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Acoustic methods of work

In Memoriam

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3rd February, 1860 – 5th March, 1923

Chief-administrator for
The Amsterdam Deli Companie, Sumatra 1896—99

Translation from the Danish by *Marie Baruël.*

Published by the author.
Copies are obtainable, post free, on application in writing to the author, Th. Kornerup, Holger Danskesvej 82, Copenhagen F., Denmark, either by Post Office Order for 4 Danish Kroner for each copy, or against cash on delivery.

ACOUSTIC METHODS OF WORK

IN RELATION TO

SYSTEMATIC COMPARATIVE MUSICOLOGY

INCLUDING SOME ACOUSTIC TABLES

BY

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COPENHAGEN F.

COPENHAGEN MCMXXXIV
PRINTED BY J. JØRGENSEN & CO.

PREFACE

The aim of this treatise is:

1. To propose a common international **terminology** of music; thus, for the white keys of the piano the names

c	d	e	f	g	a	h
bis		fes	eis			ces

and for the black keys

cis	dis	....	fis	gis	ais	....
des	es	....	ges	as	bes	....

and for double ♯♯ and ♭♭ the terminations isis and eses; for syntonic »Comma-tones« the signs »+« and »—« placed after the name of the tone to indicate increase or diminution of a Comma $= \frac{81}{80}$; as, for instance, in Indian music: a = 884 cents and a + = 906 cents, or 22 cents more.

Only Ellis's 1200 cents are used (never 1000 millioctaves, or 100 Octav-Zentimeter oz, or 600 centitones), as the number 1200 can be divided evenly by 3, 4×4 and 5×5 and their multiples (as 720 particles can be divided on a string by 3×3 and 4×4 and 5).

Below are given some further proposals concerning terminology; for instance, the expression »triple chord« is used for »triad« in opposition to the »Greek Triad« (3 neighbour-tones: des —, des and d +), and »quadruple chord« for »chord of the Seventh«; »Tonic« or starting-point for »key-note«; »Phrygian-Doric« for the Aeolian scale-type, and »Lydian-Phrygian« for the Ionic or Iastic scale-type, according to the succession of Tetrachords, Scheme 17; in other words, a difference is made between »secondary partition in particles« in the formula X, and »tertiary division« in the formula Y; further: quartary and quintary instead quaternary, etc.

The Greek spelling of names is also used, e. g. Arkytas, Plutarchos, Ptolemaios etc., and the spelling of the terms: Trichord, Tetrachord, Tritonos, Medium represents international spelling; and capital letters for »the tone Octave«, but small letters for »the interval octave«.

2. To draw attention to **3 principles** which have lately been set forth concerning systematic comparative Musicology:

a) that in the development of tone-systems from the earliest times to the present day there has, without a doubt, been a much greater **stability** than has hitherto been supposed, viz. Professor v. Hornbostel's demonstration of the stability in the length of the flute (and the gold measure) from 2000 up to 3000 B. C. till far down through the ages (Note 1). This stability we here transfer to the proportion between the **pitch** of the tones;

b) that in mentioning the ancient tone-systems it is only possible within very narrow limits to rely upon abstract Fifth-steps, but that one can rely in the highest degree on the concrete **shortening of a string** (secondary »partition into particles« formula X) cf. v. Hornbostel's remark: »One of the fundamentals of the Indian tone-system, the partition of a string, is evidently primitive« (Note 2); and v. Hornbostel's and R. Lachmann's observation in 1933: »Breloer sought to deduce Bharata's (Indian) system solely from this principle (the pure Fifths); he arrived at the 22 Sruti through a chain of pure Fifths. But it is not clear why the advance of the Fifths should stop short at 22 Sruti.« (Note 3).

c) that the principle of **Relativity** can be recognized also in relation to the tone-intervals, steps, cf. v. Hornbostel's declaration: »The investigations of recent years have .... led to surprising consequences. As is known, melodies can be displaced in their pitch as optical figures in space, without changing their shape; they may also be enlarged or diminished as figures in space without alteration of their shape if only the **relative** interval-distances are kept. In this way the structure-building intervals, as the Fifth and the Octave, also keep their function in the new formation.« (Note 4).

Indeed these 3 principles can also operatein a wider sense, as the above-mentioned authors also point out: »Thus the comparison of musical styles as well as the comparison of other musical functions — tones and instruments — might prove a valuable aid in the investigation of the history of civilization«. (Note 4).

For example: In his investigation of the length of the flute (and of the gold-measure) Professor v. Hornbostel has built up a hypothesis of an ancient **culture-stream** from China across the Pacific Ocean to Central America.

3. To add to these 3 principles the under-mentioned system of acoustic **methods of work:**

Chapter I: 5 kinds of exact interval-calculation, and:

Chapter II: 7 hypothetical principal-rules for the origin (genesis) and structure of tone-systems.

4. To make plain the advantage of having an objective means of **valuation** for ancient as well as modern or future tone-systems, which, mathematically speaking, can only occur by means of the **authoritative** golden system formed by the golden cut (super division) of the octave according to the formula: $\frac{1}{\omega} = \frac{\omega}{1-\omega}$ or

Formula:	$\omega^2 + \omega = 1$
in decimals:	$0{,}381.966 + 0{,}618.034 = 1$

or $\frac{\sqrt{5}-1}{2} = \omega$ (Omega),

Luca Pacioli's »**Divina** proportione«, Kepler's »sectio divina« and »**gemma**« i. e. precious stone (Note 13), the everlasting fundamental formula for universal power of **adaptation**, — here the formula for universal **sense** of harmony.

CHAPTER I

FIVE KINDS OF EXACT INTERVAL-CALCULATION.

Scheme I shows five methods of calculation of the intervals:

Group A. Quantitative string-partitions (Nos. 1-2), the musical faculty of vision (musikalisches Sehen).

1. **Primary** (Flageolet)-string-partition, i. e., **Partial-** or Flageolet-tones are created by partition of a string (or flute) into several equal parts, which **all** vibrate at the same time, e. g. Natural triple chord c e g c', the Partial-tones Nos. 4, 5, 6 and 8.

The difference in the first grade between the cents of the adjacent Partial-tones approximates to the golden tones (Chapter IV).

The difference in the second grade approximates to the golden Molecules:

Partial-tone	Cents
No. 3 g	701.955
— 4 c'	0.
— 5 e'	386.314
— 6 g'	701.955
— 7 ais'	968.825

1st grade	Golden tones:
498.05	503.79 f
386.31	384,86 e
315.64	311.36 es
266.87	265.93 dis

2nd grade	Golden molecules:
111.7	118.9 des
70,7	73.5 cis
48.8	45.4 deses

2. **Secondary** (ornamental, decorative) string-partition: **only part** of the string vibrates; we divide the string into 720 particles $= 3 \times 3 \times 4 \times 4 \times 5$.

The tone-fractions are indicated in the following »stable formula X«:

$$\left\{\frac{\text{the whole string as the common numerator}}{\text{the vibrating part as the denominator}}\right\} \textbf{ never} \text{ inverted.}$$

Plutarchos' median point, the secondarily halved octave, is the Fourth, the **M**ese, Indian **M**ahijama: $\frac{720}{540} = \frac{4}{3}$ in formula X: $\frac{4}{4:3.2.}$ with the common **numerator** 4.

The octave secondarily 3-parted is the small Third es $= \frac{720}{600} = \frac{6}{5}$ and the Fifth $g = \frac{720}{480} = \frac{3}{2}$, Plutarchos' **P**aramese, Indian **P**ancama, (»para« i. e. »beyond« the Mese) in formula X: $\frac{6}{6.\ 5.\ 4.\ 3.}$ with the common numerator 6.

Tautological: the Fifth secondarily halved is the tone es: $\frac{6}{6.\ 5.\ 4.}$

Group B. Qualitative divisions, (Nos. 3-5).

3. Tertiary (arithmetic) division of the **decimals** of the tone-fractions (as decimal-fractions), or division of the difference between the vibration-numbers, also indicated in the following »stable formula Y«.

The octave 2,0 tertiarily halved is just the Fifth $\frac{3}{2} = 1{,}5$ with the common **denominator** 1, or in formula Y: $\frac{2.\ 3.\ 4.}{2}$ with the common denominator 2, see **Scheme 19.**

The Fifth = 1,50 tertiarily halved is e, either 1,25 (0,50 halved) or in Y: $\frac{4.\ 5.\ 6.}{4}$ with the common denominator 4.

The number of vibrations (by Dr. Illo Peters called **objective** tone pitch) increases proportionately to the given decimal-fractions.

The difference between the decimal-fractions (or vibration-numbers) of 2 **adjacent** Partial-tones is constant 0,25; $\frac{3}{2}$ minus $\frac{5}{4} = \frac{1}{4}$; or g' minus e' = C, compare Rule 5, d.

4. Quartary (geometrical, neutral) logarithm-division, or division (and transposing) of tone-**intervals,** steps; **subjective** tone pitch.

The **product** of 2 tone-fractions is consistent with the **sum** of their logarithms; for example: 2 Fifths-steps $\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$, agree with

$$\begin{cases} 2 \times 176.0913 = 352.1826 \text{ as logarithm,} \\ 2 \times 701.9550 = 1.403.9100 \text{ as cents, minus } 1200 = 203.91 = d+ \end{cases}$$

Professor Joseph Yasser, New York, remarks »that the New York Public Library possesses a very rare and original manuscript: The Geometrical Scale in Musick, or Gam-Ut reduced to **Geometrical** proportions, and according to the statement of its author (whose name Gaudy is merely guessed from indirect indication), was written some time before the year 1705« (Note 5), — he was thus **the pioneer** before the Swiss mathematician Leonhardt Euler (1707—83), who in the year 1729 **limited** the common logarithms of tone-fractions **to one octave** (2) by dividing them by the logarithm of 2 = 0.30103.

In order to facilitate the transition to Ellis' cents as an international method of counting, an easy and practical method of logarithm-reckoning was introduced by me in 1929 by using **the Constant K,** i. e. (**Scheme 22**):

$$\begin{array}{rr} \text{Log. } 1{,}2 = \ldots\ldots\ldots\ldots\ldots & 0.079.1812 \\ \text{minus log. of (log. } 2 = 0.30103) = \text{minus} & \underline{0.478.6098 + 1} \\ \text{Difference K} = & 0.600.5714 \end{array}$$

Thus an addition and a subtraction are merged into one single addition = K.

5. Quintary structure is used in connection with the **stretching** of a string, see Scheme 1 above, $(\sqrt{2})^2 = 2$ (or, the octave: $2^2 = 4$).

By pressing the string down on the finger-board (e. g. on a violoncello) the string will be somewhat tightened, and, accordingly, the pitch of the tone will also be somewhat raised (quintarily).

Resumé: a) The methods of work set forth in Nos. 2, 3 and 4 will be recognised in the Greek **splitting of fractions,** formula Z, e. g., the Fourth divided by three: $\frac{4}{3}=\frac{12}{9}=\frac{12}{11}\times\frac{11}{10}\times\frac{10}{9}$ of which:

Form.			Cents
X	1st fraction	$\frac{12}{11}$ is **secondary** 3-partition	150.6
		common numerator 12.	
.....	2nd fraction	$\frac{11}{10}$ equals in cents:	165.0
		only little below **quartary** 3-division $=\frac{498}{3}=$	166.0
Y	3th fraction	$\frac{10}{9}$ is **tertiary** 3-division	182.4
		common denominator 9.	

The quartary cents are but little below the average number of the secondary and tertiary cents, 166.5 cents.

Should the Fourth be divided by 4, the result will be:

Formula Z.	$\frac{4}{3}=\frac{16}{12}=$	$\frac{16}{15}\times\frac{15}{14}\times\frac{14}{13}\times\frac{13}{12}$			of which:
resp. Formula................		X	$\underbrace{\dots\dots\dots}$	Y	
Cents		111.7	124.5	138.5	

1st fraction $\frac{16}{15}$ is secondary 111.7 cents, common numerator 16, X,
4th — $\frac{13}{12}$ - tertiary 138,5 — , — denominator 12, Y.

Whereas the **average** number between $\frac{15}{14}$ and $\frac{14}{13}$ comes close to the quartary cents 124.5.

b) **Twofold symmetrical calculations.** Pythagoras may have established »his small Third« es $-=\frac{32}{27}=294.1$ cents by means of

secondary 8. partition	$\frac{32}{32 \quad 27 \quad 24}$	our X.
Spaces 8	$= 5 + 3$	
tertiary 9. division	$\frac{27 \quad 32 \quad 36}{27}$	our Y.
Spaces 9	$= 5 + 4$	

Or by means of particles and decimals respectively:

secondarily	c	es —	f	
Our particles.........	720	$607\frac{1}{2}$	540	
Spaces		$112\frac{1}{2} + 67\frac{1}{2}$		= 180
abbreviated		5 + 3		= 8
tertiarily.............	c	es —	f	
Decimal-fractions	1,0	1,185	1,333	
Decimals		0,185 + 0,148		= 0.333
abbreviated		5 + 4		= 9

c) The »**Natural quadruple chord**«:

Partial-tones Nos. 4—8	c		e		g		[ais]		c'	
Primarily: Fractions	1		$\frac{5}{4}$		$\frac{3}{2}$		$\frac{7}{4}$		2	
Secondarily: our particles,	720		576		480		$411\frac{3}{7}$		360	
quickly decreasing spaces		144	+	96	+	$68\frac{4}{7}$	+	$51\frac{3}{7}$		= 360
Tertiarily: numerator in Y	4		5		6		7		8	Nos. 4—8
Quartarily: our cents,	0		386		702		969		1200	
slowly decreasing intervals		386	+	316	+	267	+	231		= 1200
Quintarily; stretching,										
2nd power of No.	16		25		36		49		64	
Increasing tension		9		11		13		15		= 48
Arithmetical progression, Andr. Kornerup's observation		 2	+	2	+	2				= 6

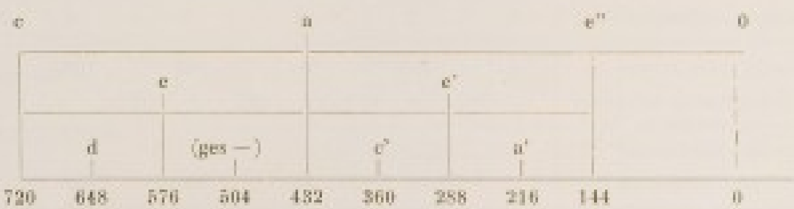
CHAPTER II

SEVEN HYPOTHETICAL PRINCIPAL RULES.

The following rules are proposed in the hope that a cooperation between philologic-historic inquiry and instrument-investigation on the one hand **and** theoretic-**acoustic** calculation on the other will lead to a clearer understanding of the **origin** and development of the ancient tone-systems than has hitherto been the case. Naturally, these rules are partly hypothetic, and are only advanced as a basis for discussion.

Group A. Presumable Construction of Scala-types and Tonics, (Rules Nos. 1—3).

Principal Rule No. 1. The stepwise progression of a scale-type is formed by secondary partition (on the string). 5-partition of the whole string gives thus (by three successive halvings) »the **ornamental** pentatonic scale«: c $d=\frac{10}{9}$ e (ges —) $=\frac{10}{7}$ a c':



Secondary 20-partition of the whole string, then partly secondary 5-partition of the Fourth, gives also 5 **ornamental Tetrachords** in oriental tuning:

1	20/19	10/9	20/17	5/4	4/3	10/7		
c	des —	. . .	es —	. . .	f	. . .	Doric	Primitive Greek before Pythagoras?
c	. . .	d	es —	. . .	f	. . .	Phrygian	
c	. . .	d	. . .	e	f	. . .	Lydian	
c	. . .	d	. . .	e	. . .	(ges —)	Tritonos	
c	des —	....		e	f	. . .	Harmonic, oriental.	

for instance: the Greek »scale-types«, ... Scheme 17, Nos. 1—7
Jewish Hedjaz-Kar, double harmonic.... — —, — 8,
— Ahavad-Rabbah, harmonic-Doric. — —, — 12,
according to some indications of A. Z. Idelsohn (Note 6).

Principal Rule No. 2. The oriental sequence of Tonics (key notes) can be formed by **tertiary** division. Three consecutive halvings of the Octave, Fifth and Third give thus the presumed primitive Greek (before Pythagoras) as well as the Indian fundamental **Tonic**-sequence: c $\left| \frac{9}{8} = \text{d}+ \right|$ e g c', (only Partial-tones):

	1	$^9/_8$	$^5/_4$	$^3/_2$	$^{27}/_{16}$	$^{15}/_8$	2	$^9/_4$
1st halving	c			g	....		c'	
2nd halving	c		e	g		b		d +
3th halving	c	d +	e	g	a +	b		d +

The result, the **oriental (Indian) Tonic**-sequence:

Roman numerals:	I	II	III.IV	V	VI	VII. I.	
Oriental Tonics.	c	d +	e. f.	g	a +	b. c'	
Indian Sruti-No.	0	4	7.9	13	17	20. 22	
Scale-Structure:	4	3	2	4	4	3	2	Ma-Grama
Scheme 2, J:		9		4		9.		= 22

In any case it agrees, mathematically, exactly with what Victor Mabillon (cf. E. v. Hornbostel and R. Lachmann's **report**, Note 7) has reported (without mentioning the source) as an **old** Indian prescription for the partition of an F string, so that seven tones can be made in **the middle third part** of the string corresponding to the number of particles on a whole string as given below (transposed from an F string to a C string by multiplying by $\frac{3}{2}$):

F. 480 particles	$426^2/_3$ ..	384 ..	360 ..	.. 320 ..	$284^4/_9$ ..	256 ..	240
C. 720 transposed	640 ..	576 ..	540 ..	.. 480 ..	$426^2/_3$ ..	384 ..	360
C. Ma-Grama, Major:	d +	e	f	g	a —	b	c'

F. Particles	288	270	240
C. transposed	432	405	360
C. Sa-Grama, Lydian-Phrygian:	a	bes —	c
Fractions:	$^5/_3$	$^{16}/_9$	2

The difference between Ma and Sa has reference to the structure of the two Indian scale-types respectively, see Rule 4.

Principal Rule No. 3. The Pythagorean and Persian Tonic-sequence is naturally formed quartarily, 6 Fifth-steps (Q Fifth, q Fourth):

	0		2Q		4Q		1q	…	1Q		3Q		5Q		—
Pythagorean Lydian scale..................	c		d +		e +		f.	....	g		a +		b +		c'
23 Pythagorean Instrument-tones: Structure		4		4		1		5		4		4		1	
19 — Song-tones —		3		4		1		3		3		4		1	
Compare: 17 Persian Sruti —		3		3		1		3		3		3		1	

Group B. Probable Construction of Ancient Tone-Material (Rules Nos. 4–7).

Principal Rule No. 4. Permanent and variable tones.

a) The transposing of secondary (ornamental, decorative) scales on the Tonics (key notes) mentioned may give several tone-systems with not a few **common permanent** tones, for instance the presumed 13 oriental permanent tones, and the 17 Persian tones with the exception of d, e, a and b:

Oriental Tonic No.	I		II		III		IV		ges —
Name............	c	des —	d +	es —	e	e +	f	ges 2 —	...
Oriental Tonic No.	V		VI		VII		I		
Name............	g	as —	a +	bes —	b	b +	c'	Scheme 2.	

The tone »ges 2 —« almost fis +.

The **13** Persian tones (with the exception of d, e, a and b) are presumably the **common** foundation for **primitive** Greek, Persian, Indian and the later Arabian tone-**material**. Continued transposition on other Tonics or with other scale-types gives **new** tones, varying both for the different tone-systems and within the same system, called »**variable** tones of the system«. The following are cited as examples:

13 + 4 = 17 Persian Sruti Nos., of which 13 are permanent, Scheme 2, P.
13 + 9 = 22 Indian Sruti Nos., of which 17 are permanent, Scheme 2, J.
22 + 2 = 24 medieval Arabian tones.

The 4 variable tones in Persian music are presumably the Persian Sruti Nos. 2, 5, 12 and 15; the 5 variable tones in Indian music: the Indian Nos. 1, 6, 10, 14 and 19.

The structure of the Tetrachords:

	C. Lydian				C. Phrygian					
Indian	Ma: g	a +	b	c'	Sa:	g	a	bes–	c'	
Intervals ...	.....	204	182	112	...	....	182	112	204	= 498
Sruti	.....	4	3	2	...	....	3	2	4	= 9

b) **The idea** in itself of a few permanent Sruti-distances as a structure cannot be thought of »as rising through a simple **advance**« of a Second only, i. e.

	c		d+		e		f		g	
Indian Nos	0		4		7		9		13	
Ma, Lydian....		4	+	3	+	2	......		......	= 9
Sa, Phrygian...	———	———	——→	3	+	2	+	4		= 9
Persian Nos. ..	3	4		6	7		10		13	
Doric.........				e+	f		g		a+	= 7
Transposition..	d+	es—	......	......	f		g			= 7

but **tertiary** 9-division of the Fourth gives also the Indian Sruti-numbers for the upper Phrygian Tetrachord in Sa, Lydian-Phrygian.

Common denominator 48:	d+ $^{9}/_{8}$		e $^{5}/_{4}$		f $^{4}/_{3}$		g $^{3}/_{2}$	
Numerators	54		60		64		72	.....
Distances	..	6	+	4	+	8		= 18
abbreviated, Indian Sa	..	3	+	2	+	4		= 9

corresponding to the tertiary **9**-division.

c) The number of **touches**: Scheme 2 P, in all $7 \times 7 = 49$, and Scheme 2 J, in all $5 \times 7 \times 2 = 70$ disperse naturally very unevenly (the columns in the middle).

The number of **Molecules** (intervals of equal value) is, according to all Sruti:

in Scheme 2, Persian 12 × 4 Commas 5 × 1 —		Total 53 Commas	In Scheme 2, Indian 7 × 4 Commas 5 × 3 — 10 × 1 —	
....	2 Molecules		....	3 Molecules
Intervals 17	....		Intervals 22	....

This condition can naturally be changed by continued transposing. The expression »Molecule« for equal intervals between tones in stepwise progression was suggested in 1923 by Professor Chr. Kroman, (1846—1925), Copenhagen.

Principal Rule No. 5. Chromatic and Enharmonic.

a) In the practice of Music even Pythagoras has not been able to carry through the »consequent« Pythagorean Fifth-system, but he has had to content himself with the 7 fundamental diatonic **Tonics**, and has presumably formed the intervening chromatic and enharmonic tones by **secondary interpolation**, for instance, the above mentioned chromatic des —, es —, etc. by secondary halving of the small Second c..d, or d+..e, whereby with great approximation the same interval is obtained from des — to d + = 113.7 cents, and from e to f = 111.7 cents — inaudible difference (Formula X):

1) c .. d $\left\{\begin{array}{ccc} & 20 & \\ \hline 20 & 19 & 18 \end{array}\right\}$	des— $= \frac{20}{19} = 88.8$ cents	Inaudible difference 1.4 cents, nearly **k**, Scheme 14.
2) Pythagorean: 5 Fifths below	$=$ des— $= \frac{256}{243} = 90.2$ »	
3) c .. d + $\left\{\begin{array}{ccc} & 18 & \\ \hline 18 & 17 & 16 \end{array}\right\}$	$= \left(\text{des}\frac{1}{3}—\right) = \frac{18}{17} = 98.9$; c .. es 3-parted	
4) syntonic, difference, Scheme 24, between the Partial-tones Nos. 16 and 15..............	$=$ des $= \frac{16}{15} = 111.7$ ct.	Inaudible **Scisma**-difference: 2.0 cents.
5) Pythagorean: 7 Fifths up.................	$=$ cis 2 + $= 113.7$—	
6) Golden des $= 5 \times 503.8$ cents	$=$ des $= 118.9$ — »c .. d« super divided.	
7) c .. e $\left\{\begin{array}{cccc} & 15 & & \\ \hline 15 & 14 & 13 & 12 \end{array}\right\}$	$=$ (des) $= \frac{15}{14} = 119.5$, c .. e 3-parted.	

The Greeks Eratosthenes and Plutarchos used the secondary fraction 20/19; Arkytas, Didymos and Ptolemaios 16/15; Plutarchos also 15/14 in Kroma **malakon** (soft, i. e. diminished Kroma).

When construing tone-systems one can for convenience overlook the small inaudible differences and, on paper, calculate by theoretical fractions. The Greeks themselves have to a large extent made use of **the splitting of fractions**, Formula Z, i. e., $d = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$, of which $\left\{\begin{array}{l}\text{the first is secondary halving} \\ \text{and the second a tertiary one.}\end{array}\right.$

By this **primitive** method of work, splitting of fractions, the legitimate tone systems have been far outstripped, by continual partition, i. e., secondary interpolation; thus Eratosthenes and Plutarchos halve des— $= \frac{20}{19} = \frac{40}{38} = \frac{40}{39} \times \frac{39}{38}$, of which the first is a secondary **enharmonic** deses $= \frac{40}{39} = 44$ cents, while Didymos halves the syntonic des $= \frac{16}{15} = \frac{32}{30} = \frac{32}{31} \times \frac{31}{30}$, of which $\frac{32}{31} = 55$ cents also is the secondary enharmonic deses, just 11 cents larger.

Curiously enough Arkytas makes use of the fraction $\frac{28}{27} = 62.9$ cents, very nearly the Molecule in the **19**-toned temperament 63.16 cents, just the Fourth **tertiarily** 9-divided (Formula Y):

$$\begin{array}{cccc} \text{c} & \text{des}— & \text{es}— & \text{f} \\ 27 & 28 & 32 & 36 \\ \hline \multicolumn{4}{c}{27} \end{array}$$

with $\frac{28}{27} =$ des— $= 62.9$ cents and $\frac{32}{27} = 294.1$ cents, see Chapter I, Resumé, b.

Plutarchos has a similar fraction $\frac{30}{29} = 58.7$ only a little lower, and (des) $= \frac{15}{14} = \frac{30}{28} = 119.5$ cents, syntonic d **secondarily** 3-parted, Formula X:

$$\begin{array}{cccc} \multicolumn{4}{c}{30} \\ \hline 30 & 29 & 28 & 27 \end{array}$$

Ptolemaios uses the fraction $\frac{45}{44} = 38.9$ cents, very nearly the Molecule in the **31**-toned temperament, 38.71 cents, the Pythagorean d + secondarily 5-parted (Formula X):

	45	
45	44	40

b) Interpolation within the **Tetrachords** is also presumably to be found to a great extent among other nations than those mentioned. Thus Helmut Ritter quotes from Rauf Bey (Note 8) for the **Turkish** scale-types »Suznak« and »Neu eser« one and two Tetrachords respectively of two unusual forms which we shall call

Scheme 17, No. 13.	cents
»**uneven** Tritonos-Tetrachord« 204 + 120 + 267 = 591, Medium 111	total 702
»**uneven** harmonic« .... 120 + 267 + ... 111	total 498
Scheme 17, Nos. 10 b and 13.	

which however, can easily be explained in the following way:

The interval on the string »d + ... fis +« (in Tritonos) is parted secondarily into 3 parts or the Fifth »d + ... a +« into 5 parts in:

	d +	es + 1/3 Comma	fis +	a +	
Particles:	640	597 1/3	512	426 2/3	
Spaces		42 2/3 +	85 1/3 +	85 1/3	= 213 1/3
Abbreviated		1 +	2 +	2	= 5

Thus the tone $\frac{720}{597^1/_3} = \frac{135}{112} = 323.4$ cents, 1/3 Comma over es, which is 315.6 cents. The two scales can hereafter be characterised as follows:

		cents		
Suznak	c d + e f Lydian Major	204	g (as) b c' uneven harmonic	Scheme 17 No. 10 b.
Neu eser	c d — (es) fis + uneven Minor Tritonos	111	g (as) b c' uneven harm. repeated	No. 13

c) In the same way some five- or six-toned scales, as for example some Indian scales may be explained by Trichords:

	Secondary partition:			Middle tone:
Trichords:	Particles	Abbreviated		
c d f	72 + 108	2 + 3 = 5		10/9
c d + f	80 + 100		4 + 5 = 9	9/8
c (es—) f	108 + 72	3 + 2 = 5		20/17
c es — f	112 1/2 + 67 1/2		5 + 3 = 8	32/27 *)
c es f	120 + 60		2 + 1 = 3	6/5
c e f	144 + 36	4 + 1 = 5		5/4

*) see Chapter I, Resumé, b.

And it may be supposed that many other riddles of a similar type can be easily cleared up as being something quite natural by secondary interpolation alone — without fantastic Fifth-steps.

d) We might probably solve Professor v. Hornbostel's theory concerning **Comma**-Fifth and **Comma**-Fourth, probably g— and f+ respectively, by means of

1) **secondary** interpolations, »4-partition of 2 Fifths«, d .. a and d+ .. a+ respectively:

g—	Formula X:	40			
		36	30	27	24
	$\frac{40}{27}$ = Comma-Fifth	d	f	g—	a
	Particles	648	540	486	432
3 proportionally abbreviated spaces		2	1	1	
		2	1	1	
f+	Particles	640	$533^1/_3$	480	$426^2/_3$
	$\frac{27}{20}$ = Comma-Fourth	d+	f+	g	a+
	Formula X:	27			
		24	20	18	16

Then Comma-Fifth:	d....... a 4-parted	or: f .... a	halved
— — -Fourth:	d+ .... g 3 —	or: d+ .... a+	

2) Further Tertiarily following divisions, see **Scheme 19**:

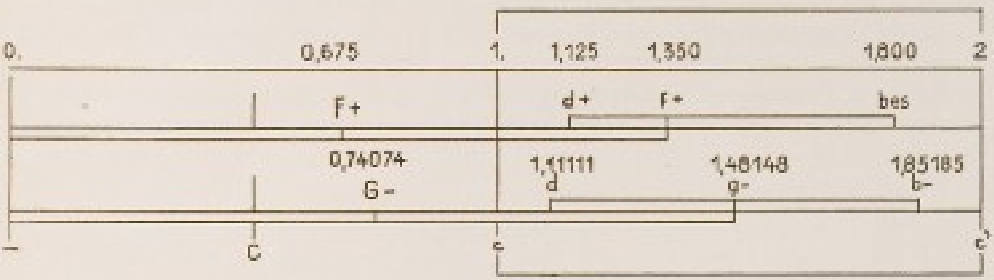
3-divided	....	4		8			
	10			5			
Formula Y	30	32	36	40	45	48	50
Denom. 27:	d	es —	f	g —	a	bes —	b —
Halved	 10			10			
	8			8			

3-divided	18		36			
5- —	18		27			
Formula Y	90	96	108	120	135	144
Denom. 80:	d+	es	f+	g	a+	bes
Halved	12		12			

3) Further »bes ... d + ... f +« and »g — ... b — ... d« are **Major** triple-chords:

f + is deep difference-tone	between d + and bes
g — — —	— d and b —

Super-Tonic bes $= \frac{9}{5} = \frac{72}{40} = 1{,}800$	resp. b— $= \frac{50}{27} = 1{,}85185$ Third
minus Third d + $= \frac{9}{8} = \frac{45}{40} = 1{,}125$	minus d $= \frac{30}{27} = 1{,}11111$ Sub-Fifth
Deep difference F + $= \frac{27}{40} = 0{,}675$	.... G— $= \frac{20}{27} = 0{,}74074$
The Fifth f+ $= \frac{27}{20} = 1{,}350$	.... g— $= \frac{40}{27} = 1{,}48148$ Tonic



Finally it may be proved by means of **other** similar difference-tones. Prof. v. Hornbostel however, gives as a **physical** explanation of this problem (Note 8), (f +) = 521.5 cents, (g —) = 678.5 cents, »die Blas-quarte und Blas-quinte« in decimalfractions: 1,3515 and 1,4798 respectively.

Principal Rule No. 6. The intervals between the tones in stepwise progression can often be grouped in several »**Greek Triads**« (3 neighbour tones) of two dimensions in two directions:

great »Triad«	direct 4 + 1 Comma	retrograde 1 + 4 Commas, Pythagorean
small »Triad«	» 3 + 1 »	» 1 + 3 » Arabian?

As it happens the octave comprises **53** Commas of 22.24 cents (about midway between the syntonic and Pythagorean Comma, respectively 21.50 and 23.46 cents); from **Scheme 3** can, for example, be constructed **Scheme 4** as an experiment: the »direct Greek Triads« with 53-toned tone-No. (Nicholas Mercator, Dane, 1675).

Perhaps the structure of the »Greek Triads« has had some importance in Melody formation.

Principal Rule No. 7. Javanese salendro and pelog Gamelan were interpreted by F. Lassen Landorph (Note 9) in 1923 as »Two tone-systems, in which two Gamelans (adapted to two different systems) cannot be played together«; we assume that they have both been built up by quartary interval-division, but in two different ways. During his stay in Sumatra (1877—99) Lassen Landorph recorded the vibration-numbers (oscillations) for the tones, and arrived at the conclusion that

1) the older form, the Salendro-octave, has 5 tones (Pentatonic) »with a distinct tendency towards five equal (quartary) intervals« within the octave, thus a 5-toned **temperament**; whilst

2) the latter form, the Pelog-octave, is also »called 5-toned, as the whole **material** of **7** tones in several places is seldom or never used, but on the other hand, various combinations on **5** tones only«.

Following the vibration numbers recorded by Lassen Landorph, **Scheme 5** has been constructed showing the Salendro-octave as a 5-toned temperament and the 7 tones of the Pelog as tone-material, i. e.

partly »a harmonic Tritonos«, in quartary 4-division with omission of the central tone: »c cisis ... eis fis +«,

partly a Pythagorean harmonic Tetrachord, »g as — .... b + c'«, in quartary 5-division with omission of the central tone. The interval »as — ... b +« is the syntonic es = 6/5.

Other intervals:

Nos.	Interval	Nos.	Inter.	Nos.	Inter.	Nos.	Inter.	Nos.	Inter.
1—2	cisis	4—5	des	5—6	des —	6—7	**es**	4—6	d +
3—4	cisis	5—6	des —	6—7	**es**	7—8	des —	6—8	e +
2—3	Sum es —	Sum	d +	Sum	e +	Sum	e +	Sum	ges —

Lassen Landorph is of opinion that the Salendro is reminiscent of the very ancient **Chinese** Tone-system, and »in its most simple composition, with reference to the Java-tradition, it is supposed to derive from the oldest Hindu period, or, more correctly, to be traced to this period, which means, from the beginning of the Christian era«; — while »Pelog must be presumed to be later, as it, with its various tones and varied musical instruments, is more fully developed«. Both are, however (partly) built up on quartary interval-division, — naturally secondary in practice.

The scale: »g as— b+ c fis+ g« may be presumed to be pure Pythagorean (fis+ = ges 2—).

In the pelog Gamelan the tone Pelog, 512 particles = 590 cents, may however easily be found on a string (or on the flute) with great approximation by the 5-partition of the octave, and the Fifth g exactly by halving, while analogously the other tones may be found by secondary 10- and 5-partition respectively, as shown in Scheme 5.

CHAPTER III

THE PYTHAGOREAN SYSTEM IN THEORY AND PRACTICE.

That the pure, i. e. consistent Pythagorean system can only be taken symbolically is evident partly by the fantastic tone-fractions formed in the outer circle of the system, partly by the Greek Schisma, for instance

Pythagorean tones:	Syntonic tones:
9 Fifths is »dis 3+« $= \frac{19.683}{16.384}$	but es $= \frac{6}{5} =$ 315.6 cents
10 — »ais 3+« $= \frac{59.049}{32.768}$	» bes $= \frac{9}{5} =$ 1017.6 »
11 — »eis 3+« $= \frac{177.147}{65.536}$	» f+ $= \frac{27}{20} =$ 519.6 »
The fractions: overstated	.... natural.

Only 6 Fifth-steps give the same tone as »2 Fifth-steps and in the opposite direction 1 syntonic Third«, that is:

6 Pythagorean Q minus 3 octaves	= 4211.730 — 3600	= 611.730	= fis 2 +
2400 minus (2 Q + 1 syntonic Third)	= 2400 — 1790.224	= 609.776	= ges —
	Difference, a Greek Schisma	= 1.954	cents.

After Pythagoras having altered the presumed **primitive** Greek

Tonic-sequence (key-notes):	c d+	e	f g	a	b+ c'
to his Diatonic (Persian) :	c d+	e+	f g	a+	b+ c'

which is also the authorised Greek Tonic-sequence, he hereby reaches his limit, and beyond that he only creates syntonic tones with the exception of one Schisma, so that the Pythagorean exaggeration recoils, in that, beyond 6 Fifths, all other Fifths **return like a boomerang**, as syntonic **interchangeable** tones. It is the playful way of Nature that all exaggeration corrects itself.

The Arabian-Persian musicians probably knew, before 636 A. D. that, for instance, 8 q Pythagorean = fes 2 — are almost identical with the syntonic e, (according to Helmholtz, as quoted by Jonquière, Note 10).

If to this be added that the 4 chromatic tones des—, es—, as— and bes— are so near to the primitive **secondary** tones that the difference of 1.4 cents is inaudible all Pythygoras's exaggerated tone-fractions may be changed to reasonable human fractions.

The Pythagorean system can thus (with the exception of the inaudible Schisma) be indicated as quoted in **Scheme 6**:

Song Lexis	Instrument Krusis		Scheme 14:
—	3	enharmonic interchangeable tones	stage No. 6 (c)
2	2	chromatic — —	
5	5	— — —	— - 7 (c)
7	7	diatonic Fifth steps	
5	5	enharmonic interchangeable tones	— - 8 (es)
—	1	— — —	
19	23	Tones in all	in 3 stages

The tones, however, are not used with equal frequency; if all the Greek scale-types be played on the following 7 Tonics (key-notes): c d + e + f g a + and **bes** — for song, the number of touches will be $7 \times 7 \times 7 = 343$, distributed as the numbers in Scheme 6, with the maximum 37 on g, which is the middle one of the 7 Tonics selected, arranged according to Fifths.

The number of Molecules for 19 song-tones will be $10 \times 90.2 + 2 \times 66.7 + 7 \times 23.5$ cents $= 1200$ cents.

The names of the song-tones are thus given in the Greek **Triads** from above downwards; for the instrument-tones a name-No. was originally given to the uppermost in the Triad (Note 11) following our hypothesis as indicated in **Scheme 7** according to triple-chords (chords of Thirds, English triads).

Scheme 17 Nos. 1—7, shows the 7 Pythagorean scale-types in syntonic or golden tuning, 6 paired off symmetrically $\begin{cases} \text{Nos. } 1 & 2 & 3 \\ \text{and } 7 & 6 & \text{and } 5. \end{cases}$

CHAPTER IV

THE AUTHORITATIVE TONE-SYSTEM, THE GOLDEN SYSTEM AS AN OBJECTIVE MEANS OF VALUATION.

However, in order to pass judgement on the various historical tone-systems, the chief defect of which is the displacement of **Commas** between for example »d and d+«, »des— and des« etc., it is necessary first to construct an authoritative tone-system **without** these Comma-displacements, which claims that many tones with plus would be made much lower, and many tones with minus much higher.

In August 1930 I found the new Fifth in two ways which gave the same result.

Group A. Arithmetic Series I—VI, retrograde and direct.

The sum of 2 golden adjacent Nos. forms the following No., and the sum of the **cents** of two golden adjacent tones forms the following cents in the same golden Series.

1. The astronomer J. Kepler (1571—1630) combined tones with the sum of the numerators and the sum of the denominators respectively of 5 tone-fractions:

$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{8}{5}$	$\frac{13}{8}$ Partial-tone 840.5 cents
C	c	g	a	as	gisis

The golden gisis .... 843.2 »

In the 19-toned and the 31-toned golden section respectively we **transform** the tone-sequence in this manner (see **Scheme 10**):

							Double super division		
Fourth-Series 1 retrograde:	19 toned	.....	.....	1 cis	2 des	3 d	5 es	8 f	13 as
	31 —	.....	1 deses	2 cis	3 des	5 d	8 es	13 f	21 as
	50 —	1 bisis	2 deses	3 cis	5 des	8 d	13 es	21 f	34 as

Ludwig Sonnenberg, principal teacher in Bonn (1820–88) called the sequence 1, 2, 3, 5, 8, 13: »Kepler's Series« (Notes 12–13), our golden **Fourth**-Series I retrograde, in 1844 extended up to No. 17 by Gabriel Lamé, in 1929 up to No. 40 by L. Kaiser.

2. Dr. Ludwig Kaiser, the mathematician, found by pure mathematics (Note 14) (without reference to tones) a similar sequence; we call the latter »Kaiser's Series«, our **Fifth**-Series II, which we use in the following manner:

			Double super division.		
Fifth-Series II retrograde	19-toned	4 dis	7 fes	11 g	18 ces
	31- —	7 dis	11 fes	18 g	29 ces

3. In May 1930 Andreas Kornerup, engineer, directed my attention to the fact that the Fifth in these systems has tone-Nos. which form similar sequences of Nos. for example $\frac{\text{Fifth}}{\text{Octave}} = \frac{7}{12}, \frac{11}{19}, \frac{18}{31}, \frac{29}{50}, \frac{47}{81}$ etc.

We name the sequence 12, 19, 31, 50, 81 the **Octave**-Series III, Andreas Kornerup's Series, the denominators of the fractions.

a) It occurred to me to calculate these fractions in cents, which I did, August 15th 1930. The value moved like a **pendulum** quickly approaching the point of balance, where the seventh decimal would be stable at the Fifth $\frac{1200 \times 3371}{6155} = 696.2145$ cents (Acoustic statics).

The temperaments, the octaves of which are the denominators of these fractions, I called the **organic** temperaments, and, at the same time, it occurred to me to construct an **authoritative** »tone-system of the Fifth« with **this** Fifth which, however, **also** generally appears by means of the super-division (golden cut) of the octave, **Scheme 10**, Series III. I calculated the Fifth on August 17th 1930, as shown below:

b) The fraction of super-division		Cents	Series III:
ω	0,618.03398 × 1200 gives	741.64078	direct ases
$1 - \omega$	0,381.96602 × 1200	458.35922	retrograde eis
Total 1,0		1200 cents	c'

Further	ases is 11 Fourths from c	and 4 octaves back	then f = 503.7855
	eis is 11 Fifths —	— 6 — —	— g = 696.2145
			Total = 1200 cents

In 19-toned and 31-toned golden sections respectively we use »Andreas Kornerup's Series« in the following manner:

			Double super division		
Octave-Series III retrograde	19-toned	0 c	7 eis	12 ases	19 c'
	31-toned	0 c	12 eis	19 ases	31 c'

The difference between golden eis = 458.4
and syntonic — = 457.0

Schemes 14 and 15: nearly k = 1/15 Comma = 1.4

4. Out of the calculated golden system I have formed some other Series for instance 31-toned see **Scheme 20**:

			Double super division.		
Great Third-Series IV:	4 cisis	6 eses	10 e	16 ges	26 hes
Tritonos — V:	. . . .		9 feses	15 **fis**	24 heses
Great Sixth- — VI:	. . . .		9 disis	14 geses	23 a

Group B. Geometrical Constructions of golden Tones.

The golden tones and intervals can also be constructed geometrically as shown in **Scheme 8**, which indicates:

1) above, part of a regular **Pentagon**, the angle of which $= 108°$ is divided into 3 parts by means of 2 chords; if the chord be equal to a Tetrachord — the interval C-F, (the Fourth f), this interval will be super-divided in »**great** double super-division«

$$\text{in}\left\{\begin{array}{l} \text{C ——— D ...... Es ——— F = Series I} \\ 192.43 + 118.93 + 192.43 = 503.79 \text{ cents} \end{array}\right.$$

2) below: a **square** placed inside a semicircle, whereby the diameter is super-divided into a »**small** double super division«

$$\text{in cents}\left\{\begin{array}{l} \text{Cis .... D ——— E ..... F} \\ \text{D ..... Es ——— F ..... Ges} \\ \text{B ..... C ——— D ..... Es} \\ 118.93 + 192.43 + 118.93 = 430.29 \end{array}\right.$$

$$\text{corresponding with: } \begin{array}{l} \text{C ..... D ——— F ..... G} \\ 192.4 + 311.4 + 192.4 = 696.2 \end{array}$$

3) further: **within** the square we can form a »**great** double super division«

$$\text{in}\left\{\begin{array}{l} \text{D ——— Dis .... Es ——— E} \\ 73.50 + 45.43 + 73.50 = 192.43 \text{ cents} \end{array}\right.$$

Scheme 9 shows the golden system constructed by means of the **Pentagram** with the same chord C .. F, so that the side is = C .. Es in the Pentagon.

Fourth-Series I: C, Deses, Cis, Des, D, Es, **F** and further As, Des'.

Further: $\frac{\text{C..(Es)}}{\text{C..E}} = \frac{\text{es}}{\text{e}} = $ Cosinus 36°, the relation between small and great Third.

Scheme 10 shows »double super-division« in 4 Series; the upper and lower edges of a square are divided:

{ from c upwards into ases, direct,
{ — c' downwards into eis, retrogade.

If we continue a single super-division,

1) direct, upwards, we will get **decreasing** intervals upwards,

2) retrograde, downwards, we will get decreasing intervals downwards. We use the oblique lines { from the corner upwards on the right / — — — downwards — left } forming 8 double super-divisions, which give the result:

	upwards, retrograde;	Cents	downwards, direct, Scheme 10:
Series I	c—es .. **f** —as	815	c—g .. a —c'
— IV	c—e .. ges—bes	1008	cis—f .. gis—c'
— II	c—fes .. **g** —ces	1126	d—fis .. as—c'
— III	c—cis ——— **c'**	1200	c ——— ases—c'

We continue double super-division only in Series I:

Series I:	c—des .. d —es	311	a —bes .. b —c'
	c—cis .. des—d	192	bes—b .. ces—c'
	c—deses .. cis —des	119	b —ces .. bis—c'

The tones are in pairs supplement-tones (making an octave together) for instance:

Series I	retrograde	deses	cis	des	d	es	**f**	as	c'
	direct	bis	ces	b	bes	a	g	e	c

Scheme 20 shows golden tones in 6 Series formed geometrically by means of parallel lines in a Pentagon, for instance:

The tones :	Eis	F	Fis	Ges	G	As	A	C'	Des'
Series I-III:	III.	I.	....	....	II.	I.	....	III.	I.
— IV-VI:	....	V.		IV.	....	....	VI.	....	....

Scheme 21 shows cents (to 4 decimals) for an organic 19-toned section of the infinite golden tone-system with two Molecules:

{ t = 73.501 cis, which is the smallest interval in this section,
v = 45.426 deses, the greatest interval in the next **organic** section on 31 golden tones.

	v.	t.	cents
The Molecules in 12-toned golden section	5 × cis	+ 7 × des	= 1200
19- — — —	7 × deses	+ 12 × cis	= 1200
31- — — —	12 × bisis	+ 19 × deses	= 1200

v. outside the section.

Group C. All Comma-displacements disappear.

Example: $d = \frac{10}{9}$ and $d+ = \frac{9}{8}$

merges into the golden tone d = 192.43 cents, approximating to the **secondary** halving of the string between d and d +, which in the Formula X gives:

		180	
	162	161	160
	d	....	d+
Particles	648	644.3	640
cents	182.4	193.1	203.9

or $\frac{180}{161} = 193.12$ cents, the difference is inaudible,

so that the golden tone can easily be found approximately on the string.

The **tertiary** halving gives a somewhat higher tone (Formula Y):

$$\frac{160 \quad 161 \quad 162}{144} \text{ or } \frac{161}{144} = 193.20 \text{ cents.}$$

The structure of the triple chord is often the golden cut directly put to use (x = great Third, y = small Third, (2) and (3) formations):

Triple chord:	Symbol	Intervals	Small Sixth with golden cut:
Major 2nd form	xy (2)	e g c	retrograde: 311 + 504 = } 815 cents
Minor 3rd —	yx (3)	g c es	direct: 504 + 311 = } 815 cents

For teachers of harmony the golden system can thus also be of use through its clear and logical construction; thus xyy (1) the Dominant quadruple chord c e g bes will, on the 1st step, be **resolved** into xy (3) = the tonic triple chord on the 3rd step: (c) f a c' (on the Tonic f) by which means the tones e, g and bes **glide** either 119 **or** 192 cents, »c des« and »c d« respectively, systematically up **or** down, with the intervals in cents:

From	0	385		696		1007	1200.	Symbols
	c	e		g		bes	c'	xyy (1)
Gliding in cents		→			←	→		
		119	←	→	119			
			192	192		192		
to	c		f		a		c'	xy (3)
	0		504		888		1200	

Since all Comma displacements disappear in the golden system, one can therefore transpose any scale whatever on any golden tone as Tonic (key-note) with the **smallest possible number** of tones; the authoritative Golden tone-system represents in this field the formula for the principle of **the smallest activity**, Nature's **economic minimum** principle. It is the system with the smallest possible number of tones (acoustic Okology), an outcome of Nature's wonderful **power of adaptation**, — the supporting principle of all life in Nature.

Andreas Kornerup has called the fraction of the golden cut 0,618.034 Omega, which is recommended as an international expression in the formula $\frac{1}{\omega} = \frac{\omega}{1 = \omega}$ or: $\omega^2 + \omega = 1$ or: cis + ases = c', Series I, or 458 + 742 = 1200 cents, which we here designate, once and for all, as the authoratitive principle of relativity in the field of acoustics: The essential formula for **the universal sense of harmony.**

Even if the composers do not know of this formula, the Danish Physicist H. C. Ørsted (1771—1851) is indeed right in saying: »The work of the composer is based on mathematics although in a **deeper** measure than has ever dawned upon us.« (Note 15). It will be the task of the future student of the theory of music to get to the bottom of this deeper-lying **law** of Nature, the golden cut, the super division — the basis of the future renaissance of harmonics.

CHAPTER V

TEMPERAMENTS.

Tone-systems with 1 Molecule only.

The Octave-Series III, Andreas Kornerup's Series, marks the boundary of the **organic** temperaments, i. e. the only **rational**, the only temperaments fit for use, namely the sequence: 12, 19, 31, 50, 81 — with 19-toned as the **practical** and with 31-toned temperament as the **Standard**-temperament, see **Schemes 12** and **18**.

How many tones will be required for pianos, organs etc. is indeed a question; but, for practical measures only the organic **19** and **31** are efficient; all the others are unworkable. This is the authoritative judgement passed on the matter in question.

»Ceterum censeo: If we wish to abandon the 12-toned pianoforte we can in no circumstances choos any other than the practical 19-tonic temperament, and for finer requirements the Standard 31- (or the 50-) toned temperament.

All the inorganic temperaments ought to be excluded.

Scheme 11 shows, by way of example, how the Fourth-Series I, Kepler's Series, is carried through logically, in 19- or 31-toned temperament, where the sum of cents for two neighbour tones gives the next tone in the Series, but is split in, for instance, the 24-tone system, which is therefore authoritatively considered to be unworkable.

Further: Dr. P. S. Wedell and N. P. J. Bertelsen, actuary, Copenhagen, in January 1915 proved, by means of »the method of the smallest squares«, that the 19-toned is better than the 12-toned temperament, and that the 31-toned, again, is better than the 19-toned. To have this judgement expressed in numbers it can easily be calculated how much the single tone in the different temperaments deviate from the golden tones, and by summing-up such deviations for 35 tones (the tones of the 7 white keys and ♯, ♭, 2♯ and 2♭) the result is found to be as follows see **Scheme 12**:

Deviation for	12-toned	Temperament	1174	cents	estimated at	100 %
	19- —	—	458	—	thus	39 —
	31- —	—	174	—	—	15 —
	50- —	—	67	—	—	6 —

In comparison it may be quoted that in the pure consistent Pythagorean system, the collective deviation for the same 35 tones is 1780 cents or 52 % **greater** than for the 12-toned temperament.

Among the numerous observations as to the change of the piano-temperaments we shall quote the following few selections:

Director Gottfred Skjerne says (1909): »We are indebted to the tempered tuning for enormous musical progress ..... but the ear is, as a matter of fact, **coarsened**.« (Note 16).

Professor Dr. José Würschmidt showed 1920—28 »that a division of the octave into 18, 24 or 36 parts can represent **no** natural extension of our tone system, but that we have before us such an extension in a division of **19** steps. (Note 17).

Professor Joseph Yasser, New York, recommends the 19-toned temperament with the purpose: »to **enrich** our musical language, particularly when one takes into account the **growing** significance of the independent twelve-tone foundation in modern music«. (Note 18).

Professor Louis Kelterborn, Neuchâtel, (Note 19) proposes, further, on practical grounds, that simultaneously with the introduction of the 19-toned temperament, the pitch of »the tone a« should be made **a little lower**, so that **c** may keep the same pitch as it has now.

Scheme 13 shows, geometrically, an equilateral hyperbole through the tones No. 5 in various organic temperaments forming the tones in the Golden Fourth-Series I, retrograde, (with approximate pitches of tone).

Scheme 18 shows lines through corresponding tones in 5 temperaments.

CHAPTER VI

CHANGE OF THE SYNTONIC SYSTEM INTO GOLDEN TONES.

a) In **Schemes 14 and 15** we have the syntonic system, built on Pythagorean Fifths (the Partial-tone = 3/2) in horizontal lines and the Thirds = 5/4 in diagonal lines with an angle of 60°, according to the proposal of the Japanese Shohé Tanaka in 1890.

The axis through »feses, c or gisis« in the schemes is called **Zero**-Axis, because the difference between the syntonic and golden feses (equalling **15** golden Fourths) is almost nil, just as is the difference between the corresponding supplement-tones, syntonic and golden gisis (equal to **15** golden Fifths), which is seen in the following table:

4 syntonic Thirds............ ......give	1545.255	cents
minus 1 — Fifth, the Partial-tone 3/2 = g	701.955	—
gives syntonic **gisis**	843.300	—
15 golden Fifths minus 8 Octaves, golden **gisis**, is	843.217	—
inaudible difference	0.083	—
Further: syntonic Third, the tone 5/4 = e	386.314	—
4 golden Fifths minus 2 Octaves: golden e.....	384.858	—
Slight deviation	1.456	—
1/15 of a syntonic **Comma** 21.5062 cents is k =	1.434	—
inaudible deviation	0.022	—

All the tones in **Scheme 14** can thus, in a practical way, be changed to golden tones by adding or subtracting a number of »k«. These numbers are regularly grouped in all the lines which can be enclosed through the tones in the Scheme, i. e. with regular rise in numbers of k, so that the Scheme will be reminiscent of mathematical number-designs, or **number-figures**, constructed by the Danish astronomer Thorvald Nicolai Thiele (1838—1910) in 1872 (Note 20).

Scheme 14 is thus divided into 9 symmetrical **figures**, each comprising 15 tones, formed on about **9** central points of, respectively + 15 or 0 or — 15 k:

	+	Zero	—
	gisis — + 15	gisis 0 0	gisis + — 15
	c — + 15	c 0	c + — 15
	feses — + 15	feses 0	feses + — 15
resp.	from + 22 to + 8	from + 7 to — 7	from — 8 to — 22

Thus, outside the Zero-Axis the tones are here changed by »addition on the left side«, »subtraction on the right side« of fifteenth parts of a Comma, increasing evenly in all directions.

Axis:	Direction:				Result: Central-points
1) Pythagorean Fifth-Axis, horizontal ...	to the right	o. c	— 4. g	— 8. d +	— 12. a +
then one Small Third-step	downward	....	.....	....	— 3. — 15. c +
2) Pythagorean Fifth-Axis, horizontal ...	to the left	o. c	— 4. f	+ 8. hes —	+ 12. es —
then one Great Sixth-step	upward	....	......	....	+ 3. + 15. c —
3) Great Third, Axis, 60° upward	to the right	o. c	— 1. e	— 2. gis	— 3 his
then one Great Sixth-step	— — left	....	......	....	+ 3 0 gisis
4) Small Sixth-Axis 60° downward	to the left	o. c	+ 1. as	+ 2. fes	+ 3 deses
then one Small-Third-step	— — right	....	......	....	— 3 0 feses
5) Small Third-Axis **through** the figure .	from »+ 15 gisis —« (through »o. c«) to »— 15 feses +«.				

If all the Third-lines, inclining 60°, be elongated they will cut the Zero-Axis in nothing but Zero-points, — ad infinitum.

Two examples of the division of 1 Comma will finally be quoted:

d + sunk	8 k = 11.4700	es sunk	3 k = 4.3 cents
d raised	7 k = 10.0362	es — raised	12 k = 17.2 —
Total	15 k = 21.5062	5 × 3 k = Total	15 k = 21.5 — = 1 Comma.

The golden d = 192.43 cents is only a trifle below the **secondary** halving of the distance on the string between d and d +, as above mentioned (Chapter IV, Group C).

Analogously the distance on the string between Pythagorean es — $= \frac{32}{27}$ and the syntonic es $= \frac{6}{5}$ is secondarily **5**-parted according to the Formula X:

	480		
	405	401	400
	es —	golden	es
with the particles	607,5	601,5	600

An extreme example:

Pythagorean 6q = ges 2 — =	588.3 cents	Syntonic fis + =	590.2 cents
Scheme 14: plus 24 k =	+ 34.4 —	Minus 9 k =	— 12.9 —
golden ges =	622.7 —	golden fis =	577.3 —
Scheme 21:		plus Molecule v =	+ 45.4 —
		ges =	622.7 —

b) **Scheme 15** shows the exact change (to three decimals) in the middle figure (15 tones) and gisis. The number of cents diminish or increase **evenly** with

{ 5.741 cents in all horizontal Fifth-Axis }
{ 1.456 — — — oblique Third — }

representing »the 2 building materials« in the syntonic mixing-system: Fifths and Thirds:

1)	2)	3)	4)
+ 1.373 cents eis	— 2.912 gis	— 2.912 gis	5 syntonic e = 378.2 feses +
— 1.456 plus e	+ 2.829 plus ζ	— 2.829 minus ζ	5 golden e = 356.8 feses
— 0.083 gisis	— 0.083 gisis	— 5.741 g	distance = 21.4 = 1 Comma

c) **Rationa lexplanation:** Super division of the octaves e … e' and as … as' (Supplement-pairs) gives »retrograde gisis« and »direct feses« respectively.

d) Similar methods (but of inferior structure) were up to 1558 used in the three famous systems of chance by: (see **Scheme 16**):

	Zero-Axis	The unit increases	The unit decreases
(X unknown author) before 1511 ……	a c es	+ 1/8 as	— 1/8 e
The Bohemian, Arnold Schlick 1511 .	as c e	+ 1/4 a	— 1/4 es
The Italian, Giosetfo Zarlino 1558 ….	ces c cis	+ 1/7 { as a	— 1/7 e — 1/7 es
here, Scheme 14 …. 1930	feses c gisis	+ 1/11 as	— 1/11 e

Scheme 16 shows the 3 Fifths oscillating about the golden g, »the truth«:

	X	Zarlino	Golden system	Schlick
the result g:	694.8	695.8	696.2	696.6
by means of:	$^5/_{15}$	$^4/_{14}$	$^4/_{15}$	$^4/_{16}$
in decimals:	= 0,333	= 0,286	= 0,267	= 0,250
Distance from the ideal:	+ 0,066	+ 0,019	**Ideal**	— 0,017

Addendum.

Scheme 17 shows diagrams of 7 Minors and 6 Majors, some of them in pairs symmetrically, see pages 10, 15 and 20.

Glareanus's erroneous **nomenclature** of the medieval scale-type should once for all be obliterated from the litterature of music, and should be replaced by that recommended in Scheme 17, compare Helmholtz's heartfelt cry: »Übrigens werde ich Glareans Namen nicht brauchen .... es wäre überhaupt besser, wenn man sie vergessen möchte,« repeated by Ellis: »But I shall not use Glareanus's names .... It would be better to forget them altogether.« (Note 21).

Scheme 18 shows lines through corresponding tones in 5 temperaments.

Scheme 19: tertiary distances as arcs in a circle, see pages 7 and 16.

Scheme 20: construction of golden tones by means of parallel lines, see page 24.

Scheme 21: Golden tones in cents with 4 decimals, see page 24.

Scheme 22: examples of logarithms as training in the use of my Constant K = 0.600.5714, see page 7.

In **Scheme 23** is set forth a proposal for a system of notation on music-paper, staff, in all other tone-systems than the 12-toned temperament, with the suggestion, offered by K. Steensen (Note 22) for a rational arrangement of the ♯ and ♭.

Scheme 24 shows how the cents can easily be calculated by taking the difference between the logarithms for Partial-tones, the numbers of which are respectively the numerator and the denominator in a tone-fraction, according to the table constructed by the organist Kai Kroman, Copenhagen, for example:

$$\text{oriental d} = \frac{10}{9} \left\{ \begin{array}{l} \text{Logarithm for Partial tone No. } 10 = 3.986.3 \\ \text{— — — — — } \quad 9 = 3.803.9 \\ \hline \text{difference ... } \quad 182.4 \text{ cents} \end{array} \right.$$

The word »Partial-tone« is preferred to »over-tone« in order to avoid the **confusion**, which Ellis characterises as »great confusion, Tyndall's **erroneous** translation« (Note 21) between

the German ober	and the English upper: adjective .....
— über	— — over: preposition, adverb.

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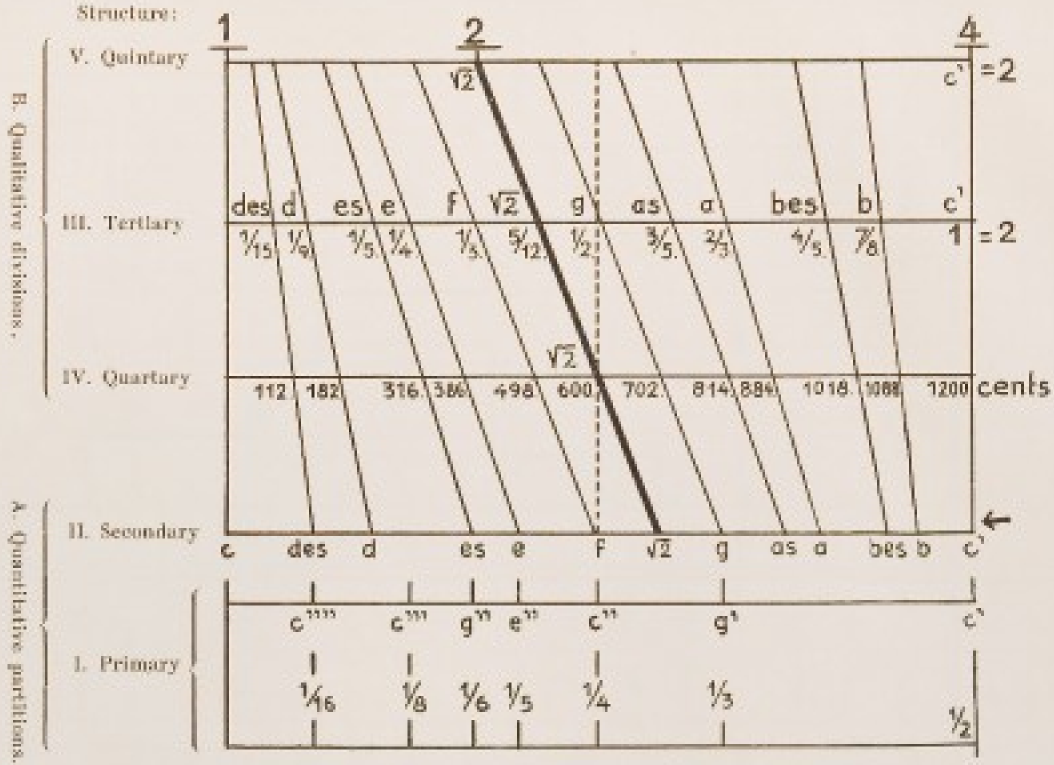


Scheme 1. Graphic sketch of 5 kinds of calculation (primary-quintary of tones.

Tertiarily: $\text{des} = 1 + \frac{1}{15} = \frac{16}{15}$.

Secondarily: $\text{des} = \frac{15}{16}$ of the whole string.

17 Persian intervals Cents.	Doric-Sruti Nos.	17 Sruti Nos. 13 permanent Nos. Cents	4 Persian blank, variable Sruti-Nos. Cents	Cents	13 permanent tones Persian Doric Minor on 7 Tonics (key-notes).							Number of touches
90	17	1200			c	c	c	c	c	c	c	7
22	16	1110					b +				b +	2
92	(15)		1088	b								—
90	14	996			bes —	bes —		bes —	bes —	bes —		5
22	13	906				a +	a +			a +	a +	4
92	(12)		884	a								—
90	11	792			as —			as —	as —			3
90	10	702			g	g	g		g	g	g	6
24	9	612		Medium							ges —	1
90	8	588						fis +				1
90	7	498			f	f	f	f	f	f		6
22	6	408					e +			e +	e +	3
92	(5)		386	e								—
90	4	294			es —	es —		es —	es —			4
22	3	204				d +	d +		d +	d +	d +	5
92	(2)		182	d								—
90	1	90			des —			des —				2
—	0	0			c		c	(c)	(c)	(c)	(c)	0
1200	Total	13	+ 4	= 17							Total	49

Scheme 2, P. The 17-toned Persian Sruti-System; fis + = ges 2 —.

Number of touches	17 permanent tones										17 permanent Nos.	53 Comma-temperament.	Indian names.	5 Indian			
	Probable Indian		Lydian—Phryian Sa Grama Double Lydian Ma Grama on 5 Tonics (key-notes)											blank, variable Sruti-Nos.			
	c		d +		f		g		a +					22 Sruti	53 Commas	Names	Cents
7	c	c		c	c	c	c				22	**53**	Sadja				
3			b +						b +		21	**49**	Nisada				
4	b			b			b				20	48					
3		bes—			bes —						18	44		(19)	45	bes	1018
7	a +		a +	a +			a +		a +		17	**40**	Dhaivata				
3		a			a						16	39					
1									as —		15	35		(14)	34	gis	772
9	g	g	g		g		g			g	13	**31**	Pancama				
1									ges—		12	27					
4			fis +				fis +			fis +	11	26		(10)	25	fis	568
5	f				f			f			9	**22**	Mahijama				
5			e +				e +		e +	e +	8	**18**	Gandhara				
4	e				e			e			7	17					
1						es —					5	13		(6)	14	es	316
9	d +		d +		d +		d +	d +	d +		4	**9**	Rsabha				
1						d					3	8					
3			des—						des —		2	4		(1)	3	cis	70
0	c			c	c	c	c				0	0	Sadja	—	—	—	—
70	Total.		Total: 17 ...... + 5 = 22.														

Scheme 2, J. The 17-toned Sruti-System, extended to 22-toned Indian-system.

53-toned No.	4	5	9	
Greek names	Iota ←	Theta	← Eta	
Pythagorean	des —	des	d +	
Cents	90	112	204	
Particles about	$683\frac{3}{4}$	675	640	
5-partition	$8\frac{3}{4}$	+	35	$= 43\frac{3}{4}$.

53-toned No.	4	5	8	
Arabian names	nim ← zirkula	.... zirkula	→ tiq zirkula	
presumably	des —	des	d	
Cents	90	112	182	
Particles	684	675	648	
4-partition	9	+	27	= 36

Scheme 3. Great and small retrograde Triads: Greek Triad.................. 5 Commas,
and presumably Arabian Triad ... 4 — .

53 Temp. Nos.	g.	as —	**a.** a +	bes —	**b.** h +	c'.	Upper Tetrachord 22 Commas.
Direct Triads	31.	35.	39. 40.	44.	48. 49.	53.	
= 5 Commas....		4 Commas	1	4 Commas	1		
Retrograde Triads..			1	4 Commas	1	4 Commas	
Persian Nos.	10.	11.	**12.** 13.	14.	15. 16.	17.	
Indian Nos.	13.	15.	16. 17.	18.	20. 21.	22.	
				f.	ges 2 — ges —	g.	Medium 9 Commas.
				22.	26. 27.	31.	
				4 Commas	1		
					1	4 Commas	
				7.	8. 9.	10.	
				9.	11. 12.	13.	
53 Temp. Nos.	c.	des —	**d.** d +	es —	**e.** e +	f.	Lower Tetrachord 22 Commas.
Direct Triads	o.	4.	8. 9.	13.	17. 18.	22.	
= 5 Commas....		4 Commas	1	4 Commas	1		
Retrograde Triads..			1	4 Commas	1	4 Commas	
Persian Nos.	0.	1.	**2.** 3.	4.	5. 6.	7.	
Indian Nos.	0.	2	3. 4.	5.	7. 8.	9.	

Total 53 Commas.

Scheme 4. The 13 permanent oriental tones and 4 variable tones (d e a h). Total 17 Persian.

		53-toned temperament = No. about	0	11	21	32	42	53	
Salendro (scale)		Malayan translation			$^1/_2$ (nem)		(2 × tengah)		
		Malayan name	barang	goeloe	tengah	lima	anam / nem	barang'	Scale
		Vibration numbers	270	310	356	409	470	540	Lassen Landorph
		Ellis cents	0	240	480	720	960	1200	Cents
	Quartary	5-toned temp.	c	eses ($^1/_2$ —)	f —	g +	ais ($^1/_2$ +)	c'	Symmetrical
		1 molecule (eses)		240	240	240	240	240	= 1200

	53-toned temperament = No. about	0	7	20	26	31	35	49	53	
					5 Pythagorean tones.					
(material)	Malayan translation	charming			exchanging	No. 5	No. 6		charming	
	Malayan name	manis	goeloe	tengah	pelog	lima	anam / nem	barang	manis'	Material
	Vibration numbers	279	304	362	395	418	443	527	558	Lassen Landorph
	Ellis cents	0	148	448	590	700	792	1110	1200	Cents
	Syntonic names	c	cisis	cis	ges 2 — / fis +	g	as —	b +	c'	

Scheme 5, see below.

	No.	1. c	2. cisis	3. eis	4. fis +	5. g	6. as —	7. b +	8. c'	
Quartary Pelog (material)	Quartary intervals:	148	200	142	110	92	**318**	90		cents = 1200
	Harmonic Tritones, intervals ...	cisis	(es —)	cisis	....	....	....	....		Total 590
	nearly 4-division	1 +	2 +	1						
	Small Medium, Greek "Diazeuxis decreased": ${}^{16}/_{15}$				des	....	....	....		— 110
	Harmonic upper Tetrachord with the small Third in tune, intervals					des —	**es**	des —		— 500
	nearly 5—division					1 +	3 +	1		....
Secondary Pelog.	10—partition........	c	cisis	eis	fis +					Material
	Particles nearly	720	658	554	512					Particles
	Spaces = 208 =	62 +	104 +	42						—
	— abbreviated	3 +	5 +	2						—
	5—partition.....................					g	as —	b +	c'	Material
	Particles nearly...					480	456	384	360	Particles
	Spaces = 120					24 +	72 +	24		—
	— abbreviated					1 +	3 +	1		—

Scheme 5, continuation. Javanese Gamelan (salendro and pelog respectively) built up upon Lassen Landorph's vibration-numbers, in "Annals for Music", Copenhagen 1923, pp. 7—23.

Presumed Fourth-steps…	12	11	10	9	8	7	(c—) unused.
Scheme 14: E-stage No. 6.	(c—)	g—	d	a	e	b	3 enharmonic
Touches …	—	—	—	—	1	3	and 2 chromatic.

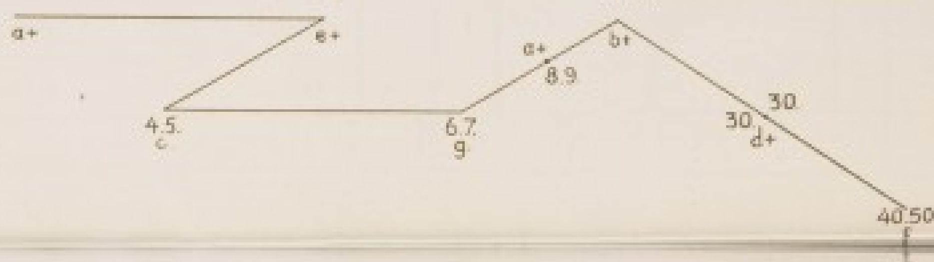
Fourth-steps	6	5	4	3	2	1	0						
Fifth-steps	—	—	—	—	—	—	0	1	2	3	4	5	5 chromatic and
Sch. 14: C-stage No. 7.	ges 2—	des—	as—	es—	bes—	f	c	g	d+	a+	e+	b+	7 diatonic tones.
Touches …	6	10	15	21	28	33	36 ←	37	36 →	33	28	21	

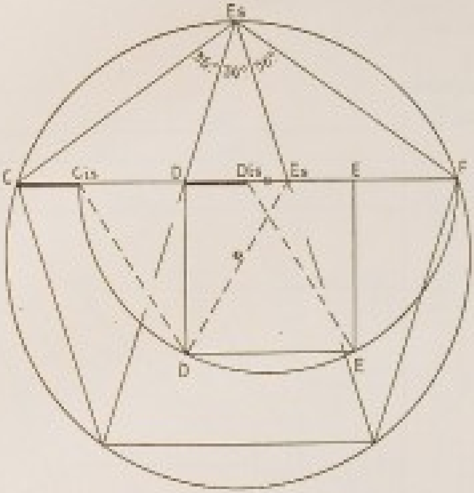
Presumed Fifth-steps	6	7	8	9	10	11	12	6 enharmonic.
Scheme 14: Es-stage No. 8.	ges—	des	as	es	bes	f+	(c+)	(c+) unused.
Touches …	15	10	6	3	1	—	—	

Scheme 6. The 23 Pythagorean instrument-tones, of which 19 are song-tones from c to bes, (from 8 Fourths to 10 Fifths).

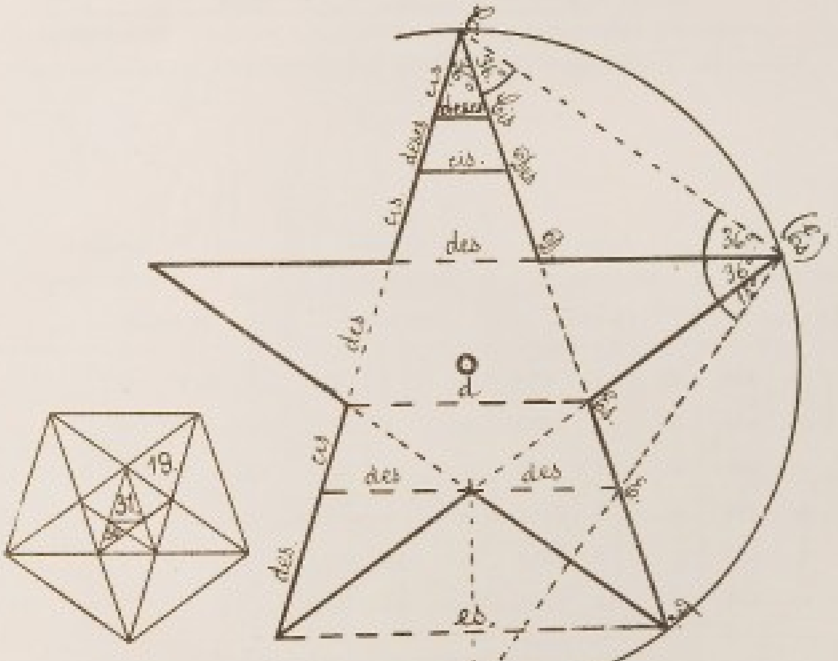
Scheme 7. The tone-Nos for 7 instrument-tones are presumably founded on triple-chords (i. e. chords of Thirds, English triads). The symbol x signifies Great Third, y Small Third.

Hyper.	Doric Minor.	Hypo.
Minor triple chord: yx	Minor triple chord: yx	Diminished triple chord: yy
a+ c e+	e+ g b+	b+ d+ f
Sub-Dominant.	Tonic.	Dominant.
1.	2.3	10.20

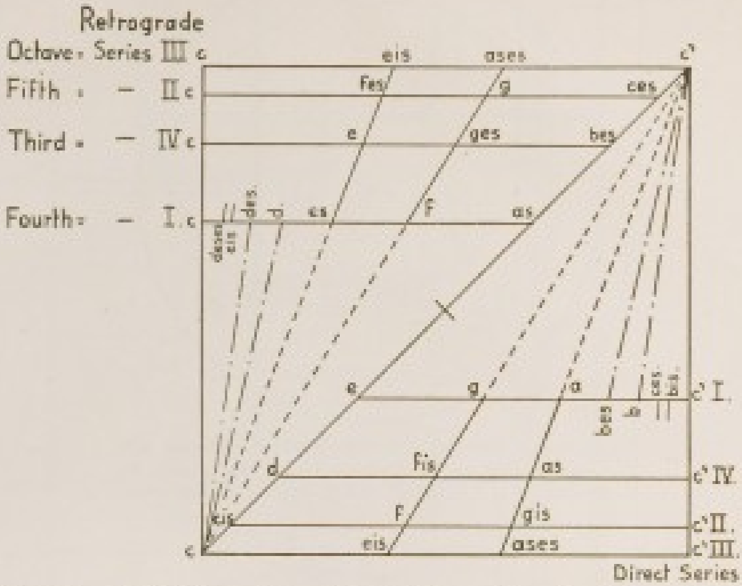




Scheme 8. Geometrical construction of great and small double super division (the golden cut), by means of a Pentagon and a square respectively.



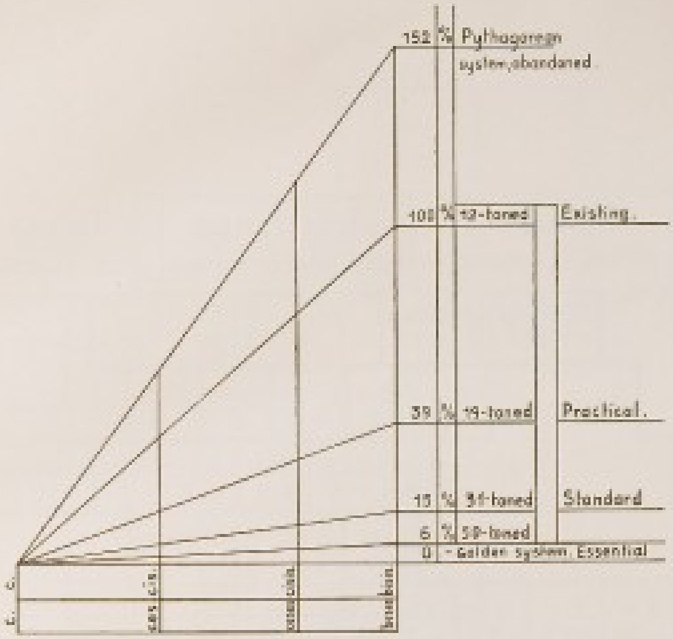
Scheme 9. The golden system constructed by means of the Pentagram.
The sides of the isosceles triangles are Molecules,
for examples deses and cis in 19-toned section.
bisis and deses » 31- » »



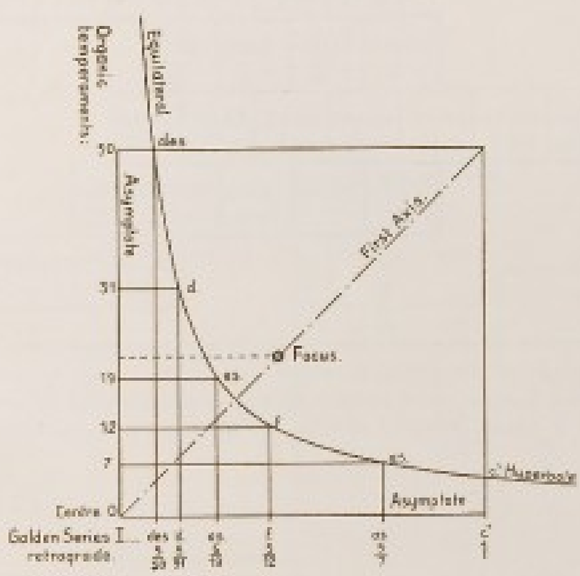
Scheme 10. Great double super division (golden cut) in 4 relations (Series).

A temperament too small 12-tones			Organic temperament 19-toned			Organic temperament 31-toned			Inorganic temperament 24-toned					
No. 1	cis.	100	cis.	No. 1	63.2	cis.	Nr. 2	77.4	cis.	No. 1	50			
	des.	100	des.	2	126.3	des.	3	116.1	des.	2	100			
2	d.	200	d.	3	189.5	d.	5	193.5	cisis.	3	150	d.	No. 4	200
3	es.	300	es.	5	315.8	es.	8	309.7				es.	6	300
5	f.	500	f.	**8**	505.3	f.	**13**	503.2				f.	10	500
8	as.	800	as.	13	821.1	as.	21	812.9				as.	16	800

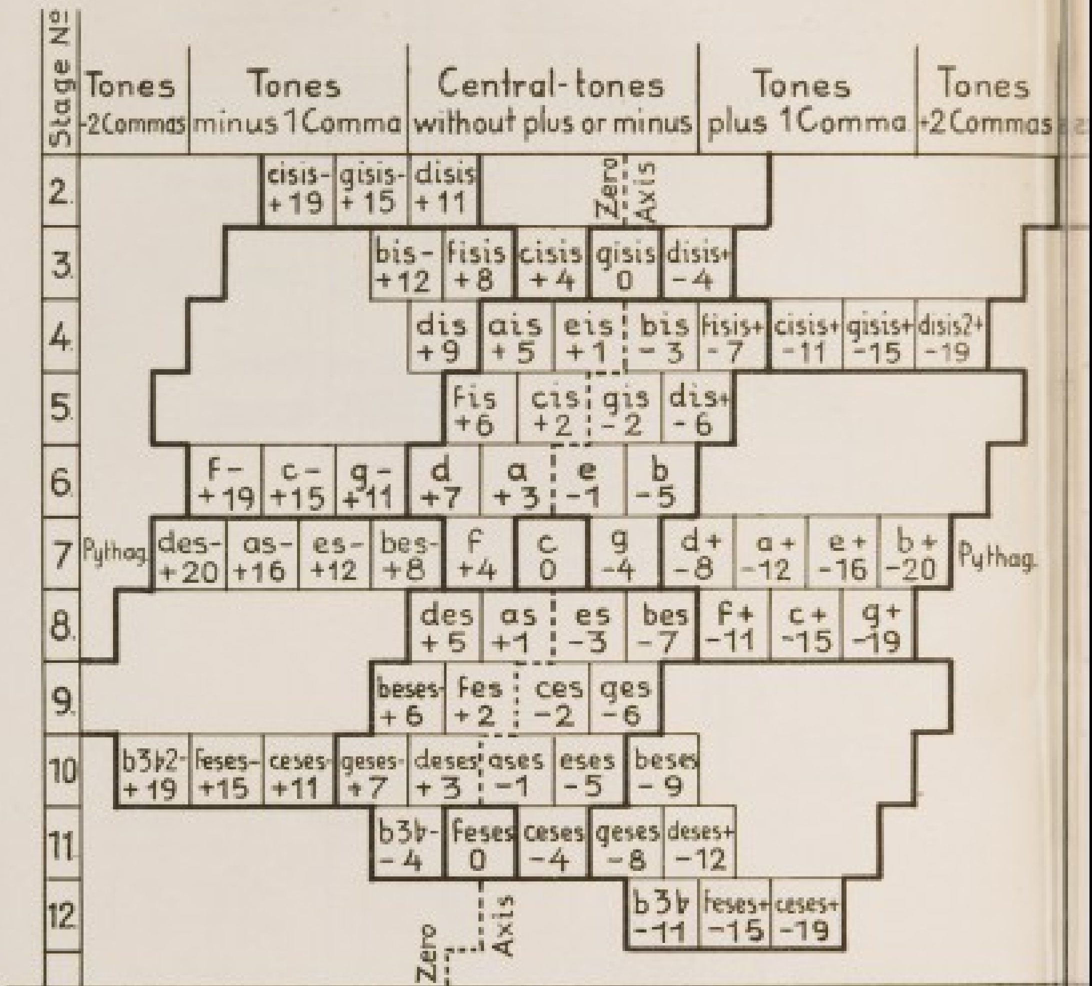
Scheme 11. Golden Series No. 1 used as a means of valuation for organic and inorganic temperaments. The lower c always No. 0.



Scheme 12. Graphic description of the deviation (falsity) of 4 organic temperaments and the Pythagorean system.



Scheme 13. An equilateral hyperbole through the tones having No. 5 in different organic temperaments: des d es f as, the golden Series I retrograde.



Scheme 14. Change of the syntonic system into golden tones by means of numbers of k (= 1/12 of a Comma), in 9 oblique figures, each containing 15 tones, formed on 9 central-points of plus 15, 0 and minus 15 k respectively.

Stage					Zero — Axis.				
3						gisis — 0.083			
4			ais + 7.114		eis + 1.373		bis — 4.367		fisis + — 10.168
5		fis + 8.569		♯ = cis + 2.829		gis — 2.912		dis + — 8.652	
6	d + 10.052		a + 4.285		e — 1.456		b — 7.196		
7		f + 5.741		c 0		g — 5.741		Pythagorean tones.	
				Zero — Axis.					

Scheme 15. The middle figure (and gisis) in Scheme 14, showing exact change into golden tuning.

0. 0. 0. Zero—Axis.

The highest cents.	+ 1/4 cis. 76.0		0 gis. 772.6
A. Schlick 1511.		0 e. 386.3	
1/4 = 5.377 cents.	0 c. 0		
Authoritative	+ 4/15 cisis. 147.6	+ 1/15 eis. 458.4	0 gisis. 843.2
golden system. k =			— 2/15 gis. 769.7
1/15 = 1.434 cents.	0 c. 0	— 1/15 e. 384.9	— 4/15 g. 696.2
The lower cents	0 cisis. 141.3		— 2/7 gisis. 837.2
G. Zarlino 1558.	0 cis. 70.7	— 1/7 eis. 453.9	— 2/7 gis. 766.5
1/7 = 3.072 cents.	0 c. 0	— 1/7 e. 383.2	— 2/7 g. 695.8
The lowest cents	— 1/3 cis. 63.5		— 2/3 gis. 758.3
	0 c. 0	— 1/3 e. 379.1	
X. before 1558.		0 es. 315.6	— 1/3 g. 694.8
1/3 = 7.169 cents.			0 ges. 631.3

Scheme 16. Three systems oscillating around the authoritative golden system. k = 1/15 of a Comma.

1. Doric-Tritonos Minor. Tritonos = ges.

2. Doric Minor.

3. Phrygian-Doric (or Aiolian) Minor, our descending melodic Minor.

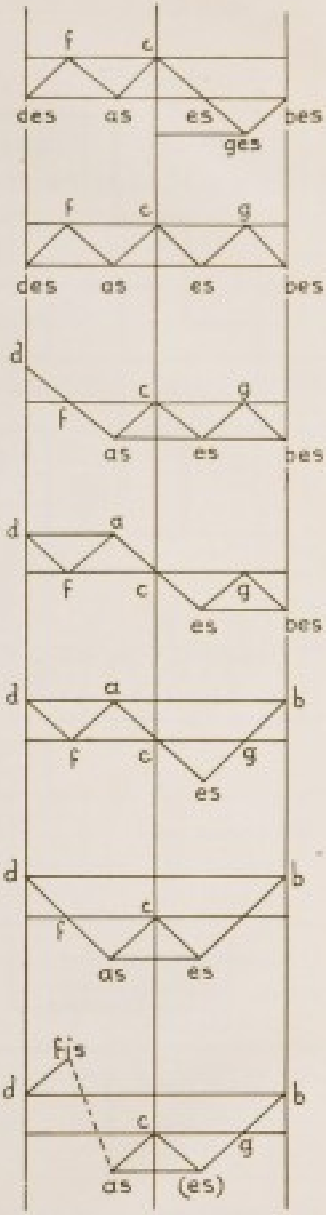
4. Phrygian Minor, double symmetric.

---

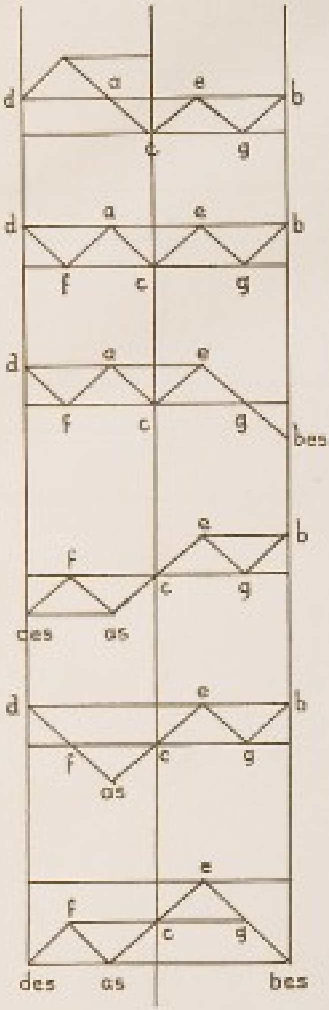
9. Phrygian-Lydian Minor, our ascending melodic Minor.

11. Phrygian-harmonic Minor, Ernst F. E. Richter 1853.

13. Turkish »uneven Tritonos ... uneven harmonic« Minor, Neu-eser (Rauf Yekta Bey).



Scheme 17.



7. Tritones-Lydian Major. Tritones = fis.

6. Lydian Major. Indian Ma.

5. Lydian-Phrygian (or Jonic, or Jastic) Major, Indian Sa.

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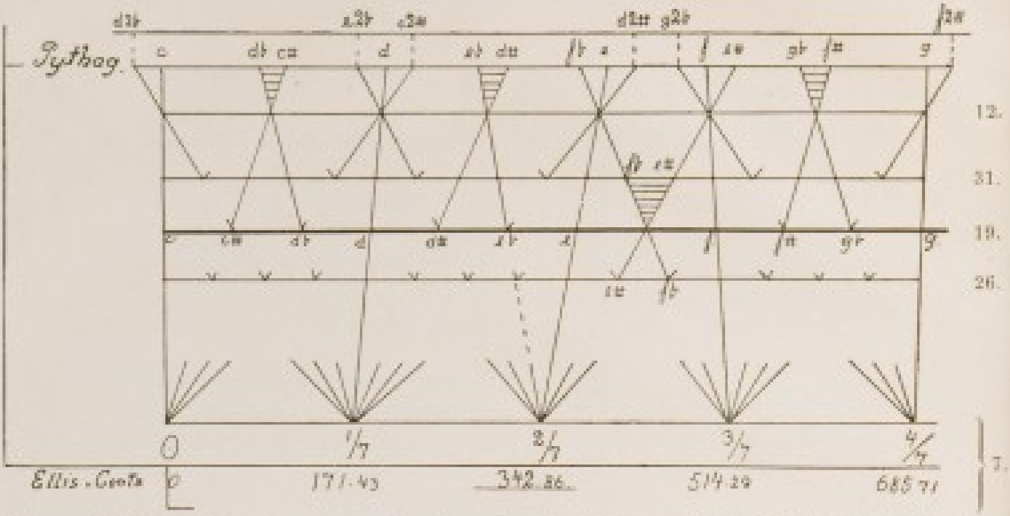
8. Double harmonic Major, Hedjaz-Kar (A. Z. Idelsohn).

10a. Lydian-harmonic Major, Rimsky Korssakoff 1898.

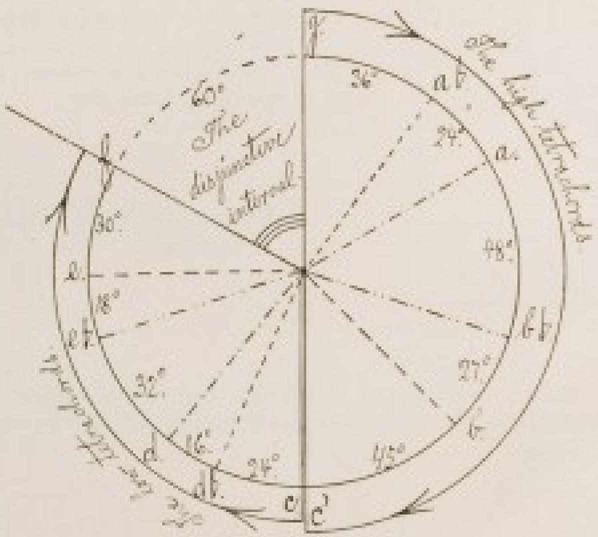
10b. »Lydian-uneven harmonic«, Turkish Suznak. (Rauf Yekta Bey).

12. Harmonic-Doric Major, Ahavah Rabbah, (A. Z. Idelsohn).

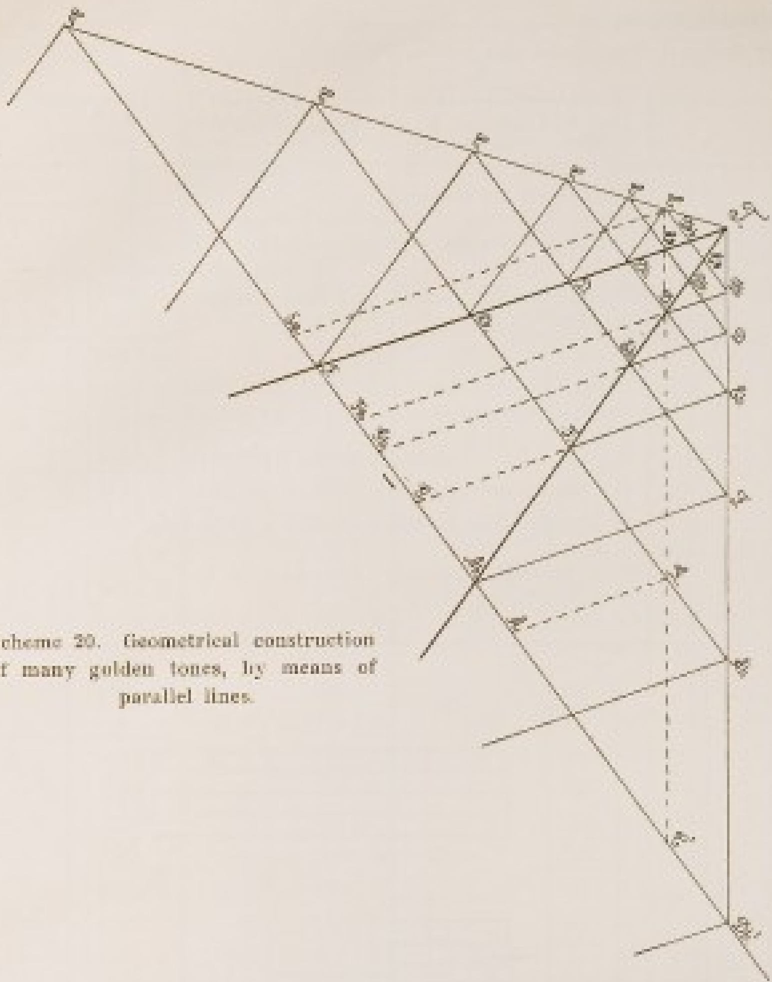
Scheme 17. Names and diagrams of 13 scale-types, on the Tonic c, in **syntonic** or **golden** tuning on c; 7 Minors and 6 Majors; No. 1—7 Pythagorean.



Scheme 18. Lines through corresponding tones in the Pythag. system and 5 temperaments.



Scheme 19. Two Tetrachords tertiarily graduated as a semi-circle and a sector.



Scheme 20. Geometrical construction of many golden tones, by means of parallel lines.

No.		Cents		Dec.	No.		Cents		Dec.	No.		Cents		Dec.
26	16	bes	1007	5711	28	17	b	1081	0724	30	18	bis	1154	5737
21	13	as	815	1421	23	14	a	888	6434	25	15	ais	962	1447
16	10	ges	622	7132	18	11	g	696	2145—	20	12	gis	769	7158
11	7	fes	430	2842	13	8	f	503	7855+	15	9	fis	577	2868
8	5	es	311	3566	10	6	e	384	8579	12	7	eis	458	3592
3	2	des	118	9276	5	3	d	192	4289	7	4	dis	265	9303
29	18	ces	1126	4987	**31**	**19**	c	0	—	2	1	cis	73	5013
					or No. 0					Two 19-toned Molecules.		t = cis		
												v = deses	45	4263

Scheme 21. The 19-toned golden section, with **19**-toned and **31**-toned Nos.

Logarithmic Examples:

1) The decimal fraction of the golden Fifth?

Log. 0.696.2175 (g) = ... 0.842.7431 — 1
minus Constant K.. = minus 0.600.5714

Log. difference .... = 0.242.1717 — 1
to which corresponds ~ 0.174.6512

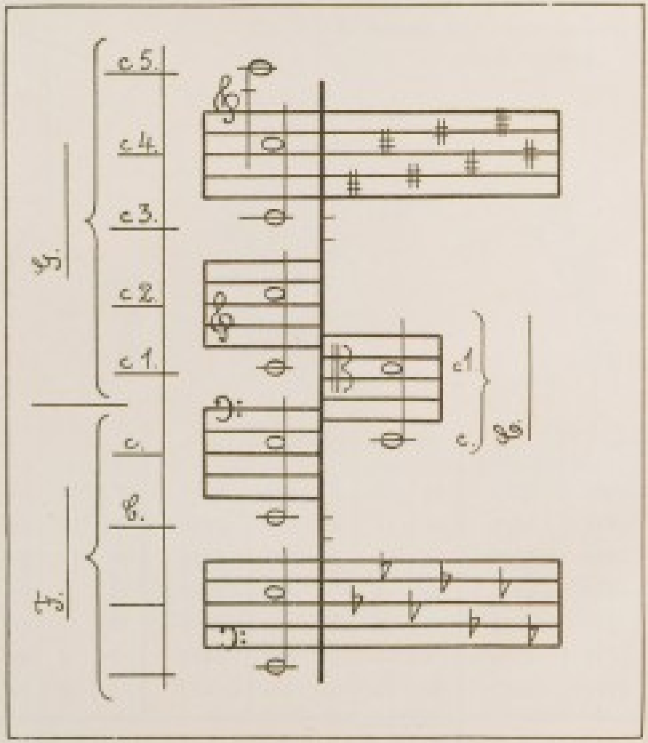
to which again corresponds the decimal fraction 1,495.34
Syntonic g = 3/2 = 1,5 is a little larger.

How many particles?

Log. 720 particles 2.857.3323
minus the above ~ 0.174.6512

Difference ... 2.682.6813
to which corresponds ~ ... 481,594
For »syntonic g« 480 particles a little less.

Scheme 22, compare Chapter I, B, 4.



Scheme 23. B new 5-lined staff (1931) applicable to other temperaments than the 12-toned temperament.

The first 5 octaves				The 6th octave				The 7th octave							
	Cents				Cents			c — g	Cents			g — c'	Cents		
				$2^5=32$	6	000	0	$2^6=64$ c	7	200	0	96 g	7	902	0
C 1		0	0	3		053	3	5		26	8	7		19	9
c. $2^1=2$	1	200	0	4		105	0	6		53	3	8		37	7
g 3	1	902	0	5		155	1	7		279	3	9		55	2
c. $2^2=4$	2	400	0	d + 6	6	203	9	8		305	0	100		72	6
e...... 5	2	786	3	7		251	3	9		30	2	1		989	9
g 6	3	102	0	8		297	5	70		55	1	2	8	006	9
7	3	368	8	9		342	5	1		379	7	3		23	8
c. $2^3=8$	3	600	0	e. 40	6	386	3	2	7	403	9	4		40	5
d +.. 9	3	803	9	1		429	1	3		27	8	5		57	1
e..... 10	3	986	3	2		470	8	4		51	3	6		73	5
11	4	151	3	3		511	5	5		74	6	7		089	8
g..... 12	4	302	0	4		551	3	6		497	5	8		105	9
13	4	440	5	f♯ + 5	6	590	2	7		520	1	9		21	8
14	4	568	8	6		628	3	8		42	5	110		37	6
b 15	4	688	3	7		665	5	9		64	5	1		53	3
c. $2^4=16$	4	800	0	g. 8	6	702	0	80 e		586	3	2		68	8
17	4	905	0	9		737	7	1	7	607	8	3		84	2
d +.. 18	5	003	9	50		772	6	2		29	1	4		199	5
19		097	5	1		806	9	3		50	0	5		214	6
e..... 20	5	186	3	2		840	5	4		70	8	6		29	6
1		270	8	3		873	5	5		691	3	7		44	4
2		351	3	4		905	9	6		711	5	8		59	2
3		428	3	5		937	6	7		31	5	9		73	8
g..... 4	5	502	0	6		968	8	8		51	3	120 b	8	288	3
5		572	6	7		999	5	9		70	9	1		302	6
6		640	5	8	7	029	6	90	7	790	2	2		16	9
7		705	9	9		059	2	1		809	4	3		31	0
8		768	8	b. 60	7	088	3	2		28	3	4		45	0
9		829	6	1		116	9	3		47	0	5		58	9
b 30	5	888	3	2		145	0	4		65	5	6		72	7
31		945	0	3		172	7	5		883	8	7		386	4
c. $2^5=32$	6	000	0	$2^6=64$	7	200	0	96 g	7	902	0	128 c'	8	400	0

Scheme 24. Special cents-table, showing 6 octaves of the Partial-tones 1—128, built up upon the original table by K. Kroman, Organist, Copenhagen.

Example: b. Partial-tone No. ...... 15 .... 30 .... 60 .... 120 ..

cents .... 4.6883 = 5.888.3 = 7.088.3 = 8.288.3

Scheme-List.

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11	—	2	The 17toned Persian Sruti-system, 13 permanent, 4 variable tones.
—	—	2	— 22 — Indian — — , 17 — , 5 — —
17	—	3	Great and small retrograde »Triads«, Greek and presumably Arabian.
—	—	4	The 13 permanent oriental tones and 4 variable tones as »Triads«. Total 17 Persian.
—	—	5	Javanese Gamelan built up upon Lassen Landorph's vibration-numbers 1923.
			Chapter III—IV.
20	—	6	The 23 Pythagorean instrument-tones, Krusis; 7 diatonic, 7 chromatic and 5 enharmonic = 19 song-tones, Lexis.
—	7	—	The Nos. for 7 instrument-tones, presumably founded on triple-chords.
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—	9	—	The golden system constructed by means of the Pentagram.
—	10	—	Great double super-division (golden cut) in 4 relations (Series).
			Chapter V—VI.
27	—	11	Golden Series No. I used as a means of valuation for organic and inorganic temperaments.
—	12	—	Graphic description of the deviation (falsity) of 4 organic temperaments and the Pythagorean system.
29	13	—	An equilateral hyperbole through the tones having No. 5 in different organic temperaments: des, d, es, f and as, the golden Series No. I retrograde.
—	14	—	Change of the syntonic system into golden tones by means of xk (k = $^1/_{11}$ of a Comma), in 9 oblique figures, each containing 15 tones, formed on 9 central-points of plus 15, 0, minus 15 k respectively.
31	—	15	The middle figure (and gisis) in Scheme 14, showing exact change.
32	—	16	Three systems formed by means of $^1/_2$, $^1/_3$, $^1/_4$ of 1 Comma, oscillating around the authoritative golden system, the ideal.

Addendum.

Text-Page	No. Cliché	No. Table	
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—	18	—	Lines through corresponding tones in 5 temperaments.
—	19	—	Tetrachords tertiarily graduated as a semi-circle and a sector.
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—	—	21	The 19-toned golden section, with 19-toned and 31-toned Nos.
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