  
(0 4)

← 70 variat.

5  
(1 5)  
3  
(3 3)

10  
(2 5)  
3  
(2 3)

10  
(3 5)  
3  
(1 3)

5  
(4 5)  
1  
(0 3)

← 56 variations

15  
(2 6)  
1  
(2 2)

20  
(3 6)  
2  
(1 2)

15  
(4 6)  
1  
(0 2)

← 28 var

1  
(0, 0)  
20  
(3, 6)

← 1 var.

1  
(0, 0)  
6  
(2, 4)

← 1 var.

35  
(3 7)  
1  
(1 1)

35  
(4 7)  
1  
(0 1)

← 8 var

1  
(0, 1)  
10  
(3, 5)

1  
(1, 1)  
10  
(2, 5)

← 6 var.

1  
(0 1)  
3  
(2 3)

1  
(1 1)  
3  
(1 3)

← 4 var.

70  
(4 8)  
1  
(0 0)

← 1 var.

1  
(0, 2)  
4  
(3, 4)

2  
(1, 2)  
6  
(2, 4)

1  
(2, 2)  
4  
(1, 4)

15 var.

1  
(0 2)  
1  
(2 2)

2  
(1 2)  
2  
(1 2)

1  
(2 2)  
1  
(0 2)

← 6 var

Cross-sets  
of Hebdony

1  
(0, 3)  
1  
(3, 3)

3  
(1, 3)  
3  
(2, 3)

3  
(2, 3)  
3  
(1, 3)

1  
(3, 3)  
20  
(0, 3)

1 var

3  
(1 3)  
1  
(1 1)

3  
(2 3)  
1  
(0 1)

← 4 var

Mekontany

Fig. 7

4  
(1, 4)  
1  
(2, 2)

6  
(2, 4)  
2  
(1, 2)

4  
(3, 4)  
1  
(0, 2)

15 var

Fig. 5

6  
(2 4)  
1  
(0 0)

← 1 var

Fig. 6

10  
(2, 5)  
1  
(1, 1)

10  
(3, 5)  
1  
(0, 1)

← 6 var.

Cross-sets of  
Hexany

20  
(3, 6)  
1  
(0, 0)

← 1 var.

Cross-sets of Fikosany