PART III

HYPOTHETICAL AND EXPERIMENTAL MATERIALS

CHAPTER 14

A THEORY OF HARMONY BASED ON SYMMETRICAL SCALES WITH QUASI-EQUAL INTERVALS IN NINETEEN-TONE TEMPERAMENT

In this section, I present what I consider, from careful examination of the proposals already made and from my own laboratory experiments, to be a particularly fruitful line of inquiry into the possibilities of 19-tone temperament. It is, needless to say, subject to disagreement at any point, and it makes no pretense at exhausting any but one branch of the possibilities inherent within 19-tone temperament. It is hoped that the inclusion of such a theory will further stimulate thought and experimentation in 19-tone temperament.

THE GAMUT

The gamut is drawn from natural intervals, which are tempered to facilitate transposition and modulation. In its origin the system is bi-intervalic (tri-intervalic counting the octave). The two basic intervals are the perfect fifth (3:2) chosen because of its dominant position in the overtone series, and the minor third (6:5) chosen because among the intervals of the senario it is most superbly suited to the peculiar features of 19-tone temperament, falling short of the interval created by five units of the temperament by only 0.15 cent. Starting from a central tone, the gamut is created by adding single fifths above and below, and by adding two superimposed minor thirds above and three below each tone thereby obtained. Eighteen tones, including the central tone, are produced in this manner. The nineteenth tone may be derived in any of several ways. The 19-tone system derived in this manner is shown, above, in Example 47, p 324 Chapter 12. Its intervals are compared with those of 19-tone equal temperament in Example 45. p 314

See also p 324

CONSONANCE AND DISSONANCE

The two basic intervals are consonances, as are their octave inversions. Also consonant are all combinations of the basic intervals used singly, and their inversions. All other intervals are either neutral or dissonant.[1] The eight consonances within the 19-tone system are:

3:2 Perfect fifth (11/19)
4:3 Perfect fourth (8/19)

6:5 Minor third (5/19)
5:3 Major sixth (14/19)

---

[1]There are several reasons for the selection of this particular group of intervals as consonant. They are, (1) structural: since these are the intervals made from the starting point of two basic intervals each used singly; (2) acoustic: these intervals represent unquestionably the simplest ratios which can be approximated with any degree of accuracy in 19-tone temperament; and (3) evolutional: these are the intervals which most closely resemble their counterparts in 12-tone temperament, counterparts which themselves are regarded as the least dissonant intervals in 12-tone temperament.

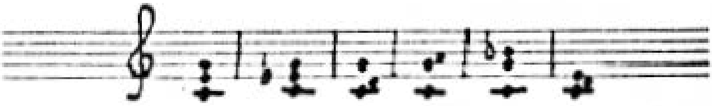
9:5 Minor seventh (16/19) (Perfect fifth plus minor third)
10:9 Major second (3/19)

5:4 Major third (6/19) (Perfect fifth minus minor third)
8:5 Minor sixth (13/19)

No differentiation is made between perfect and imperfect consonances as concerns the single intervals.

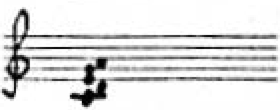
HARMONIC SONORITIES

All chords containing only consonant intervals are considered perfect. This includes the following six triads, with their inversions,



the following four tetrads with their inversions,

and a single pentad, representing what is generally considered to be the pentatonic scale. The perfect sonorities all have close equivalents in music based on 12-tone temperament.



Dissonant are the two smallest intervals of the system and their inversions. On these Yasser and Ariel agree, and through an agreement to treat them as dissonances the basic melodic and harmonic units are made distinct.[2] The remaining six intervals, representing 4/19, 7/19, and 9/19 and their inversions, are regarded as neutral. Sonorities containing neutral intervals but no dissonant intervals shall be regarded as imperfect consonances, while those containing dissonant intervals shall be regarded as dissonances. The imperfect consonances, which include sonorities with those intervals unique to 19-tone temperament, are the characteristic sounds of this temperament which distinguish it from other musical systems. They will be analyzed later in some detail.

THE FORMATION OF SCALES

Scales can be formed, of course, by any number of tones within the octave, at any sequence of distances from one another. The systems considered here will all have in common that they are based on scales containing intervals as equal as is possible within a 19-tone system. Since 19 is a prime number, 19-tone temperament contains no equal-interval scale within it such as might be likened to the

---

[2]Just as the major and minor seconds in 12-tone temperament are distinguished by the intensity of their dissonances (the latter considered the more severe), 1/19 and its inversion are considered to be more dissonant than 2/19 and its inversion.

whole-tone scale in 12-tone temperament. But, as the diatonic scale in 12-tone temperament is composed only of the intervals 1/12 and 2/12 (as opposed to the harmonic minor scale which also contains an interval 3/12), so all of the scales to be considered here contain a range of two intervals of adjacent size; the size of the intervals will depend on the number of tones in the scale. This is what is meant by the term "quasi-equal" in the title of this supplement. "Symmetrical" means that where two different sizes of scale intervals are used they will be spaced as evenly as possible throughout the scale. This is how the standard diatonic scale differs from the melodic minor. In the former, the semitones are separated by 2 and 3 whole steps, while in the latter they are separated by 1 and 4 whole steps. The scales considered in this section will be symmetrical in this special sense whereby the major scale is symmetrical but the melodic minor is not.

In 19-tone temperament, quasi-equal-interval-symmetrical scales are formed by the superposition of a single interval. It is one of the basic theorems of the system that when a given scale is quasi-equal-interval-symmetrical, the tones which are not used, taken in their totality, also create a scale which is quasi-equal-interval-symmetrical. Such a pair of scales will be considered complementary. The sum of the number of tones in any two complementary scales is always 19. The two complementary scales of which Yasser writes, involving 12 and 7 tones respectively, are quasi-equal-interval-symmetrical. They are created by the superposition of perfect fifths until a quasi-equal-interval-symmetrical pattern is attained. Example 54 shows the 7- and 12-tone scales of 19-tone temperament resulting from the superposition of perfect fifths and the number of units of the system in each of their intervals.

Example 54: The 7- and 12-Tone Scales

Fb Cb Gb Db Ab Eb Bb F C G D A E B F# C# G# D# A#

C D E F G A B C	C C# D D# E F F# G G# A A# B C
3 3 2 3 3 3 2	1 2 1 2 2 1 2 1 2 1 2 2

The harmonic units of which Yasser writes belong not to the system "12 + 7", but rather to the system "13 + 6", which is the second scale group of the system and is based on the superposition of major seconds. Example 55 shows the scale-group "13 + 6" with enharmonic pitch names freely used. It should be remarked that since the scale formations rest purely on a tempered basis, the enharmonic names are perfectly suitable.

Example 55: Scale Group "13 + 6"

```
C# D# Fb Gb Ab Bb C  D  E  F# G# A# Cb Db Eb F  G  A  B
```

```
C  Db D  Eb E  F  F# G  G# A  A# B  Cb C    C D E F# G# A# C
 2  1  2  1  2  1  2  1  2  1  2  1  1        3 3 3  3  3  4
```

It will be noted that the six-tone scale above forms a Yasser-hexad; this hexad would appear to be a reasonable basis for harmony involving the 13-tone scale. The consonances are imperfect, owing to the presence of all of the "neutral" but none of the "dissonant" intervals in the hexads.

The third system is comprised of the 11- and 8-tone scales, and is derived from the augmented third (diminished fourth), a neutral interval, totally unfamiliar to present-day ears. The characteristic harmony of this scale-system is dissonant to neutral. The 11-tone scale shows a preponderance of "whole-steps" over "half-steps" not unlike that of our present diatonic scale.

Example 56: Scale Group "11 + 8"

```
D  Gb B  Eb G# C  Fb A  Db F# Bb D# G  Cb E  Ab C# F  A#
```

```
C  Db D  Eb Fb F# Gb G# A  Bb B  C    Cb C# D# E F G Ab A# Cb
 2  1  2  2  2  1  2  2  2  1  2        2  3  2 2 3 2  2  3
```

The fourth scale system is comprised of the 14- and 5-tone scales. The traditional Western pentatonic scale is, of course, a satisfactory scale, and is the basis for consonant harmony in the 19-tone system. However, it is not a quasi-equal-interval scale, since it possesses intervals of 5- and 3- units. While it is possible to build such a pentatonic scale, and the 14-tone scale which is its complement, by juxtaposing perfect fifths, such a group of scales, although excellent, falls outside of the scope of this particular theoretical study. The quasi-equal-interval 5-tone scale is composed of intervals of 3 and 4 units, one of the former and four of the latter. It sounds nearly equal-tempered and suggests the Indonesian salendro scale. The subtended interval is the augmented second (diminished third) which does not exist as an independent interval in 12-tone temperament, and possesses a strikingly novel sound.

Example 57: Scale Group "14 + 5"

B Db E Gb A Cb D Fb G A# C D# F G# Bb C# Eb F# A

C C# D D# Eb Fb F F# G G# Ab A# Bb Cb C   B Db E Gb A B
1 2 1 1 2 1 1 2 1 1 2 1 2 1   4 4 4 4 3

The fifth scale system is comprised of the 10- and 9-tone scales. The two component scales are almost identical. The smaller scale can be produced from the larger by the omission of a single tone. The intervals of the two scales are almost entirely equal as well, suggesting a rather limited possibility of use, except for a single effect or color. This scale system might play a role in 19-tone music somewhat akin to the role of the whole-tone scale in 12-tone temperament. It might also be quite effective for serial exploitation since the intervals are larger than those of 12-tone temperament and the serial pattern is likely to be at once less dissonant and easier to follow.

Example 58: Scale Group "10 + 9"

Fb F# G Ab A# B C Db D# E F Gb G# A Bb Cb C# D Eb

C Db D# Fb F# G Ab A# B C	C# D Eb E F Gb G# A Bb Cb C#
2 2 3 2 2 2 2 2 2	2 2 1 2 2 2 2 2 2 2

The sixth scale system is comprised of the 15- and 4-tone scales. It is probably less useful than the preceding systems because of the greater disparity between its component scales. It is produced by a series of minor thirds, and takes on additional significance from the importance of this interval in 19-tone temperament. It is the only one of the scale systems which may be considered to be just, as well as tempered, in 19-tone temperament. When minor thirds are subtended, the first four tones produce a diminished seventh chord (albeit quite different in sound from the diminished seventh of 12-tone temperament); the fifth tone exceeds the octave, however, by one unit of the system. The resulting sonority is quite effective, provided that the first and fifth tones are not sounded within the same octave. Since the minor third represents the basic unit in the scale, pentachordal configurations whose outer tones form that interval might well be the principal melodic materials of this scale system.

Example 59: Scale Group "15 + 4"

C# B D F Ab Cb D# F# A C Eb Gb A# C# E G Bb Db Fb

C C# D D# Eb E F F# Gb G# Ab A A# B Cb C   Db Fb G Bb Db

The remaining scales are probably of interest only as curiosities, involving as they do so many or so few tones. The 16- and 3-tone scales result from the juxtaposition of major thirds, the 17- and 2- from combining augmented fourths, and, if one wishes to call them such, the 18- and 1-tone scales result from single units of the temperament, the diminished seconds.

It will be noted from the above that 19-tone temperament contains nine different scale classes, just as there are nine different intervals (plus their octave inversions). For each interval there is a scale class. For each scale class there is, correspondingly, an interval. Because 19 is a prime number, any interval in the system creates a complete cycle of 19 tones, when repeated, before returning to a replica of the original tone.

The correspondence of interval to scale class is significant, because the interval which, juxtaposed, produces a given scale class, is the basic interval of modulation for music composed utilizing the scale class in question.[3] Example 60 shows the distance in number of steps, ascending, from C to each member of the system, given each of the various possible intervals as the basis for scale and modulation.

Example 60 shows the multiplicity of function possible for each tone with respect to a central tone C. Each of the 18 tones other than C can be any of the first 18 members of a cycle, depending on the interval chosen as the basis for the cycle. There is no duplication of function whatsoever.

---

[3]On this basis the fifth is, of course, the determining interval for diatonic music in both 12- and 19-tone systems, as well as for the 12-tone scale in 19-tone temperament and for certain 14-tone scales in the latter temperament. It follows that these are likely to be the most consonant scale systems in 19-tone temperament; more dissonant intervals produce scale systems requiring more dissonant idiomatic harmonic treatment. The one-to one correspondence of scale to interval to harmonic type is one of the most challenging possibilities of this theoretical system.

Example 60: Distance from C of Each of the Tones by Cycles of the Various Intervals

Basic Interval	C#	Db	D	D#	Eb	E	Fb	F	F#	Gb	G	G#	Ab	A	A#	Bb	B	Cb	C
dim. 2nd	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
minor 2nd	10	1	11	2	12	3	13	4	14	5	15	6	16	7	17	8	18	9	19
major 2nd	13	7	1	14	8	2	15	9	3	16	10	4	17	11	5	18	12	6	19
aug. 2nd	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19
minor 3rd	4	8	12	16	1	5	9	13	17	2	6	10	14	18	3	7	11	15	19
major 3rd	16	13	10	7	4	1	17	14	11	8	5	2	18	15	12	9	6	3	19
aug. 3rd	11	3	14	6	17	9	1	12	4	15	7	18	10	2	13	5	16	8	19
perf. 4th	12	5	17	10	3	15	8	1	13	6	18	11	4	16	9	2	14	7	19
aug. 4th	17	15	13	11	9	7	5	3	1	18	16	14	12	10	8	6	4	2	19
dim. 5th	2	4	6	8	10	12	14	16	18	1	3	5	7	9	11	13	15	17	19
perf. 5th	7	14	2	9	16	4	11	18	6	13	1	8	15	3	10	17	5	12	19
aug. 5th	8	16	5	13	2	10	18	7	15	4	12	1	9	17	6	14	3	11	19
min. 6th	3	6	9	12	15	18	2	5	8	11	14	17	1	4	7	10	13	16	19
maj. 6th	15	11	7	3	18	14	10	6	2	17	13	9	5	1	16	12	8	4	19
aug. 6th	14	9	4	18	13	8	3	17	12	7	2	16	11	6	1	15	10	5	19
minor 7th	6	12	18	5	11	17	4	10	16	3	9	15	2	8	14	1	7	13	19
major 7th	9	18	8	17	7	16	6	15	5	14	4	13	3	12	2	11	1	10	19
aug. 7th	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	19

A MORE DETAILED TREATMENT OF HARMONIC SONORITIES

The number of possible tone combinations in 12-tone equal temperament is sufficiently large to fill an entire book,[4] and, needless to say, the number of possible combinations of tones in 19-tone temperament is far larger. There are, for example, 43 distinct tetrads possible in 12-tone temperament, with inversions and transpositions excluded. In 19-tone temperament there are 201. In order that any harmonic order be made of so myriad a field of possibilities, a drastic reduction must be made in the same way that harmonic usage cut the field of tetrads in traditional 12-tone music to the seven seventh chords and the French sixth.

Because of the increase in the number of available tones and because of the general tendency toward increased complexity, it seems reasonable to assume that the function of the simple triad will be taken over by a more complex chord (Yasser has suggested the hexad). The four triads generally accepted in traditional 12-tone music possess one essential characteristic: no interval is smaller than 3 units of the system. The four established triads represent every possible combination of three intervals within the octave such that no interval is smaller than 3 units. The pattern 6-3-3 produces the diminished triad, the patterns

---

[4] I am thinking of Howard Hanson's Harmonic Materials of Modern Music, and there may well be others.

5-4-3 and 4-5-3 produce major and minor triads, while the pattern 4-4-4 yields the augmented triad. There is no other triad in the 12-tone system without an interval smaller than 3 units.[5]

Were 3 units of 19-tone temperament to be used as the smallest interval in the construction of triads in that system, however, no fewer than 22 distinct triads would become part of the harmonic vocabulary, including one (13-3-3) whose largest interval would exceed four times the size of the smallest. With tetrads the number is increased; there are 30. Classification into chord types, whereby all chords consisting of a given group of intervals (such as 6, 5, 4, and 4 units) would be treated together, would reduce to a manageable 11 the number of distinct classifications.

Hexads, on the other hand, offer too few possibilities; there is only one hexad (4-3-3-3-3-3) which, with its inversions, is possible without the use of intervals smaller than 3 units. Furthermore, there is no consonant hexad according to our theory of consonance, although the hexad cited here is the generating basis for consonance according to Yasser.

Offering the best compromise between too few and too many possibilities is the pentad devoid of intervals smaller

---

[5]It should be noted that in forming triads, I use three rather than two intervals, thereby completing the octave. Triads such as C-E-B are eliminated from consideration by the smallness of that third, octave-completing interval. A similar procedure is used for the harmonic sonorities of 19-tone temperament, causing the sum of the intervals therein represented always to equal 19.

than three units. There are five types of pentads, containing 14 possible chords. Of these, as will be seen, 2 types, containing 5 chords, can to all intents and purposes be eliminated. Here are the five types of pentads:

7-3-3-3-3	yielding one chord
6-4-3-3-3	yielding four chords (6-4-3-3-3, 6-3-3-3-4, 6-3-4-3-3, 6-3-3-4-3)
5-5-3-3-3	yielding two chords (including the consonant 5-3-5-3-3)
5-4-4-3-3	yielding six chords (5-4-4-3-3, 5-4-3-4-3, 5-4-3-3-4, 5-3-3-4-4, 5-3-4-3-4, 5-3-4-4-3)
4-4-4-4-3	yielding one chord

The first two types can be eliminated because they represent essentially incomplete hexad structures wherein the range of sizes of the intervals equals or exceeds two to one. Such chords may be used as neutral pentads in some instances, but essentially they belong not to the realm of pentads but rather of hexads. The remaining three pentad types yield only nine different chords plus their inversions. These can be considered the nucleus of non-dissonant harmony in 19-tone temperament.

The determination of the possible roots of these chords offers one of the greatest obstacles to the development of a theory of harmony. Three possibilities exist: that these chords have no roots; that they have variable roots which are determined by the position of the chord, as is often the case in Hindemith's theory; and that they have distinct roots as do the triads in Rameau's theory. For experimental purposes I have tentatively adopted the third alternative, seeking to assign to a single tone in each sonority the function of the root, regardless of the inversion. Since no such simple matter as tertian construction is available as a basis, the selection of the root depends on Hindemith's procedure up to a point.

The procedure is as follows: (1) Determine which members of the sonority possess perfect fifths above or perfect fourths below. (2) Where one member of the sonority is so endowed it is the root. (3) Where two or more members of the sonority are accompanied by the perfect fifth above or the perfect fourth below, eliminate all other tones from consideration; the root is the remaining sonority which also possesses a major third above. (4) Where no perfect fifths or fourths exist, the root shall be that member of the sonority which possesses a major third above. (5) Where it is still impossible to determine the root after consideration of the major thirds, continue the process of considering the lesser intervals eliminating from consideration, however, those tones which lack the aforementioned consonances that are possessed by other tones. (6) The other intervals shall be considered in the following order: minor third above, perfect fourth above, major second above.

The nine chords, stated numerically by intervals taken upwards from the root are:

a	3-3-5-5-3	d	4-3-4-3-5	g	5-3-3-4-4
b	3-3-5-3-5	e	4-3-4-5-3	h	3-3-4-5-4
c	3-3-5-4-4	f	3-5-3-4-4	i	4-4-3-4-4

The spelling of the same nine pentads, considering C to be the root, is as follows:

a	C-D-E-G-Bb	d	C-D#-Fb-G-A	g	C-Eb-F-G-A#
b	C-D-E-G-A	e	C-D#-Fb-G-Bb	h	C-D-E-Gb-A#
c	C-D-E-G-A#	f	C-D-F-G-A#	i	C-D#-F-G-A#

Of the above, pentad b is the most consonant, while pentads h and i are probably the least stable, h alone possessing no perfect fifth whatsoever.

The pentads are most effective when used in open position. Because the roots are less pronounced in their character than in traditional triadic harmony, it is advisable that any composer wishing to induce a sense of root progression in these harmonies be careful to use a preponderance of pentads in root position, at least for the present.

THE TENDENCY OF INTERVALS

In traditional harmony dissonances are said to create tendencies toward intervals of resolution. The tritone and its inversion, in particular, are said to have strong outward and inward tendencies respectively, while seconds and sevenths create (in one of the voices only, as a rule) outward and inward tendencies respectively. While it is generally assumed, especially in the case of the tritone, that harmonic context creates the specific tendency of an otherwise neutral interval (by determining whether it will be, for example, an augmented fourth or a diminished fifth), it is necessary, in 19-tone music, to have a concept of tendency which exists independent of total chordal context. This is because the traditional enharmonic relationship between such intervals as the augmented fourth and the diminished fifth no longer exists in 19-tone temperament. The tritone and the diminished fifth are two different intervals. In order to provide some basis for the consideration of tendency in temperaments other than 12-tone, I have evolved the following theory of tendency, which I believe to be applicable to traditional practice in 12-tone temperament as well as to a possible practice in 19-tone temperament.

This theory might be called the theory of repulsion. When a consonance is perfect (or very, very close to perfect) a sense of rest is achieved. When a consonance is substituted for by a near-lying dissonance, the implied consonance repells the closely approximating dissonance, causing the dissonance to possess a tendency <u>away</u> from the said consonance.[6] Two principles affect the strength of this magnetic repulsion. The first is that the more perfect the consonance

---

[6] It is this tendency which may well be the reason why players tend to sharpen their thirds and upward-tending leading tones. The nearby consonant third imparts to the Pythagorean third an outward tendency through "repulsion."

the greater its repelling force on nearby dissonances. The second is that except for intervals so close to the consonance that the ear substitutes the consonance for the actual dissonant interval played, the closer the interval to the consonance, the greater the force of the repulsion.

In 12-tone music the tritone is more or less neutral until it is harmonized, for the inward repulsion created by the fifth is all but cancelled out by the outward repulsion created by the fourth. The dissonant interval is suspended between these two consonances and the tendencies neutralize, requiring the creation of harmonic context for the development of a tendency. In 19-tone temperament, the augmented fourth, being closer to the perfect fourth than to the perfect fifth, has a natural outward tendency, while the diminished fifth has a natural inward tendency. These tendencies are independent of the harmonization which may either reinforce them or weaken them at the discretion of the composer.

The second, especially the minor second, obtains its outward tendency from its nearness to the prime, while the seventh is repelled inward by the octave. With these intervals, 19-tone temperament possesses the same tendencies as 12- except that since the major second and minor seventh are treated as consonances, no tendencies are considered to affect them.

The augmented second in 19-tone temperament might better be called the diminished third, as it derives its tendency from its nearness to the minor third. Since the major second is too weak to offer a counter-repulsion and too close to serve as a target for the resolution, the diminished third (4/19) tends to want to resolve inward to the prime. The dissonant interval 7/19 is, like the tritone in 12-tone temperament, suspended between two relatively equal consonances, the major third and perfect fourth. It can resolve with equal effectiveness in either direction. Somewhat different, however, is the tendency of this interval when it is inverted. 12/19 is repelled with the greatest strength by the perfect fifth and so tends to resolve outward.

It is postulated that where any of the above dissonances are used, their resolutions should, where possible, follow the patterns as set forth above.

CONCLUSION

It is hoped that the above suggested principles may offer the experimental composer a meaningful starting point for his exploration of 19-tone temperament. In the following supplement, some but not all of the principles are illustrated in the compositions. It will require, of course, a far larger body of works to evolve a workable language, let alone an accurate measure of the aesthetic wealth of 19-tone temperament. It is hoped that those who feel impelled to write additional music in 19-tone temperament will inform me, and if possible send scores and tapes, in order that a central source be available to which those seeking information about progress in 19-tone temperament might come.