MULTIPLE DIVISION OF THE OCTAVE AND THE TONAL RESOURCES OF 19-TONE TEMPERAMENT

BY

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PREFACE

The proper role of the music theorist and the proper relationship of musical theory to practice have been subjects of frequent controversy. Much of the preoccupation of music theory is with the music of the past. Speculation on a possible future course for music involves the risk of serious error. The breaking of ground for new musical patterns of construction seems a particularly speculative preoccupation, and yet a number of theorists have accepted just this as their primary task.

It is the basic presupposition behind this study that inquiry into possible new directions for musical development represents theory in its most creative and constructive role. It would seem reasonable to suppose that in a field as vast and relatively unexplored as the multiple division of the octave there must be misconceptions to be swept away and complications to be simplified so that the creative musician might eventually be able to make a choice based neither on ignorance nor on fear of the unknown.

It was the initial purpose of this study to make a thorough investigation of one of the possible kinds of multiple division, 19-tone equal temperament, only. It became apparent on investigation, however, that the entire field of multiple division had received insufficient comprehensive examination or evaluation since Bosanquet in 1875, and that as specific a study as was contemplated would be without any link to the body of general knowledge. There is almost nothing in recent literature more specific than the single-chapter references of J. Murray Barbour[1] yet more general than the passionate advocacy of individual systems by such writers as Würschmidt, Wyschnegradsky, Fokker, and Partch.[2]

My purpose is, therefore, to present a general picture of the field of multiple division of the octave as it exists today. In partial retention of the original intent of the study, 19-tone temperament will be emphasized and studied in more detail than the other systems.

WHAT IS 19-TONE TEMPERAMENT?

19-tone temperament, as its name indicates, is the division of the octave into nineteen equal parts instead of the twelve equal parts into which it is divided in common

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[1] J. Murray Barbour, Tuning and Temperament, the most widely recognized work in this country on tuning systems in general.

[2] The works of these writers are listed in the bibliography and are studied in the various chapters of this text. It should be noted that several books have dealt at greater than one-chapter length with multiple division. Augusto Novarro's Sistema Natural de la Música contains many interesting insights into a number of recently explored systems of multiple division, while Partch and Fokker have looked with considerable sympathy into proposed systems of multiple division other than their own. Their works are useful to the student of multiple division as primers, but they explore at best only a few of the problems and possibilities inherent in the field as a whole.

practice. Since the octave is an abstract quality produced by two sounds whose vibration frequencies are to one another as the relationship 2 to 1, it follows that the octave can be divided into as many parts as one wishes, just as it is possible to divide the abstract number 2 into as many fractional parts as one wishes.[3]

The vibration ratio of the smallest interval in 19-tone temperament is approximately equal to the superparticular ratio 28:27, and exactly equal to the repeating decimal fraction $\overline{63.1578947368421052}$ cents.[4] The interval in 12-tone temperament which has its closest equivalent in 19-tone temperament is the perfect fifth (together with its inversion the perfect fourth), which is approximately 5¼ cents larger than its 19-tone equivalent (which will also be called a "perfect fifth"). To find the discrepancy between corresponding intervals of the two temperaments it is possible to use the circle of fifths, adding 5¼ cents for each fifth used.

There are many ways the intervals might be named and

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[3]These subdivisions of the number 2 may be equal arithmetically, geometrically, or exponentially, or they may be not equal by any method. In 19-tone temperament the subdivisions are geometrically equal, since the systems represents the 19-fold geometric division of the ratio 2:1. The smallest unit is $\sqrt[19]{2}$.

[4]This figure is given by Charles S. Kent in *An Introduction to Tuning and Temperament*. He gives the value in cents for the smallest interval of all of the commonly proposed temperaments, in every case obtaining a repeated decimal fraction.

the pitches notated. As it would seem particularly to be desired that this study be easily understood by musicians whose entire experience has been with standard diatonic nomenclature, this same nomenclature will be applied to 19-tone temperament throughout this study. Traditional enharmonic equivalences must, in this process, be cancelled out while other enharmonic equivalences are created. C# and Db are, in 19-tone temperament, distinct pitches, while E# and Fb are one and the same. In Example 1, below, the names of pitches and intervals in the 19-tone system are compared to those of 12-tone temperament.

PRELIMINARY ABBREVIATIONS AND DEFINITIONS

It has proved necessary to use a few uncommon abbreviations or new words for concepts which must be referred to repeatedly and the full description of which in customary language would be most cumbersome.

The smallest interval in any equal-tempered system will be referred to as a unit of that particular system, since the term "semitone" is ambiguous where more than 12 tones are used.

Since it is often necessary to refer to intervals both as units of a temperament and as just ratios, the following procedure has been adopted. Whenever a just ratio is intended, the two numbers are separated by a colon:

EXAMPLE 1

19-TONE

Name of tone	Interval from C	Cents
C	perfect prime	00
C#	aug.prime-dim. 2nd	63
Db	minor second	126
D	major second	189
D#	aug.2nd-dim. 3rd	253
Eb	minor third	316
E	major third	379
E#-Fb	aug.3rd-dim. 4th	442
F	perfect fourth	505
F#	augmented fourth	568
Gb	diminished fifth	632
G	perfect fifth	695
G#	aug.5th-dim. 6th	758
Ab	minor sixth	821
A	major sixth	884
A#	aug.6th-dim. 7th	947
Bb	minor seventh	1011
B	major seventh	1074
B#-Cb	aug.7th-dim. 8th	1137
c	perfect octave	1200

12-TONE

Cents	Interval from C	Name of tone
00	perfect prime	C
100	minor second	Db (C#)
200	major second	D
300	minor third	Eb (D#)
400	major third	E
500	perfect fourth	F
600	aug. 4th; dim. 5th	F#-Gb
700	perfect fifth	G
800	minor sixth	Ab (G#)
900	major sixth	A
1000	minor seventh	Bb (A#)
1100	major seventh	B
1200	perfect octave	c

3:2. Whenever a logarithmic fraction is intended, the two numbers are separated by a virgule: 7/12. In the absence of any designation of which interval such a configuration of numbers is the logarithmic fraction, it may be assumed that it is of an octave.

Terms pertaining to specific chapters will be introduced at the beginnings of those chapters, and I have tried to include all unfamiliar concepts and terms in a glossary which appears on page 443.