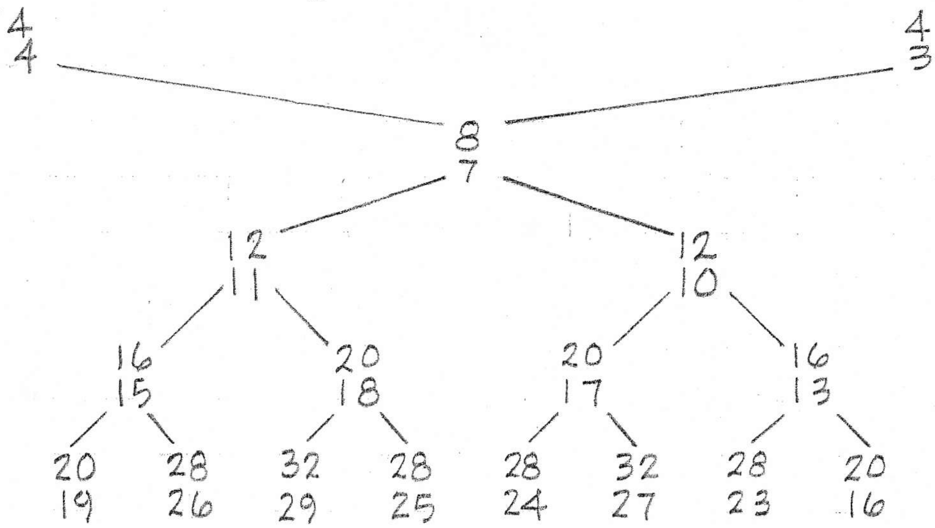
$\left(\frac{4}{4}, \frac{4}{3}\right)$ Scale-Tree, and $\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangle

* "$\frac{c-a}{c-b} = \frac{b}{a}$", Neo-Pythagorean School/Ghyka 1977

Scale-Tree

(Stern-Brocot, Peirce)

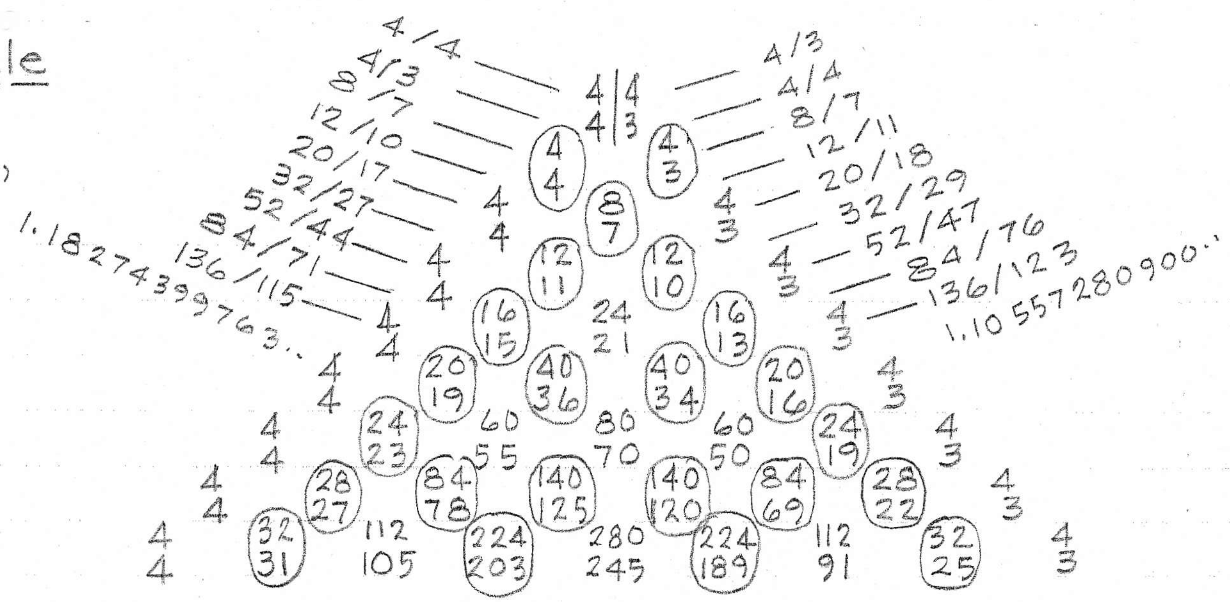


reduced: 1/1 20/19 16/15 14/13 12/11 32/29 10/9 28/25 8/7 7/6 20/17 32/27 6/5 28/23 16/13 5/4 4/3

epimoria: 20/19 76/75 105/104 78/77 88/87 145/144 126/125 50/49 49/48 120/119 136/135 81/80 70/69 92/91 65/64 16/15

Triangle

(Pascal, Meru)



reduced; 4/4 32/31 28/27 24/23 20/19 16/15 14/13 12/11 32/29 10/9 28/25 8/7 7/6 20/17 32/27 6/5 28/23 16/13 5/4 24/19 14/11 32/25 4/3

epimoria; 32/31 217/216 162/161 115/114 76/75 105/104 78/77 88/87 145/144 126/125 50/49 49/48 120/119 136/135 81/80 70/69 92/91 65/64 96/95 133/132 176/175 25/24

* Example; $\frac{8-4}{8-4} = \frac{4}{4}$, and $\frac{7-3}{7-4} = \frac{4}{3}$.

THE TRIANGLE/LAMBDA EQUIVALENCE

work in progress –

3Nov03·EW

Archtypal Triangle $\{\frac{0}{1}, \frac{1}{0}\}$

$$\frac{0}{1} \Big| \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{1} \quad \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{2} \quad \frac{2}{1} \quad \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{3} \quad \frac{3}{3} \quad \frac{3}{1} \quad \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{4} \quad \frac{4}{6} \quad \frac{6}{4} \quad \frac{4}{1} \quad \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{5} \quad \frac{5}{10} \quad \frac{10}{10} \quad \frac{10}{5} \quad \frac{5}{1} \quad \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{6} \quad \frac{6}{15} \quad \frac{15}{20} \quad \frac{20}{15} \quad \frac{15}{6} \quad \frac{6}{1} \quad \frac{1}{0}$$
$$\frac{0}{1} \quad \frac{1}{7} \quad \frac{7}{21} \quad \frac{21}{35} \quad \frac{35}{35} \quad \frac{35}{21} \quad \frac{21}{7} \quad \frac{7}{1} \quad \frac{1}{0}$$

Archtypal Lambda $\{\frac{0}{1}, \frac{1}{0}\}$

$$\frac{0}{0}$$
$$\frac{0}{1} \quad \frac{1}{0}$$
$$\frac{0}{2} \quad \frac{1}{1} \quad \frac{2}{0}$$
$$\frac{0}{3} \quad \frac{1}{2} \quad \frac{2}{1} \quad \frac{3}{0}$$
$$\frac{0}{4} \quad \frac{1}{3} \quad \frac{2}{2} \quad \frac{3}{1} \quad \frac{4}{0}$$
$$\frac{0}{5} \quad \frac{1}{4} \quad \frac{2}{3} \quad \frac{3}{2} \quad \frac{4}{1} \quad \frac{5}{0}$$
$$\frac{0}{6} \quad \frac{1}{5} \quad \frac{2}{4} \quad \frac{3}{3} \quad \frac{4}{2} \quad \frac{5}{1} \quad \frac{6}{0}$$
$$\frac{0}{7} \quad \frac{1}{6} \quad \frac{2}{5} \quad \frac{3}{4} \quad \frac{4}{3} \quad \frac{5}{2} \quad \frac{6}{1} \quad \frac{7}{0}$$
$$\frac{0}{8} \quad \frac{1}{7} \quad \frac{2}{6} \quad \frac{3}{5} \quad \frac{4}{4} \quad \frac{5}{3} \quad \frac{6}{2} \quad \frac{7}{1} \quad \frac{8}{0}$$

Reduced Triangle & Lambda $\left\{\frac{0}{1}\ \frac{1}{0}\right\}$ 5Nov03·EW

$$\frac{0}{1}\quad \frac{1}{0}$$

$$\frac{0}{1}\quad \frac{1}{1}\quad \frac{1}{0}$$

$$\frac{0}{1}\quad \frac{1}{2}\quad \frac{2}{1}\quad \frac{1}{0}$$

$$\frac{0}{1}\quad \frac{1}{3}\quad \frac{1}{1}\quad \frac{3}{1}\quad \frac{1}{0}$$

$$\frac{0}{1}\quad \frac{1}{4}\quad \frac{2}{3}\quad \frac{3}{2}\quad \frac{4}{1}\quad \frac{1}{0}$$

$$\frac{0}{1}\quad \frac{1}{5}\quad \frac{1}{2}\quad \frac{1}{1}\quad \frac{2}{1}\quad \frac{5}{1}\quad \frac{1}{0}$$

$$\frac{0}{1}\quad \frac{1}{6}\quad \frac{2}{5}\quad \frac{3}{4}\quad \frac{4}{3}\quad \frac{5}{2}\quad \frac{6}{1}\quad \frac{1}{0}$$

$$\frac{0}{1}\quad \frac{1}{7}\quad \frac{1}{3}\quad \frac{3}{5}\quad \frac{1}{1}\quad \frac{5}{3}\quad \frac{3}{1}\quad \frac{7}{1}\quad \frac{1}{0}$$

Decimal Equivalences;

0 ∞

0 1 ∞

0 .500 2.000 ∞

0 .333 1 3.000 ∞

0 .250 .667 1.500 4.000 ∞

0 .200 .500 1 2.000 5.000 ∞

0 .167 .400 .750 1.333 2.500 6.000 ∞

0 .143 .333 .600 1 1.667 3.000 7.000 ∞

Triangle $\{\frac{1}{1}, \frac{2}{1}\}$ † — 9Nov03·EW

† ref Kraig Grady 1995

$\frac{1}{1} \cdot \frac{2}{1}$

$\frac{1}{1} \quad \frac{2}{1}$

$\frac{1}{1} \quad \frac{3}{2} \quad \frac{2}{1}$

$\frac{1}{1} \quad \frac{4}{3} \quad \frac{5}{3} \quad \frac{2}{1}$

$\frac{1}{1} \quad \frac{5}{4} \quad \frac{9}{6} \quad \frac{7}{4} \quad \frac{2}{1}$

$\frac{1}{1} \quad \frac{6}{5} \quad \frac{14}{10} \quad \frac{16}{10} \quad \frac{9}{5} \quad \frac{2}{1}$

$\frac{1}{1} \quad \frac{7}{6} \quad \frac{20}{15} \quad \frac{30}{20} \quad \frac{25}{15} \quad \frac{11}{6} \quad \frac{2}{1}$

$\frac{1}{1} \quad \frac{8}{7} \quad \frac{27}{21} \quad \frac{50}{35} \quad \frac{55}{35} \quad \frac{36}{21} \quad \frac{13}{7} \quad \frac{2}{1}$

$\frac{1}{1} \quad \frac{9}{8} \quad \frac{35}{28} \quad \frac{77}{56} \quad \frac{105}{70} \quad \frac{91}{56} \quad \frac{49}{28} \quad \frac{15}{8} \quad \frac{2}{1}$

Lambda $\{\frac{1}{1} \cdot \frac{2}{1}\}$

$\frac{0}{0}$

$\frac{1}{1} \quad \frac{2}{1}$

$\frac{2}{2} \quad \frac{3}{2} \quad \frac{4}{2}$

$\frac{3}{3} \quad \frac{4}{3} \quad \frac{5}{3} \quad \frac{6}{3}$

$\frac{4}{4} \quad \frac{5}{4} \quad \frac{6}{4} \quad \frac{7}{4} \quad \frac{8}{4}$

$\frac{5}{5} \quad \frac{6}{5} \quad \frac{7}{5} \quad \frac{8}{5} \quad \frac{9}{5} \quad \frac{10}{5}$

$\frac{6}{6} \quad \frac{7}{6} \quad \frac{8}{6} \quad \frac{9}{6} \quad \frac{10}{6} \quad \frac{11}{6} \quad \frac{12}{6}$

$\frac{7}{7} \quad \frac{8}{7} \quad \frac{9}{7} \quad \frac{10}{7} \quad \frac{11}{7} \quad \frac{12}{7} \quad \frac{13}{7} \quad \frac{14}{7}$

$\frac{8}{8} \quad \frac{9}{8} \quad \frac{10}{8} \quad \frac{11}{8} \quad \frac{12}{8} \quad \frac{13}{8} \quad \frac{14}{8} \quad \frac{15}{8} \quad \frac{16}{8}$

Reduced Triangle/Lambda $\{\frac{1}{1}, \frac{2}{1}\}$ 9Nov03·EW

ref; Augusto Novaro 1927

† Kraig Grady 1995

$$
\begin{array}{ccccccccc}
\frac{1}{1} & \frac{2}{1} \\
\frac{1}{1} & \frac{3}{2} & \frac{2}{1} \\
\frac{1}{1} & \frac{4}{3} & \frac{5}{3} & \frac{2}{1} \\
\frac{1}{1} & \frac{5}{4} & \frac{3}{2} & \frac{7}{4} & \frac{2}{1} \\
\frac{1}{1} & \frac{6}{5} & \frac{7}{5} & \frac{8}{5} & \frac{9}{5} & \frac{2}{1} \\
\frac{1}{1} & \frac{7}{6} & \frac{4}{3} & \frac{3}{2} & \frac{5}{3} & \frac{11}{6} & \frac{2}{1} \\
\frac{1}{1} & \frac{8}{7} & \frac{9}{7} & \frac{10}{7} & \frac{11}{7} & \frac{12}{7} & \frac{13}{7} & \frac{2}{1} \\
\frac{1}{1} & \frac{9}{8} & \frac{5}{4} & \frac{11}{8} & \frac{3}{2} & \frac{13}{8} & \frac{7}{4} & \frac{15}{8} & \frac{2}{1}
\end{array}
$$

Decimal Equivalences;

1.000 2.000

1.000 1.500 2.000

1.000 1.333 1.667 2.000

1.000 1.250 1.500 1.750 2.000

1.000 1.200 1.400 1.600 1.800 2.000

1.000 1.167 1.333 1.500 1.667 1.833 2.000

1.000 1.143 1.286 1.429 1.571 1.714 1.857 2.000

1.000 1.125 1.125 1.375 1.500 1.625 1.750 1.875 2.000

† Kraig Grady noticed a Novaro/Pascal link, 1995 verbal communication

$\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangularis (Truncate Lambdoma)

* ROW 0	$\frac{0}{0}$
1	$\frac{4}{4}$ $\frac{4}{3}$
2	$\frac{8}{8}$ $\frac{8}{7}$ $\frac{8}{6}$
3	$\frac{12}{12}$ $\frac{12}{11}$ $\frac{12}{10}$ $\frac{12}{9}$
4	$\frac{16}{16}$ $\frac{16}{15}$ $\frac{16}{14}$ $\frac{16}{13}$ $\frac{16}{12}$
5	$\frac{20}{20}$ $\frac{20}{19}$ $\frac{20}{18}$ $\frac{20}{17}$ $\frac{20}{16}$ $\frac{20}{15}$
6	$\frac{24}{24}$ $\frac{24}{23}$ $\frac{24}{22}$ $\frac{24}{21}$ $\frac{24}{20}$ $\frac{24}{19}$ $\frac{24}{18}$
7	$\frac{28}{28}$ $\frac{28}{27}$ $\frac{28}{26}$ $\frac{28}{25}$ $\frac{28}{24}$ $\frac{28}{23}$ $\frac{28}{22}$ $\frac{28}{21}$
8	$\frac{32}{32}$ $\frac{32}{31}$ $\frac{32}{30}$ $\frac{32}{29}$ $\frac{32}{28}$ $\frac{32}{27}$ $\frac{32}{26}$ $\frac{32}{25}$ $\frac{32}{24}$

reduced; $\frac{4}{4}$ $\frac{32}{31}$ $\frac{28}{27}$ $\frac{24}{23}$ $\frac{20}{19}$ $\frac{16}{15}$ $\frac{14}{13}$ $\frac{12}{11}$ $\frac{32}{29}$ $\frac{10}{9}$ $\frac{28}{25}$ $\frac{8}{7}$ $\frac{7}{6}$ $\frac{20}{17}$ $\frac{32}{27}$ $\frac{6}{5}$ $\frac{28}{23}$ $\frac{16}{13}$ $\frac{5}{4}$ $\frac{24}{19}$ $\frac{14}{11}$ $\frac{32}{25}$ $\frac{4}{3}$

epimoria; 32/31 217/216 162/161 115/114 76/75 105/104 78/77 88/87 145/144 126/125 50/49 49/48 120/119 136/135 81/80 70/69 92/91 65/64 96/95 133/132 176/175 25/24

Compare this with the $\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangle. There are direct ties.

* Note; The horizontal rows display the aliquot (subharmonic) divisions of the 1/4-string (tetrachord). Augusto Novaro 1927 & 1951.

(

15SEP2000·EW

$\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangularis (Truncate Lambdoma)

0/0, 4/4, 12/11, 20/19, 36/33, 48/45, 72/66, 88/82, 120/110

* ROW 0	0/0								
1	4/4	4/3							
2	8/8	8/7	8/6						
3	12/12	12/11	12/10	12/9					
4	16/16	16/15	16/14	16/13	16/12				
5	20/20	20/19	20/18	20/17	20/16	20/15			
6	24/24	24/23	24/22	24/21	24/20	24/19	24/18		
7	28/28	28/27	28/26	28/25	28/24	28/23	28/22	28/21	
8	32/32	32/31	32/30	32/29	32/28	32/27	32/26	32/25	32/24

reduced; 4/4 32/31 28/27 24/23 20/19 16/15 14/13 12/11 32/29 10/9 28/25 8/7 7/6 20/17 32/27 6/5 28/23 16/13 5/4 24/19 14/11 32/25 4/3

epimoria; 32/31 217/216 162/161 115/114 76/75 105/104 78/77 88/87 145/144 126/125 50/49 49/48 120/119 136/135 81/80 70/69 92/91 65/64 96/95 133/132 176/175 25/24

Compare this with the $\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangle. There are direct ties.

* Note; The horizontal rows display the aliquot (subharmonic) divisions of the 1/4-string (tetrachord). Augusto Novaro 1927 & 1951. Barbara Hero.

15SEP2000·EW

$\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangularis (Truncate Lambdoma)

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ROW	
* ROW 0	$\frac{0}{0}$
1	$\frac{4}{4}$ $\frac{4}{3}$
2	$\frac{8}{8}$ $\frac{8}{7}$ $\frac{8}{6}$
3	$\frac{12}{12}$ $\frac{12}{11}$ $\frac{12}{10}$ $\frac{12}{9}$
4	$\frac{16}{16}$ $\frac{16}{15}$ $\frac{16}{14}$ $\frac{16}{13}$ $\frac{16}{12}$
5	$\frac{20}{20}$ $\frac{20}{19}$ $\frac{20}{18}$ $\frac{20}{17}$ $\frac{20}{16}$ $\frac{20}{15}$
6	$\frac{24}{24}$ $\frac{24}{23}$ $\frac{24}{22}$ $\frac{24}{21}$ $\frac{24}{20}$ $\frac{24}{19}$ $\frac{24}{18}$
7	$\frac{28}{28}$ $\frac{28}{27}$ $\frac{28}{26}$ $\frac{28}{25}$ $\frac{28}{24}$ $\frac{28}{23}$ $\frac{28}{22}$ $\frac{28}{21}$
8	$\frac{32}{32}$ $\frac{32}{31}$ $\frac{32}{30}$ $\frac{32}{29}$ $\frac{32}{28}$ $\frac{32}{27}$ $\frac{32}{26}$ $\frac{32}{25}$ $\frac{32}{24}$

4-fold Lambdoma

reduced; $\frac{4}{4}$ $\frac{32}{31}$ $\frac{28}{27}$ $\frac{24}{23}$ $\frac{20}{19}$ $\frac{16}{15}$ $\frac{14}{13}$ $\frac{12}{11}$ $\frac{32}{29}$ $\frac{10}{9}$ $\frac{28}{25}$ $\frac{8}{7}$ $\frac{7}{6}$ $\frac{20}{17}$ $\frac{32}{27}$ $\frac{6}{5}$ $\frac{28}{23}$ $\frac{16}{13}$ $\frac{5}{4}$ $\frac{24}{19}$ $\frac{14}{11}$ $\frac{32}{25}$ $\frac{4}{3}$

epimoria; 32/31 217/216 162/161 115/114 76/75 105/104 78/77 88/87 145/144 126/125 50/49 49/48 120/119 136/135 81/80 70/69 92/91 65/64 96/95 133/132 176/175 25/24

Compare this with the $\left(\frac{4}{4}, \frac{4}{3}\right)$ Triangle. There are direct ties.

* Note; The horizontal rows display the aliquot (subharmonic) divisions of the 1/4-string (tetrachord). Augusto Novaro 1927 & 1951.

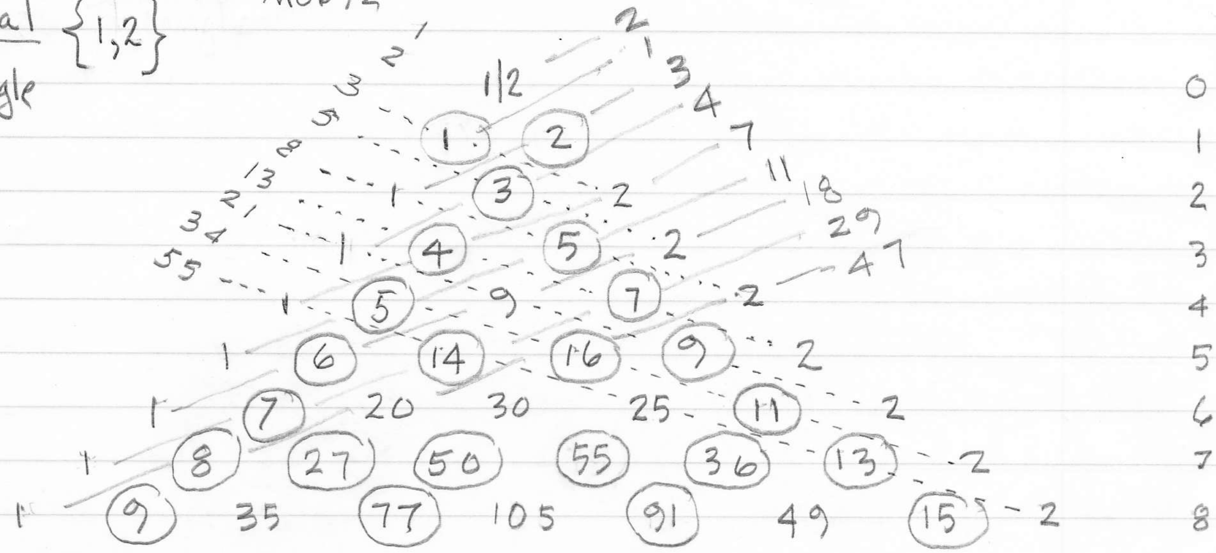
15SEP2000·EW

split header

Prolog

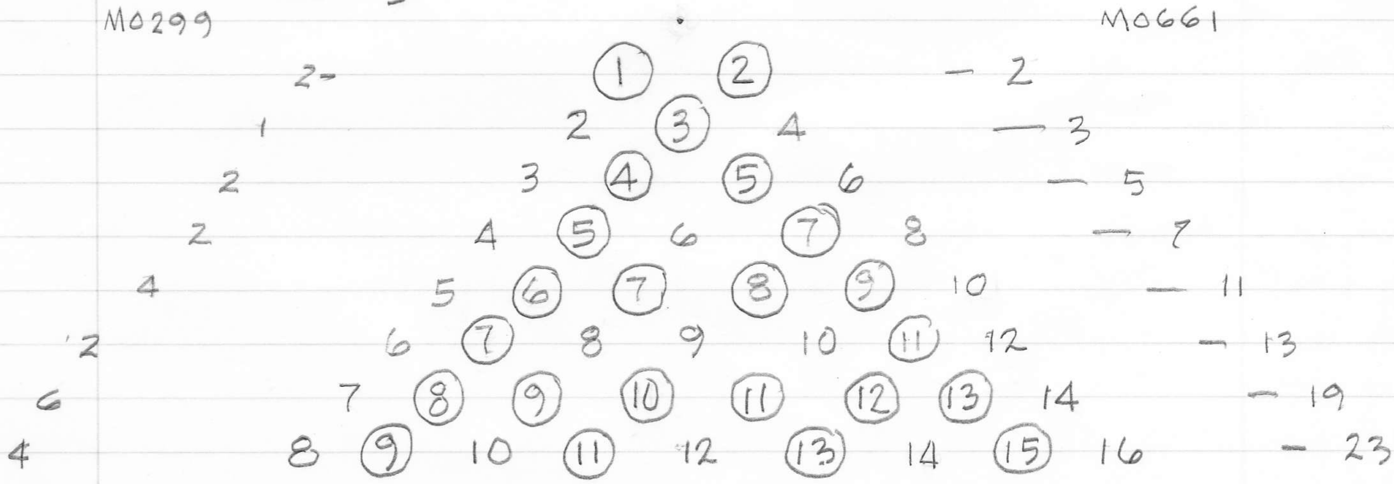
Triangle
Pascal $\{1,2\}$
Triangle

M0692 M2341



Lambda
Novaro $\{1,2\}$

M0299 M0661



$\frac{1}{1}\ \frac{9}{8}\ \frac{8}{7}\ \frac{7}{6}\ \frac{6}{5}\ \frac{5}{4}\ \frac{9}{7}\ \frac{4}{3}\ \frac{11}{8}\ \frac{7}{5}\ \frac{10}{7}\ \frac{3}{2}\ \frac{11}{7}\ \frac{8}{5}\ \frac{13}{8}\ \frac{5}{3}\ \frac{12}{7}\ \frac{7}{4}\ \frac{9}{5}\ \frac{11}{6}\ \frac{13}{7}\ \frac{15}{8}\ \frac{2}{1}$

$\frac{9}{8}\ \frac{64}{63}\ \frac{49}{48}\ \frac{36}{35}\ \frac{25}{24}\ \frac{36}{35}\ \frac{28}{27}\ \frac{33}{32}\ \frac{56}{55}\ \frac{50}{49}\ \frac{21}{20}\ \frac{22}{21}\ \frac{56}{55}\ \frac{65}{64}\ \frac{40}{39}\ \frac{36}{35}\ \frac{49}{48}\ \frac{36}{35}\ \frac{55}{54}\ \frac{78}{77}\ \frac{105}{104}\ \frac{16}{15}$

Novaro nested Escalas Harmonicas. Where $\frac{a}{b}\ \frac{c}{d}$ adjacent, $bc-ad=1$.
(Diaphantus)

3Mar04·EW

0

1 2

2 3 4

3 4 5 6

4 5 6 7 8

5 6 7 8 9 10

6 7 8 9 10 11 12

7 8 9 10 11 12 13 14

8 9 10 11 12 13 14 15 16

0

16 15

32 31 30

48 47 46 45

64 63 62 61 60

80 79 78 77 76 75

96 95 94 93 92 91 90

112 111 110 109 108 107 106 105

128 127 126 125 124 123 122 121 120

Continuing The Triangle / Lambda Equivalence (work in progress)

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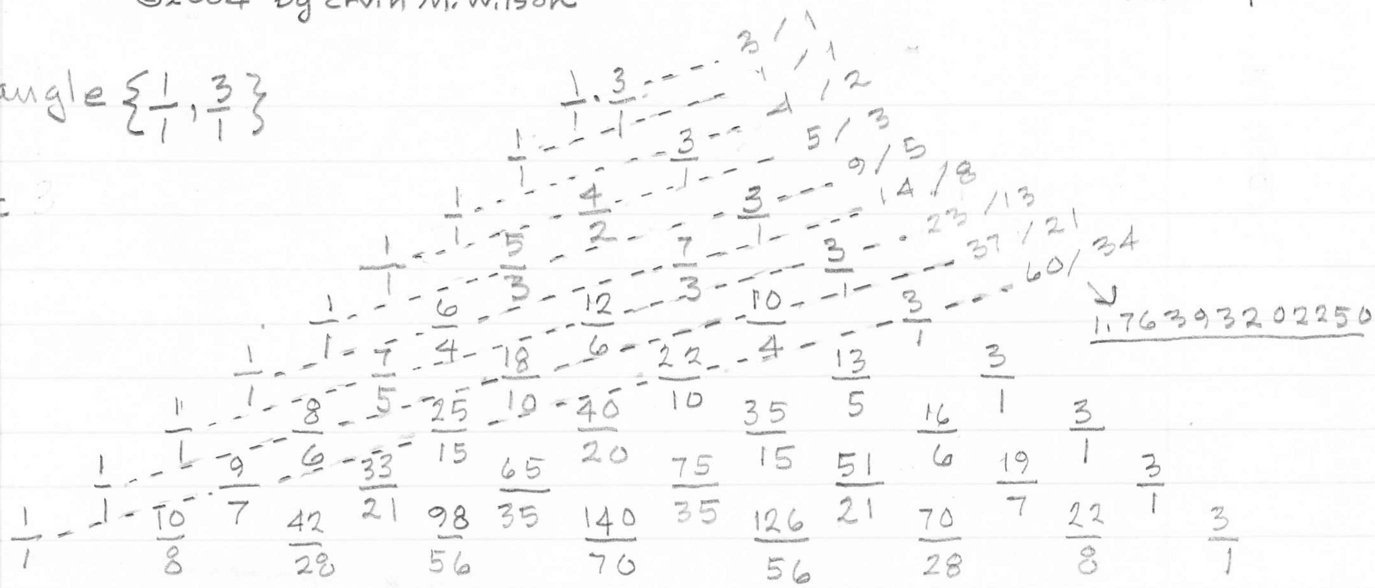
14 Mar 04. EW

Triangle $\{\frac{1}{1}, \frac{3}{1}\}$

1/x	
1.763932	
1	.7639
1	.3090
3	.2360
4	.2360
4	.2360
4	.2360
4	.2360

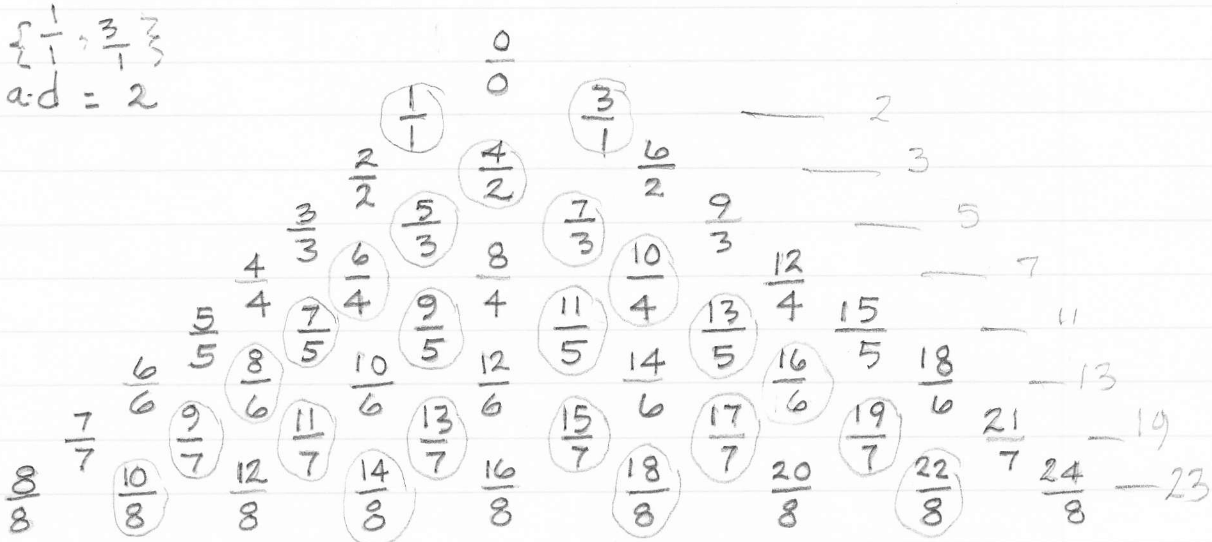
etc (.236067978)



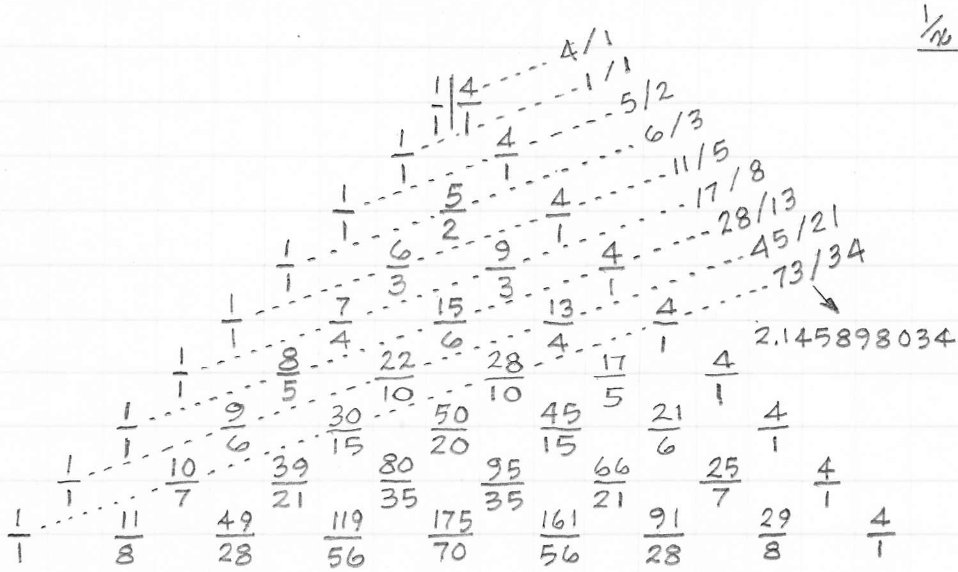
Ref. Simple Repeating ZigZag Patterns On The Wilson Scale-Tree, 1991 & 1993 by Larry A. Hanson

Lambda $\{\frac{1}{1} \cdot \frac{3}{1}\}$

$b \cdot c - a \cdot d = 2$

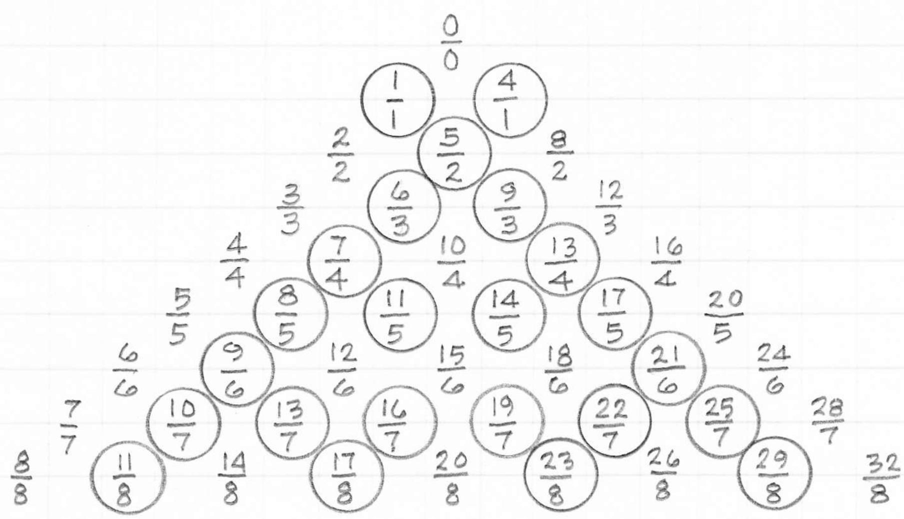


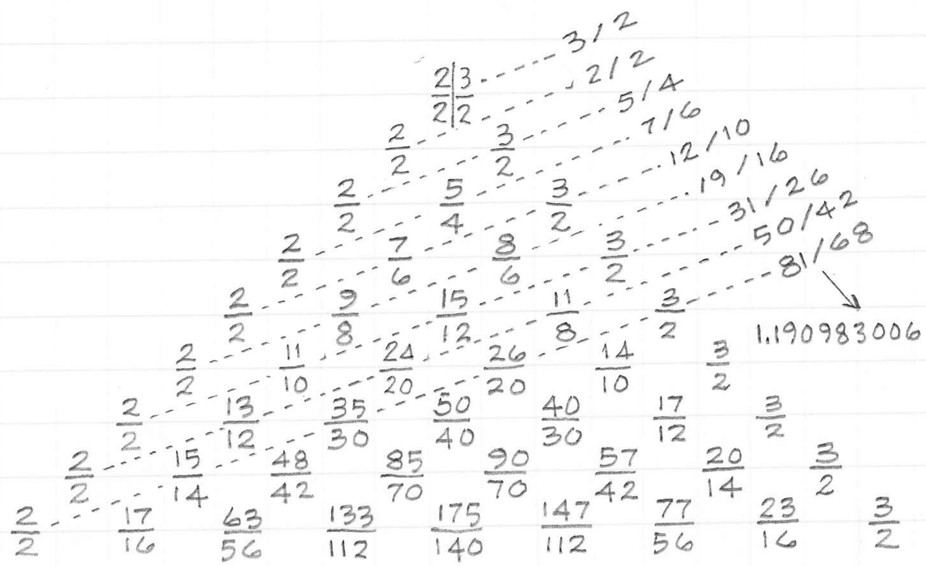
Ref. Sistema Natural de la Música, 1951 Augusto Novaro, p29



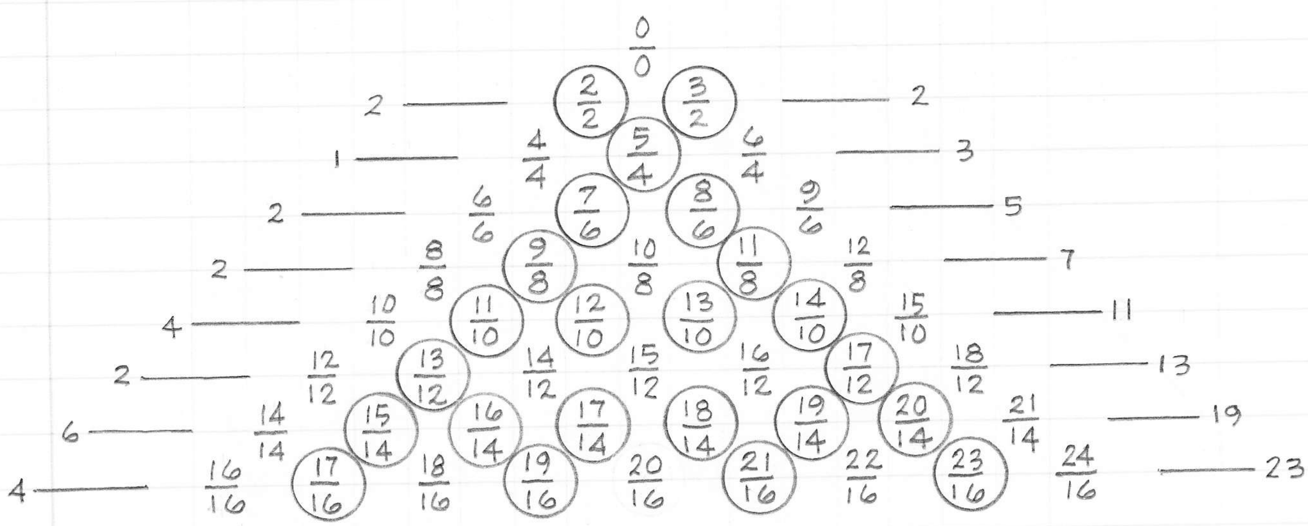
$1/n$ Pattern

2	.145
6	.854
1	.170
5	.854
1	.170
5	.854
1	.170
5	.854
1	.170
5	.854





1/n pattern	
1	.190
5	.236
4	.236
4	.236
4	.236
4	.236
4	.236



2/2 | 5/2

2/2   5/2

2/2   7/4   5/2

2/2   9/6   12/6   5/2

2/2   11/8   21/12   17/8   5/2

2/2   13/10   32/20   38/20   22/10   5/2

2/2   15/12   45/30   70/40   60/30   27/12   5/2

2/2   17/14   60/42   115/70   130/70   87/42   32/14   5/2

2/2   19/16   77/56   175/112   245/140   217/112   119/56   37/16   5/2

5/2   2/2   7/4   9/16   16/10   25/16   41/26   66/42   107/68

1.572949017

1/x Pattern	
1	.572
1	.745
1	.341
2	.927
1	.078
12	.708
1	.412
2	.427
2	.341
2	.927
1	.078

0/0

2/2   5/2

4/4   7/4   10/4

6/6   9/6   12/6   15/6

8/8   11/8   14/8   17/8   20/8

10/10   13/10   16/10   19/10   22/10   25/10

12/12   15/12   18/12   21/12   24/12   27/12   30/12

14/14   17/14   20/14   23/14   26/14   29/14   32/14   35/14

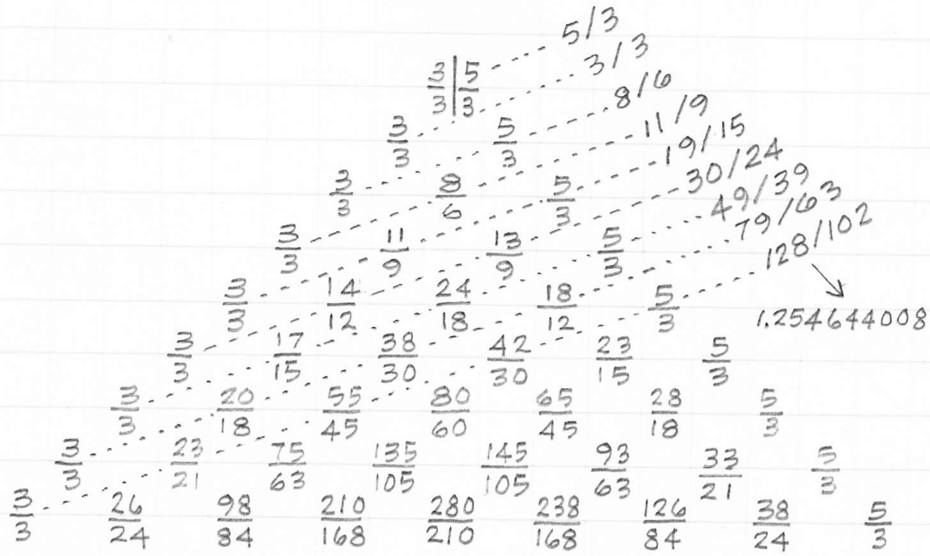
16/16   19/16   22/16   25/16   28/16   31/16   34/16   37/16   40/16

36/   39/18   42/18

44/20

etc

5/3



1/x Pattern	
1	.254
3	.927
1	.078
12	.708
1	.412
2	.427
2	.341
2	.927
1	.078
12	.708

Diaphonic cycles p. 158

Expansions p. 135

Summation Tetrachords p. 32

Diagram 2-4 p p. 12 is Novaroan ☆ 1927 1953

Bingo <u>Indexed</u> <u>Genera</u> P27 is Novaroan ★ 1927 1953

Note: <u>Arabic</u> theorists described the Tetrachords from the simplest string length. Example: 21 22 24 28 which is found at <u>28</u> 27 26 25 <u>24</u> 23 <u>22</u> <u>21</u>. Which is part of the series: (a Novaroan Triangle)

2	4								3	2
	8			7					6	3
2	12	11 11			10				9	5
2	16	15	14		13				12	7
4	20	19	18	17		16			15	11
2	24	23	22	21	20	19			18	13
6	28	27	26	25	24	23	22		21	← 19
4	32	31	30	29	28	27	26	25	24	23
	36								27	
	40								30	
	44								33	
	48								36	

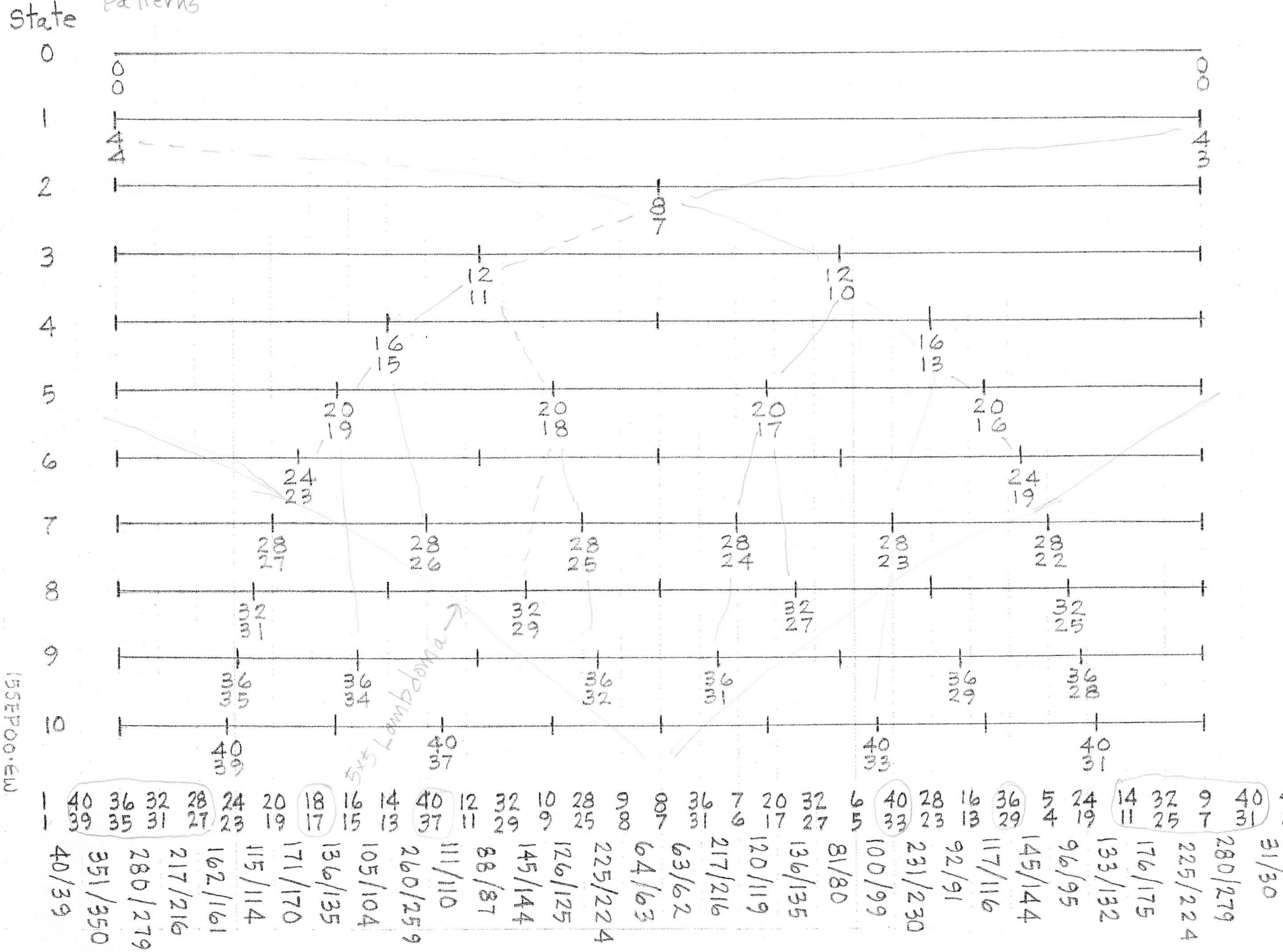
and so forth

The co-prime pattern is as shown encircled O.

1 Feb 2004 · EW } about
2 Feb } midnight

Neo-Pythagorean string-length theorists set the stage for Lambdoma, Pascals Triangle (Meru) Farey Series, Peirce/Stern-Brocot Series (Scale-Tree), Fibonacci Series, & Biaxial Co-prime Patterns

Aliquot Divisions of 1/4-String, thru 10



Continuing The Triangle/Lambda Equivalence

15Feb04.EW

Lambda $\left\{\frac{15}{15}, \frac{16}{15}\right\}$

$\frac{0}{0}$

2 $\quad \frac{15}{15} \quad \frac{16}{15} \quad$ 2

1 $\quad \frac{30}{30} \quad \frac{31}{30} \quad \frac{32}{30} \quad$ 3

2 $\quad \frac{45}{45} \quad \frac{46}{45} \quad \frac{47}{45} \quad \frac{48}{45} \quad$ 5

2 $\quad \frac{60}{60} \quad \frac{61}{60} \quad \frac{62}{60} \quad \frac{63}{60} \quad \frac{64}{60} \quad$ 7

4 $\quad \frac{75}{75} \quad \frac{76}{75} \quad \frac{77}{75} \quad \frac{78}{75} \quad \frac{79}{75} \quad \frac{80}{75} \quad$ 11

2 $\quad \frac{90}{90} \quad \frac{91}{90} \quad \frac{92}{90} \quad \frac{93}{90} \quad \frac{94}{90} \quad \frac{95}{90} \quad \frac{96}{90} \quad$ 13

4 $\quad \frac{105}{105} \quad \frac{106}{105} \quad \frac{107}{105} \quad \frac{108}{105} \quad \frac{109}{105} \quad \frac{110}{105} \quad \frac{111}{105} \quad \frac{112}{105} \quad$ 19

4 $\quad \frac{120}{120} \quad \frac{121}{120} \quad \frac{122}{120} \quad \frac{123}{120} \quad \frac{124}{120} \quad \frac{125}{120} \quad \frac{126}{120} \quad \frac{127}{120} \quad \frac{128}{120} \quad$ 23

Triangle $\left\{\frac{15}{15}, \frac{16}{15}\right\}$

$\frac{15}{15}, \frac{16}{15}$

$\frac{15}{15} \quad \frac{31}{30} \quad \frac{16}{15}$

$\frac{15}{15} \quad \frac{46}{45} \quad \frac{47}{45} \quad \frac{16}{15}$

$\frac{15}{15} \quad \frac{61}{60} \quad \frac{93}{90} \quad \frac{63}{60} \quad \frac{16}{15}$

$\frac{15}{15} \quad \frac{76}{75} \quad \frac{154}{150} \quad \frac{156}{150} \quad \frac{79}{75} \quad \frac{16}{15}$

$\frac{15}{15} \quad \frac{91}{90} \quad \frac{230}{225} \quad \frac{310}{300} \quad \frac{235}{225} \quad \frac{95}{90} \quad \frac{16}{15}$

$\frac{15}{15} \quad \frac{106}{105} \quad \frac{321}{315} \quad \frac{540}{525} \quad \frac{545}{525} \quad \frac{330}{315} \quad \frac{111}{105} \quad \frac{16}{15}$

$\frac{15}{15} \quad \frac{121}{120} \quad \frac{427}{420} \quad \frac{861}{840} \quad \frac{1{,}085}{1{,}050} \quad \frac{875}{840} \quad \frac{441}{420} \quad \frac{127}{120} \quad \frac{16}{15}$

Continuing The Triangle/Lambda Equivalence

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work in progress

Ref Novaro 1927 & 1951, Escalas Harmonicas

15Feb04.ew

Lambda $\left\{\frac{15}{15}, \frac{16}{15}\right\}$

bc−ad = 15 which reduces to bc−ad = 1

This figure is very similar to the Farey Series and to the Scale Tree (Peirce Series)

0/0
15/15 16/15
30/30 31/30 32/30
45/45 46/45 47/45 48/45
60/60 61/60 62/60 63/60 64/60
75/75 76/75 77/75 78/75 79/75 80/75
90/90 91/90 92/90 93/90 94/90 95/90 96/90
105/105 106/105 107/105 108/105 109/105 110/105 111/105 112/105
120/120 121/120 122/120 123/120 124/120 125/120 126/120 127/120 128/120

2 2 2 4 2 4 4

2 3 5 7 11 13 19 23

15/15 121/120 106/105 91/90 76/75 61/60 107/105 46/45 123/120 77/75 108/105 31/30 109/105 79/75 125/120 47/45 110/105 63/60 79/75 95/90 111/105 127/120 16/15

121/120 848/847 637/636 456/455 305/304 428/427 322/321 369/368 616/615 540/539 217/216 218/217 546/545 625/624 376/375 330/329 441/440 316/315 475/474 666/665 889/888 128/127

Triangle $\left\{\frac{15}{15}, \frac{16}{15}\right\}$

Triangle $\left\{\frac{15}{15}, \frac{16}{15}\right\}$

This figure is similar to Pascals triangle

15/15 16/15 … 16/15
… 15/15
15/15 16/15 … 31/30
15/15 31/30 16/15 … 46/45
15/15 46/45 47/45 16/15 … 77/75
15/15 61/60 93/90 63/60 16/15 … 123/120
15/15 76/75 154/150 156/150 79/75 16/15 … 200/195
15/15 91/90 230/225 310/300 235/225 95/90 16/15 … 323/315
15/15 106/105 321/315 540/525 545/525 330/315 111/105 16/15 … 523/510
16/15 121/120 427/420 861/840 1,085/1,050 875/840 441/420 127/120 16/15

etc to 1,025464401

Novaroan Triangle

2-4. *Genesis of the enharmonic pykna by katapyknosis. In principle, all pyknotic divisions can be generated by this process, although very high multipliers may be necessary in some cases. The ones shown are merely illustrative. See the Catalogs for the complete list. (1x) The basic form is the enharmonic trichord, or major third pentatonic, often ascribed to Olympos. (2x) Didymos's enharmonion, a "weak" form. (3x) Ptolemy's enharmonion, a "strong" form. To comply with Greek melodic canons, it was reordered as 46/45 · 24/23 · 5/4. (4x) Serre's enharmonic, sometimes attributed to Tartini, and discussed by Perrett (1926, 26). Pachymeres may be the earliest source. (5x) Author's enharmonic, also on Hofmann's list of superparticular divisions. (6x) Wilson's enharmonic, also on Hofmann's list of superparticular divisions.*

INDEX	NUMBERS	PYKNA
1x	16 15	16/15
2x	32 31 30	32/31 · 31/30
3x	48 47 46 45	24/23 · 46/45
4x	64 63 62 61 60	64/63 · 21/20
5x	80 79 78 77 76 75	20/19 · 76/75
6x	96 95 94 93 92 91 90	96/95 · 19/18

This is a very Novaroan image.
See Lambda $\{\cancel{16}\ \frac{15}{15}, \frac{16}{15}\}$

Aristoxenos's hemiolic chromatic and may descend from neutral third pentatonics such as Winnington-Ingram's reconstruction of the *spondeion* or libation mode (Winnington-Ingram 1928 and chapter 6), if Sachs's ideas on the origin of the genera have any validity (Sachs 1943). In any case, the scale is a beautiful sequence of intervals and has been used successfully by both Harry Partch *(Windsong, Daphne of the Dunes)* and Lou Harrison, the latter in an improvisation in the early 1970s.

Ptolemy returned to the use of the number seven in his chromatic and soft diatonic genera and introduced ratios of eleven in his intense chromatic and equable diatonic. These tetrachords appear to be in agreement with the musical reality of the era, as most of the scales described as contemporary tunings for the lyra and kithara have septimal intervals (6-4).

Ptolemy's intense diatonic is the basis for Western European just intonation. The Lydian or C mode of the scale produced by this genus is the European major scale, but the *minor mode* is generated by the intervallic retrograde of this tetrachord, 10/9 · 9/8 · 16/15. This scale is not identical to the Hypodorian or A mode of 12-tone equally tempered, meantone, and Pythagorean intonations. (For further discussion of this topic, see chapters 6 and 7.)

The numerical technique employed by Eratosthenes, Didymos, and Ptolemy to define the majority of their tetrachords is called *linear division* and may be identified with the process known in Greek as *katapyknosis*. Katapyknosis consists of the division, or rather the filling-in, of a musical interval by multiplying its numerator and denominator by a set of integers of increasing magnitude. The resulting series of integers between the extreme terms generates a new set of intervals of increasingly smaller span as the multiplier grows larger. These intervals form a series of microtones which are then recombined to produce the desired melodic division, usually composed of epimore ratios. The process may be seen in 2-4 where it is applied to the enharmonic *pyknotic* interval 16:15. By extension, the pyknon may also be termed the katapyknosis (Emmanuel 1921). It consists of three notes, the *barypyknon*, or lowest note, the *mesopyknon*, or middle note, and the *oxypyknon*, or highest.

The harmoniai of Kathleen Schlesinger are the result of applying katapyknosis to the entire octave, 2:1, and then to certain of the ensuing intervals. In chapter 4 it is applied to the fourth to generate *indexed genera*.

The divisions of Eratosthenes and Didymos comprise mainly 1:1 divi-

Indexed Genera used by Novaro!

is the 3/2's complement of 11/9 and 52/45 the 4/3's complement of 15/13. Various genera were then constructed by dividing the pykna or apykna by linear division into two or three parts to produce 1:1, 1:2, and 2:1 divisions. Both the 1:2 and 2:1 divisions were made to locate genera composed mainly of superparticular ratios. Even Ptolemy occasionally had to reorder the intervals resulting from triple division before recombining two of them to produce the two intervals of the pyknon (2-2 and 2-4). More complex divisions were found either by inspection or by katapyknosis with larger multipliers.

4-2. *Indexed genera. The terms 4 and 3 which represent the 1/1 and 4/3 of the final tetrachord are multiplied by the index. The lefthand sets of tetrachords are those generated by selecting and recombining the successive intervals resulting from the additional terms after the multiplication. The righthand sets of tetrachords have been reduced to lowest terms and ordered with the CI uppermost.*

MULTIPLIER: 4	TERMS: 16 15 14 13 12
16/15 · 15/14 · 14/12	16/15 · 15/14 · 7/6
16/15 · 15/13 · 13/12	16/15 · 13/12 · 15/13
16/14 · 14/13 · 13/12	14/13 · 13/12 · 8/7
MULTIPLIER: 5	TERMS: 20 19 18 17 16 15
20/19 · 19/18 · 18/15	20/19 · 19/18 · 6/5
20/19 · 19/17 · 17/15	20/19 · 19/17 · 17/15
20/19 · 19/16 · 16/15	20/19 · 16/15 · 19/16
20/18 · 18/17 · 17/15	18/17 · 10/9 · 17/15
20/18 · 18/16 · 16/15	16/15 · 10/9 · 9/8
20/17 · 17/16 · 16/15	17/16 · 16/15 · 20/17
MULTIPLIER: 6	TERMS: 24 23 22 21 20 19 18
24/23 · 23/22 · 22/18	24/23 · 23/22 · 11/9
24/23 · 23/21 · 21/18	24/23 · 23/21 · 7/6
24/23 · 23/20 · 20/18	24/23 · 10/9 · 23/20
24/23 · 23/19 · 19/18*	24/23 · 19/18 · 23/19
24/22 · 22/21 · 21/18	22/21 · 12/11 · 7/6
24/22 · 22/20 · 20/18	12/11 · 11/10 · 10/9
24/22 · 22/19 · 19/18	19/18 · 12/11 · 22/19
24/21 · 21/20 · 20/18	21/20 · 10/9 · 8/7
24/21 · 21/19 · 19/18	19/18 · 21/19 · 8/7
24/20 · 20/19 · 19/18	20/19 · 19/18 · 6/5

* *see Catalog number 536.*

* Indexed genera

One useful technique, originated by Ervin Wilson, is a variation of the katapyknotic process. In 4-2 this technique is applied to the 4/3 rather than to the pyknon (as it was in 2-4). The 1/1 and 4/3 of the undivided tetrachord are expressed as 3 and 4, and are multiplied by a succession of numbers of increasing magnitude. The new terms resulting from such a multiplication and all the intermediate numbers define a set of successive intervals which may be sequentially recombined to yield the three intervals of tetrachords. I have termed the multiplier, the index, and the resulting genera *indexed genera*. The intermediate terms are a sequence of arithmetic means between the extremes.

The major shortcoming of this procedure is that the number of genera grows rapidly with the index. There are 120 genera of index 17, and not all of these are worth cataloguing, since other genera of similar melodic contours and simpler ratios are already known and tabulated. The technique is still of interest, however, to generate sets of tetrachords with common numerical relations for algorithmic composition.

Pentachordal families

Archytas's genera were devised so that they made the interval 7/6 between their common first interval, 28/27, and the note a 9/8 below the first note of the tetrachord (Erickson 1965; Winnington-Ingram 1932; see also 6-1). Other first intervals *(x)* may be chosen so that in combination with the 9/8 they generate harmonically and melodically interesting intervals. These intervals may be termed *pentachordal intervals (PI)* as they are part of a pentachordal, rather than a tetrachordal tonal sequence. Three such groups or families of tetrachords are given in 4-3 along with their initial and pentachordal intervals.

* A long time ago I recall seeing the 6-fold permutations of the tetrachords each ~~all~~ spelled out in the most economical string-lengths — That was at UCLA music library and possibly in Erlanger. This would tend to give rise to "indexed genera", (and to my facility in that idiom). E.W.